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On the MILP Modeling of Remote-Controlled Switch and Field Circuit Breaker Malfunctions in Distribution System Switch Placement

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ABSTRACT Installing sectionalizing switches and field circuit breakers (FCBs) is vital for the fast restoration of customer electricity supply in distribution systems. However, the high capital costs of these protection devices, especially remote-controlled switches (RCSs) and FCBs, necessitate finding a trade-off between their costs and financial benefits. In this study, we propose a mixed-integer linear programming (MILP) model for optimizing switch planning in distribution systems. The proposed model determines the optimal allocation of manual switches, RCSs, and FCBs to minimize the costs of switches and the reliability-oriented expenses. While the former includes the costs of installing and operating the switches, the latter consists of the distribution company’s lost revenue due to the undelivered energy and the regulatory incentives (or penalties) associated with service reliability indices. Two penalty-reward mechanisms are used to account for the financial benefits of increasing the service reliability through reducing the duration and frequency of interruptions. Proposing a novel reliability assessment model, we consider the possibility of malfunctions in both RCSs and FCBs in a highly efficient manner, which is a major contribution of this work. The proposed MILP model is applied to three test networks to validate its applicability and efficacy. The results show the importance of considering the possibility of switch malfunctions in distribution networks.

INDEX TERMS Distribution system reliability, field circuit breaker, mixed-integer linear programming, optimal switch placement, remote-controlled switch.

NOMENCLATURE

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<td>Index for indicating sending or receiving end of feeder sections.</td>
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\[ R_n \] Subset of \( R \) that consists of the tie line, which can be used to restore load node \( n \).

\[ \Gamma_{l,n} \] Set of feeder sections connecting feeder section \( l \) to load node \( n \).

\[ \Omega \] Set of load nodes.

\[ \Omega_{l,d}^n, \Omega_{l,u}^l \] Sets of load nodes downstream and upstream of feeder section \( l \), respectively.

**PARAMETERS**

\( g \) Annual load growth rate.

\( IC^k \) Investment cost for a switch of type \( k \).

\( N_n \) Number of customers connected to load node \( n \).

\( OC^k \) Operation and maintenance cost for a switch of type \( k \).

\( P_n \) Power demand at load node \( n \).

\( s^M, S^R \) Switching times for manual and remote-controlled switches, respectively.

\( T \) Demand growth period.

\( U \) Useful lifetime of the switches.

\( \alpha \) Annual interest rate.

\( \delta^b \) Annuity factor for investment costs.

\( \delta^f \) Annualizing factor for revenue lost due to the undelivered energy during power cuts.

\( \lambda_l \) Failure rate of feeder section \( l \).

\( \Pi^F, \Pi^R \) Malfunctioning probabilities for a field circuit breaker and a remote-controlled switch, respectively.

\( \rho \) Distribution company’s revenue lost for a unit of energy not delivered during network contingencies.

\( \Delta \) Repair time for feeder sections.

**VARIABLES**

\( EENS \) Expected energy not supplied.

\( Inv, Op \) Investment and operating costs, respectively.

\( PRS \) Cost imposed by the reward-penalty schemes.

\( SAIDI \) System average interruption duration index.

\( x^R_l \) Binary investment variable that is 1 if a remote-controlled switch is installed in tie line \( r \), being 0 otherwise.

\( x^k_{l,e} \) Binary investment variables for switches, which is equal to 1 if a switch of type \( k \) is installed at location \( e \) of feeder section \( l \), being 0 otherwise.

\( v_{l,n} \) Annual frequency of interruptions for customers connected to node \( n \) due to the failures in feeder section \( l \).

\( t_{l,n} \) Annual interruption duration for customers connected to node \( n \) due to the failures in feeder section \( l \).

\( u^k_{l,n} \) Binary auxiliary variable that becomes 1 if there exists only one switch of type \( k \) between feeder section \( l \) and load node \( n \), being 0 otherwise.

\( \varphi^k_{l,n} \) Binary auxiliary variable that is 1 if there are two or more switches of type \( k \) installed between feeder section \( l \) and load node \( n \), being 0 otherwise.

\( \psi^M_{l,n} \) Binary auxiliary variable that is equal to 1 if there are at least one manual switch between feeder section \( l \) and load node \( n \), being 0 otherwise.

