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Supporting platelet inventory management decisions: What is the effect of extending platelets’ shelf life?

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1. Introduction

Blood products such as platelets, red blood cells, and plasma are crucial for maintaining the health and well-being of a population. Because blood products save lives, it is critically important to ensure the blood supply chain’s proper functioning. This supply chain, however, is notoriously difficult to manage. Firstly, blood products are highly perishable; platelets have a maximum shelf life of five to seven days, red blood cells of 45 days, and frozen plasma of 24–36 months but, once thawed, has to be used within four to five days if stored between 2 and 8 °C and within eight hours if stored at room temperature. Secondly, both the supply and demand for blood products are uncertain. In particular, there may be significant variability in supply depending on the time of year: for instance, donations are typically lower in December in the US (American Red Cross, 2020) and in May in Finland (Finnish Red Cross Blood, 2019). Furthermore, demand can also vary depending on the season, month, or day of the week. Finally, in Finland and in many other countries, all blood donation is voluntary and non-renumerated. Hence, there is a high social cost associated with attempts to avoid unmet demand with excessive supply: if blood is often disposed of instead of used to help someone, people may be less inclined to donate in the future, thus causing a knock-on effect to the supply.

Platelets, in particular, are a challenging inventory to manage. Platelets are used in preventing and treating any cases of major bleeding, in cancer treatment, and recently, in many types of in vitro production processes such as the development of the Coronavirus vaccine (Chang, Yan, & Wang, 2020). However, compared with red blood cells, their demand is less frequent. In Finland, platelet sales to hospitals are approximately 17% of the red blood...
cells sales (Finnish Red Cross Blood Service, 2021). This, together with their short shelf life increases the difficulty of efficiently managing platelet inventories. Inventories with intermittent or sporadic demand are a much-researched field in forecasting and inventory control, dating back to Croston (1972). Nevertheless, most theory is based on non-perishable items, such as spare car parts (Snyder, Ord, & Beaumont, 2012) or non-perishable grocery items (Sillanpää & Liesiö, 2018).

Balugani, Lolli, Gamberiní, Rimini, & Babai (2019) state the difficulties to inventory managers of perishable items with a limited lifespan and intermittent demand and, to address these difficulties, propose a modification of the periodic order-up-to-level inventory policy in view of minimising operational costs. Yet, this approach cannot be readily applied in the context of blood supply chains, in which the objectives of minimising shortage and expiry hold equal importance with the minimisation of costs. Importantly, given the uncertain demand, these three objectives are typically conflicting with one another: a supply chain cannot achieve 100% met demand with no outdating and minimal cost unless the demand is known beforehand. As platelets are used in organ transplants, cancer treatment, and other prearranged procedures, a proportion of their demand is known. Nevertheless, daily platelet usage is highly variable, and there is limited literature on platelet demand forecasting (Motamedi, Li, Down, & Heddle, 2021). Flint et al. (2020) reinforce this point, highlighting that considerable interest has instead been devoted to modelling platelet systems.

In this paper, we develop a method for optimising platelet inventory management in view of minimising operational costs while ensuring that unmet demand and expiry are minimised. We develop a two-stage stochastic programming model to determine optimal platelet inventory order-up-to levels in Finnish hospitals and the Finnish Red Cross Blood Service (FRCBS) central blood bank. To accurately represent the decentralised decision process between hospitals and the FRCBS, we take a two-levelled approach. The first level generates inventory order-up-to levels and daily order quantities for each hospital individually. The second level uses the aggregated hospital order quantities to generate the FRCBS’s inventory order-up-to level. To circumvent the computational challenge posed by the proposed model, we use the Progressive Hedging algorithm to decompose the problem into computationally manageable sub-problems and ultimately recover tractability.

We use our model to investigate the effect of extending the shelf life of platelets in Finland. Without such a model, it would be difficult to assess the impact of extending the shelf lives on the national supply chain. There have been recent analytical studies investigating the benefits of extending platelet shelf life (see, for example, Earnshaw et al. 2021; Ejohwomu, Too, & Edwards 2021; Prioli et al. 2018; Rebulla & Prati 2022). Yet, these approaches either fail to accommodate for demand uncertainties, or to determine optimal inventory management policies. To the best of our knowledge, we are the first to approach the impacts of shelf life extension techniques employing a stochastic programming approach, which makes it possible to take into account the uncertain nature of the demand while also defining optimal policies for managing the platelet inventories. Finland currently limits platelets to five days shelf life, but the FRCBS wants to investigate extending it to seven days using bacterial cultures or pathogen inactivation, which reduce the infection risk for older platelets. We compute the optimal policies for both five- and seven-day shelf lives and show that the extension would (i) decrease unmet demand, expiry, and operational costs for hospitals and (ii) reduce the order-up-to inventory level at the FRCBS.

The contribution of this paper to the literature is three-fold: (i) We propose a decentralised two-levelled approach to the inventory management problem that reflects the relationship between the FRCBS and hospitals. (ii) Combining the stochastic programming model with the Progressive Hedging solution approach allows us to accommodate a sufficiently large scenario set using real-world data to produce implementable policy recommendations. (iii) We are the first to assess real-world inventory control and cost implications of extending the shelf life of platelets using a stochastic programming model that is solved by employing the Progressive Hedging algorithm.

