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Stochastic equilibria with capacity expansion: Increasing expected profit with risk aversion

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ABSTRACT

Profit-maximizing firms hedge risk from uncertainty by deciding on capacity investment and production. Typically, risk-averse firms monotonically forgo expected profit in exchange for an improved risk measure, e.g., conditional value-at-risk (CVaR). However, the stochastic-equilibrium literature exhibits non-monotonicities, i.e., both CVaR and expected profit increase with risk aversion. We prove that this result arises because oligopolistic firms account for the price impacts of their own decisions but ignore those of other firms. Consequently, firms reduce capacity "too much" with risk aversion.

1. Introduction

Conventional wisdom and much of the literature on risk aversion indicates that insurance against undesirable future outcomes involves a tradeoff. In particular, risk-averse capacity investment is driven by returns in less-favorable scenarios, thereby effectively forgoing high but uncertain returns in more-favorable scenarios. Consequently, risk-averse hedging decisions result in lower expected benefits or higher expected costs in exchange for less exposure to uncertainty [1]. This intuitive outcome from stochastic programming originates from single-agent settings assuming that uncertain market prices are exogenous. Alternatively, a single-agent setting wherein demand is treated as exogenous and stochastic with fixed prices and costs will lead to the same outcome [2].

Recently, however, several authors have observed situations in which risk-averse behavior actually led to higher expected profits. More generally, such non-monotonicity occurs in the tradeoff between the expected profit and the conditional value-at-risk (CVaR) in multi-agent games with risk-averse agents [3,4]. The CVaR is a coherent risk measure that reflects the conditional expected profit in the worst α% of cases [5]. Similar risk measures have applications in contexts beyond finance, e.g., energy [6,7] and health [8–10]. In a variety of settings, risk aversion reduces (expected) profit in different ways. In classical financial markets such as the stock market, risk-averse investors require a risk premium for taking on (non-diversifiable) risk, which effectively lowers stock prices [11]. In capacity-investment problems, risk aversion will lead to lower investment and lower output [7,12]. Risk-averse retailers with a finite time horizon (or, equivalently, faced with high enough discount rates) to sell off a fixed inventory will set lower prices to avoid the risk of unsold wares [13]. Forward trading, for instance in energy and commodity markets, commonly leads to forward contract prices that are lower than expected spot prices due to risk aversion [4]. In loss-aversion settings, the willingness to invest in insurance increases with risk aversion. Classical examples include health insurance [14,15] and travel insurance [16]. In governance and corporate-social-responsibility settings, measures are invested in to prevent and to insure against the consequences of reputation-damaging risk from non-compliance with social or judicial rules [17]. Higher risk aversion or perceived risk will lead to higher costs and, consequently, lower profits [18]. Similarly, risk reduction in off-grid energy systems will lead to higher costs due to higher capacity investment [19].

In early game-theoretic models, risk aversion gave rise to the concepts of minimax and maximin strategies [20], which eventually culminated in the stream of literature concerning robust optimization [21–23]. A focus on a mere worst-case scenario is extremely risk averse and often hard to reconcile with rational behavior. More recent work extends and nuances optimality criteria in game-theoretic settings [24]. Risk and loss aversion in the economic literature are the main explanatory components for many behavioral decision-making models. Risk aversion is often considered in terms of expected utility and diminishing marginal utility. Any risky decision can be formulated as maximizing the expected utility for an appropriately defined lottery [25]. However, when uncertainty and, consequently, up- and downside risk is (relatively) small, decision-making behavior is less driven by risk aversion and may be adequately represented by expected value optimization [26]. The specific choice of risk measure

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in multi-stage settings can prevent the existence of subgame perfect equilibria [27], thereby invalidating the time consistency of optimal strategies [28]. In the quest to address challenges such as tractability and how much knowledge may be assumed concerning underlying probabilities, the number of methods available to deal with risk aversion in the field of stochastic programming is ever expanding [29–34]. Recent societal developments encompass the platform economy, sharing economy, and green supply-chain management [35–39] to yield a multitude of new decision types in investment and pricing, which integrate increasing numbers of agents with heterogeneous risk attitudes. Hence, methods developed in fields such as stochastic programming and game theory will have to be expanded to analyze and inform optimal decision making in such new and relevant settings.

Given this background, intuitively, a risk-averse firm that includes the CVaR in its objective function uses risk-adjusted probabilities, e.g., to focus on the less-profitable outcomes (c.f. [40]), and *should* lower its expected profit in exchange for a higher CVaR as risk aversion increases. In this work, we show that this intuition may not hold when considering price-elastic demand in an equilibrium framework. While there are various risk measures deployed in the literature besides CVaR, e.g., VaR [1], the main reason why we use CVaR is because it is a coherent risk measure with an intuitive economic interpretation and mathematical tractability. In particular, CVaR respects the subadditivity property, i.e., the CVaR of a portfolio of several assets is always less than or equal to the sum of CVaRs of all assets considered individually. By contrast, this property does not hold for VaR. Moreover, VaR reflects a threshold value, which means that it does not reveal information about the magnitude of the gain or loss beyond that level. This is in contrast to CVaR. As we also mentioned earlier, CVaR is the risk measure used in the literature that exhibits a non-monotonic relationship between risk and return that we wish to explore [3,4]. Finally, as CVaR can be incorporated in linear programming, it is amenable for a host of optimization problems, including the ones that we study here.

Using this approach, [41]'s stochastic-equilibrium analysis of power-sector capacity expansion distills how greater risk aversion leads to higher equilibrium prices. This arises due to the capacity limitations that result from risk-averse firms' adoption of less capital-intensive technologies. In a similar vein, a stochastic-equilibrium model of the Northern European gas market finds that lower supply volumes stem from risk-averse investment [42]. This causes prices to go up and sometimes offsets the profit loss from lower output.

While such non-monotonic results are observed in the literature, a rigorous explanation for why they occur has been lacking. Based on this research gap, we provide a mathematical underpinning for why both expected profit and CVaR may increase with risk aversion. Toward that end, we use a stylized model with either price-taking or Cournot oligopolistic behavior along with a benchmark monopoly model as a reference for full exertion of market power. This framework allows us to convert the resulting equilibrium problem into an optimization one via the extended-cost term [43–45]. Our objective here is *not* to prove that the non-monotonic relationship between the expected profit and the CVaR holds in general. Instead, it is to provide a conceptual explanation for why such seemingly counterintuitive results arise. For this purpose, a stylized model that abstracts from certain real-world attributes, such as asymmetric firms, financial hedging, and startup constraints, suffices.

We consider three cases in an investment-and-operational stochastic-equilibrium problem: perfect competition (PC), Cournot oligopoly (CO), and monopoly (MO). We find that greater risk aversion under PC induces a firm to reduce its capacity investment. Yet, such a firm overlooks the impact of both its own decisions as well as other firms' decisions on the equilibrium price, i.e., the price-taking conjecture. Under CO, greater risk aversion leads to less of a reduction in a firm's capacity investment because it accounts for its own impact on the market-clearing price while still ignoring the effects of competitors' decisions. However, a monopolist fully internalizes the price effect of capacity reduction. Consequently, its capacity reduction corresponds to its *overall* level of risk aversion. Since PC and CO firms reduce their capacities "too much," this leads to significantly higher prices and higher expected profits that may actually increase with risk aversion. We show analytically that this underestimation of price impacts is linked to assumptions about the elasticity of (residual) market demand. By contrast, the expected monopoly profit monotonically decreases with risk aversion.

