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MOOCs in First Year Engineering Mathematics

Experiences and Future Aims

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INTRODUCTION

MOOCs (Massive Open Online Courses) are open and free Internet courses in which everyone can participate: there are neither tuition fees nor entry requirements. An experimental MOOC on Matrix Algebra was opened at Aalto University in 2016. The MOOC was arranged in connection with regular teaching for a period of six weeks from January to February, and it followed the curriculum of the usual first-year engineering students' Matrix Algebra course. High school math on vector calculus (such as the course MAA5 in Finnish high-school curriculum) or a similar level of knowledge was a sufficient prerequisite, and earlier studies of university level mathematics were not expected.

The purpose of the course was learning to use matrices to present and solve systems of linear equations, to carry out arithmetical operations with matrices, and to learn about matrix decompositions. The course was arranged in Finnish, and there were 192 students of Aalto University and a group of 33 active participants who were not students of Aalto University. A larger number of students enrolled to the course, but many of them did not actually submit any solutions to assignments.

The mathematics e-learning core of the Matrix Algebra MOOC is STACK automatic assessment system, originally developed by Chris Sangwin, University of Edinburgh [3]. It is an open source software package that integrates into the popular Moodle

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Virtual Learning Environment. A customized version of STACK was first used at Aalto University in autumn of 2006, and Aalto University has been active to further develop capabilities and the user experience of the system since then. Currently, STACK has a significant development community in several countries [11], for example the MITO project [12,13] at IP Leiria, Portugal. A large community of STACK assignment developers is building around the Abacus material bank [14].

The MOOC was organized as a pure distance learning course for participants coming from outside the Aalto University. For students of Aalto University, the course was arranged in blended learning style: traditional lectures and problem solving sessions were given as usual to support the web-based MOOC materials. There was a weekly schedule for studies provided by the organizers, which followed a regular six-week model used by our 5 ECTS credit courses. In addition to using the electronic materials, students could interact via social learning environment Piazza.

1 MOOC MATERIALS AND THE E-LEARNING ENVIRONMENT

There were several reasons for choosing Matrix Algebra as the topic for the pilot course. For example, the first-year Matrix Algebra course does not require a background in university mathematics, and the topic is useful for various applications. Further, Matrix Algebra is particularly suitable for automatic assessment, allowing the course to be arranged with the reasonable number of teachers.

Typical MOOCs may have thousands of participants, and studying takes place in some e-learning environment. In view of this, our course was a quite small, and we did not know about the number of participants beforehand. We used Aalto OpenLearning platform based on Moodle and STACK 3. In addition to these resources, we used web-based materials implemented by using HTML with MathJax for displaying mathematical formulas, GeoGebra animations for visualization, and screencasts for short lecture videos. The graphical design of the MOOC was made emotionally and visually appealing by using illustrations that were created by a professional cartoonist.

2 AUTOMATIC ASSESSMENT IN LEARNING CONCEPTS AND PROCEDURES

It is a somewhat unique aspect of mathematics as a subject of studies that learning new and more abstract topics always involves a substantial amount of communication with lower level abstractions. The foundation of learning new mathematical concepts rests on already known concepts, but learning a new concept also requires practicing on a suitable collection of examples. This is how a student becomes familiar with new concepts and is eventually able to either assimilate or accommodate it [5, 6].

In traditional classroom education, mathematical concepts are usually implicitly embedded in lectures and exercise assignments that explicitly involve mostly procedural skills. Conceptual learning is not regarded as a matter requiring particular attention or treatment in teaching. Since automatic assessment technology as such calls for smaller and more specific assignments, it makes much sense to train the most important concepts more explicitly (see [1]). Therefore, we consider the desired conceptual and procedural learning outcomes separately. They are collectively called the mastery skills as they are crucial prerequisites for further studies [7].

2.1 Case 1: Systems of linear equations and Gaussian elimination

As a mathematical concept, the system of linear equations builds on the equation of the line that the students learn in high school. Unfortunately, school teaching tends to emphasize procedural skill of solving an equation of first order; something that almost all first year engineering students can do (see [8]).