The outline of this paper is as follows. Section 2 reviews the recent literature in stochastic programming for blood inventories and solution approaches in inventory management models. Section 3 presents the problem description and the mathematical model. Section 4 describes the solution approach. Section 5 discusses the results and provides policy recommendations for both five- and seven-day shelf lives. The final section provides conclusions and directions for future research.

2. Literature review

In recent years, there has been an increase in research into the blood supply chain. Reviews by Beliën & Forcé (2012), Osorio, Brailsford, & Smith (2015), Pirabân, Guerrero, & Labadie (2019), and Eghtesadifard & Jozan (2021) provide a detailed account of the literature. Based on these reviews, two methodological streams can be highlighted: those employing optimisation- and simulation-based approaches. Simulation can be used to represent, in a realistic way, the system features and flows of donors, blood products, and information throughout the supply chain. On the other hand, optimisation is best suited to define the optimal parameters of the whole supply chain such as collection and production policies. According to Pirabân et al. (2019) and Eghtesadifard & Jozan (2021), research efforts are increasingly being focused on optimising the performance of the blood supply chain by using methods such as two- or multi-stage stochastic programming (Dillon, Oliveira, & Abbasi, 2017; Osorio, Brailsford, Smith, & Blake, 2018; Zahiri, Torabi, Mohammadi, & Aghabegloo, 2018), robust optimisation (Heidari-Fathian & Pasandideh, 2018; Kazemi, Rabbaní, Tavakkoli-Moghaddam, & Shahreza, 2017), and Markov decision processes (Ayer et al., 2018; Civelek, Karaesmen, & Scheller-Wolf, 2015). In light of the scope of our work, we concentrate on surveying applications employing optimisation in the context of the blood supply chain. We refer the reader to Pirabân et al. (2019) for a recent survey on research using alternative analytical techniques.

Optimisation efforts have been targeted to each of the four main echelons of the blood supply chain, namely: collection, production, inventory, and distribution (Osorio et al., 2015). More attention has been given to the interactions between two or more echelons of the blood supply chain over the past few years. In particular, many researchers are interested in network design, where the strategic planning of the blood supply chain is modelled. For example, Guppinar & Centeno (2015) present stochastic integer programming models in the inventory and distribution echelons to minimise total costs, shortage, and wastage of blood products at one hospital and one blood bank. Furthermore, Hamdan & Diabat (2019) consider all four echelons of the blood supply chain and present a two-stage stochastic programming model for red blood cells to optimise production, inventory, and location decisions.

One limitation of the above approaches is that the supply chain is assumed to work as a centralised decision-making unit, i.e., the decision-maker has an overview of all modelled nodes (blood banks and hospitals) in the supply chain. For example, Hosseini-Motlagh, Samani, & Homaei (2020) implement a bi-objective two-stage stochastic programming model for red blood cells in a blood supply chain with disruptions. The problem setting includes nine hospitals, one regional blood bank, five local blood banks, and 12 candidates for mobile blood banks. The aim is to minimise the total supply chain costs while providing safer blood transfusion ser-
vices in disruption risks. Dehghani, Abbasi, & Oliveira (2019) consider a network of hospitals that allows for transshipments between hospitals to avoid shortages and mitigate wastage. Yet, a centralised approach may not be realistic in situations such as the one considered in this paper, where hospitals have their independent inventory management and blood banks react to the demand of the hospitals. Therefore, we take a decentralised approach assuming that each node in the supply chain (blood banks and hospitals) is an individual decision-making unit. To the best of our knowledge, decentralised decision-making is a novel approach in the blood supply chain inventory management literature.

The most commonly used optimisation technique for modelling the blood supply chain under demand uncertainty is stochastic programming (see Pirabán et al., 2019, Table 24). Stochastic programming, either two- or multi-stage, lends itself well to this type of problem due to the uncertainty present in the blood supply chain. The overarching goal of stochastic programming is to make “here-and-now” decisions before the uncertainty is revealed and “wait-and-see” decisions after the uncertainty is revealed. In the blood supply chain, these uncertainties are typically supply and demand levels. The uncertainties in stochastic programming are often represented as scenarios, for example, red blood cell demand scenarios over a given period. The aim is to make “here-and-now” decisions that perform well across all scenarios. Thus, the larger the scenario set, the more robust the decision.

In the blood supply chain management literature, scenarios are often generated by assuming a defined distribution. For instance, Gumninar & Centeno (2015) use Poisson and Gamma distributions for the demand of red blood cells and platelets. Najafi, Ahmadi, & Zolfagharian (2017) use Poisson distributed demand with two demand types (fresh units and any age units), and utilise chance constraints to deal with the uncertainties within the model to obtain risk averse decision recommendations. Dillon et al. (2017) use a Negative Binomial distribution to optimise red blood cell inventory replenishment control policies in a hospital, where multiple blood types and substitution were included in the model. The authors also state that Poisson and Normal distributions are often too simplistic to render meaningful models in practice. In contrast, Dehghani et al. (2019) employ what may be viewed as a more realistic Zero-inflated Negative Binomial demand distribution of red blood cells.