While a risk-averse agent does not aim to maximize expected profit, the conventional wisdom in stochastic programming has been that there is an inherent tradeoff between profit maximization and risk control. Yet, this perspective presupposes exogenous prices and neglects endogenous price formation as a consequence of the risk-averse agents' collective behavior. Meanwhile, the literature on game theory has observed in passing seemingly counterintuitive results whereby risk-averse agents actually enjoy higher expected profit while also lowering their risk exposure. A coherent explanation for this finding was missing in the literature, which we have now rendered by linking the agents' incentives to market outcomes.

The rest of this paper is organized as follows. Section 2 formulates the problems and obtains analytical solutions. Section 3 conducts comparative statics to test our hypotheses, while Section 4 provides numerical illustrations. Section 5 summarizes our findings and charts future research topics. The Appendix contains proofs for all propositions.

2. Problem formulation

2.1. Modeling assumptions

We use a stylized market setting with a number of symmetric firms all supplying a single market for a homogeneous good with stochastic demand. We assume a two-stage problem in which all firms invest in capacity in the first stage, and, in the second stage, production is adapted to the demand scenarios. Capacity investment is the here-and-now decision, and production is the wait-and-see decision. The demand scenarios represent high- and low-price outcomes and differ by the value of the intercept of the inverse-demand curve, which reflects consumers' willingness to pay for the good. Risk-neutral firms maximize expected profits, whereas risk-averse firms maximize the CVaR, a risk measure that considers only part of the profit in the high-price scenario. This gives more weight to the lower-profit outcomes, thereby reducing the risk-adjusted profitability from production, which will dampen capacity investment. Extremely risk-averse firms may not consider even all the profit in the low-price scenario.

We assume open-loop decisions, i.e., investment and production decisions are treated as if they were made simultaneously [46]. There are $n = 1, \ldots, N$ identical firms, and the production period has $m = 1, \ldots, M$ scenarios with corresponding probabilities $0 \leq Q_m \leq 1$ and $\sum_m Q_m = 1$. Production by firm n in scenario m is $0 \leq q_{n,m} \leq k_n$ [units]. The linear inverse-demand function in scenario m, $P_m\left(q_m\right)$, has intercept parameter $A_m > 0$ [\$/unit] and slope parameter $B > 0$ [\$/unit^2], i.e., $P_m\left(q_m\right) = A_m - B\sum_n q_{n,m}$, where $q_m \equiv \sum_n q_{n,m}$. Each firm's marginal cost of investment is $I > 0$ [\$/unit], whereas its quadratic production cost has parameters $C > 0$ [\$/unit] and $D > 0$ [\$/unit^2]. Investment by firm n, $k_n \geq 0$ [units], has $\lambda_{n,m} \geq 0$ [\$/unit] as the shadow price of the associated capacity constraint. We write the equilibrium price in scenario m as π_m [\$/unit]. Without loss of generality, we set $M = 2$ with $Q_1 = Q_2 = \frac{1}{2}$ and $A_2 > A_1 + \frac{I}{Q_2}$ to ensure that capacity does not restrict production in (the low-price) scenario 1. Such restrictiveness may be due to an irrationally high level of risk aversion. Note that $A_1 > C$ also ensures that scenario-1 production is strictly positive. Since we model a stylized setting, our parameters are notional and merely need to ensure economically meaningful solutions. In effect, the analytical results hold for any parameter choices that satisfy these general conditions.

To reflect risk aversion, $0 \leq R_2 \leq Q_2$ is each firm's subjective probability about the high-demand scenario. Since the CVaR captures the conditional expected profit in the worst α% of the cases, we likewise reweight those relevant outcomes, i.e., via a risk-adjusted probability for the high-demand scenario. This approach has similarities to the concept of risk-adjusted discount rates that has been developed and applied in various settings [47–50]. In the following subsections, we first lay out the risk-neutral decision-making problems for the three market structures, CO, PC, and MO, as stepping stones for the risk-averse setups that follow thereafter.

2.2. Risk-neutral setting

2.2.1. Cournot oligopoly

Firm n's optimization problem is:

$$\max_{k_n \geq 0, q_{n,m} \geq 0} \sum_m Q_m \left[A_m - B \sum_{n'} q_{n',m}\right] q_{n,m} - I k_n - \sum_m Q_m \left[C q_{n,m} + \frac{1}{2} D q_{n,m}^2\right] \quad (1)$$

$$\text{s.t.} \quad q_{n,m} \leq k_n \, : \, \lambda_{n,m}, \forall m \quad (2)$$

Here, the first term is the expected revenue, the second term is the investment cost, and the third term is the expected production cost. Since (1)–(2) is a convex optimization problem, its Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for optimality:

$$0 \leq k_n \quad \perp I - \sum_m \lambda_{n,m} \geq 0 \quad (3)$$

$$0 \leq q_{n,m} \;\perp -Q_m \left[A_m - B \sum_{n'} q_{n',m} - B q_{n,m} - C - D q_{n,m}\right] + \lambda_{n,m} \geq 0, \forall m \quad (4)$$

$$0 \leq \lambda_{n,m} \;\perp k_n - q_{n,m} \geq 0, \forall m \quad (5)$$

Assuming interior solutions for primary decision variables, k_n and $q_{n,m}$, and a binding capacity only in scenario 2, we obtain via symmetry $q_m \equiv \sum_{n'} q_{n',m} = N q_{n,m}$ and $q_m^- = (N-1) q_{n,m}$. KKT condition (4) yields:

$$q_{n,m}^{CO} = \frac{A_m - C - \frac{\lambda_{n,m}}{Q_m}}{(N+1)B + D}, \forall m \quad (6)$$

Since $\lambda_{n,1}^{CO} = 0$ and $\lambda_{n,2}^{CO} = I > 0$ (from (3)):

$$q_{n,1}^{CO} = \frac{A_1 - C}{(N+1)B + D} \quad (7)$$

$$q_{n,2}^{CO} = \frac{A_2 - C - \frac{I}{Q_2}}{(N+1)B + D} \quad (8)$$

$$k_n^{CO} = \frac{A_2 - C - \frac{I}{Q_2}}{(N+1)B + D} \quad (9)$$

Consequently, equilibrium prices are $\pi_1^{CO} = C + (B+D) q_{n,1}^{CO}$ and $\pi_2^{CO} = C + (B+D) k_n^{CO} + \frac{I}{Q_2}$, which are obtained by substituting (7) and (8) into the inverse-demand function, $A_m - B \sum_{n'} q_{n',m}$.