However, it is not always clear if engineering students can deeply understand the mathematical concept it invokes, i.e., the connection between the formula and the geometric object. Many school teachers choose not to emphasise the geometric meaning since it is not strictly required in solving procedural assignments involving equations of first order, and it is difficult to test or detect conceptual understanding in traditional examinations. Without deeper conceptual understanding mathematics, however, becomes a collection of seemingly arbitrary, poorly motivated rules that are difficult to remember. Not having learned to see, or find, simplicity in complications is a poor foundation for higher education in engineering and science. Therefore, we first presented the system of linear equations as a geometric concept, which is illustrated in Figure 1.

The basic procedural skill related to the concept of linear system equations is the Gaussian elimination algorithm. Understanding the concept is not strictly a requirement for solving a basic elimination assignment. This task is a straightforward application of the manipulations that students learn to do for equations in the high school. The main challenge here is in ensuring that all students repeat the solution process sufficiently many times to learn the algorithm, which is easy do with an automatic assessment system. On the other hand, the process itself is somewhat dull and uninspiring, in particular, for a student who does not understand the underlying concepts to the degree of really understanding the meaning of the assignment. Therefore, conceptual assignments support and motivate learning of procedural skills as well.

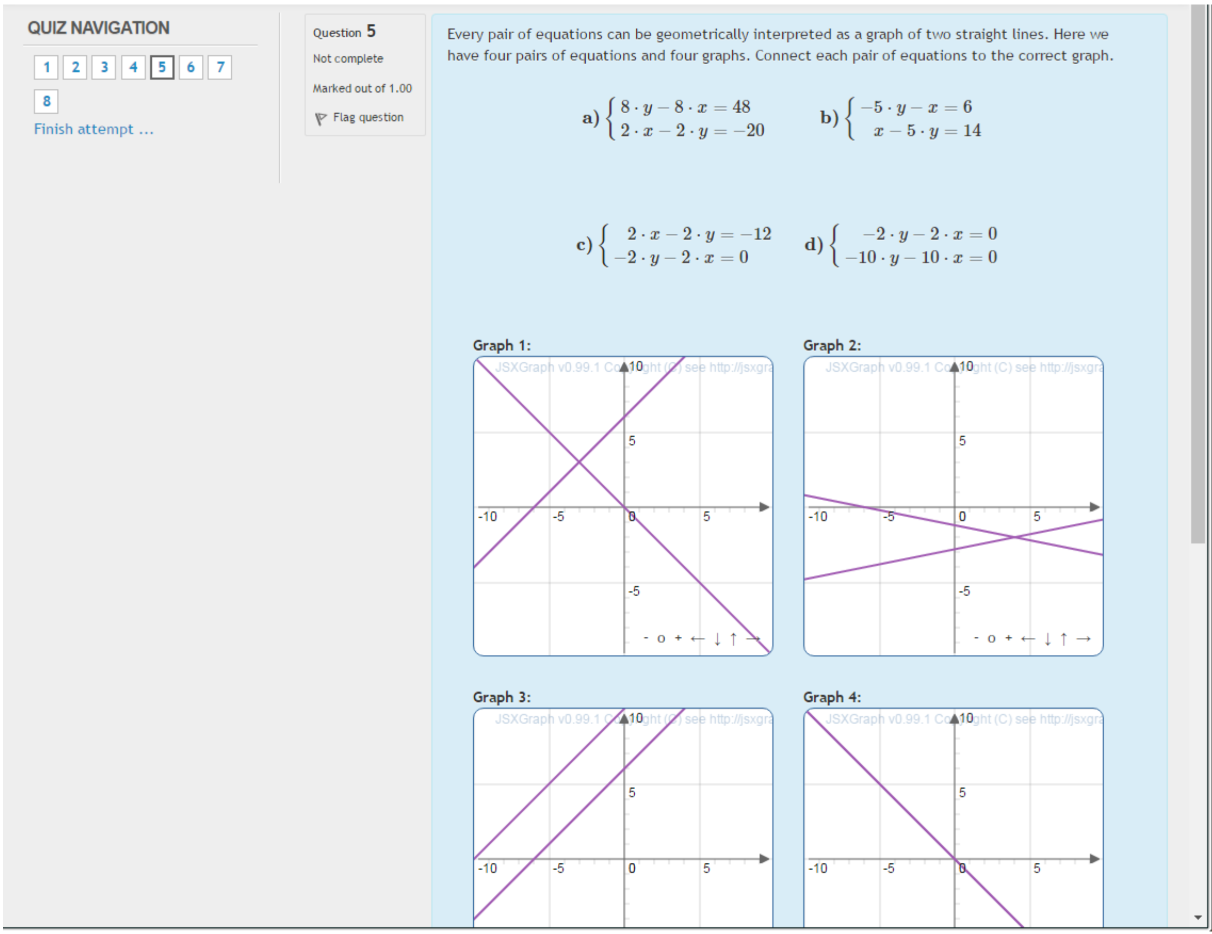

Fig. 1. Systems of linear equations.