Assuming a demand distribution has its drawbacks, as it may not closely reflect reality and, thus, can lead to unreliable decision recommendations. Some authors have sought to overcome this problem by augmenting the distribution assumptions based on real data. For example, Rajendran & Srinivas (2020) propose two new variants of a review ordering policy for platelet inventory management in hospitals. In doing so, they create demand scenarios from a Poisson distribution to be used as inputs for a stochastic mixed-integer linear programming (MILP) model. Despite the assumption of Poisson demand, they also included real-life hospital settings such as weekday/weekend demand fluctuation and varying ages on arrival. In contrast, Rajendran & Ravindran (2017) use real data as a basis for creating 35 demand scenarios to be used as inputs in a MILP model to develop ordering policies for platelets to reduce outdated and shortages under demand uncertainty. Yet, this model includes a single hospital and, therefore, lacks practical relevance in the platelet supply chain. Rajendran & Ravindran (2019) seek to improve the problem setting in Rajendran & Ravindran (2017) by extending it to include a central blood bank and two hospitals. However, the resulting increase in the complexity of the MILP model necessitates the reduction of the number of demand scenarios to 15 to maintain computational tractability. Such a small number of scenarios may result in non-robust decision recommendations.

More generally, there is a trade-off between the accuracy of the stochastic representation (i.e., the number of scenarios) and the practical relevance of the problem setting (i.e., the complexity of the model). For instance, Hamdan & Diabat (2019) introduce a model for optimising the blood supply chain between 22 hospitals, one regional blood bank, three local blood banks, and 36 mobile blood locations. Due to the complexity of the model, the problem can only be solved with six scenarios. The authors state the need for a specialised solution algorithm to include more scenarios. Similarly, in Hossein-Motlagh et al. (2020), including all echelons of the blood supply chain is hugely computationally heavy; thus, only nine scenarios can be utilised. On the other hand, Dillon et al. (2017) and Dehghani et al. (2019) employ 100 scenarios in their models. However, the problem settings in these studies include only one and four hospitals, respectively.

The complexity of practically relevant blood supply chain models comes from their size and combinatorial nature (the models include integer decision variables), posing challenges even to state-of-the-art commercial solvers such as CPLEX or Gurobi (Hamdan & Diabat, 2019). Consequently, utilising models that consider larger numbers of demand scenarios requires the development of specialised solution algorithms that can cope with their large-scale nature. Specifically, one can rely on the idea of breaking the problem into more manageable parts (sub-problems) that can be solved separately and recombined into a solution for the original problem; an approach often referred to as decomposition. Such methods exploit the fact that the computational requirements to solve such problems increase much faster than their size. Thus, it is often more efficient to solve several smaller problems and combine these solutions than tackling the full-scale counterpart at once.

Decomposition methods that have been proposed for stochastic programming problems in particular include Lagrangian Relaxation and Decomposition (Boujema, Jebali, Hammami, & Ruiz, 2020; Oliveira, Gupta, Hamacher, & Grossmann, 2013; Zanjani, Bajgiran, & Nourelfath, 2016), Dual Decomposition (Carae & Schultz, 1999; Lubin, Martin, Petra, & Sandvik, 2013), Benders Decomposition (Cheramin, Saha, Cheng, Paul, & Jin, 2021; Rodríguez, Anjos, Côté, & Desaulniers, 2021; Soares, Canizes, Ghavini, Vale, & Venayagamoorthy, 2017) and Progressive Hedging (Rashiri, Nikzad, Eberhard, Hearne, & Oliveira, 2021; Løkketangen & Woodruff, 1996; Veliz, Watson, Weintrab, Wets, & Woodruff, 2015). Recently, Lagrangian Relaxation has been employed in efficiently solving supply chain optimisation problems in contexts such as inventory-location-routing problem for a perishable product under uncertainty (Rafiei-Majd, Pasandideh, & Naderi, 2018), a green-blood supply chain network design problem (Heidari-Fathian & Pasandideh, 2018), a two-stage stochastic programming model for a blood supply chain under risk of disruptions (Hamdan & Diabat, 2020), and a multi-objective sustainable supply chain problem (Tautenhain, Barbosa-Povoa, Mota, & Nascimento, 2021).

In this paper, we utilise the Progressive Hedging (PH) algorithm as a decomposition method. The choice of utilising PH is justified by its simplicity of implementation, reliability, and successful employment in other application areas (García-Gonzalo, Pais, Bachmatiuk, Barreiro, & Weintrab, 2020; Nikzad, Bashiri, & Abbasi, 2021). The PH algorithm enables to overcome issues of computation tractability even with a fairly complex problem setting and a large number of demand scenarios. To the best of our knowledge, Progressive Hedging has not been applied to perishable supply chain inventory management problems modelled using two-stage stochastic programming, including the blood supply chain.