For two scenarios, as it will never be optimal to invest in capacity that is not used in the high-price scenario, i.e., we will have $q_{n,2}^{CO} = k_n^{CO}$, the problem (1)–(2) may alternatively be reformulated as follows:

$$\max_{k_n \geq 0, q_{n,1} \geq 0} Q_1 \left[A_1 - \frac{(N+1)}{2} B q_{n,1} - C - \frac{D}{2} q_{n,1}\right] q_{n,1} + Q_2 \left[A_2 - \frac{(N+1)}{2} B k_n - \frac{I}{Q_2} - C - \frac{D}{2} k_n\right] k_n \quad (10)$$

Again, assuming interior solutions, we obtain from the first-order necessary conditions the same results for $q_{n,1}^{CO}$ and k_n^{CO} as in (7) and (9), respectively. For example, the partial derivative of (10) with respect to $q_{n,1}$ set equal to zero yields $Q_1 \left(A_1 - N B q_{n,1} - B q_{n,1} - C - D q_{n,1}\right) = 0$, i.e., the marginal revenue of firm n equals its marginal cost.

2.2.2. Perfect competition

Using the reformulated problem from the risk-neutral Cournot oligopoly (10) and recognizing the role of N, the number of firms, we have the following problem for a risk-neutral perfectly competitive firm:

$$\max_{k_n \geq 0, q_{n,1} \geq 0} Q_1 \left[A_1 - \frac{N}{2} B q_{n,1} - C - \frac{D}{2} q_{n,1}\right] q_{n,1} + Q_2 \left[A_2 - \frac{N}{2} B k_n - \frac{I}{Q_2} - C - \frac{D}{2} k_n\right] k_n \quad (11)$$

Note that when the partial derivative of (11) with respect to $q_{n,1}$ is set equal to zero, we have $Q_1 \left(A_1 - N B q_{n,1} - C - D q_{n,1}\right) = 0$, i.e., price equals marginal cost. Thus, optimal decisions are as follows:

$$q_{n,1}^{PC} = \frac{A_1 - C}{NB + D} \quad (12)$$

$$k_n^{PC} = \frac{A_2 - C - \frac{I}{Q_2}}{NB + D} \quad (13)$$

Based on the results (12) and (13), perfectly competitive equilibrium prices are $\pi_1^{PC} = C + D q_{n,1}^{PC}$ and $\pi_2^{PC} = C + D k_n^{PC} + \frac{I}{Q_2}$.

2.2.3. Monopoly

A monopoly serves as a counterfoil to perfect competition since it exhibits full exercise of market power. Its *total* capacity and scenario-1 production decisions are $\hat{k}$ and $\hat{q}_1$, respectively. Assuming that capacity is again binding only in scenario 2, we render the monopolist's profit-maximization problem in a similar vein as (10). However, in order to have comparable industry-wide supply curves between a monopoly and an oligopoly, we modify the monopolist's production-cost parameters to $\hat{C} \equiv C$ and $\hat{D} \equiv \frac{D}{N}$. Thus, the profit-maximization problem under a monopoly is:

$$\max_{\hat{k} \geq 0, \hat{q}_1 \geq 0} Q_1 \left[A_1 - B \hat{q}_1 - \hat{C} - \frac{\hat{D}}{2} \hat{q}_1\right] \hat{q}_1 + Q_2 \left[A_2 - B \hat{k} - \frac{I}{Q_2} - \hat{C} - \frac{\hat{D}}{2} \hat{k}\right] \hat{k} \quad (14)$$

By partially differentiating (14) with respect to $\hat{q}_1$ and setting the resulting expression equal to zero, we have $Q_1 \left(A_1 - B \hat{q}_1 - B \hat{q}_1 - \hat{C} - \hat{D} \hat{q}_1\right) = 0$, i.e., the monopolist's marginal revenue equals its marginal cost. The resulting optimal quantities under a monopoly are:

$$\hat{q}_1^{MO} = \frac{A_1 - \hat{C}}{2B + \hat{D}} \quad (15)$$

$$\hat{k}^{MO} = \frac{A_2 - \hat{C} - \frac{I}{Q_2}}{2B + \hat{D}} \quad (16)$$

Upon substitution of (15) and (16) into the inverse-demand function, the monopoly equilibrium prices are $\pi_1^{MO} = \hat{C} + \left(B + \hat{D}\right) \hat{q}_1^{MO}$ and $\pi_2^{MO} = \hat{C} + \left(B + \hat{D}\right) \hat{k}^{MO} + \frac{I}{Q_2}$.

2.3. Risk-averse setting

We now use risk-adjusted probabilities such that $0 \leq R_2 \leq Q_2$. There are two possibilities per case based on the threshold, $\overline{R} \equiv \frac{I}{A_2 - A_1} \geq 0$, below which capacity binds in both scenarios (cf. Proposition 1 in Section 3). Note that a high (low) value of R_2 corresponds to low (high) risk aversion because it weights profit more (less) in the high-price scenario.

2.3.1. Cournot oligopoly

Low risk aversion ($R_2 \geq \overline{R}$) The setup of the problem is identical to that in (10) but with the scenario-2 probability, Q_2, replaced by the subjective scenario-2 probability, R_2:

$$\max_{k_n\geq 0, q_{n,1}\geq 0} Q_1\left[A_1-\frac{(N+1)}{2}Bq_{n,1}-C-\frac{D}{2}q_{n,1}\right]q_{n,1} + R_2\left[A_2-\frac{(N+1)}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n - Ik_n \quad (17)$$

Unaffected by the change in probability measure, $q_{n,1}^{CO}$ is the same as in (7), while k_n^{CO} becomes:

$$k_n^{CO}=\frac{A_2-C-\frac{I}{R_2}}{(N+1)B+D} \quad (18)$$

Note that for $R_2 = Q_2$, (18) is identical to (9), while (18) also exhibits that optimal capacity, k_n^{CO}, decreases with increasing risk aversion, i.e., smaller R_2 (cf. Proposition 2 in Section 3).

High risk aversion ($R_2 < \overline{R}$) Since k_n^{CO} decreases monotonically as R_2 decreases, i.e., the firm becomes more risk averse, under extremely high levels of risk aversion, the capacity constraint may become binding in both scenarios. This leads to the following special case of problem (17) where $q_{n,1} = k_n$:

$$\max_{k_n\geq 0} Q_1\left[A_1-\frac{(N+1)}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n + R_2\left[A_2-\frac{(N+1)}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n - Ik_n \quad (19)$$

This yields $q_{n,1}^{CO} = q_{n,2}^{CO} = k_n^{CO}$, where:

$$k_n^{CO}=\frac{\left(\frac{Q_1}{Q_1+R_2}\right)A_1+\left(\frac{R_2}{Q_1+R_2}\right)A_2-C-\frac{I}{Q_1+R_2}}{(N+1)B+D} \quad (20)$$

2.3.2. Perfect competition

Low risk aversion ($R_2 \geq \overline{R}$) The setup of the problem is similar to that in (11):

$$\max_{k_n\geq 0, q_{n,1}\geq 0} Q_1\left[A_1-\frac{N}{2}Bq_{n,1}-C-\frac{D}{2}q_{n,1}\right]q_{n,1} + R_2\left[A_2-\frac{N}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n - Ik_n \quad (21)$$

