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V-notched components under non-localized creeping condition: numerical evaluation of stresses and strains

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Abstract

Geometrical discontinues such as notches need to be carefully analysed by engineers because of the stress concentration generated by them. Notches become even more important when the component is subjected, in service, to very severe conditions, such as the high temperature fatigue and imposed visco-plastic behaviour such as creep.
The aim of the paper is to present an improvement and extension of the existing notch tip creep stress-strain analysis method developed by Nuñez and Glinka, validated for U-notches only, to a wide variety of blunt V-notches.
The key in getting the extension to blunt V-notches is the assumption of the generalized Lazzarin-Tovo solution that allows a unified approach to the evaluation of linear elastic stress fields in the neighbourhood of both cracks and notches. Numerous examples have been analysed up to date, and the stress fields obtained according to the proposed method were compared with appropriate finite element data, showing a very good agreement.
In view of the promising results, authors are considering possible further extension of the method to sharp V-notches and cracks introducing the concept of the Strain Energy Density (SED).

Keywords: Creep, V-notches, stress fields, sress evaluation, strain energy density;

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Nomenclature

a	notch depth
C_p	plastic zone correction factor
d	distance from the coordinate system origin at which the far field contribution is evaluated
E	Young's modulus
K_Ω	strain energy concentration factor
K_t	stress concentration factor
K_I	mode I stress intensity factor
r	radial coordinate
r_0	distance within notch tip and coordinate system origin
r_p	plastic zone radius
t	time
2α	notch opening angle
$\Delta\varepsilon_{22}^{c_n}$	creep strain increment at the notch tip at step n
$\Delta\varepsilon_{22}^{cf_n}$	incremental far field creep strain
$\Delta\varepsilon_{22}^{t_n}$	increment of total strain
Δr_p	plastic zone increment
$\Delta\sigma_{22}^{t_n}$	stress decrement at the notch tip at step n
Δt_n	time increment
ε^{p0}	plastic strain at time t=0
$\varepsilon_{22}^{c_f}$	creep strain at the far field
$\varepsilon_{22}^{c_n}$	creep strain at the notch tip
ε_{22}^{t}	time dependent notch tip strain
ε_{ij}^{0}	actual elastic-plastic strain
ε_{ij}^{e}	hypothetical strain components obtained from linear elastic analysis
θ	angular coordinate
λ_1	mode I eigenvalue
μ_1	mode I second order eigenvalue
ρ	notch tip radius
σ_{max}	maximum stress at the notch tip
σ_{nom}	applied nominal stress
σ_{22}^{f}	far field stress
σ_{22}^{f0}	far field stress, t=0
σ_{22}^{t}	time dependent notch tip stress
σ_{ij}^{0}	actual elastic-plastic stress
σ_{ij}^{e}	hypothetical stress components obtained from the linear elastic analysis
χ_1	mode I associated constant

1. Introduction

Because of technological progress demanding service conditions, engineering components are becoming more complex geometry-wise including various geometrical discontinuities (e.g. notches) that generate localized high stress concentration zones (Berto et al. (2015); He et al. (2015); Sih (2015)). Therefore geometrical discontinuities in a component are regions which have to be carefully considered by the engineers. They become even more important when, in operating conditions, the component is subjected to very demanding conditions such as high temperature fatigue.

The high temperature environment induces time and temperature dependent deformations resulting in a nonlinear stress-strain response such as creep (visco-plasticity). When the creep phenomena are localized or concentrated in a small region near the notch root, they can be considered as localized-creep cases. Non-localized (or gross) creep condition, instead, refers to situations in which the far stress field also experiences some creep and this may contribute to more intense creeping around the notch tip. To the best of the authors' knowledge only a limited number of solutions concerning localized time-dependent creep-plasticity problems are available in literature.

Nuñez and Glinka (2004) have recently presented in one of their papers a solution for non-localized creep strains and stresses at the notch root, based on the linear-elastic stress state, the constitutive law and the material creep model. The method was derived by using the Neuber (1961) total strain energy density rule. This approach yielded very good results when applied to U-notches (2α=0 and ρ≠0).