Table 1 illustrates our positioning and contribution to the literature. In particular, we explore a lesser-studied blood product (platelets), using a specialised solution algorithm to allow the accommodation of a large number of demand scenarios based on real data, acknowledging the decentralised decision-making used
in Finland. In addition to this, our work aids in answering a real-world problem faced by the Finnish Red Cross Blood Service: what are the effects of extending the shelf life of platelets? To our knowledge, no results to support this important policy decision that are based on a stochastic programming model have previously been presented in the literature.

3. Model for optimal inventory management

3.1. Problem and model description

The Finnish Red Cross Blood Service (FRCBS) is in charge of all blood management in Finland: from donation and manufacturing to delivery to hospitals and medical manufacturers. Finland has ten permanent donor centres located mainly in its largest cities. Additionally, the FRCBS organise donation events every weekend across the country. All blood donations are sent to the central manufacturing and storage site in Helsinki, where whole blood donations are separated into blood products, stored, and distributed. Of the total number of platelet units produced in 2020, 90% were used in blood transfusions, with the remaining 10% expired or supplied for medicines manufacture (Finnish Red Cross Blood Service, 2021). Thus, this study limits the platelet supply chain to only include deliveries to hospitals, not medical manufacturers. In 2020, 31,381 platelet units were sold to hospitals, down 0.8% from the previous year. It should also be noted that unlike red cells, platelet deliveries were not much affected by the Covid-19 pandemic.

The FRCBS delivers platelets to 62 national hospitals, of which 6 are large hospitals, 19 are medium-sized hospitals, and 37 are small hospitals. The large hospitals account for 77% of all platelet deliveries from 2018 to 2020, the medium hospitals 21%, and the small 2%. As the demand from the medium and especially the small hospitals is proportionally low, the data from the individual hospitals are sparse. Therefore, we include the six individual large hospitals (named H1, . . . , H6) in our dataset; however, we combine the demand of the 19 medium hospitals into one representative hospital (H7) and the demand of the 37 small hospitals to form another representative hospital (H8).

In this paper, we are interested in the final two echelons for the Finnish platelet supply chain, namely inventory and distribution. Currently, the FRCBS production director places a weekly order to the blood donation organisation based on rule-of-thumb estimates and “gut feeling”. The ordered inventory must satisfy national demand from the 62 hospitals. Our aim is to optimise the Finnish platelet supply chain focusing on the FRCBS and the national hospitals. To do this, we take a two-levelled approach, as illustrated in Fig. 1. First, we model the hospital-level inventory to minimise costs, unmet demand, and expiry of units. In this, we define an inventory order-up-to level for each hospital under demand uncertainty. The hospital-level model is run once for each hospital independently. The resulting order quantities for each hospital are then aggregated to obtain the demand input for the FRCBS central blood bank-level model. The FRCBS inventory control model has the same formulation as the hospital-level model but with different associated costs and lead times. The FRCBS model’s outputs also include an inventory order-up-to level and the number of units needed to be produced in a given period from whole blood collections (the first echelon in the supply chain) to satisfy national demand. The overall objective is to minimise total operational costs to the FRCBS and hospitals while also minimising unmet demand and wastage of platelet units past their shelf life.

Due to the demand uncertainty, we employ a two-stage stochastic programming formulation to minimise the operational costs in the supply chain. The stochasticity is represented by a set of scenarios for platelet demand at each hospital. These scenarios are generated from observed delivery and return data of the 62 national hospitals forming our eight representative hospitals. In the problem setting for both the hospital and FRCBS models, the first-stage decision is to set an optimal order-up-to inventory level prior to observing the uncertain demand. The second-stage decisions, which are made after having observed the demand, relate to allocating the platelet units to minimise operational costs and fulfill demand. Fig. 2 illustrates the first- and second-stage decisions of each hospital and the FRCBS.

The FRCBS is interested in two methods to reduce the chance of transfusion-transmitted bacterial infections and consequently extend the platelets shelf lives to seven days. The first method is a bacteria culture that detects contamination – however, early sampling is required to allow for microbial growth and this comes with a risk of sampling error. The second is a pathogen inactivation method that uses ultraviolet light to not only target bacteria but also reduce the risk associated with other viruses (Levy, Neal, & Herman, 2018). A drawback of this method is that the wavelength used has the potential of causing damage to the platelets. In this paper, we are solely focused on the effects of each extension method to the inventory control management and total costs, whereby the risks associated with these methods are not explicitly considered. To this end, we will use the two-stage stochastic programming model to determine optimal platelet inventory management policies for both five- and seven-day shelf lives.

3.2. Two-stage stochastic programming model

This section presents a mixed-integer linear programming formulation of the Finnish platelet supply chain’s inventory management problem. A table with the notation used in this formulation is presented in Appendix A. The MILP model is represented.
as the deterministic equivalent (Birge & Louveaux, 2011) two-stage stochastic programming (2SSP) model based upon models presented in Dillon et al. (2017) and Dehghani et al. (2019). In this model, we implement the classic order-up-to inventory control policy (Nahmias et al., 2011) for one demand level per decision-making unit with fixed periodic (i.e., daily) review, fixed lead time, no backlog fulfillment, and allowance for emergency orders. In this policy, an order is placed to replenish the inventory to the target level S whenever the current inventory position is below this target1.