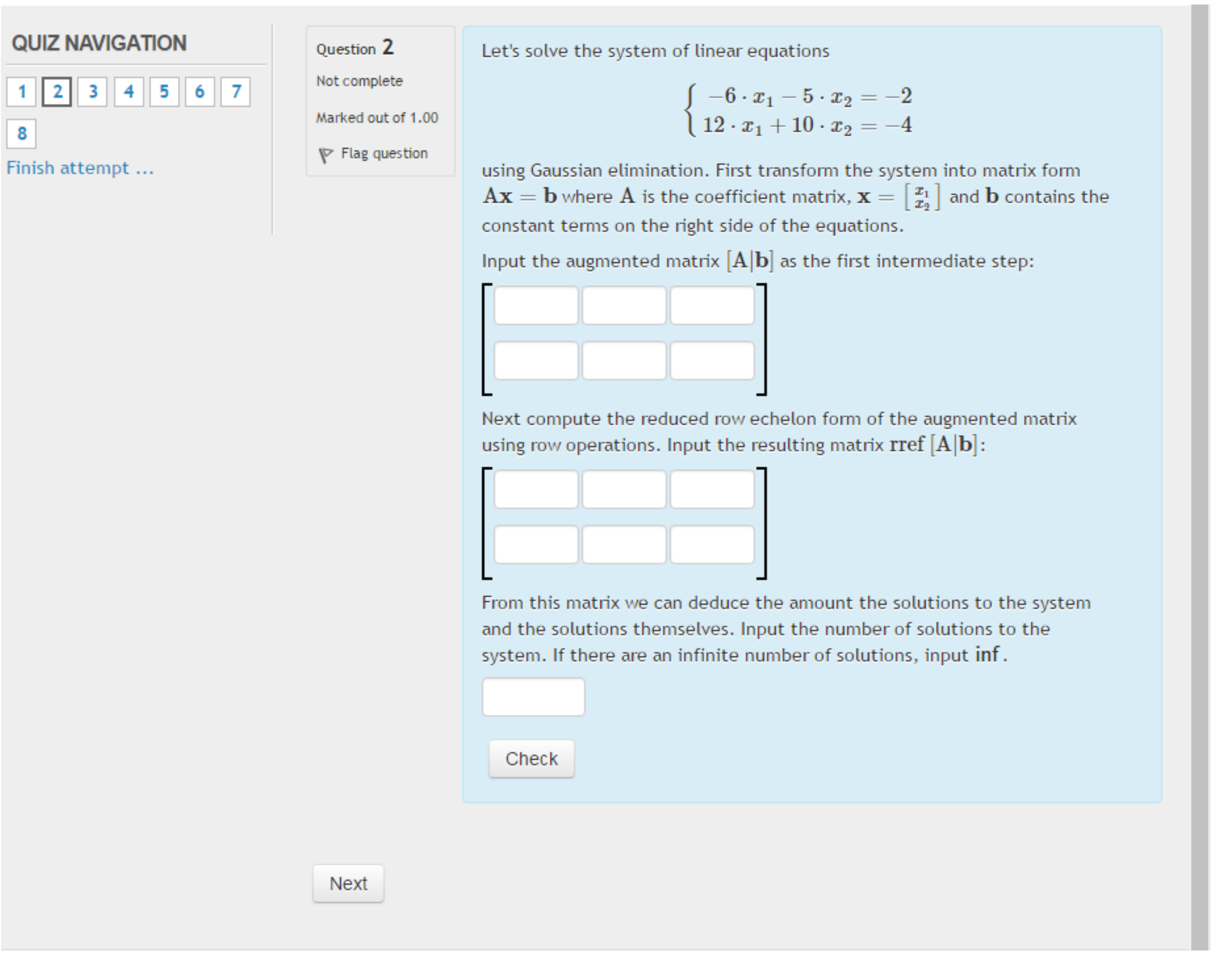

Fig. 2. Gaussian elimination.

2.2 Case 2: Linear independence and linear combinations

Linear independence is another important concept that arises in mathematical and engineering applications. The challenge in teaching this concept is that its level of abstraction is rather high, and many relevant examples involving linear independence are not closely related to each other. Indeed, such examples tend to be either too trivial to properly illustrate the point of the concept, or the mathematical idea is overly difficult to grasp because disorienting or misleading information is presented (Skemp [6] calls these high noise examples). One solution is to aim at the same target from many directions; i.e., to have the student practise with several examples, each illustrating different aspects of the same concept.

One such assignment, created with GeoGebra, is illustrated in Figure 3. The present implementation does not give student feedback or make use of automatic assessment at all. In future, we expect to have an extension of STACK for better handing of geometric information. The illustrated assignment is, however, a good example of a use case for the required future functionality.

Again, there are procedural tasks associated with the concept of linear independence. One simple example is presenting a vector as linear combinations of other vectors, which reduces into solving a system of linear equations. Another, more involved example is finding an orthonormal basis of a vector space by using the Gramm-Schmidt process. Both of these examples are readily implementable as automatically assessable problems by using the current version STACK.