The aim of the present work is to introduce an extension of the method proposed by Nuñez and Glinka to blunt V-notches. The base of the extension is the substitution of the Creager and Paris (1967) equations with the more general Lazzarin and Tovo (1996) equations. The aim is to propose a method that permits a fast evaluation of the stresses and strains at notches under non-localized creeping condition, without the use of complex and time-consuming FE non-linear analyses. The obtained stresses and strains can be used as input parameters for life prediction creep models based on local approaches. Some comments on the extension of the method to sharp V-notches and cracks based on the average strain energy density concept, as well as on the applicability of linear elastic approaches under creeping conditions, are discussed at the end of the paper.

2. Evaluation of stresses and strains under non-localized creeping condition for blunt V-notches

Nuñez and Glinka (2004) presented a method for the estimation of stress and strain at U-notch tip, subjected to non-localized creep. The method was based on the Neuber (1961) concept extended to time dependent plane stress problems and on the introduction of K_Ω parameter introduced by Moftakhar et al. (1994). It can be assumed in fact that the total strain energy density changes occurring in the far field produce magnified effects at the notch tip. For this reason, the total strain energy density concentration factor is introduced in order to magnify the energy at the notch tip. The introduction of this parameter and of the far field stress and strain contribution in the Neuber's time dependent formulation is the main difference within the non-localized and localized creep formulation that, instead, can be easily derived directly by extending the Neuber's rule. Details about the original formulation can be found in the original works Nuñez and Glinka (2004) and in Gallo et al. (2016).

The key to extend the Nuñez-Glinka method to blunt V-notches is the assumption of the Lazzarin and Tovo (1996) equations to describe the early elastic state of the system.

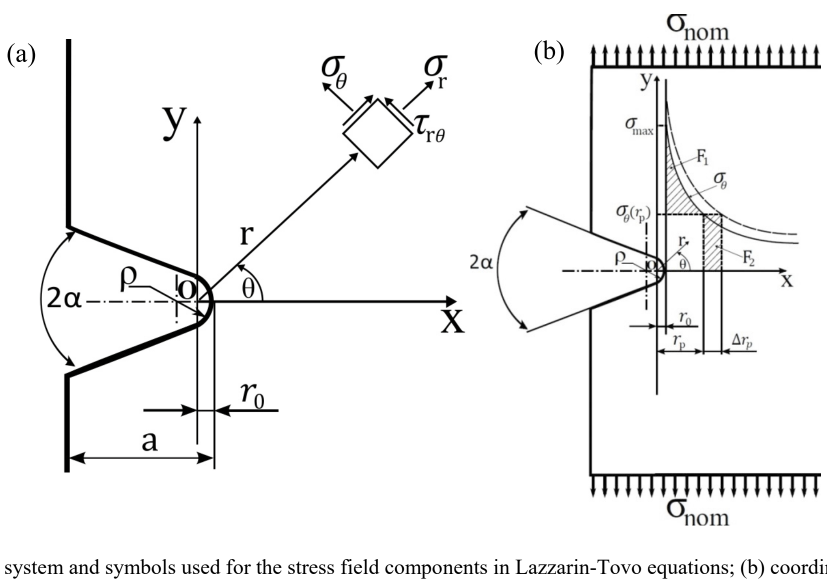

Fig. 1. (a) Coordinate system and symbols used for the stress field components in Lazzarin-Tovo equations; (b) coordinate system and symbols used for the elastic stress field redistribution for blunt V-notches.

The Lazzarin-Tovo equations, in the presence of a traction loading, along the bisector (x axis), can be expressed as follows, as a function of the maximum stress (see Fig. 1):

$$\begin{Bmatrix} \sigma_\theta \\ \sigma_r \end{Bmatrix} = \frac{\sigma_{max}}{4}\left(\frac{r}{r_0}\right)^{\lambda_1-1} \begin{bmatrix} (1+\lambda_1)+\chi_1(1-\lambda_1)+\left(\frac{r}{r_0}\right)^{\mu_1-\lambda_1}\left[(3-\lambda_1)-\chi_1(1-\lambda_1)\right] \\ (3-\lambda_1)-\chi_1(1-\lambda_1)-\left(\frac{r}{r_0}\right)^{\mu_1-\lambda_1}\left[(3-\lambda_1)-\chi_1(1-\lambda_1)\right] \end{bmatrix} \tag{1}$$

Where σ_{max} can be expressed as a function of stress concentration factor K_t (evaluated through linear elastic finite element analysis) and the applied load σ_{nom},

$$\sigma_{\max} = K_t \sigma_{nom} \tag{2}$$

Employing the more general conformal mapping of Neuber (1958) that permit a unified analysis of sharp and blunt notches, the notch radius, ρ, and the origin of the coordinate system, r_0, are related by the following equation on the basis of trigonometric considerations:

$$\rho = \frac{q \cdot r_0}{q-1} \tag{3}$$

where $q = \dfrac{2\pi - 2\alpha}{\pi}$.