1 Since our order-up-to policy is periodic, we have no order-triggering minimum level (typically represented by s in the classical (s, S) policy).
In our model, we do not differentiate demand by the age of units. This choice is based on discussions with the FRCBS, who noted that platelet orders for a specific age are uncommon in practice and that they are reluctant to allow such orders. A formulation capable of handling different demand policies at hospitals such as a new-inventory-to-S policy (a fixed-order-quantity replenishment policy) and a two-level demand policy, both found in Civelek et al. (2015), is provided in Appendix B. Furthermore, we do not differentiate between the two main methods used to produce platelets in Finland: pooled platelets produced from whole blood collection, and via apheresis. In our setting, there are two different uses for apheresis platelets. The majority of apheresis units (80–90%) are used to complement the pooled platelet inventory, in which case they do not need to be differentiated from pooled platelets in inventory management. The remaining 10–20% of apheresis platelets are used when a patient needs a specific platelet phenotype, typically human leukocyte antigens (HLA) matched or, more rarely, human platelet antigens (HPA) matched. These products have a specific logistic arrangement and, thus, the modelling of which is not considered in this article.

Moreover, our model does not consider distinct ABO or RhD types. As is the case in other countries, such as the UK (NHS, 2017) and Australia (Australian Red Cross, 2021), ABO types are not mandatorily matched in practice for the transfusion of platelets in Finland. This is because in the majority of cases, platelets are not at risk for intravascular hemolysis when transfused out of group, differently from red blood cells (Dunbar, 2020). On the other hand, RhD types should be differentiated because they do represent a considerable risk for alloimmunization to the RhD antigen (Dunbar, 2020). Yet, due to the lack of data regarding RhD types, such differentiation is not included in our model. Nevertheless, we assume that inventory capacity is not an issue at the hospitals and that there is no substitution between RhD-positive and RhD-negative patients. Consequently, the aggregated ordering sizes and inventory levels we obtain would be similar to the aggregation of the values observed with inventories differentiated by RhD types.

Let us denote the decision-making units in our problem setting by \( r \in \mathbb{R} = \{H_1, \ldots, H_8, \text{FRCBS}\} \), where \( H_1, \ldots, H_8 \) refer to the hospitals and FRCBS to the Finnish Red Cross Blood Service central blood bank. The first-stage decision of each decision-making unit \( r \) is to define an order-up-to level \( s_r \) for platelets prior to observing platelet demand. Belief about this demand is captured by scenario \( \xi \in \Xi \). Consider time periods \( t \in T = \{1, \ldots, T\} \) (representing days). Let us denote by \( q(\xi)_{t,r} \) the total inventory level at the end of period \( t \) and by \( g(\xi)_{t,r} \) the ordering quantity (or variable production quantity in the case of \( r = \text{FRCBS} \)) for period \( t \) in decision-making unit \( r \) under scenario \( \xi \). If the total inventory level \( v(\xi)_{t,r} \) is lower than the order-up-to level \( s_r \), then the order-up-to inventory policy sets \( q(\xi)_{t,r} \) for time period \( t \) equal to the difference between \( s_r \) and \( v(\xi)_{t,r} \):

\[
q(\xi)_{t,r} = \max \{0, s_r - v(\xi)_{t,r}\}, \quad \forall \, \xi, \, r, \, t.
\]  

Constraint (1) can be linearised through Eqs. (2)–(7), where \( S_r \) is the maximum inventory capacity for decision-making unit \( r \) and \( b(\xi)_{t,r} \) is a binary variable whose value is equal to 1 if and only if the ordering quantity \( q(\xi)_{t,r} \) is positive.

\[
q(\xi)_{t,r} \geq s_r - v(\xi)_{t,r}, \quad \forall \, \xi, \, r, \, t
\]  

(2)

\[
q(\xi)_{t,r} \leq s_r - v(\xi)_{t,r} + S_r(1 - b(\xi)_{t,r}), \quad \forall \, \xi, \, r, \, t
\]  

(3)

\[
q(\xi)_{t,r} \leq S_r b(\xi)_{t,r}, \quad \forall \, \xi, \, r, \, t
\]  

(4)

Let \( is(\xi)_{t,r,m} \) be the starting inventory level of platelet units in scenario \( \xi \) at decision-making unit \( r \) at the beginning of time period \( t \) with remaining shelf life \( m \in M = \{1, \ldots, M\} \) (in days). The initial inventory \( is(\xi)_{t,r,m} \) of units with remaining shelf life \( m \) in the beginning of period \( t = 1 \) is set by the parameter \( b_{m,r} \), as stated in constraint (8). For the remaining periods, \( is(\xi)_{t,r,m} \) is defined by the quantity \( a(\xi)_{t,r,m} \) of units with shelf life \( m \) used to fulfill demand at decision-making unit \( r \) in period \( t \) plus the ending inventory level \( ie(\xi)_{t,r,m} \) in that same period, as per constraint (9). The lead time of an order (in days) is denoted by \( L \). The starting inventory \( is(\xi)_{t+1,L} \) in period \( t+1 \) of platelet units with the maximum shelf life remaining is equal to the quantity of units ordered (or produced when \( r = \text{FRCBS} \)) in \( t \), as set by constraint (10).