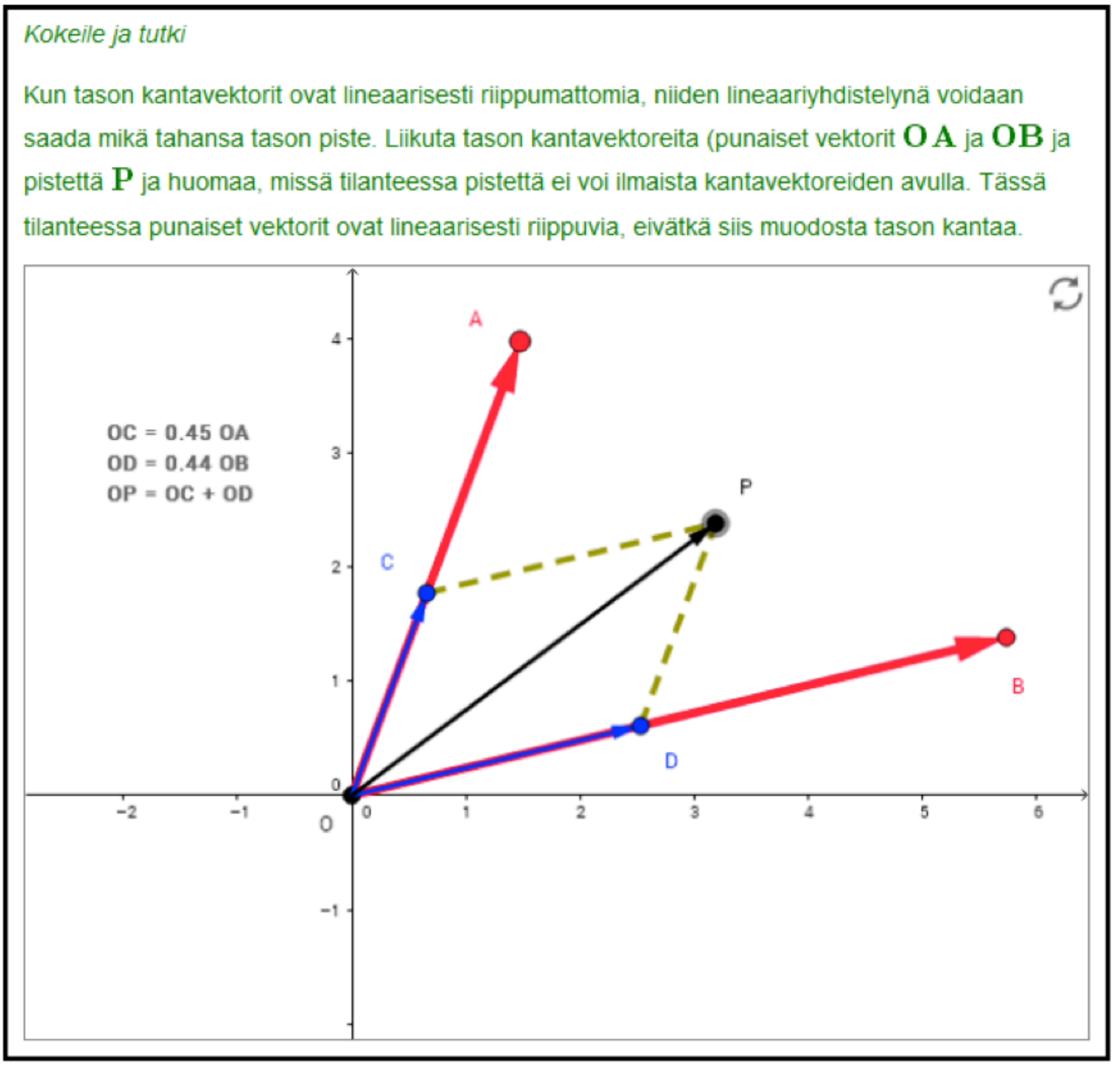

Fig. 3. GeoGebra assignment involving linear combinations of vectors.

3 EVALUATION OF THE PILOT COURSE

The main purpose of the pilot MOOC was to study to what extent it is possible to organize such a course by using the available technology. In particular, we were interested in how much support the online tools require and how much resources are needed.

As an overall evaluation of this experiment, it can be said is that organizing MOOC on Matrix Algebra is both possible and practical. The required e-learning technology is mature enough for use in significant scale with only a few technical problems. Required amount of human support and supervision was significantly lower than initially expected. A complete evaluation of experiences is given in [2].

Unfortunately, there were fewer participants from outside the university than expected. Not many participants were high school students, the original target audience. This is probably due to the inconvenient schedule dictated by university period system; the MOOC had to be arranged in parallel with the six-week Matrix Algebra lecture course in Aalto University. A better time for MOOC would be late spring or summer, and the timetable should be more relaxed as well.

As a summary, it can be said that the pilot course was quite successful from the point of view of the technology. The success is partly explained by clever choices made in developing the pilot course: the e-learning materials were designed to play on current strengths and engineer around the inherent limitations of the on-line learning technologies, including the automatic assessment system STACK.

The learning outcomes of the MOOC-type Matrix Algebra course were substantially better than we have seen on comparable courses based on mostly traditional university teaching. In particular, we found that practically all of the active students had learned the basic skills required for passing the course in the examinations, such as solving a system of linear equations and computing the product of matrices. This result further underlines the observation already made in [9] that automatic assessment is particularly useful in helping lower performing students. The student feedback from the course was overwhelmingly positive, and the course materials and web assignments earned the most of praise (see [2]).

Interestingly, students also commented that the course arrangements offered them total academic freedom, which they greatly appreciated. This shows that academic freedom and good learning outcomes should not always be seen contradictory even for lower performing students.

