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Tunable localization of light using nested invisible metasurface cavities

Abstract: An invisible cavity is an open resonant device that confines a localized field without producing any scattering outside of the device volume. By exploiting the scatter-less property of such device, it is possible to nest two invisible cavities, as the outer cavity would simply not notice the presence of the inner one, regardless of their relative position. As a result, the position of the inner cavity becomes a means to easily control the field localized inside the cavity and its quality factor. In this paper, we discuss the properties of nested invisible cavities as a simple method to achieve stronger localized fields and high tunable quality factor. Furthermore, we show that in optics, these cavities can be implemented using nanodisk-based dielectric metasurfaces that operate near their electric resonances.

Keywords: cavity; field localization; metasurface; quality factor; tunable.

1 Introduction

Light localization plays an important role in modern physics and applications, particularly, in nanophotonics. It is the basis for creating nonlinear devices, sensors, waveguides, resonators, photonic crystals, interferometers, optical fibers, lasers, etc. With the advent of new nanofabrication technologies, optical systems enabling efficient and controllable light concentration at the nanoscale are on demand [1–3]. Foreseeable applications of such systems include integrated circuitry for silicon photonics, optical computing, solar technologies, and biosensing.

Recently, a possibility for light localization in a given volume with simultaneous partial or complete light scattering suppression from that volume was discussed in multiple contexts, such as bound states in the continuum (BICs) [4–9], anapole modes [10–13], and virtual absorption [14–16]. Such nonradiating light concentration enables nonobstructive detection, enhancement and suppression of radiation, and energy storing. Probably, the most pronounced synergy between light localization and scattering suppression is achieved in the so-called invisible metasurface-based cavities [17, 18]. These cavities allow to generate strong field maxima and deep minima inside them when illuminated by incident light, while they remain invisible to observers, both in the backward and forward directions. Such a regime is achieved due to the full suppression of light scattering from the cavity into outside space. It is in sharp contrast to conventional Fabry–Perot resonators made of symmetric combinations of two partially transparent mirrors where strong forward scattering is produced (resulting in a nonzero phase shift of the transmitted wave) [19]. The invisible metasurface cavities have an antisymmetric configuration of semi-transparent mirrors: one with weak inductive and one with weak capacitive response. An illustration of such an invisible cavity is shown in Figure 1(a). Due to the antisymmetric (dual) and balanced electromagnetic response of the two sheets, the cavity remains invisible, being to some extent related to parity-time symmetric systems [20–24]. In this work, we consider invisible cavities formed by a pair of two electrically polarizable metasurfaces. It is worth mentioning that the present study can be also extended to invisible cavities formed by one electric and one magnetic, two magnetic [17], or two bianisotropic metasurfaces [18].

Due to this nonscattering nature, it was suggested that the invisible cavities can be “nested” [Figure 1(b)], allowing further enhancing the field localization, still preserving the overall invisibility [17, 18]. However, implementation for such structures was conceptualized only for the microwave frequency range using metallic patterns. Simple scaling-down of this design to the optical frequency range is not

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The grid impedances of the walls are complement in the form of the inner cavity implies the following [17]: the complex magnitude of the electric field inside the inner cavity does not produce any scattering outside the operational free-space wavelength. The grid (or sheet) that operate near the electric resonance [25–31]. In order to provide required capacitive and inductive responses of the two metasurfaces, we slightly detune the dimensions of their unit-cell elements away from the electric-dipole resonance condition. Furthermore, we design a nested configuration of two cavities that provides quadratic improvement of the quality factor as compared to that of a single cavity. Finally, we demonstrate that due to the invisibility property, the nested cavity enables a simple mechanism for tuning its quality factor (alternatively, the strength of field localization) in a wide and continuous range of values.