**I. INTRODUCTION**

Delivering electrical power to customers with the least interruptions has become increasingly important. Considering that over 80% of customer interruptions are due to failures in electricity distribution networks [1], this sector has tremendous potential for enhancing the continuity of electricity supply to the end-users. A fundamental approach to improving distribution system reliability is the deployment of protection and sectionalizing equipment to reduce the frequency and duration of customer interruptions through fault isolation and feeder segmentation for service restoration [2]. To this end, various research studies have proposed methods for the optimal placement of protection devices in electricity distribution networks. Such devices are installed in order to protect the network assets and ensure the continued supply or fast restoration of electrical power to as many customers as possible. These devices, if installed and used properly, can significantly reduce the average duration and the number of customer interruptions in distribution networks, thereby enhancing the service reliability [3].

Sectionalizing switches (SSs) are among the most important protection devices. These switches are installed in the network in order to isolate the faulty section and facilitate the restoration of electricity to the rest of the network. In previous decades, due to its complexity, optimal distribution system switch placement could only be solved by employing heuristic methods. For instance, researchers employed graph-based, ant colony optimization, particle swarm optimization, and non-dominated sorting genetic algorithm II (NSGA-II) methods to solve this problem in [4], [5], [6], and [7], respectively. Metaheuristic optimization algorithms were also utilized in [8] and [9] to solve the optimal planning of protection and sectionalizing equipment considering their malfunctioning. Nevertheless, with the recent improvements in mixed-integer program (MIP) solvers and computing performance, numerous research studies have proposed mixed-integer linear programming (MILP) models for the optimal switch placement problem. The MILP models have attracted significant attention, since MIP solvers can guarantee the convergence to the global optimal solution and provide a measure of the distance from the optimal solution during the solving process [10], [11], [12], in contrast to the heuristic and metaheuristic methods.

In this regard, one of the first MILP models for the optimal SS placement in distribution networks was proposed in [13]. Many followed their approach, extending
that basic model to a more general one. A mixed-integer model developed in [13] determined the optimal allocation of remote-controlled switches (RCSs) in distribution networks to improve reliability. Siirto et al. in [14] extended the MILP model proposed in [13] by considering earth faults in the system. The authors in [15] developed their MILP model for optimal SS placement, such that the distributed generation impact was also taken into account. In [16], the proposed MILP model aimed at finding the optimal allocation of fault indicators, manual switches (MSs), and RCSs simultaneously, so as to improve the system reliability. Following the same MILP approach for the optimal RCS placement problem in distribution networks, Izadi and Safdarin developed a technique to assess the financial risk of the RCS deployment in [17], and proposed a model that considers such risks in [18]. The authors in [19] and [20] extended the previous MILP models, considering tie line installation together with optimal SS placement to enhance the system reliability. An MILP model was also proposed in [21] for optimally upgrading MSs to RCSs in a distribution network under various objectives, where a novel transformation was proposed to reduce the number of binary decision variables, thereby lowering the computational burden of the problem. Shahbazi et al. proposed a novel MILP formulation in [22] to efficiently determine the optimal allocation of MSs and RCSs in distribution networks. The authors in [23] derived a formulation to concurrently optimize the placement of switches and preventive maintenance scheduling. In [24], an MILP model for simultaneous planning of RCSs, dispatchable distributed generation units, and tie lines was presented. Lastly, two improved MILP models for optimal placement of MSs, RCSs, and tie lines in distribution systems with complex topology were developed in [25] and [26]. Nonetheless, all these studies have focused on the placement of remote-controlled and manual SSs, or only one of them, ignoring other important protection devices, including field circuit breakers (FCBs). Therefore, while these models are efficient in finding the optimal allocation of SSs, they can not be used for installing FCBs.

To address this problem, the authors in [27] proposed an MILP model for the simultaneous placement of MSs, RCSs, and FCBs, aiming at improving the system reliability metrics, namely system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI), and expected energy not supplied (EENS). Of major concern with protection devices is the possibility of their malfunctioning, i.e., their not operating when required. This is more probable in harsh weather conditions, such as the freezing weather conditions that prevail in Nordic countries such as Finland. In addition, RCS malfunction might also happen due to failures in communication systems [28]. In this regard, the authors in [28] showed that malfunction in the operation of RCSs can significantly compromise the system reliability, thereby increasing the risks of using them. To capture these impacts accurately, authors in [27], [29], and [30] developed MILP models for optimal planning of SSs while considering the possibility of the protection devices malfunctioning.