The main steps to extend the method to blunt V-Notches can be summarised as follows:

- Assumption of Lazzarin-Tovo equations to describe the stress distribution ahead the notch tip instead of Creager-Paris equations;
- Calculation of the origin of the coordinate system, r_0, as a function of the opening angle and notch radius, as described by Eq. (3);
- Re-definition of the plastic zone correction factor C_p that is a function of plastic zone size r_p and plastic zone increment Δr_p;

The definition of the parameters C_p, r_p and Δr_p is very similar to that clearly reported by Glinka (1985), except for the assumption of different elastic stress distribution equations. Definition of these variables is briefly reported hereafter. Referring to Fig. 2, considering the Von Mises (1913) yield criterion in polar coordinate:

$$\sigma_{ys} = \sqrt{\sigma_r^2 - \sigma_r \sigma_\theta + \sigma_\theta^2} \tag{4}$$

and introducing Eqs. (1) into Eq. (4), a first approximation of r_p that can be solved numerically is obtained.

Once r_p is known, the force F_1 can be evaluated as follows:

$$F_1 = \int_{r_0}^{r_p} \sigma_\theta dr - \sigma_\theta(r_p)\cdot(r_p - r_0) =$$

$$\frac{K_t\sigma_{nom}}{4}\left\{(r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\left[(\lambda_1-1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right] - \frac{[(\lambda_1+1)+\chi_1(1-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1} + \frac{[\chi_1(1-\lambda_1)-(3-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\right\} \tag{5}$$

The stress $\sigma_y(r_p)$ is considered to be constant inside the plastic zone, that means elastic-perfectly plastic behavior is assumed. The lower integration limit is r_0, that depends on the opening angle and notch tip radius. Due to the plastic yielding at the notch tip, the force F_1 cannot be carried through by the material in the plastic zone r_p . But in order to satisfy the equilibrium conditions of the notched body, the force F_1 has to be carried through by the material beyond the plastic zone r_p . As a result, stress redistribution occurs, increasing the plastic zone r_p by an increment Δr_p. If the plastic zone is small in comparison to the surrounding elastic stress field, the redistribution is not significant, and it can be interpreted as a shift of the elastic field over the distance Δr_p away from the notch tip. Therefore the force F_1 is mainly carried through the material over the distance Δr_p, and therefore the force F_2 (represented by the area depicted in the Fig. 1-b) must be equal to F_1. For this reasons, $F_1 = F_2 = \sigma_\theta(r_p) \cdot \Delta r_p$, and the plastic zone increment can be expressed as the ratio between F_1 and σ_θ evaluated (through Lazzarin-Tovo equations) at a distance equal to the previously calculated r_p:

$$\Delta r_p = \frac{F_1}{\sigma_\theta(r_p)} \tag{6}$$

Substituting in Eq. (6) the formula given by Eq. (5) for F_1 and the explicit form of σ_θ, the expression for the evaluation of Δr_p is obtained:

$$\begin{aligned}\Delta r_p = &\left\{\left(\frac{r_p}{r_0}\right)^{1-\lambda_1}\left[(r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]\right.\right.\right.\\ &\left.+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right] - \frac{[(\lambda_1+1)+\chi_1(1-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1}\\ &\left.\left.+\frac{[\chi_1(1-\lambda_1)-(3-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\right]\right\}\\ &/\left\{(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right\}\end{aligned} \tag{7}$$