2 Nested invisible cavities

The simplest nested invisible cavity consists of two open cavity resonators, where one of the cavities is placed inside the other one, as portrayed in Figure 1(b). In this setup, since the inner cavity does not produce any scattering outside of it, its placement inside the outer cavity does not modify the standing wave inside the inner cavity. The invisibility of the inner cavity implies the following [17]: the complex grid impedances of the walls are complement in the form $Z_{e3} = -Z_{e4} = Z_{in}$, and they are separated by a distance $d_{in} = n\lambda_0/2$ [see Figure 1(b)], where $n$ is an integer and $\lambda_0$ is the operational free-space wavelength. The grid (or sheet) impedance of a metasurface is the ratio between the averaged tangential electric field at the metasurface and the induced electric surface current density [32, 33]. Likewise, invisibility of the outer cavity occurs when $Z_{e1} = -Z_{e2} = Z_{out}$ and $d_{out}$ is proportional to half-wavelength at the operational frequency. By assuming that the metasurfaces have a uniaxial symmetry, with $z$ as the propagation direction of the incident wave, the standing wave of the outer cavity can be decomposed into its “forward” and “backward” components:

$$E_{\text{F, out}} = E_1 \left(1 - \frac{\eta_0}{Z_{\text{out}}^2}\right) e^{-j\kappa 2 a^x}, \quad (1a)$$

$$E_{\text{B, out}} = E_1 \left(\frac{\eta_0}{Z_{\text{out}}^2}\right) e^{j\kappa 2 a^x}, \quad (1b)$$

where $\kappa_0$ and $\eta_0$ are the free-space wavenumber and wave impedance, and $E_1$ is the amplitude of the incident electric field. Here, we assume the time harmonic convention in the form of $\exp(j\omega t)$. The field maximum of the standing wave $E_{\text{max, out}}$ corresponds to the constructive combination of the forward and backward waves. The normalized field maximum $A_{\text{max, out}}$ is then given by

$$A_{\text{max, out}} = \frac{E_{\text{max, out}}}{|E_1|} = \frac{\eta_0 + |\eta_0 - 2Z_{\text{out}}|}{2Z_{\text{out}}} \quad (2)$$

The standing wave at the inner cavity is the result of the coherent illumination by the outer forward and backward waves (see Supplementary Sections S2 and S3). Therefore, the inner fields read

$$E_{\text{F, next}} = \frac{E_1 \eta_0}{Z_{in}} \left[2Z_{in} - \left(1 + \frac{\eta_0 e^{2j\kappa \delta}}{2Z_{\text{out}} - \eta_0}\right)\right] e^{-j\kappa (z - \delta)} a^x, \quad (3a)$$

$$E_{\text{B, next}} = \frac{E_1 \eta_0}{Z_{in}} \left[1 + \left(\frac{2Z_{in} + \eta_0}{2Z_{\text{out}} - \eta_0}\right) e^{2j\kappa \delta}\right] e^{j\kappa (x - \delta)} a^x. \quad (3b)$$

It can be appreciated that both inner fields in Eqs. (3) are periodic functions that depend on the inner cavity position $\delta$. As a result, the amplitude of the inner standing wave can be tuned by moving the inner cavity within the outer one, as presented in Figure 2.

The nested cavity concept offers a flexible design framework, depending on the combination of cavities with different $Z_{in}$ and $Z_{out}$, relative position of the inner cavity $\delta$, and the order of the sheets. This work will focus on a quite simple but attractive setup where both cavities have negligible losses ($Z_{in} = jX_{in}$ and $Z_{out} = jX_{out}$), and their individual sheets have the same reactance values, but alternating between inductive and capacitive layers ($X_{out} = -X_{in}$). The first sheet of the system (the first one to be illuminated by the incident wave) was chosen arbitrarily to be inductive.
In terms of frequency bandwidth, the combination of two nested invisible cavities can increase the quality factor of the combined structure, compared to the quality factor of each individual cavity. For the purpose of analyzing the quality factor, the transmitted wave amplitude was calculated analytically by satisfying the boundary conditions for each sheet at an arbitrary frequency (see Supplementary Section S4). The quality factor, for this study, is defined as the ratio of resonant frequency to the bandwidth between the resonant frequency and the bandpass bandwidth BW, the region where the power of the transmitted wave is at least half of the power coming from the incident wave. Due to the complexity of the analytical expression for the transmitted wave, the half-power passband was estimated numerically using an inner cavity with $d_{in} = \frac{\lambda_0}{2}$, while the outer cavity measures $d_{out} = 3\lambda_0/2$. The resonant frequency was arbitrarily defined as $f_0 = 1$ GHz. Both cavities use the same pair of grid impedances, with reactances fluctuating between $X_{out} = 11.925\ \Omega$ ($A_{max,out}^2 \approx 1002$) and $X_{out} = 29.161\ \Omega$ ($A_{max,out}^2 \approx 3.37$).