In this respect, while FCBs are not modeled in [29] and [30], the authors in [27] considered their optimal placement together with the possibility of their malfunctioning in their model. However, these studies all utilized a similar scenario-enumeration-based concept to model the impact of switch malfunctions on system reliability, i.e., incorporating all switch malfunction scenarios into the optimization model [27], [29], [30]. In such models, a protection device, which may malfunction, is ignored in each malfunctioning scenario, i.e., it is assumed that the corresponding switch does not exist, even if its binary investment variable is equal to one, implying that the switch is installed [29]. Considering a set of equations for each malfunction scenario, the optimal protection device problem is solved in [27], [29], and [30]. This modeling approach, despite being conceptually straightforward, introduces an excessive number of binary decision variables to the optimization model, which may not only considerably increase the computational burden of the optimization problem, but can also lead to intractability, particularly for real-size distribution networks. This is because the computational burden of an MILP problem has an almost exponential relationship with the number of binary variables. Inefficient modeling can not only lead to longer solution times, but might also render the problem intractable for real-world distribution networks. As an example, the authors in [31] showed that utilizing the failure-scenario-enumeration-based technique in a distribution expansion planning setting can cause intractability, even for small grids. As a result, it is imperative that we must develop an MILP model that introduces the least number of binary variables, in order to make the MILP model practical for such networks, which usually have more than 100 load nodes (i.e., distribution transformers).

Aiming to address the aforementioned caveats, the main contributions of this work are as follows:

1) Proposing an efficient MILP model for optimal protection device placement, while considering the possibility of malfunctioning in the automatic and remote-controlled protection devices. To consider the impact of potential malfunctions in RCSs or FCBs in an efficient way, the MILP model proposed in this paper uses a novel approach, in which a set of auxiliary binary variables and mixed-integer linear expressions are used to capture the impact of switch malfunctions on the load restoration following a failure in the network. More specifically, the auxiliary binary variables indicate whether or not a given load point is impacted by the malfunctioning of FCBs and RCSs during the isolation and restoration stages following a network contingency. This approach does not require the exhaustive integration of failure-effect-analysis constraints for all of the malfunction scenarios, as was implemented in [27], [29], and [30], which drastically increases the
dimension of the resulting optimization model and can cause intractability.

2) In addition to both MSs and RCSs, the proposed model considers installing a third type of distribution network protection device, not taken into account in [29] and [30], namely FCBs. As an advantage over RCSs and MSs, FCBs are capable of improving the interruption frequency-based reliability metrics, such as the system average interruption frequency index (SAIFI). Thus, the proposed model will find the optimal allocation of both SSs and FCBs for decreasing both the frequency and duration of the system’s customer interruptions, aiming at minimizing the system costs.

The rest of this paper is organized as follows. In the next section, the optimal protection device placement problem is first described, and then the proposed MILP formulation is presented in detail. In Section III, the proposed model is implemented in several test distribution networks to show its applicability. In Section IV, we conclude the study.

II. OPTIMAL PROTECTION DEVICE PLACEMENT PROBLEM

This section is devoted to explaining the proposed formulations for the objective function and the reliability assessment model for optimizing distribution system switch planning.

A. OBJECTIVE FUNCTION

The objective of the proposed model is minimizing the total installation, operation, and maintenance costs of switches, together with the reliability-related costs, over the project lifetime. It is worth noting that the reliability-oriented costs consist of the distribution company’s (Disco’s) lost revenue due to undelivered energy and regulatory incentives associated with the service reliability. In order to determine such incentives, we considered two penalty-reward mechanisms, which evaluate the incentives or penalties of the Disco based on the SAIFI and SAIDI of the corresponding network. Also, according to the EENS of the network, the Disco’s lost revenue is determined. As a result, in order to quantify SAIFI, SAIDI, and EENS for the distribution network, an efficient reliability assessment method is developed that considers the possibility of both faults in the network feeders and malfunctioning in the operation of the RCSs and FCBs. It is worth noting that MS malfunctions are not taken into consideration, because such incidents are rare in comparison to FCBs and RCSs. Additionally, the crew sent to open manual switches can typically deal with MS malfunctions promptly, for example, by unbolting jumper lines.

Expression (1) minimizes the objective function, which consists of the annualized investment cost of the switches, the operational and maintenance cost of the switches, and the revenue lost due to undelivered energy.

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Expression (1) minimizes the objective function, which consists of the annualized investment cost of the switches, $\delta^S Inv$, the operational and maintenance cost of the switches, $Op$, the annualized value of the lost revenue due to undelivered energy, $\delta^T \rho EENS$, and the cost imposed by the penalty-reward schemes, $PRS$. Equations (2) and (3) determine the investment and annual operational costs of the switches installed in the network feeders, and the tie lines connecting them. As can be inferred from indices $l \in L$ and $e \in E = \{es, er\}$ of the binary decision variables $x_{l,e}^k$, both the sending and receiving ends – denoted by $es$ and $er$, respectively – of each feeder section $l$ are considered candidate locations for installing different switch types, expressed by the index $k$.