4 FUTURE STEPS: TOWARDS TRACING THE SOLUTION PROCESS

The current version of STACK has certain limitations that are evident when expanding its use in MOOCs outside the relatively straightforward topics such as the core Matrix Algebra. Presently, STACK works very for teaching procedural and algorithmic tasks, but it is not particularly suitable for assignments where choice of a solution strategy is required, e.g., for certain assignments in Calculus courses [1].

An important source of these limitations is the so-called function model that STACK (and other automated assessment systems) use for grading the student answers. In grading based on the function model, the grade given for a particular student's solution is just a pre-defined function of some parameters that contain the student's answer. Superficially, this process of grading appears similar to the work done by a human teacher when grading examinations, but actually it is quite different for, at least, three reasons.

Firstly, a human teacher is able to do more delicate and sophisticated decisions in grading an assignment. For example, if a student makes a simple mistake on the first line of the solution, but then solves the rest of the problem correctly, a human teacher is able to give an appropriate small penalty and grade the rest of the problem accordingly. However, an automatic assessment system is only able to see that solution is wrong, and ask the student to try again. At its best, the system is able to show the student where the mistake was made but only after the last part of the solution was submitted to the system. While this behaviour will undoubtedly encourage students to learn careful and error-free working practices, it can also be very frustrating. A better approach would be to track the student's solution process in real time, and give the student a possibility to correct a mistake right after it was made, instead of only giving final feedback when the whole solution process is completed.

Secondly, an automatic assessment system is usually restricted to smaller amount of information than a human teacher. It is not practical to require the student to type in the entire solution process on the computer, rather than the solution and some particularly important intermediate steps, only. This restriction limits the possibility of giving detailed feedback.

Thirdly, the student's solution process may be influenced by the questions. If the student is asked to provide some specific intermediate steps, it may already provided significant help in solving the problem. This is particularly true for multiple choice questions where it is often a clear sign of intelligence to approach the problem through excluding impossible choices rather than finding the right solution directly. However, the basically same problem may arise in more sophisticated assignments. While guiding the student step by step through the solution process can be good pedagogy in certain situations, it sometimes provides too much information, and there should be an option not to show the remaining steps. In order to address these limitations, the plan is to improve STACK by adding two advanced features discussed in the next sections.

5 WORKING WITH EQUATIONS

There is a requirement for free-form manipulations of equivalent equations with intermediate steps. The student should be able to work in the manner similar to using pen and paper; an important goal in mathematics education even today. Limited ability of the current STACK to handle intermediate steps in algebraic expressions makes it too easy for a student to use solution directly from systems such as Wolfram Alpha. We would like to point out that using Wolfram Alpha in solving STACK assignments may very well be recommendable, intelligent action but then it should rise above the level of just copying and pasting the final result. This functionality would allow, for example, completely computerized and even web-based examinations along the lines discussed in the paper [10]. On the other hand, asking the student to type all intermediate steps the solution process, is probably be too complex and cumbersome for most purposes.

6 STATE VARIABLES

We are developing an architecture that allows the use of both internal and external state variables within STACK assignments [15]. External state variables make it possible for the assignment to communicate effectively with the “ambient system” carrying out, e.g., learning analytics, or with “sibling systems” (such as GeoGebra) whose functionality complements that of STACK. Even more importantly, the internal state variable extension allows elegant implementation of two types of assignments, at least:

- Assignments whose solution process is game-like: It may be open-ended or return to an earlier part of the same problem with small modifications depending on earlier input.
- Assignments that do not necessarily repeat exactly the same steps or in a specific order for all students that get different versions of the assignment because of parameter randomization.

The internal state variables let STACK keep track of and remember the student’s solution process at each the moment. The question type may be dynamically changed inside a single assignment, depending on the student’s answer history. This will allow elegant implementation of classical calculus problems such as repeated integrations by parts, proofs by mathematical induction, and techniques involving integral transforms that are difficult to properly implement by using existing e-assessment systems. An example of such assignment is illustrated in Figure 4.