Figure 3 provides comparison of the quality factor of the nested cavity and the values achieved by the inner and outer cavities individually. For simplicity in numerical calculations, we consider nondispersive sheets. Due to the resonant nature of the standing wave located at the inner cavity, it is expected that the alignment that produces strong field localization will also offer a high quality factor. Therefore, by placing the inner cavity at $\delta_{max}$, the nested cavity becomes highly resonant, achieving its peak quality factor. Then, the quality factor can be controlled at will, decreasing down to its minimum value located at $\delta_{min}$. In general terms, the lower boundary for the nested cavity quality factor is the one achieved for the inner cavity only. One exception is found for high reactance values of $X_{out}$, where neighbor four-layer resonant modes combine with the main resonant mode, creating a broader band-pass region.

### 4 Implementation with dielectric nanodisks

As a proof of concept, a nested cavity is designed to operate in the near-IR range (with the design wavelength $\lambda_0 = 1550$ nm and frequency $f_0 = 193.41$ THz). The target field enhancement in the inner cavity, when it is aligned at $\delta_{max}$, is chosen as nine times the incident field amplitude ($A_{max,nest} = 9$). This relatively small number was chosen deliberately to speed up three-dimensional frequency-domain simulations (systems with high quality factors generally require denser simulation meshes). It should be noted that, in principle, the field enhancement ratio is limited only by long simulation time or possible fabrication constraints. According to Eq. (4), this enhancement can be
not qualitatively affects the presented results. In order to reduce computational complexity during design, both metasurfaces have meta-atoms with the same square lattice with period \( \alpha = \lambda_d / 2 \). This particular value allows the excitation of electric dipole moments in the nanodisks, while preventing generation of diffraction orders for normally incident waves. To design metasurfaces with small impedances \( Z_e = \pm j0.375\eta_0 \), we first implement a resonant metasurface that emulates a perfect electric conductor (PEC) sheet with \( Z_e = 0 \) (\( \tilde{\tau} = 0 \) and \( \tilde{\Gamma} = -1 \)). This can be done by choosing a combination of \( r \) and \( h \) that produces an electric dipole resonance at the design frequency [32]. It is worth mentioning that similarly one can realize invisible cavities formed by two magnetic [34] or two bianisotropic [18] metasurfaces. While in the former case, one needs to exploit the magnetic resonance of the nanodisks, the latter scenario can be reached with asymmetric nanodisks [35]. Next, we slightly scale up (scale down) the radius in order to depart from its dipole resonance and reach the desired response for the inductive (and capacitive) nanodisk layers (see Supplementary Material S5). Next, each kind of metasurface was optimized separately using the built-in methods in CST (trust region algorithm for the inductive nanodisk and genetic algorithm for the capacitive one), with the required \( \tilde{\tau} \) and \( \tilde{\Gamma} \) as the optimization targets. In contrast to the inductive nanodisk, whose optimization was relatively easy (as its resonance frequency was far from the design frequency at 189.43 THz), the capacitive nanodisk (with the resonance frequency of 193.65 THz) required additional optimization steps, using CST built-in genetic algorithm. For instance, the capacitive nanodisk was optimized using first the outer cavity setup, with invisibility (unitary transmission with zero phase shift) at the design frequency as the optimization target. Then, the capacitive layers were tuned in the nested configuration by placing the inner cavity at \( \delta_{\text{max}} = 1937.5 \) nm (about 1.25 wavelengths), also with invisibility as the optimization target. This value of \( \delta \) was purposely chosen to maximize the field localized inside the cavity and overall quality factor. Because of that, any small variations in the capacitive metasurfaces scattering would result in large variations in both localized field and the overall transmission coefficient. During the single cavity and nested cavity optimizations, the inductive layers remained unchanged, as their individual transmission and reflection coefficients were close to the optimization target from the beginning.