Finally, (4) and (5) calculate the annuity factors for the investment and the revenue lost due to undelivered energy, respectively.

\[
\text{Minimize } \left( \delta^S Inv + Op + \delta^T \rho EENS + PRS \right) 
\]

\[
Inv = \sum_{l \in L} \sum_{e \in E} \sum_{k \in K} x_{l,e}^k IC^k 
+ \sum_{r \in R} \left( x_r^R IC^R + (1 - x_r^R) IC^M \right)
\]

\[
Op = \sum_{l \in L} \sum_{e \in E} \sum_{k \in K} x_{l,e}^k OC^k 
+ \sum_{r \in R} \left( x_r^R OC^R + (1 - x_r^R) OC^M \right)
\]

\[
\delta^S = \frac{1}{1 - (1 + \alpha)^{-U}}
\]

\[
\delta^T = \alpha \left( \frac{1 + \frac{1}{1 + \alpha}}{g - \alpha} + \frac{1 + g}{\alpha (1 + \alpha)^T} \right)
\]

B. NETWORK RELIABILITY EVALUATION

In order to determine the reliability indices used for determining the reliability-related costs, a new reliability assessment model is developed that not only considers the installation of the FCBs, but also takes into account the possibility of malfunctioning in RCSs and FCBs. The proposed model only considers the first-order switch malfunctioning events; i.e., contingencies involving the malfunctioning of two or more switches at the same time are disregarded considering their low probability of occurrence. Moreover, it is assumed that RCSs and FCBs can be operated manually even if their automatic or remote-operated disconnecting mechanism is not functioning properly.

As noted previously, the reliability-related costs are determined according to the values of SAIDI, SAIFI, and EENS, which are calculated in (6), (7), and (8), respectively.

\[
SAIDI = \sum_{l \in L} \sum_{n \in \Omega} \frac{\tau_{l,n} N_n}{\sum_{n \in \Omega} N_n}
\]

\[
SAIFI = \sum_{l \in L} \sum_{n \in \Omega} \frac{\tau_{l,n} P_n}{\sum_{n \in \Omega} N_n}
\]
national regulatory authorities to pragmatically reflect the financial consequences associated with power interruptions caused by network failures [32], [33]. The general structure of each reward-penalty scheme is represented in Fig. 1, which shows the relation between the amount of penalty or reward and the reliability index (i.e., SAIDI or SAIFI). As per the figure, a lower value of the reliability index brings more reward (or less penalty) to the Discos. However, in order to restrict the financial risks associated with the reward-penalty schemes, it is common practice to limit the reward and the penalty to definite levels [32]. The reward-penalty function presented in Fig. 1 is nonlinear and non-convex. In [20], the authors presented a set of mixed-integer linear expressions to model this function. By utilizing the MILP model proposed in [20], the rewards or penalties imposed by the schemes are calculated, the sum of which is \( PRS \).

As can be seen in (6)–(8), the reliability indices are determined based on two groups of variables, \( \tau_{l,n} \) and \( \nu_{l,n} \), the annual duration and the annual frequency of interruptions for the customers at load node \( n \) due to faults in feeder section \( l \). In order to efficiently calculate these variables while considering the possibility of malfunctioning in RCSs and FCBs, three groups of binary auxiliary variables, i.e., \( \psi_{l,n}^k \), \( \psi_{l,n}^{M} \), and \( \psi_{l,n}^{M} \), are exploited in the reliability assessment model. The first is set to one when there is only one switch of type \( k \) (i.e., FCB or RCS) between feeder section \( l \) and load node \( n \), while the second is one in cases where two or more switches of type \( k \) exist between feeder section \( l \) and node \( n \). The third one corresponds to MSs, and is set to one when one or more MSs are installed between feeder section \( l \) and node \( n \).

In this respect, (9)–(13) jointly determine the values of \( u_{l,n}^k \) and \( \psi_{l,n}^k \). To be more specific, (9) ensures that both of the two variables for a feeder section, a load node and a switch type, are not set to one simultaneously. Expressions (10) and (11) specify that only in the case that one or more switches of type \( k \) are installed between feeder section \( l \) and load node \( n \), \( u_{l,n}^k \) can be one. Similarly, (12) and (13) ensure that only if two or more installed switches of type \( k \) exist between feeder section \( l \) and load node \( n \), \( \psi_{l,n}^k \) can be one. It is worth noting that even though it may seem that either \( u_{l,n}^k \) or \( \psi_{l,n}^k \) can be one when two or more switches of type \( k \) exist between \( l \) and \( n \), \( \psi_{l,n}^k \) being one is preferable to setting \( u_{l,n}^k \) to one, because the reliability indices would be better in that case and therefore, the reliability-related costs would be lower.