The last step consists in the definition of the plastic zone correction factor C_p, which is according to Glinka (1985) but introducing the Lazzarin-Tovo equations:

$$\begin{aligned}C_p = 1 + \frac{\Delta r_p}{r_p} = 1 + &\left\{\left(\frac{r_p}{r_0}\right)^{1-\lambda_1}\left[(r_0 - r_p)\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]+\right.\right.\right.\\ &\left.\left.\left.(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right] - \frac{[(\lambda_1+1)+\chi_1(1-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\lambda_1-1}\right]}{\lambda_1} + \frac{[\chi_1(1-\lambda_1)-(3-\lambda_1)]\left[r_0 - r_p\left(\frac{r_p}{r_0}\right)^{\mu_1-1}\right]}{\mu_1}\right]\right\}/\\ &\left\{r_p\left[(\lambda_1+1)+\chi_1(1-\lambda_1)\left[1-\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}+(3-\lambda_1)\left(\frac{r_p}{r_0}\right)^{\mu_1-\lambda_1}\right]\right]\right\}\end{aligned} \tag{8}$$

At this point, the general stepwise procedure to be followed to generate a solution is identical to that proposed by Nuñez and Glinka (2004):

1. Determine the notch tip stress, σ_{22}^e, and strain, ε_{22}^e, using the linear-elastic analysis.

2. Determine the elastic-plastic stress, σ_{22}^{0}, and strain, ε_{22}^{0}, using the Neuber (1961) rule (or other methods e.g. ESED by Molski and Glinka (1981), finite element analysis).
3. Begin the creep analysis by calculating the increment of creep strain, $\Delta\varepsilon_{22}^{c_n}$, for a given time increment Δt_n. The selected creep hardening rule has to be followed.

$$\Delta\varepsilon_{22}^{c_n} = \Delta t_n \cdot \dot{\varepsilon}_{22}^{c}\left(\sigma;t\right) \tag{9}$$

4. Determine the decrement of stress, $\Delta\sigma_{22}^{t_n}$, from Eq. (10), due to the previously determined increment of creep strain, $\Delta\varepsilon_{22}^{c_n}$:

$$\Delta\sigma_{22}^{t_n} = \frac{(K_\Omega C_p)\sigma_{22}^{f0}\Delta\varepsilon_{22}^{cf_n} - \sigma_{22}^{t_{n-1}}\Delta\varepsilon_{22}^{c_n}}{\frac{2}{E}\sigma_{22}^{t_{n-1}} + \varepsilon^{p0} + \varepsilon_{22}^{c_n}} \tag{10}$$

5. For a given time increment Δt_n, determine from Eq. (11) the increment of the total strain at the notch tip, $\Delta\varepsilon_{22}^{t_n}$:

$$\Delta\varepsilon_{22}^{t_n} = \Delta\varepsilon_{22}^{c_n} - \frac{\Delta\sigma_{22}^{t_n}}{E} \tag{11}$$

6. Repeat steps from 3 to 5 over the required time period.

3. Results

The proposed new method has been applied to a hypothetical plate weakened by lateral symmetric V-notches, under Mode I loading; see Fig 1b. The notch tip radius ρ and the opening angle 2α have been varied, while for the notch depth *a*, a constant value equal to 10 mm has been assumed. Three values of the opening angle 2α have been considered: 60°, 120° and 135°. The notch tip radius assumes for every opening angle three values: 0.5, 1 and 6 mm. The plate has a constant height, H, equal to 192 mm and a width, W, equal to 100 mm. The numerical results have been obtained thanks to the implementation of the new developed method and its equations in MATLAB®. In the same time, a 2D finite element analysis has been carried out through ANSYS. The Solid 8 node 183 element has been employed and plane stress condition is assumed. The material elastic (E, ν, σ_{ys}) and Norton Creep power law (n, B) properties are reported in Table 1.

For the sake of brevity, only few examples are reported in Fig. 2 (a-b) considering different opening angles and notch root radius. All the other cases presented the same trend of Fig. 2. The theoretical results are in good agreement with the numerical FE values. All the stresses and strains as a function of time have been predicted with acceptable errors. In detail, maximum discrepancy in modulus of about 20% has been found for both quantities, with a medium error about 10%. The error, as clearly depicted in Fig. 2, increases when considering "long time" while it remains limited when considering a time lower than 5h. This results suggested that, after 5h, large plastic strains are occurring.

In detail, considering the example given in Fig. 2(a), the maximum error for the strain and stress evolution is 9% and 20%, respectively.

Figure 2(b) reports instead the strain evolution against time for different notch radius and constant opening angle 2α equal to 135°. The discrepancy, in absolute value is 10%, negligible and 20% for a notch radius of 0.5 mm, 1 mm and 6 mm, respectively. Their associated stresses (not reported here for the sake of brevity) presented a percentage error varying from 2% (ρ=6 mm) to 17% (ρ=0.5 mm).