Simultaneous optimization of both inductive and capacitive nanodisks was avoided to prevent other possible scenarios with different field localization strengths. The separation between the metasurfaces was chosen sufficiently large to prevent near-field coupling between contiguous layers.

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**Figure 3:** Quality factor of a nested cavity behaves similarly to its electric field maximum, being more resonant for lower values of the grid impedance, while the locations for maximum and minimum values are the same for the electric field maximum. Compared to the individual quality factor of the outer and the inner cavities, a nested cavity can offer a variable quality factor that can be similar to the individual cavities or become more narrowband. The drop of \( Q_{\text{max}} \) for high \( X_{\text{out}} \) values is the result of neighbor four-layer resonant modes that extend the band-pass region.

Achieved by nesting two individual cavities each of which provides maximum standing field amplitude of \( A_{\text{max, out}} = 3 \). It can be found that these individual cavities can be formed from inductive and capacitive metasurfaces with grid impedances \( Z_e = \pm j0.375\eta_0 \). In optics, it is more convenient to describe metasurfaces by their individual transmission (\( \tilde{\tau} \)) and reflection (\( \tilde{\Gamma} \)) coefficients, which are related to the grid impedance \( Z_e \) as [17, 32]

\[
\tilde{\tau} = \frac{2Ze}{2Ze + \eta_0}, \quad \tilde{\Gamma} = \frac{-\eta_0}{2Ze + \eta_0}.
\]  

(5)

For the inductive metasurfaces \( (Z_e = j0.375\eta_0) \), the scattering coefficients are written as \( \tilde{\tau} = 0.6\angle53.13^\circ \) and \( \tilde{\Gamma} = 0.8\angle143.13^\circ \). Likewise, the required capacitive metasurfaces \( (Z_e = -j0.375\eta_0) \) should have equivalent transmission \( \tilde{\tau} = 0.6\angle-53.13^\circ \) and reflection \( \tilde{\Gamma} = 0.8\angle-143.13^\circ \) coefficients.

Implementation of such nested cavities in optics can be conveniently done using dielectric nanodisks [27, 29, 31], as depicted in Figure 1. The meta-atom of Figure 4(d) represents a nanodisk made of silicon (refractive index \( n = 3.46 \) near the design wavelength), surrounded by free space. The nanodisk has radius \( r \) (diameter \( D = 2r \)) and height \( h \). Its axis is oriented orthogonally to the metasurface plane. Here, for clarity of the analysis, we omit dielectric spacers between the individual metasurfaces that does...
that regards, the inner cavity was designed with $d_{\text{in}} = 2\lambda_0$, while the outer cavity had $d_{\text{out}} = 5\lambda_0$.

After the optimization, the geometry of the nested cavity was slightly modified, considering fabrication constraints, as summarized in Table 1. This modification is possible since the nested cavity is able to tolerate fabrication inaccuracies at the cost of shifting the operational frequency. In Supplementary Material Section S6, we performed a tolerance study for the variation of nanodisk radii due to fabrication inaccuracies. For a typical variation of 4 nm [36], a minor frequency shift in the operation of the cavity occurs at the level of 1% of its operational frequency. Importantly, the transmission coefficient amplitude remains close to unity (with nearly zero phase) within the mentioned deviation range. The performance of the designed cavity is shown in Figure 4 for different values of the separation distance $\delta$ between the inner and outer cavities. The most significant effect is a shift of the operational frequency away from the design frequency $f_0$ (where the nested cavity remains invisible) toward $f = 193.695$ THz (a 0.2% variation with respect to the design wavelength), as shown in Figure 4(a). Invisibility is verified by the results shown in Figure 4(b), where the phase difference between the incident and transmitted fields is close to zero around the operational frequency range. It can be noticed that the operational frequency remains almost stable regardless of the position of the inner cavity. However, for some discrete values of $\delta$, the resonant frequency of the nested cavity is moved. This effect takes place because the inner layers, which have thicknesses comparable to the wavelength and support Fabry–Perot-like modes [37], suffer from destructive coherent illumination. By shifting the frequency, the single layers have a better suitable excitation for the standing wave inside the outer cavity. In terms of field localization, Figure 4(c) reveals that this cavity produces field maxima at $\delta_{\text{max}} = 1.68\lambda_0$ close to $E_{\text{max}} = 9.9839E_0$, which is an increment of 10.93% with respect to the target amplification. The strong field located in the right side of Figure 4(c)
Table 1: Nanodisks dimensions (diameter $D = 2r$, height $h$, and period $a$) of the inductive and capacitive metasurfaces with the most significant digit with respect to the design wavelength $\lambda_0 = 1550$ nm. It is also included the transmission $\tilde{t}$ and reflection $\tilde{\Gamma}$ coefficients produced by a single metasurface at the design frequency $f_0$.