\[
is(\xi)_{t,r,m} = \begin{cases} b_{m,r} & \text{if } t = 1 \text{ and } m = 1 \\ is(\xi)_{t+1,m} & \text{otherwise} \end{cases}, \quad \forall \, \xi, \, r, \, m
\]  

(8)

\[
is(\xi)_{t,r,m} = is(\xi)_{t+1,m} - ie(\xi)_{t,r,m} = a(\xi)_{t,r,m}, \quad \forall \, \xi, \, r, \, t, \, m
\]  

(9)

\[
is(\xi)_{t+1,L} = q(\xi)_{t,r}, \quad \forall \, \xi, \, t \leq T - L.
\]  

(10)

Constraint (11) sets the balance between the ending inventory \( ie(\xi)_{t,r,m} \) in a given period and the starting inventory \( is(\xi)_{t+1,m} \) in the next. The total inventory \( v(\xi)_{t,r} \) at the end of period \( t \) is equal to the sum of all platelet units with a remaining shelf life greater than 1 day, as per constraint (12). The amount \( e(\xi)_{t,r} \) of expired units at decision-making unit \( r \) in period \( t \) is defined in constraint (13) as the ending inventory level of units with a remaining shelf life of 1 at the end of that period, i.e., units that will have expired by the next period.

\[
ie(\xi)_{t,r,m+1} = ie(\xi)_{t+1,m} - e(\xi)_{t,r}, \quad \forall \, \xi, \, r, \, t \in T \setminus \{T\}, \, m \in M \setminus \{1\}
\]  

(11)

\[
v(\xi)_{t,r} = \sum_{m \in M \setminus \{1\}} \begin{cases} ie(\xi)_{t,r,m} & \text{if } t = T \\ is(\xi)_{t+1,m} - e(\xi)_{t,r} & \text{otherwise} \end{cases}, \quad \forall \, \xi, \, r, \, t \in T
\]  

(12)

\[
e(\xi)_{t,r} = \begin{cases} e(\xi)_{t,r+1} & \text{if } t = T - L \\ 0 & \text{otherwise} \end{cases}, \quad \forall \, \xi, \, r, \, t \leq T - L
\]  

(13)

Scenarios \( \xi \in \Xi \) capture the uncertain demand. Specifically, these scenarios set values for parameters \( D(\xi)_{t,r} \) for \( r \in \{H_1, \ldots, H_8\} \) representing the demand at each hospital in period \( t \) under scenario \( \xi \). The demand at \( r = \text{FRCBS} \) is the sum of each hospital's order quantity \( D(\xi)_{t,\text{FRCBS}} = \sum_{r=H_1}^{H_8} q(\xi)_{t,r}, \forall \, \xi, \, t \in T \). Constraint (14) states that the total demand in any period and decision-making unit \( r \) is equal to the fulfilled \( a(\xi)_{t,r,m} \) and unmet \( f(\xi)_{t,r} \) demand for that period. Constraint (15) ensures that a minimum service level (A) is met at the hospital and FRCBS level, i.e., all fulfilled demand must be above a specified proportion of the total demand. Notice that this is stricter than imposing a level constraint that applies to all hospitals and FRCBS at once. On the other hand, since the medium and small hospitals are not considered individually (but via representative hospitals H7 and H8), the service level is not, in principle, guaranteed for the individual hospitals represented by H7 and H8.

\[
\sum_{m \in M} a(\xi)_{t,r,m} + f(\xi)_{t,r} = D(\xi)_{t,r}, \quad \forall \, \xi, \, r, \, t
\]  

(14)

\[
\sum_{t \in T} \sum_{m \in M} a(\xi)_{t,r,m} \geq A \sum_{t \in T} D(\xi)_{t,r}, \quad \forall \, \xi, \, r.
\]  

(15)
The model objective is to minimise the expected costs associated with: ordering or production \( q(ξ)_{r,t} \), holding of inventory \( v(ξ)_{r,t} \), expiry \( e(ξ)_{r,t} \), and unmet demand \( f(ξ)_{r,t} \) across all scenarios \( ξ ∈ Ξ \) and for each decision-making unit \( r ∈ R \). The corresponding unit costs for decision-making unit \( r \) are denoted by \( O_r, H_r, E_r, G_r \), respectively. Denoting the probability of scenario \( ξ \) by \( P(ξ) \), the optimal inventory policies for each decision-making unit \( r \) can be found by solving the following optimisation problem:

\[
\begin{align*}
\text{minimise} \quad & \sum_{ξ ∈ Ξ} P(ξ) \left[ \sum_{r ∈ R} \left( O_r q(ξ)_{r,t} + H_r v(ξ)_{r,t} + E_r e(ξ)_{r,t} + G_r f(ξ)_{r,t} \right) \right] \\
\text{subject to} \quad & (2) - (15) \\
& b(ξ)_{r,t,m} = i(ξ)_{r,t,m} + v(ξ)_{r,t} + e(ξ)_{r,t}, \quad a(ξ)_{r,t,m}, \quad f(ξ)_{r,t} ∈ Z_{≥0}, \quad ∀ ξ, r, t, m.
\end{align*}
\]