Again, $q_{n,1}^{PC}$ is the same as in (12), while k_n^{PC} becomes:

$$k_n^{PC}=\frac{A_2-C-\frac{I}{R_2}}{NB+D} \quad (22)$$

High risk aversion ($R_2 < \overline{R}$) The monotonic decrease in k_n^{PC} with R_2 again may cause the capacity constraint to bind in both states, thereby leading to the following special case of problem (21):

$$\max_{k_n\geq 0} Q_1\left[A_1-\frac{N}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n + R_2\left[A_2-\frac{N}{2}Bk_n-C-\frac{D}{2}k_n\right]k_n - Ik_n \quad (23)$$

Thus, we have $q_{n,1}^{PC} = q_{n,2}^{PC} = k_n^{PC}$, where:

$$k_n^{PC}=\frac{\left(\frac{Q_1}{Q_1+R_2}\right)A_1+\left(\frac{R_2}{Q_1+R_2}\right)A_2-C-\frac{I}{Q_1+R_2}}{NB+D} \quad (24)$$

2.3.3. Monopoly

Low risk aversion ($R_2 \geq \overline{R}$) The setup of the problem follows from that in (14):

$$\max_{\hat{k}\geq 0, \hat{q}_1\geq 0} Q_1\left[A_1-B\hat{q}_1-\hat{C}-\frac{\hat{D}}{2}\hat{q}_1\right]\hat{q}_1 + R_2\left[A_2-B\hat{k}-\hat{C}-\frac{\hat{D}}{2}\hat{k}\right]\hat{k} - I\hat{k} \quad (25)$$

Independent of the probability measure, $\hat{q}_1^{MO}$ is the same as in (15). However, $\hat{k}^{MO}$ becomes:

$$\hat{k}^{MO}=\frac{A_2-\hat{C}-\frac{I}{R_2}}{2B+\hat{D}} \quad (26)$$

High risk aversion ($R_2 < \overline{R}$) As before, $\hat{k}^{MO}$ decreases monotonically as R_2 decreases, and the capacity constraint may bind in both states. This leads to a special case of problem (25):

$$\max_{\hat{k}\geq 0} Q_1\left[A_1-B\hat{k}-\hat{C}-\frac{\hat{D}}{2}\hat{k}\right]\hat{k} + R_2\left[A_2-B\hat{k}-\hat{C}-\frac{\hat{D}}{2}\hat{k}\right]\hat{k} - I\hat{k} \quad (27)$$

Consequently, $\hat{q}_1^{MO} = \hat{q}_2^{MO} = \hat{k}^{MO}$, where:

$$\hat{k}^{MO}=\frac{\left(\frac{Q_1}{Q_1+R_2}\right)A_1+\left(\frac{R_2}{Q_1+R_2}\right)A_2-\hat{C}-\frac{I}{Q_1+R_2}}{2B+\hat{D}} \quad (28)$$

3. Comparative statics

Here, we present our main results in the form of five propositions for which the proofs are given in the Appendix. The risk-adjusted probability, $\overline{R}$, corresponds to the threshold at which (18) is greater than (20).

Proposition 1. *The critical threshold is* $\overline{R} = \frac{I}{A_2-A_1} \geq 0$.

In effect, $R_2 < \overline{R}$ defines a "high" level of risk aversion in which the capacity constraint binds in both scenarios.

Next, a more risk-averse firm reduces optimal capacity investment, which is intuitive.

Proposition 2. *Optimal capacity investment decreases with risk aversion, i.e.,* $\frac{\partial k_n^{CO}}{\partial R_2} > 0$, $\frac{\partial k_n^{PC}}{\partial R_2} > 0$, *and* $\frac{\partial \hat{k}^{MO}}{\partial R_2} > 0$.

Interestingly, in an extremely risk-averse outcome where $R_2 = 0$, capacity investment is determined just by the scenario-1 probability:

Proposition 3. *The minimum installed capacity per firm is* $\frac{A_1-C-\frac{I}{Q_1}}{(N+1)B+D}$ *and* $\frac{A_1-C-\frac{I}{Q_1}}{NB+D}$ *under Cournot oligopoly and perfect competition, respectively. Meanwhile, the minimum installed capacity for the entire industry is* $\frac{A_1-\hat{C}-\frac{I}{Q_1}}{2B+\hat{D}}$ *under monopoly.*

It may be verified that the capacities in (18) and (20) for Cournot oligopoly (or (22) and (24) for perfect competition or (26) and (28) for monopoly) converge as $R_2 \to \overline{R}$:

Proposition 4. *The installed capacity per firm when* $R_2 = \overline{R}$ *is* $\frac{A_1-C}{(N+1)B+D}$ *and* $\frac{A_1-C}{NB+D}$ *for Cournot oligopoly and perfect competition, respectively. The installed capacity for the entire industry when* $R_2 = \overline{R}$ *is* $\frac{A_1-\hat{C}}{2B+\hat{D}}$ *under monopoly.*

We next investigate how it may be possible for greater risk aversion to increase the *expected profit* per firm. As before, there are two possibilities.

Low risk aversion ($R_2 \geq \overline{R}$) Let the *maximized objective function value* of an oligopolistic firm be denoted by $F_n^{CO}(R_2)$, which is obtained by inserting the optimal solutions, (7) and (18), into (17):

$$F_n^{CO}(R_2) = Q_1\left[A_1-\frac{(N+1)}{2}Bq_{n,1}^{CO}-C-\frac{D}{2}q_{n,1}^{CO}\right]q_{n,1}^{CO} + R_2\left[A_2-\frac{(N+1)}{2}Bk_n^{CO}(R_2)-C-\frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) - Ik_n^{CO}(R_2) \quad (29)$$

Note that we explicitly indicate the dependence of the optimal capacity on the risk-aversion parameter, R_2. Next, we apply the envelope theorem [51] to determine $\frac{\partial F_n^{CO}}{\partial R_2}$:

$$\frac{\partial F_n^{CO}}{\partial R_2} = \left[A_2-\frac{(N+1)}{2}Bk_n^{CO}(R_2)-C-\frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) \quad (30)$$

It should be emphasized that $F_n^{CO}(R_2)$ is *not* the expected profit of an oligopolistic firm, which is $V_n^{CO}(R_2)$. In fact, the connection between the expected profit per firm, $V_n^{CO}(R_2)$, and the maximized objective function value of the firm under risk aversion, $F^{CO}(R_2)$, is established by inserting the optimal solutions, (7) and (18), into (1) with $M = 2$ and a binding capacity constraint only in scenario 2:

$$V_n^{CO}(R_2) = Q_1\left[A_1 - NBq_{n,1}^{CO} - C - \frac{D}{2}q_{n,1}^{CO}\right]q_{n,1}^{CO} + Q_2\left[A_2 - NBk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) - Ik_n^{CO}(R_2) \quad (31a)$$

$$\Rightarrow V_n^{CO}(R_2) = F_n^{CO}(R_2) - Q_1\frac{(N-1)}{2}B\left(q_{n,1}^{CO}\right)^2 + \left(Q_2 - R_2\right)\left(A_2 - C - \frac{D}{2}k_n^{CO}(R_2)\right)k_n^{CO}(R_2) - \left(Q_2N - R_2\frac{(N+1)}{2}\right)Bk_n^{CO}(R_2)^2 \quad (31b)$$

$$\Rightarrow \frac{\partial V_n^{CO}}{\partial R_2} = \left\{\left(Q_2 - R_2\right)\left(A_2 - C - Dk_n^{CO}(R_2)\right) - B\left[2NQ_2 - (N+1)R_2\right]k_n^{CO}(R_2)\right\}\frac{\partial k_n^{CO}}{\partial R_2} \quad (31c)$$