<table>
<thead>
<tr>
<th></th>
<th>Inductive</th>
<th>Capacitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$ [nm]</td>
<td>604.5 (0.39$\lambda_0$)</td>
<td>573.5 (0.37$\lambda_0$)</td>
</tr>
<tr>
<td>$h$ [nm]</td>
<td>542.5 (0.35$\lambda_0$)</td>
<td>573.5 (0.37$\lambda_0$)</td>
</tr>
<tr>
<td>$a$ [nm]</td>
<td>775 (0.5$\lambda_0$)</td>
<td>775 (0.5$\lambda_0$)</td>
</tr>
<tr>
<td>$\tilde{\Gamma}$</td>
<td>$0.8015 \pm 142.82^\circ$</td>
<td>$0.8586 \pm -141.79^\circ$</td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>$0.5980 \pm 53.82^\circ$</td>
<td>$0.5127 \pm -52.34^\circ$</td>
</tr>
</tbody>
</table>

is the near field excited at one of the nanodisk interfaces. Field localization for alternative alignments at $\delta = 1.25 \lambda_0$ and $\delta = 2.25 \lambda_0$ do not reach the same levels found for $\delta_{\text{max}} = 1.68 \lambda_0$ due to near-field interaction between consecutive layers, which feed one of the inner cavity walls with fields different from the expected standing wave. The frequency response in Figure 4(a)–(c) also demonstrates that the quality factor can be tuned by simply shifting the position of the inner cavity, as locating the inner cavity at the position of the maximum field localization also grants a high quality factor with a narrow pass-band region. A cross section of the field distribution along the nested cavity, as portrayed in Figure 4(e), shows the presence of both standing waves (inner and outer). The additional space between metasurfaces has been proven effective, as the strong near fields around the nanodisks remain localized close to the metasurfaces.

5 Conclusions

In this work, we presented and discussed the properties of nested invisible cavities and their implementations in the near infrared region. Each invisible cavity was formed by inductive and capacitive metasurfaces. Both metasurfaces were designed from nanodisk meta-atoms with an electric-dipole resonance. The desired inductive and capacitive response was achieved by detuning the nanodisk dimensions to shift the dipole resonance. The resulting nested cavity offered a quadratic improvement of the quality factor achievable by the individual cavities. Similar property was observed in the field localization inside the nested cavity: with the proper alignment, the inner field intensity increases by a power of two. Finally, we demonstrated that using an inner invisible cavity, which does not perturb the outer cavity, the resulting nested combination enables a simple means for tuning the quality factor and the field localized strength in a wide and continuous range of values. We note that it is possible to increase the number of nested cavities allowing further increase of localized field strength.

In practice, if the cascaded metasurfaces are fabricated as four independent layers, the distance between the inner and outer cavities can be controlled using microelectromechanical systems (MEMS) nanopositioners [38, 39]. Alternatively, the optical distance $k_0 \delta$ can be tuned through thermal modulation [40] or using laser pulses [41–44]. The proposed cavities can be used for cloaking sensors and obstacles, enhancement of emission, tunable resonators for axion dark-matter haloscopes [45], and creating exotic waveguides, which are invisible in the direction orthogonal to their walls [17].

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References


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