(16)

subject to

(2) – (15)

\[
\begin{align*}
& b(ξ)_{r,t,m} = i(ξ)_{r,t,m} + v(ξ)_{r,t} + e(ξ)_{r,t}, \quad a(ξ)_{r,t,m}, \quad f(ξ)_{r,t} ∈ Z_{≥0}, \quad ∀ ξ, r, t, m.
\end{align*}
\]

(17)

3.3. Data

Data for generating demand scenarios and setting the parameters of the model were provided by the FRCBS. In particular, to reflect the current Finnish setting, the planning horizon was set at \( T = 14 \) days for both the FRCBS and hospitals. Minimum service levels for FRCBS central blood bank and the hospitals were set at \( A = 95% \) and \( A = 90% \), respectively. The lead time was assumed to be \( L = 0 \) days for the FRCBS blood bank (blood bank to blood bank) and \( L = 1 \) day for the hospitals (blood bank to hospital). The assumption of \( L = 0 \) for the central blood bank is based on the fact that the units are available on the same day as they are requested, a premise we have validated with the FRCBS. Specifically, the newly collected whole blood is transported to the FRCBS central location, where it is processed and tested within 24 to 28 hours from donation, it is then released to fulfill the order from the FRCBS blood bank (in the same location). The lead time of one day for the hospitals \( L = 1 \) represents the fact that orders placed to the FRCBS by the end of business hours are fulfilled the following morning, an assumption also validated by the FRCBS.

The raw data for generating demand scenarios consisted of deliveries and returns of platelet units of the 62 national hospitals from December 2016 to January 2020. Due to confidentiality concerns, any delivery or return of fewer than five units appeared as a range 1–5. To replace this range with a numerical value, the FRCBS provided the mode values of deliveries and returns within the 1–5 range for each hospital and year. To use the delivery and return data to calculate the demand of each hospital, the age of the units on delivery was first to be estimated. From discussions with the FRCBS, we agreed upon the following time-to-events: one day from collection to separation and testing, and one day in storage at the FRCBS central blood bank. Admittedly, this can only provide us with an approximation of the real demand. On the other hand, since the inventories are rather short-lived and small at the hospitals, the distortions that such approximation causes are most likely negligible.

These assumptions, together with the lead time of \( L = 1 \) day from FRCBS central blood bank to hospital imply that all platelets are of age three days upon arrival at the hospitals and, thus, have three days of shelf life. The demand was then calculated as the number of units delivered to the hospital minus the number returned once expired. Any negative demand values due to this assumption were replaced with a zero. In total, 52 demand scenarios \( D(ξ)_{r,t} \) each with a length of \( T = 14 \) time periods, were generated for each (representative) hospital \( r ∈ \{H1, \ldots, H8\} \). These scenarios sets represent the historical demand for each hospital from Monday, 26 December 2016 until Sunday, 12 January 2020. For the FRCBS model, the corresponding national demand scenarios were obtained as outputs from the eight hospital-level models through \( D(ξ)_{\text{FRCBS},t} = \sum_{r ∈ R} D(ξ)_{r,t} \), \( ∀ ξ ∈ Ξ, \ t ∈ T \).

4. Solution approach

In order to solve the model (2)–(17), we settled upon Progressive Hedging (PH). A decomposable version of the model is achieved by replacing the first-stage decision variables with scenario-dependent copies. A further set of constraints (herein referred to as non-anticipativity constraints) are required to enforce all scenario-dependent first-stage variables to be of equal value regardless of the scenario (Birge & Louveaux, 2011). Hereinafter, let us consider a more compact form of problem (2)–(17):

\[
\begin{align*}
& z = \min \sum_{ξ ∈ Ξ} P(ξ) \left[ c^T X + \sum_{ξ ∈ Ξ} P(ξ) q(ξ)^T y(ξ) \right] \\
& : (x, y(ξ)) ∈ K(ξ), \ ∀ ξ ∈ Ξ,
\end{align*}
\]

(18)

where \( K(ξ) = \{ (x, y(ξ)) : W(ξ) y(ξ) = h(ξ) − T(ξ)x, \ x ∈ X, y(ξ) ∈ Y(ξ) \} \). In this compact form, first-stage decisions are represented by \( x \), whereas \( y(ξ) \) represents the second-stage decisions, such as ordering amounts, inventory, unmet demand and expiry. Notice that in our case, \( c \) is the null vector, as no costs are incurred from setting the value of the order-up-to levels \( S \), given by the decision variable \( s \). \( W(ξ) \) is the coefficient matrix of \( y(ξ) \) and \( T(ξ) \) is the coefficient matrix of \( x \) in the constraints in which both the first- and second-stage variables are presented (e.g., Eqs. (2) and (3)). The set \( X \) represents the constraints only related to \( x \), similar as Eq. (6), and \( Y(ξ) \) represents the constraints on the second-stage variables not involving \( x \), such as Eq. (10). Finally, \( h(ξ) \) represents the demands.