Following similar steps for a perfectly competitive firm and a monopolistic industry, we insert (12) and (22) into (21) to obtain $F_n^{PC}(R_2)$ and (15) and (26) into (25) to obtain $F^{MO}(R_2)$:

$$F_n^{PC}(R_2) = Q_1\left[A_1 - \frac{N}{2}Bq_{n,1}^{PC} - C - \frac{D}{2}q_{n,1}^{PC}\right]q_{n,1}^{PC} + R_2\left[A_2 - \frac{N}{2}Bk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) - Ik_n^{PC}(R_2) \quad (32)$$

$$F^{MO}(R_2) = Q_1\left[A_1 - B\hat{q}_1^{MO} - \hat{C} - \frac{\hat{D}}{2}\hat{q}_1^{MO}\right]\hat{q}_1^{MO} + R_2\left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) - I\hat{k}^{MO}(R_2) \quad (33)$$

Application of the envelope theorem to (32) and (33) yields:

$$\frac{\partial F_n^{PC}}{\partial R_2} = \left[A_2 - \frac{N}{2}Bk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) \quad (34)$$

$$\frac{\partial F^{MO}}{\partial R_2} = \left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) \quad (35)$$

The connection between the maximized objective function value under risk aversion and the expected profit for PC is derived by inserting the optimal solutions, (12) and (22), into (1) with $M = 2$ and a binding capacity constraint only in scenario 2:

$$V_n^{PC}(R_2) = Q_1\left[A_1 - NBq_{n,1}^{PC} - C - \frac{D}{2}q_{n,1}^{PC}\right]q_{n,1}^{PC} + Q_2\left[A_2 - NBk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right] \times k_n^{PC}(R_2) - Ik_n^{PC}(R_2) \quad (36a)$$

$$\Rightarrow V_n^{PC}(R_2) = F_n^{PC}(R_2) - Q_1\frac{N}{2}B\left(q_{n,1}^{PC}\right)^2 + \left(Q_2 - R_2\right)\left(A_2 - C - \frac{D}{2}k_n^{PC}(R_2)\right)k_n^{PC}(R_2) - \left(Q_2N - R_2\frac{N}{2}\right)Bk_n^{PC}(R_2)^2 \quad (36b)$$

$$\Rightarrow \frac{\partial V_n^{PC}}{\partial R_2} = \frac{\partial k_n^{PC}}{\partial R_2}\left\{\left(Q_2 - R_2\right)\left(A_2 - C - Dk_n^{PC}(R_2)\right) - \left[2Q_2 - R_2\right]BNk_n^{PC}(R_2)\right\} \quad (36c)$$

Likewise, for MO, the expected profit is defined by inserting the optimal solutions, (15) and (26), into (14):

$$V^{MO}(R_2) = Q_1\left[A_1 - B\hat{q}_1^{MO} - \hat{C} - \frac{\hat{D}}{2}\hat{q}_1^{MO}\right]\hat{q}_1^{MO} + Q_2\left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) - I\hat{k}^{MO}(R_2) \quad (37a)$$

$$\Rightarrow V^{MO}(R_2) = F^{MO}(R_2) + \left(Q_2 - R_2\right)\left(A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right)\hat{k}^{MO}(R_2) \quad (37b)$$

$$\Rightarrow \frac{\partial V^{MO}}{\partial R_2} = \left(Q_2 - R_2\right)\left(A_2 - 2B\hat{k}^{MO}(R_2) - \hat{C} - \hat{D}\hat{k}^{MO}(R_2)\right)\frac{\partial \hat{k}^{MO}}{\partial R_2} \quad (37c)$$

High risk aversion ($R_2 < \overline{R}$) In a similar vein, we insert the optimal solution, (20), into (19):

$$F_n^{CO}(R_2) = Q_1\left[A_1 - \frac{(N+1)}{2}Bk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) + R_2\left[A_2 - \frac{(N+1)}{2}Bk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) - Ik_n^{CO}(R_2) \quad (38)$$

Application of the envelope theorem yields:

$$\frac{\partial F_n^{CO}}{\partial R_2} = \left[A_2 - \frac{(N+1)}{2}Bk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) \quad (39)$$

Again, exploiting the relationship between the expected profit and the maximized objective function value under risk aversion, we use (20) in (1) with $M = 2$ and a binding capacity constraint in both scenarios to yield:

$$V_n^{CO}(R_2) = Q_1\left[A_1 - NBk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) + Q_2\left[A_2 - NBk_n^{CO}(R_2) - C - \frac{D}{2}k_n^{CO}(R_2)\right]k_n^{CO}(R_2) - Ik_n^{CO}(R_2) \quad (40a)$$

$$\Rightarrow V_n^{CO}(R_2) = F_n^{CO}(R_2) - Q_1\frac{(N-1)}{2}Bk_n^{CO}(R_2)^2 + \left(Q_2 - R_2\right)\left(A_2 - C - \frac{D}{2}k_n^{CO}(R_2)\right)k_n^{CO}(R_2) - \left(Q_2N - R_2\frac{(N+1)}{2}\right)Bk_n^{CO}(R_2)^2 \quad (40b)$$

$$\Rightarrow \frac{\partial V_n^{CO}}{\partial R_2} = \frac{\partial k_n^{CO}}{\partial R_2}\left\{\left(Q_2 - R_2\right)\left(A_2 - C - Dk_n^{CO}(R_2)\right) - B\left[Q_1(N-1) + 2NQ_2 - (N+1)R_2\right]k_n^{CO}(R_2)\right\} \quad (40c)$$

Similarly, for a perfectly competitive firm and monopolistic industry, we insert (24) into (23) and (28) into (27), respectively, to obtain analogous maximized objective function values under risk aversion:

$$F_n^{PC}(R_2) = Q_1\left[A_1 - \frac{N}{2}Bk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) + R_2\left[A_2 - \frac{N}{2}Bk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) - Ik_n^{PC}(R_2) \quad (41)$$

$$F^{MO}(R_2) = Q_1\left[A_1 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) + R_2\left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) - I\hat{k}^{MO}(R_2) \quad (42)$$

Application of the envelope theorem to (41) and (42) leads to the following:

$$\frac{\partial F_n^{PC}}{\partial R_2} = \left[A_2 - \frac{N}{2}Bk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) \quad (43)$$

$$\frac{\partial F^{MO}}{\partial R_2} = \left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) \quad (44)$$

Subsequently, the expected profit for a PC firm may be derived by utilizing (24) in (1) with $M = 2$ and a binding capacity constraint in both scenarios:

$$V_n^{PC}(R_2) = Q_1\left[A_1 - NBk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) + Q_2\left[A_2 - NBk_n^{PC}(R_2) - C - \frac{D}{2}k_n^{PC}(R_2)\right]k_n^{PC}(R_2) - Ik_n^{PC}(R_2) \quad (45a)$$

$$\Rightarrow V_n^{PC}(R_2) = F_n^{PC}(R_2) - Q_1\frac{N}{2}Bk_n^{PC}(R_2)^2 + \left(Q_2 - R_2\right)\left(A_2 - C - \frac{D}{2}k_n^{PC}(R_2)\right)k_n^{PC}(R_2)$$