Similar to (10) and (11), expressions (14) and (15) specify that \( \psi_{l,n}^{M} \) can be set to one if one or more MSs exist between feeder section \( l \) and load node \( n \). As can be inferred from the equations, there is no difference between the existence of one and two or more MSs because the possibility of malfunctioning in MSs is not considered. Lastly, expressions (16) and (17) represent the binary nature of \( v_{l,n}^k \), \( \psi_{l,n}^k \), and \( \psi_{l,n}^{M} \).

$\begin{align*}
  u_{l,n}^k + \psi_{l,n}^k & \leq 1; \quad \forall l \in L, \forall n \in \Omega, \quad k \in K^a \\
  u_{l,n}^k & \leq x_{l,es}^k + \sum_{i \in I \cap \Omega} \sum_{e \in E} x_{i,e}^k; \quad \forall l \in L, \forall n \in \Omega^l, \quad k \in K^a \\
  v_{l,n}^k & \leq \frac{x_{l,er}^k + \sum_{i \in I \cap \Omega} \sum_{e \in E} x_{i,e}^k}{2}; \quad \forall l \in L, \forall n \in \Omega_{Up}^l, \quad k \in K^a \\
  \psi_{l,n}^{M} & \leq \frac{x_{l,er}^M + \sum_{i \in I \cap \Omega} \sum_{e \in E} x_{i,e}^M}{2}; \quad \forall l \in L, \forall n \in \Omega_{Up}^l \\
  \psi_{l,n}^{M} & \leq x_{l,es}^M + \sum_{i \in I \cap \Omega} \sum_{e \in E} x_{i,e}^M; \quad \forall l \in L, \forall n \in \Omega_{Up}^l \\
  \psi_{l,n}^{M} & \in \{0,1\}; \quad \forall l \in L, \forall n \in \Omega, \quad k \in K^a \\
  \psi_{l,n}^{M} & \in \{0,1\}; \quad \forall l \in L, \forall n \in \Omega 
\end{align*}$

Based upon the values of the three groups of auxiliary variables, (18)–(30) determine the value of \( \tau_{l,n} \), which is the annual interruption duration of customers at load node \( n \) due to a fault in feeder section \( l \). In this regard, (18) shows that the value of \( \tau_{l,n} \) for every load node and feeder section is non-negative. Nonetheless, (19)–(24) and (25)–(30) determine the tightest lower bound of \( \tau_{l,n} \) for every load node, due to faults in their downstream and upstream feeder sections, respectively. In each equation, the lower bound is determined by calculating the expected value of \( \tau_{l,n} \) for that specific switch placement. As a result, (18)–(30) can jointly calculate \( \tau_{l,n} \) for every possible switch placement in the problem. It is also worth noting that since the objective function is monotonically increasing with respect to \( \tau_{l,n} \), it would be set to its tightest lower bound for all load nodes and feeder sections.

![FIGURE 1. Reward-penalty scheme.](image-url)
As mentioned above, (19)–(24) determine the minimum annual interruption duration of customers at load node $n$ due to a fault in their downstream feeder section $l$ (i.e., the load nodes are located upstream of the faulted sections). Accordingly, (19) determines the lower bound of $\tau_{l,n}$ in the case that there are one or more RCSs and only one FCB between a faulted feeder section $l$ and its upstream load node $n$. Equation (20) specifies this lower bound if one FCB but no RCS is installed between the load node and the section. Constraint (21) determines the minimum interruption duration of customers at load node $n$ due to its downstream faulted section $l$, whenever two or more RCSs but no FCB exist between the node and the feeder section. Equation (22) enforces the tightest lower bound of $\tau_{l,n}$ when there is no FCB and only one RCS between node $n$ and feeder section $l$. If there are no FCBs and RCSs, but at least an MS is installed between the faulted feeder section $l$ and the upstream node $n$, (23) determines the tightest lower bound. Lastly, for cases where no switch is installed between feeder section $l$ and upstream node $n$, (24) imposes the tightest lower bound for $\tau_{l,n}$.