Table 1. Mechanical properties.

E (MPa)	ν	σ_{ys} (MPa)	n	B (MPa^{-n}/h)
191000	0.3	275.8	5	1.8

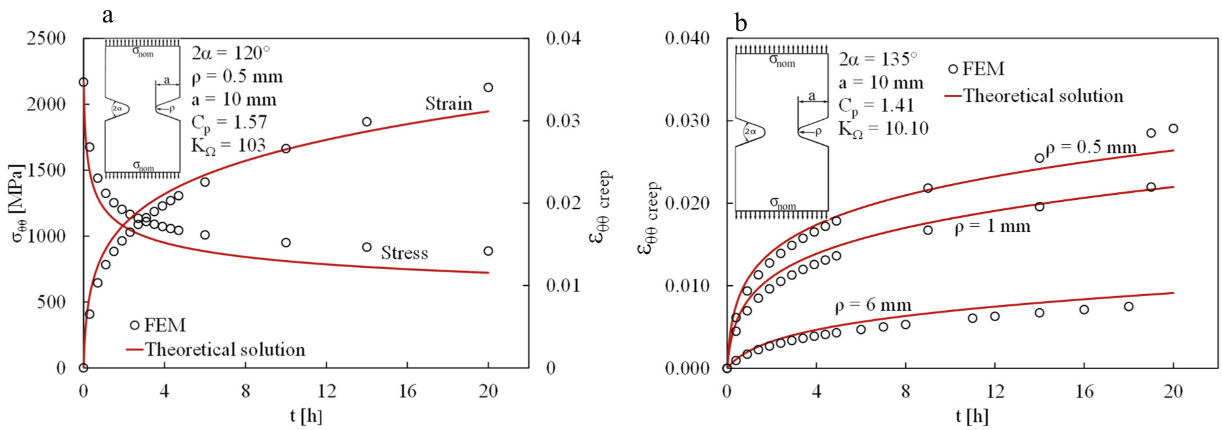

Fig. 2. Comparison between theoretical and FEM evolution of stress and strain as a function of time for V-notch geometry: (a) ρ=0.5 mm, 2α=120°; (b) 2α=135° and ρ=0.5, 1, 6 mm.

4. Conclusions

The present paper proposed an extension of the method presented by Nuñez and Glinka (2004) for blunt U-notches, to a blunt V-notches. The key to getting the extension to blunt V-notches is the substitution of the Creager and Paris (1967) equations with the more general Lazzarin and Tovo (1996) equations that allow an unified approach to the evaluation of linear elastic stress fields in the neighborhood of crack and notches. The main advantage of the new formulation is that it permits a fast evaluation of the stresses and strains at notches under creep conditions, without the use of complex and time-consuming FE non-linear analyses. It is presented for blunt V-notches but also valid for U-notches. Moreover, the localized creep formulation can be easily derived neglecting the contribution of the far field.

The results have shown a good agreement between numerical and theoretical results. Thanks to the extension to blunt V-notches, all geometries can be easily treated with the aim of the numerical method developed.

Although Lazzarin and Tovo equations are valid also in case of sharp V-notches (i.e. for a notch radius that tends to be zero), the values of stress and strain are no longer suitable as characteristic parameters governing failure. As well known, in fact, these local approaches failed when the stress fields tends toward infinity (such as for crack or sharp notches), and the development of alternative solutions becomes crucial. The evaluation of stress and strain at some points ahead of the notch tip may be a possible way to address the problem. Different methods are available in literature dealing with this matter, for example based on energy local approaches such as Strain Energy Density (Berto and Gallo (2015); Gallo and Berto (2015); Gallo (2015)). This parameter could be useful also to characterize creeping conditions if combined with the present model, giving the possibility to include in the analysis also cracks and sharp V-notches. However, some points remain open:

-order singularity variation with time: when considering creeping conditions, the singularity order does not assume a constant value, but varies with time.

-evolution against time from elastic to elastic-plastic or fully plastic state of the system, especially when dealing with high temperature.

Because of the promising results showed in the preliminary analyses, the authors still devoting effort to overcome the problems cited previously and to combine successfully the proposed model for the prediction of stresses and strain with the SED averaged over a control volume, in order to give a useful and more general tool when dealing with notches subjected to creep, regardless of the specimen geometries.

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