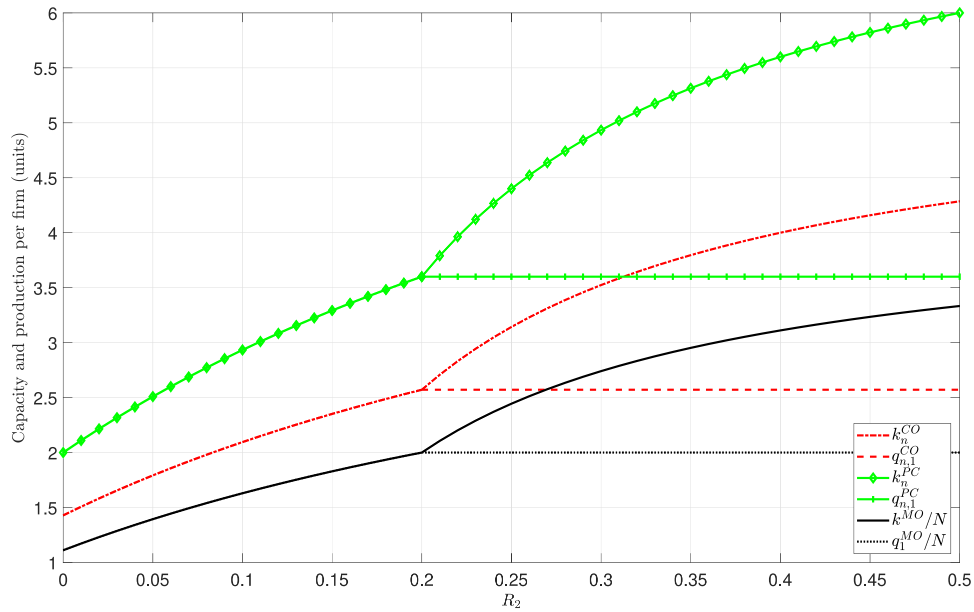

Fig. 1. Capacity and production per scenario per firm with respect to risk aversion, R_2.

$$-\left(Q_2 N - R_2 \frac{N}{2}\right) B k_n^{PC}(R_2)^2 \tag{45b}$$

$$\Rightarrow \frac{\partial V_n^{PC}}{\partial R_2} = \left\{\left(Q_2 - R_2\right)\left(A_2 - C - D k_n^{PC}(R_2)\right) - \left[Q_1 + 2Q_2 - R_2\right] N B k_n^{PC}(R_2)\right\} \frac{\partial k_n^{PC}}{\partial R_2} \tag{45c}$$

Applying the same principle, the expected profit for a monopoly results from inserting the optimal solution (28) into (14) with $\hat{q}_1^{MO} = \hat{k}^{MO}(R_2)$:

$$V^{MO}(R_2) = Q_1\left[A_1 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) + Q_2\left[A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right]\hat{k}^{MO}(R_2) - I\hat{k}^{MO}(R_2) \tag{46a}$$

$$\Rightarrow V^{MO}(R_2) = F^{MO}(R_2) + \left(Q_2 - R_2\right)\left(A_2 - B\hat{k}^{MO}(R_2) - \hat{C} - \frac{\hat{D}}{2}\hat{k}^{MO}(R_2)\right)\hat{k}^{MO}(R_2) \tag{46b}$$

$$\Rightarrow \frac{\partial V^{MO}}{\partial R_2} = \frac{\partial \hat{k}^{MO}}{\partial R_2}\left(Q_2 - R_2\right)\left(A_2 - 2B\hat{k}^{MO}(R_2) - \hat{C} - \hat{D}\hat{k}^{MO}(R_2)\right) \tag{46c}$$

It can now be proven that it is possible for an oligopolistic firm to *increase* its expected profit while becoming more risk averse. Furthermore, this seemingly counterintuitive result also holds for a perfectly competitive firm. However, a monopolist's expected profit *decreases* monotonically as risk aversion increases.

Proposition 5. *The expected profit of an oligopolistic or a perfectly competitive firm increases with risk aversion when* $R_2 = Q_2$, *i.e.,* $\left.\frac{\partial V_n^{CO}}{\partial R_2}\right|_{R_2=Q_2} < 0$ *or* $\left.\frac{\partial V_n^{PC}}{\partial R_2}\right|_{R_2=Q_2} < 0$. *The expected profit of a monopolist monotonically decreases with risk aversion, i.e.,* $\frac{\partial V^{MO}}{\partial R_2} > 0$.

Note that in order to prove that the expected profit of an oligopolistic firm does not monotonically decrease as risk aversion increases, it suffices to evaluate its gradient with respect to R_2 at some point $0 \leq R_2 \leq Q_2$ and to verify that it is negative. Thus, we choose $R_2 = Q_2$ and confirm that as the oligopolistic firm becomes infinitesimally more risk averse from a position of risk neutrality, its expected profit increases.

4. Numerical examples

The parameter values used for the examples are: $N = 2$, $M = 2$, $Q_1 = 0.5$, $Q_2 = 0.5$, $A_1 = 10$, $A_2 = 20$, $B = 1$, $I = 2$, $C = 1$, $\hat{C} = 1$, $D = 0.5$, $\hat{D} = 0.25$, $0 \leq R_2 \leq 0.5$. In the figures, MO capacity, production, and profit values are divided by two to facilitate comparison. Vertical axes are truncated accordingly to focus on the main insights. Note that from Proposition 1, we have $\overline{R} = 0.2$, which explains the kinks in Figs. 1–3. This phenomenon is precisely an illustration of Proposition 1's main result: a high level of risk aversion, i.e., low R_2, leads to such an extensive restriction in investment that the capacity constraint binds in both scenarios. By contrast, a relatively high value of R_2, i.e., low risk aversion, results in expanded capacity investment. However, this additional capacity will be utilized fully only in the high-demand scenario since a lower production level maximizes profit in the low-demand scenario, which causes the production levels in the two scenarios to diverge. The critical threshold of R_2, i.e., $\overline{R}$, depends on the capacity-investment cost and consumers' relative willingness to pay in the different scenarios.

Fig. 1 illustrates the monotonicity of optimal investment with respect to risk aversion as proven in Proposition 2. Meanwhile, the minimum installed capacity per firm from the same figure is also evident as 1.43, 2.00, and 1.11 under Cournot oligopoly, perfect competition, and monopoly, respectively, cf. Proposition 3. Moreover, Proposition 4 is also illustrated by Fig. 1 as the optimal investment per firm under Cournot oligopoly, perfect competition, and monopoly at $R_2 = \overline{R}$ is 2.57, 3.60, and 2.00, respectively. Intriguingly, although the installed capacity decreases monotonically and equilibrium prices increase monotonically with risk aversion, cf. Figs. 1 and 2, the expected profit, $V_n^{CO}(R_2)$ and $V_n^{PC}(R_2)$, may increase under Cournot oligopoly and perfect competition, respectively, but monotonically decreases with risk aversion for a monopoly (see Fig. 3), which is our main result and proven rigorously in Proposition 5.