\[
\begin{align*}
\tau_{l,n} & \geq 0; \quad \forall l \in L, \forall n \in \Omega \\
\tau_{l,n} & \geq \pi^F \lambda l S^R (v_{l,n}^F + v_{l,n}^R + \phi_{l,n}^R - 1); \\
& \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \pi^F \lambda l S^M (v_{l,n}^F - v_{l,n}^R - \phi_{l,n}^R); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \lambda l S^R (\phi_{l,n}^R - v_{l,n}^F - \phi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \lambda l \left(1 - \pi^R S^R + \pi^R S^M\right) (v_{l,n}^F - v_{l,n}^R - \phi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \lambda l S^M (\phi_{l,n}^M - v_{l,n}^R - \phi_{l,n}^R - v_{l,n}^F - \phi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \lambda l \Delta (1 - \psi_{l,n}^F - v_{l,n}^R - \phi_{l,n}^F - v_{l,n}^F - \psi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\tau_{l,n} & \geq \lambda l \left(1 - \pi^R S^R + \pi^R S^M\right) (v_{l,n}^F - v_{l,n}^R - \phi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\end{align*}
\]

Expressions (31)–(34) determine the lower bound of $\psi_{l,n}$, which is the annual interruption frequency of the customers at load node $n$ due to failures in feeder section $l$. Accordingly, (31) enforces the nonnegativity of $\psi_{l,n}$. Equation (32) determines the tightest lower bound of $\psi_{l,n}$ whenever there is only one FCB between the faulted section and its upstream load node, whereas (33) specifies a tighter lower bound for $\psi_{l,n}$ if no FCB is installed between the section and the upstream node. Lastly, constraint (34) sets the lower bound of $\psi_{l,n}$ to $\lambda l$ for all load nodes, in the case of failures in their upstream feeder sections. This is because any failure in the feeder sections upstream of a load node interrupts the power of the customers located at the load node (since the power should be restored through a tie line).

\[
\begin{align*}
\psi_{l,n} & \geq 0; \quad \forall l \in L, \forall n \in \Omega \\
\psi_{l,n} & \geq \lambda l \pi^F v_{l,n}^F; \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\psi_{l,n} & \geq \lambda l (1 - v_{l,n}^F - \phi_{l,n}^F); \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\psi_{l,n} & \geq \lambda l; \quad \forall l \in L, \forall n \in \Omega_{l}^{Up} \\
\end{align*}
\]

### III. NUMERICAL RESULTS

In this section, we discuss the results of implementing the proposed model on three test distribution networks, namely the distribution system connected to bus 5 of the Roy Billinton Test System (RBTS-5) [34], an 83-node grid, and a 135-node grid.
distribution network. The technical input data of all the three test systems are available in [35].

The annual interest rate is set at 3% in the simulation studies, with a 30-year useful lifetime for the switches. The sum of the investment and present worth of operation costs of an MS, an RCS, and an FCB are assumed to be $6,100, $13,200, and $26,700, respectively. The switching times of RCSs and MSs are assumed to be 0.1 and 1.5 hours respectively, while the repair time of a faulted section would be 5 hours. Additionally, we assumed that all loads in the systems will increase for 20 years with a 1% annual growth rate. The expected lost revenue of undelivered electrical energy to the customers is equal to $120/MWh. Lastly, we assumed that the chance of malfunctioning in installed RCSs and FCBs are respectively 0.10 and 0.08.

It is worth noting that we implemented the proposed model in the General Algebraic Modeling System (GAMS) 27.3 and solved the problem using IBM CPLEX 12.9, with the optimality gap set to 0 for the RBTS-5 and 83-node systems, and 1% for the 135-node network. All the simulations were carried out on a Dell Precision 3650 Tower PC with a 6 Core 2.80 GHz Intel Core i5-11600 processor and 32 GB of RAM.

Using the three test distribution networks, we carried out the optimization for four different cases:  

Case I: Only MSs can be installed.  
Case II: Both MSs and RCSs are candidates for installation, while considering RCS malfunctioning.  
Case III: MSs, RCSs, and FCBs are all considered in the optimization, where malfunctioning of RCSs and FCBs is taken into account.  
Case IV: MSs, RCSs, and FCBs are considered, but the possibility of malfunctions is disregarded in the optimization (similar to Case III but without switch malfunctions).

The solutions obtained from solving the proposed optimization model for the above-mentioned cases in the three test systems are represented in Tables 1 and 2, which will be discussed in detail in the next subsections.