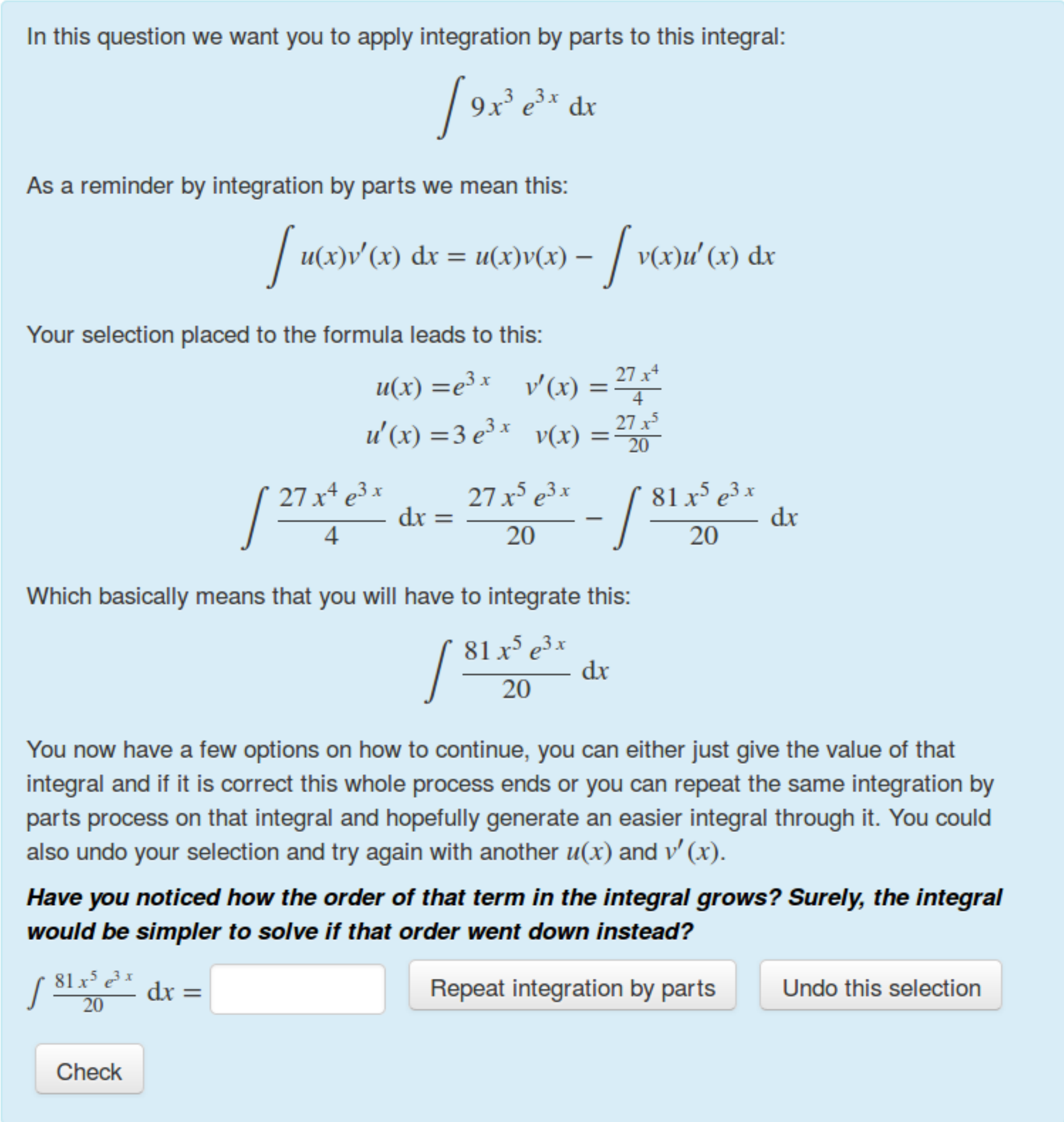

Fig. 4. An example of a STACK assignment with state variables by Matti Harjula [15].