The expected profit in Fig. 3 (likewise the prices in Fig. 2) is an output as a result of making the optimal decisions in Fig. 1. The purpose of these illustrations is not only to demonstrate that the conventional wisdom about the monotonic tradeoff between expected profit and CVaR in stochastic programming may not hold in an equilibrium framework but also to provide a rigorous proof for why and how it may arise. Toward this end, consider Figure 5.16 from [1] (reproduced here as Fig. 4) that is a standard result in stochastic programming when prices

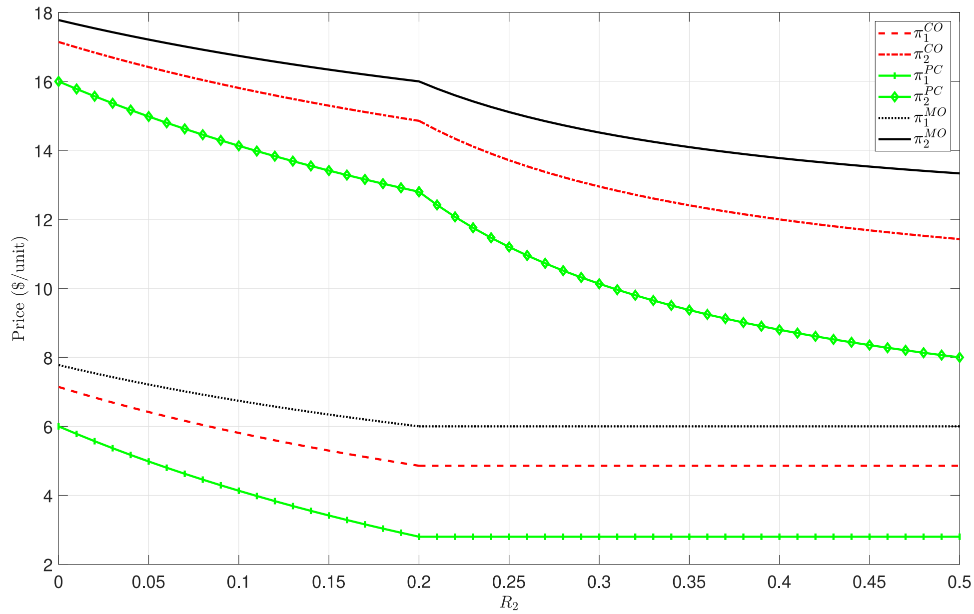

Fig. 2. Prices per scenario with respect to risk aversion, R_2.

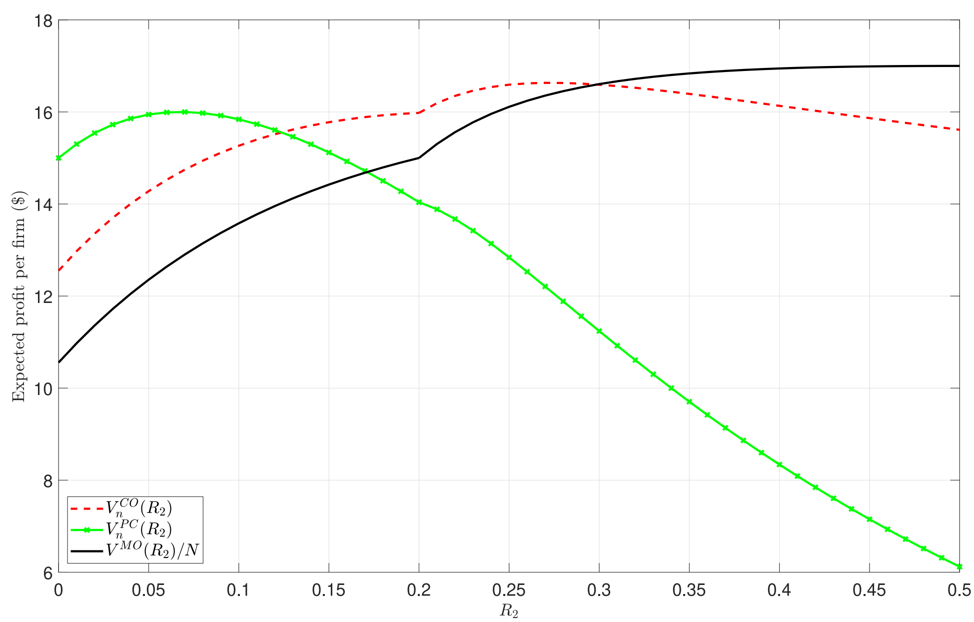

Fig. 3. Expected profit per firm with respect to risk aversion, R_2.

are taken as exogenous. This efficient frontier is obtained precisely by solving the deterministic equivalent of a stochastic programming problem for different values of β, i.e., the risk-aversion parameter. In particular, when $\beta = 0$, the agent maximizes expected profit, but when $\beta = 1$, it maximizes CVaR. Therefore, it is standard to obtain optimal decisions for different levels of risk aversion and subsequently to compare metrics of interest, viz., expected profit and CVaR. Our approach is no different and is fully aligned with that of the literature.

5. Conclusions

Via a Nash–Cournot framework, we provide a rigorous proof for why risk aversion may increase expected profit in capacity-investment equilibria under uncertainty. In particular, oligopolistic and perfectly competitive firms underestimate the impact of their own decisions on price. Thus, they reduce installed capacity "too much" considering their *actual* level of risk aversion.

Given the deregulation of industries such as energy and telecommunications, firms increasingly make decisions under imperfect competition and uncertainty. In such potential oligopolies, where barriers to entry exist due to high capital intensities, the conventional wisdom about a monotonic tradeoff between the expected profit and the CVaR is likely to be affected, thereby complicating investment planning. Indeed, this has been demonstrated in more detailed analyses of the energy sector [41,42]. Consequently, policymakers should anticipate such effects when designing markets and crafting policy. In effect, policy has conventionally been informed by the view that more uncertainty will curb investment by producers as well as their expected

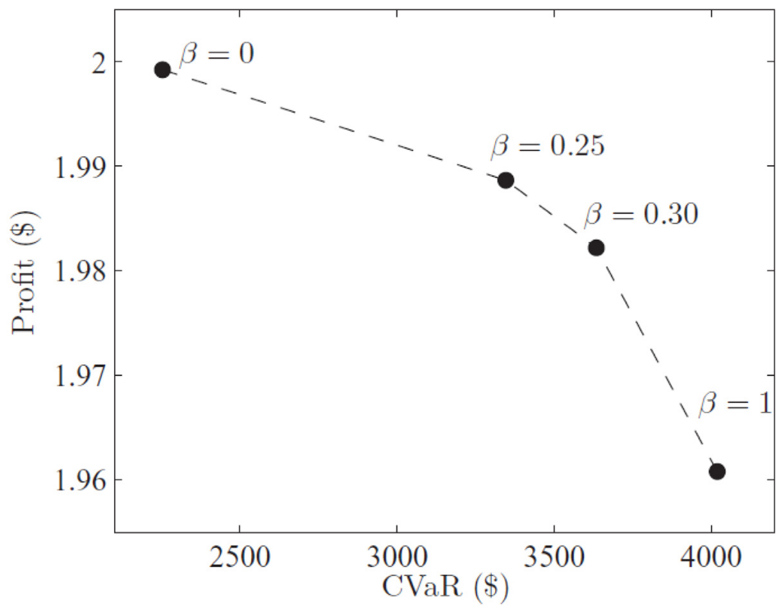

Fig. 4. Efficient frontier from stochastic programming [1].

profit. In order to mitigate the undesirable consequences of reduced investment, policymakers could incentivize greater capacity adoption, e.g., via subsidies. However, the effectiveness of those subsidies would be assessed under the assumption of a single welfare-maximizing entity that fully accounts for the impact of its decisions on prices, which does not hold in a decentralized industry, cf. Proposition 2. Hence, welfare-enhancing subsidies would have to be tuned to the level of risk-averse agents' responses in PC and CO settings.