### A. VALIDATING THE APPLICABILITY OF THE PROPOSED MODEL

To assess the applicability of the proposed model, we analyze and compare the simulation results for Cases I, II, and III in this subsection. In this regard, by considering more diverse protection devices, we see that the objective function (i.e., the total cost) decreases, while the reliability indices improve, which is consistent with our expectations. To be more specific, by taking into account both MS and RCS installations in Case II, the EENS and the SAIDI of the system improve significantly compared to Case I, where only MSs can be installed. This means that optimal installation of RCSs together with MSs can effectively decrease the duration of the interruptions, as well as the energy that could not be served due to faults in the distribution networks. However, when the FCBs are also added to the available protection devices for installation in the network in Case III, the SAIDI and EENS do not change considerably. In contrast, the SAIFI, which was not improved from Case I to Case II, shows a considerable improvement in Case III, compared to Cases I and II. This is due to the fact that although the installed MSs and RCSs can decrease the duration of the interruptions caused by faults in the network, they are not able to change the frequency of the interruptions in the customers’ electricity supply. On the other hand, FCBs can effectively decrease the frequency of interruptions, which is reflected in the improved SAIFI of the system in Case III. Overall, we can observe that the best reliability condition in all three distribution networks is achieved for Case III, when MSs, RCSs, and FCBs are all considered as available protection devices in the optimization problem.

Another important point in the results is the high efficiency of the proposed model. The simulation times for all the solved optimization models are shown in Table 1. Despite considering the possibility of malfunctioning in both RCSs and FCBs for Case III, the high efficiency of the proposed model means the solution can be reached in a reasonable amount of time for all the test networks. In other words, as the proposed method for considering the malfunction possibility does not increase the number of binary and continuous variables significantly, the optimization can be solved in about half an hour for Case III of the 135-node test distribution network, which has the highest computation burden. This is significant, because only efficient MILP models, such as the one proposed in this paper, can be utilized for real-world distribution networks, due to their high number of nodes.

Other worthwhile points in the simulation results can be found in the allocation of installed protection devices in the distribution networks for different cases. For this purpose, we have shown the optimal allocation of the installed

---

**Table 1. Optimal solution results for the four studied cases.**

<table>
<thead>
<tr>
<th>Case</th>
<th>RBTS-5</th>
<th>83-Node</th>
<th>135-Node</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SAIFI</td>
<td>SAIDI</td>
<td>EENS</td>
</tr>
<tr>
<td>Case I</td>
<td>0.200</td>
<td>0.739</td>
<td>1.656</td>
</tr>
<tr>
<td>Case II</td>
<td>0.297</td>
<td>1.293</td>
<td>2.566</td>
</tr>
<tr>
<td>Case III</td>
<td>4.416</td>
<td>27.655</td>
<td>50.571</td>
</tr>
<tr>
<td>Case IV</td>
<td>1.305</td>
<td>1.265</td>
<td>171.163</td>
</tr>
</tbody>
</table>

Objective Function (k$)  
Simulation Time (s)  

EENS: MWh/year; SAIDI: hours/customer/year; SAIFI: interruptions/customer/year.
TABLE 2. Number of installed switches in different cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>RBTS-5 switches</th>
<th>83-Node switches</th>
<th>135-Node switches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>17 - -</td>
<td>67 - -</td>
<td>94 - -</td>
</tr>
<tr>
<td>Case II</td>
<td>9 8 -</td>
<td>30 36 -</td>
<td>32 50 -</td>
</tr>
<tr>
<td>Case III</td>
<td>9 6 2</td>
<td>29 32 5</td>
<td>32 38 13</td>
</tr>
<tr>
<td>Case IV</td>
<td>8 7 2</td>
<td>29 28 5</td>
<td>35 37 11</td>
</tr>
</tbody>
</table>

switches in the RBTS-5 test network for Cases I, II, and III in Figs. 2, 3, and 4, respectively. First, in all figures, we can see that the switches are allocated in a way that the nodes with the highest number of customers, such as n1, n2, n3, n11, and n14, benefit from the fastest restoration when a fault occurs. Second, we can see that the number and location of installed protection devices are the same in all three figures. In other words, when installing RCSs are considered as an option in Case II, eight RCSs are installed in the same locations that eight MSs were installed in Case I. Also, by considering the installation of FCBs in Case III, two FCBs are installed in the places where the installation of RCSs was the optimal solution in Case II. Based upon these results, one may conclude that concurrent optimization of the three types of switches, as done in this paper, is not required, and that the problem could be tackled via sequentially solving three computationally light models, namely, MS placement, MS upgrading to RCS, and RCS upgrading to FCB. Nonetheless, such an approach would not yield the global optimal solution in all scenarios as evidenced by the results provided in Table 2, where the total number of switches for the 83- and 135-node systems are different in various cases. This means that if the optimization of all the three types of protection devices was not done concurrently (e.g., if the three optimization problems are solved sequentially for different switch types), the distribution utility would encounter significant redundant costs due to unnecessary installation of switches. Lastly, we can see that the FCBs in Case III are only installed in the two feeder that have the highest number of customers. In other words, due to their high investment cost, installing them in other feeders is not optimal. Nevertheless, these two FCBs could significantly improve the SAIFI of the system, as shown in Table 1.