State-awareness also allows problems to be changed according to the data stored in the system. For example, the problem assignment could ask the student to type some intermediate steps of the solution process and, if the student can do them without errors, ask for fewer details in later attempts. The randomization of STACK problem's parameters values may be carried out in the course of student's progress, and it may take into consideration student's past input. Another option is to first ask only for the full solution, and only in case of an error further details would be asked in order to pinpoint the mistake.

A particularly interesting application utilizing state-awareness are learning games, where a student is given a large number of similar tasks. Instead of obtaining a grade from an individual assignment, the player (student) has to accumulate certain minimum score in order to move forward (pass the course), see [1].

7 CONCLUSIONS

Automatic assessment can be a powerful tool in reducing teacher's workload and making online distance learning more attractive. It is a valuable tool for organizing MOOCs, where challenges of traditional mass education meet those of distance learning.

We have demonstrated that an attractive online course on Matrix Algebra can be organized in a highly automated manner by using current technologies. Learning outcomes and student satisfaction on this course were comparable to or even better than those on traditional courses. While the technology is, at the moment, not able to deal with most challenging types of problem assignments, its scope of applications is rapidly widening. We expect that the vast majority of the content in Bachelor level engineering mathematics is suitable for automatic assessment within a few years.

REFERENCES

[1] Rasila, A., Malinen, J., & Tiitu, H. (2015). On automatic assessment and conceptual understanding. Teaching Mathematics and its Applications 34 (3), pp. 149–159.

[2] Rasila, A., Savela, J., & Tiitu, H. (2016). Learning Matrix Algebra on a MOOC. Accepted to ITK 2016 Tutkijatapaaminen.

[3] Sangwin, C. (2013). Computer aided assessment of mathematics. OUP Oxford.

[4] Rasila, A., Harjula, M., & Zenger, K. (2007). Automatic assessment of mathematics exercises: Experiences and future prospects. In Yanar, A. & Saarela-Kivimäki, K. (eds.) ReflekTori 2007. Symposium of Engineering Education, December 3–4, 2007. Helsinki University of Technology Teaching and Learning Development Unit Publication 1/2007. Espoo: Helsinki University of Technology, pp. 70–80. Available at http://matta.math.aalto.fi/publications/Reflektori2007_70-80.pdf

[5] Piaget, J. (1929). The Child's Conception of the World. London: Routledge & Kegan Paul.

[6] Skemp, R. R. (1987). The Psychology of Learning Mathematics. Expanded American Edition. New York: Routledge.

[7] Rasila, A. & Sangwin C. J. (2016). Development of STACK Assessments to Underpin Mastery Learning. To appear in Proceedings of the 13th International Congress on Mathematical Education, Hamburg, 24-31 July 2016.

[8] Havola, L. (2012). Assessment and learning styles in engineering mathematics education. Licentiate thesis. Aalto University. http://urn.fi/URN:NBN:fi:aalto-201209193106

[9] Rasila, A., Havola, L., Majander, H., and Malinen, J. (2010). Automatic assessment in engineering mathematics: evaluation of impact. In Myller, E. (ed.), ReflekTori 2010 Symposium of Engineering Education, Aalto University School of Science and Technology, 37-45.

[10] Sangwin, C., & Kocher, N. (2016). Automation of mathematics examinations. Computers & Education, 94, 215-227. 10.1016/j.compedu.2015.11.014

[11] Sangwin, C. J. (2015). Who uses STACK? A report on the use of the STACK CAA system, Loughborough University, UK.

[12] Paiva, R. C., Ferreira, M. S., Mendes, A. G., & Eusébio, A. M. J. (2015). Interactive and multimedia contents associated with a system for computer-aided assessment. Journal of Educational Computing Research, 52(2).

[13] Paiva, R. C., Ferreira, M. S. & Frade, M. M. (2016). Intelligent tutorial system based on personalized system of instruction to teach or remind mathematical concepts. Submitted.

[14] Rasila, A. (2016). E-Assessment Material Bank Abacus. To appear EDULEARN16 Proceedings.

[15] Harjula, M., Malinen, J., & Rasila, A. (2016). STACK with state. Under preparation.