Future work may build upon our framework to assess how risk aversion in bi-level problems would affect closed-loop investment decisions [46] in a dynamic framework [52] and policy measures [53]. Considering multi-stage investment problems and games, it would be interesting to analyze interactions with time consistency [28] and subgame perfect equilibria [54].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix. Proofs of propositions

Proof of Proposition 1. Equate (18) and (20) with $R_2 = \overline{R}$ and solve for $\overline{R}$. □

Proof of Proposition 2. There are two possibilities depending on the value of R_2:

Low risk aversion ($R_2 \geq \overline{R}$) Partially differentiate (18), (22), and (26) with respect to R_2 to yield $\frac{IR_2^{-2}}{(N+1)B+D} > 0$, $\frac{IR_2^{-2}}{NB+D} > 0$, and $\frac{IR_2^{-2}}{2B+\hat{D}} > 0$, respectively.

High risk aversion ($R_2 < \overline{R}$) Partially differentiate (20), (24), and (28) with respect to R_2 to yield $\frac{I+Q_1(A_2-A_1)}{[(N+1)B+D](Q_1+R_2)^2} > 0$, $\frac{I+Q_1(A_2-A_1)}{(NB+D)(Q_1+R_2)^2} > 0$, and $\frac{I+Q_1(A_2-A_1)}{(2B+\hat{D})(Q_1+R_2)^2} > 0$, respectively. □

Proof of Proposition 3. Taking $\lim_{R_2\to 0} k_n^{CO}$ in (20), $\lim_{R_2\to 0} k_n^{PC}$ in (24), and $\lim_{R_2\to 0} \hat{k}^{MO}$ in (28) yields the result. □

Proof of Proposition 4. There are two possibilities depending on the value of R_2:

Low risk aversion ($R_2 \geq \overline{R}$) Use (18) in $\lim_{R_2\to\overline{R}} k_n^{CO}$ to obtain $\frac{A_2-C-\frac{I}{\frac{I}{A_2-A_1}}}{(N+1)B+D}$, (22) in $\lim_{R_2\to\overline{R}} k_n^{PC}$ to obtain $\frac{A_2-C-\frac{I}{\frac{I}{A_2-A_1}}}{NB+D}$, and (26) in $\lim_{R_2\to\overline{R}} \hat{k}^{MO}$ to obtain $\frac{A_2-\hat{C}-\frac{I}{\frac{I}{A_2-A_1}}}{2B+\hat{D}}$, which yields $\frac{A_1-C}{(N+1)B+D}$, $\frac{A_1-C}{NB+D}$, and $\frac{A_1-\hat{C}}{2B+\hat{D}}$, respectively.

High risk aversion ($R_2 < \overline{R}$) Use (20) in $\lim_{R_2\to\overline{R}} k_n^{CO}$ to obtain $\frac{\frac{Q_1A_1(A_2-A_1)+IA_2-I(A_2-A_1)}{(A_2-A_1)Q_1+I}-C}{(N+1)B+D}$, (24) in $\lim_{R_2\to\overline{R}} k_n^{PC}$ to obtain $\frac{\frac{Q_1A_1(A_2-A_1)+IA_2-I(A_2-A_1)}{(A_2-A_1)Q_1+I}-C}{NB+D}$, and (28) in $\lim_{R_2\to\overline{R}} \hat{k}^{MO}$ to obtain $\frac{\frac{Q_1A_1(A_2-A_1)+IA_2-I(A_2-A_1)}{(A_2-A_1)Q_1+I}-\hat{C}}{2B+\hat{D}}$, which yields $\frac{A_1-C}{(N+1)B+D}$, $\frac{A_1-C}{NB+D}$, and $\frac{A_1-C}{2B+\hat{D}}$, respectively. □

Proof of Proposition 5. Evaluate (31c) at $R_2 = Q_2$ using (18):

$$\left.\frac{\partial V_n^{CO}}{\partial R_2}\right|_{R_2=Q_2} = -B\left[2NQ_2 - (N+1)Q_2\right] k_n^{CO}(Q_2) \left.\frac{\partial k_n^{CO}}{\partial R_2}\right|_{R_2=Q_2}$$
$$= -B(N-1)Q_2\left(\frac{IQ_2^{-2}}{(N+1)B+D}\right)\left(\frac{A_2-C-\frac{I}{Q_2}}{(N+1)B+D}\right) < 0$$

Evaluate (36c) at $R_2 = Q_2$ using (22):

$$\left.\frac{\partial V_n^{PC}}{\partial R_2}\right|_{R_2=Q_2} = -Q_2 B N k_n^{PC}(Q_2) \left.\frac{\partial k_n^{PC}}{\partial R_2}\right|_{R_2=Q_2}$$
$$= -Q_2 B N \left(\frac{I Q_2^{-2}}{NB+D}\right)\left(\frac{A_2 - C - \frac{I}{Q_2}}{NB+D}\right) < 0$$

For the monopolist, there are two possibilities depending on the level of risk aversion, R_2, and its relation to the critical threshold, $\overline{R}$, as given in Proposition 1.

Low risk aversion ($R_2 \geq \overline{R}$) Evaluate (37c) using (26):

$$\frac{\partial V^{MO}}{\partial R_2} = (Q_2 - R_2)\left[A_2 - \hat{C} - (2B+\hat{D})\frac{\left(A_2 - \hat{C} - \frac{I}{R_2}\right)}{2B+\hat{D}}\right]\frac{\partial \hat{k}^{MO}}{\partial R_2}$$
$$= (Q_2 - R_2)\left(\frac{I}{R_2}\right)\left(\frac{I R_2^{-2}}{2B+\hat{D}}\right) > 0$$

High risk aversion ($R_2 < \overline{R}$) Evaluate (46c) using (28):

$$\frac{\partial V^{MO}}{\partial R_2} = (Q_2 - R_2)\left[A_2 - \hat{C} - (2B+\hat{D})\frac{\left(A_1 Q_1 + A_2 R_2 - I - \hat{C}(Q_1+R_2)\right)}{(2B+\hat{D})(Q_1+R_2)}\right]\frac{\partial \hat{k}^{MO}}{\partial R_2}$$
$$= (Q_2 - R_2)\left(\frac{(A_2 - A_1)Q_1 + I}{Q_1 + R_2}\right)\left(\frac{(A_2 - A_1)Q_1 + I}{(2B+\hat{D})(Q_1+R_2)^2}\right) > 0 \quad \square$$

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