B. IMPACT OF CONSIDERING MALFUNCTION POSSIBILITY

To investigate the impact of considering the possibility of RCS and FCB malfunctions on the optimization results, we compare the outcomes of Cases III and IV for the test distribution networks. First of all, we can see that the optimal solutions obtained for the three test networks are different when the malfunctioning of FCBs and RCSs are considered. Therefore, to obtain the optimal solution for the allocation of switches in practical applications, we have to take into account the possibility of switch malfunction. Second, considering this aspect of switch behaviour affects the optimal solutions differently in the test distribution networks. For...
example, while the number of installed RCSs and FCBs increases when malfunctioning is considered in the 83- and 135-node distribution networks, the opposite happens in RBTS-5. Therefore, the only way to determine the optimal solution for a real-world system is to carry out the optimization while considering the possibility of malfunctioning in switches. Despite all the benefits of considering this important aspect in the optimization model, the computational burden of the problem increases, which is reflected in higher simulation times for Case III than those of Case IV for all the test systems. Nevertheless, thanks to the high efficiency of the MILP model proposed in this paper, the optimal solutions can still be found in a reasonable amount of time, even for a relatively large distribution network.

C. SENSITIVITY ANALYSIS ON THE MALFUNCTION PROBABILITY

To analyze the impact of different probabilities of malfunctioning in RCSs and FCBs on optimal switch allocation, we carried out a sensitivity analysis on this parameter, using the RBTS-5 test network. In the analysis, the RCS and FCB malfunction probabilities were multiplied by a scaling factor ranging from 0 to 2 with a step of 0.2. Figs. 5 and 6 respectively show the values of the reliability indices and the number of installed switches for different values of the scaling factor. As shown in Fig. 5, increasing the malfunction probability leads to optimal solutions with higher reliability indices (i.e., worse reliability), which is mainly due to lower benefit from the installed protection devices in the distribution network. This can be easily understood by taking the cases with a scaling factor of 0.6 to 1.6 into consideration. In these cases, while the number of different types of installed switches is constant, as shown in Fig. 6, the projected reliability indices become worse with the increase in the malfunction probability. Nevertheless, the reason for accurately estimating the malfunction probability is that different estimates lead to different optimal solutions for the switch allocation problem, which can be seen in Fig. 6. This means that not considering the malfunction in switches or even considering an inaccurate estimate for its probability can lead to non-optimal switch allocation in distribution networks. Therefore, to obtain the optimal switch placement for ensuring the highest possible reliability of the customers’ energy supply in a distribution network, we must consider the malfunction probability of RCSs and FCBs, as was done in this study. If sufficiently accurate values for the malfunction probabilities cannot be estimated, a sensitivity analysis on these parameters can be performed to assist the decision makers. As an example, it can be implied from Fig. 6 that the optimal switch investment plan can be robust to the variations of the malfunction probability in specific intervals. For instance, the number of switches with a specific type installed in the network remains identical if changing the scaling factor from 0 to 0.4 or from 0.6 to 1.6. However, if the estimated malfunction probability falls in the borders of such intervals, a risk-averse planner might prefer to choose the switch investment plan based on a higher malfunction probability.

IV. CONCLUSION

We presented an MILP model for optimizing switch planning in distribution systems, considering malfunctioning of remote-operated and automatic switches in an efficient manner. In the proposed formulation, three types of switches, namely MS, RCS, and FCB, were considered as the candidates for installation. To quantify the reliability-oriented financial benefits of the switches, we utilized two penalty-reward schemes based on SAIFI and SAIDI, while also accounting for the lost revenue associated with the energy not delivered to the customers during contingencies, determined according to the system EENS. The proposed model was applied to three test distribution systems, and the outcomes of the optimizations were thoroughly investigated for various cases. The results revealed the significant impact of RCSs on the interruption duration and FCBs on the frequency metrics, the importance of modeling switch malfunction in distribution switch planning, and the applicability of the presented approach for large distribution networks. Although the model presented in this paper is applicable in the presence of non-dispatchable distributed generation (DG) units, further work is required for including the impact of dispatchable DG on the optimal switch planning.
REFERENCES


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