To expose the separable structure of Problem (18), we reformulate it equivalently as

\[
\begin{align*}
& z = \min \sum_{ξ ∈ Ξ} P(ξ) \left[ \sum_{ξ ∈ Ξ} P(ξ) q(ξ)^T y(ξ) \right] \\
& : (x(ξ), y(ξ)) ∈ K(ξ), x(ξ) = x, \ ∀ ξ ∈ Ξ, x ∈ R^n,
\end{align*}
\]

(19)

where \( x(ξ) = x \) are the non-anticipativity constraints and \( n \) is the dimension of the original decision variable vector \( x \). If one relaxes the non-anticipativity constraints, it is possible to decompose the problem into scenario-wise independent sub-problems, one for each \( ξ ∈ Ξ \), which can then be solved independently, and possibly in parallel. This is how the computational burden is lessened compared with solving the full-scale (non-decomposed) problem, as solving each scenario sub-problem is several orders of magnitude less computationally demanding than solving Problem (18).

A suitable mechanism that allows for decomposition is the employment of Lagrangian duality. Specifically, by applying Lagrangian relaxation to the non-anticipativity constraints, we obtain the Lagrangian dual function

\[
\begin{align*}
& \phi(μ) = \min \sum_{ξ ∈ Ξ} P(ξ) \left[ \sum_{ξ ∈ Ξ} P(ξ) q(ξ)^T (y(ξ) + \mu (ξ) (x(ξ) − x)) \right] \\
& : (x(ξ), y(ξ)) ∈ K(ξ), \ ∀ ξ ∈ Ξ, x ∈ R^n
\end{align*}
\]

(20)

where \( μ = (μ(1), \ldots, μ(|Ξ|)) ∈ \prod_{ξ ∈ Ξ} R^n \) is the vector of Lagrangian multipliers associated with the relaxed non-anticipativity constraints \( x(ξ) = x, \ ∀ ξ ∈ Ξ \). Denoting \( λ(ξ) = μ(ξ) / P(ξ) \), we can
rewrite the Lagrangian dual function (20) as
\[
\phi(\lambda) = \min_{\xi \in \Xi} \left\{ \sum_{\xi \in \Xi} P(\xi) L\left(x(\xi) , y(\xi), \lambda(\xi) \right) : \left(x(\xi) - x, y(\xi) - y\right) \in K(\xi), \forall \xi \in \Xi, x, y \in \mathbb{R}^n \right\},
\]
(21)
where \(L(x(\xi), y(\xi), \lambda(\xi)) = c^T x(\xi) + q(\xi)^T y(\xi) + \lambda(\xi)^T (x(\xi) - x)\). Since \(x\) is unconstrained in Problem (21), \(\sum_{\xi \in \Xi} P(\xi) \lambda(\xi) = 0\) is required as a dual feasible condition to ensure that the Lagrangian dual function \(\phi(\lambda)\) is bounded from below. Using this assumption, the function \(L(x(\xi), y(\xi), \lambda(\xi))\) may be redefined as
\[
L(x(\xi), y(\xi), \lambda(\xi)) = c^T x(\xi) + q(\xi)^T y(\xi) + \lambda(\xi)^T x(\xi).
\]
(22)
This, in turn, allows for the decomposition of the Lagrangian dual function into separate functions
\[
\phi(\lambda) = \sum_{\xi \in \Xi} P(\xi) \phi_\xi(\lambda(\xi)),
\]
(23)
where, for each \(\xi \in \Xi\), \(\phi_\xi(\lambda(\xi)) = \min_{\xi \in \Xi} \left\{ L(x(\xi) , y(\xi), \lambda(\xi)) : (x(\xi), y(\xi)) \in K(\xi) \right\}\).

The reformulation in Eq. (23) is the basis for separable approaches for solving Problem (18). However, as Eq. (23) is a relaxation, \(\phi(\lambda)\) can only be expected to be a lower bound on the optimal value of Problem (18) for any value of the Lagrangian multipliers \(\lambda\). The problem of finding the best of such bounds consists of the Lagrangian dual problem
\[
z^{LD} = \max_{\xi \in \Xi} \left\{ \phi(\lambda) : \sum_{\xi \in \Xi} P(\xi) \lambda(\xi) = 0 \right\}.
\]
(24)

PH is a method to solve the Lagrangian dual problem (24) that relies upon the employment of augmented Lagrangian dual functions of the form
\[
\phi^\rho(\lambda) = \min_{\xi \in \Xi} \left\{ \sum_{\xi \in \Xi} P(\xi) L^\rho\left(x(\xi) , y(\xi), \lambda(\xi) \right) : \left(x(\xi) - x, y(\xi) - y\right) \in K(\xi), x(\xi) = x, y(\xi) = y \right\},
\]
(25)
where \(L^\rho(x(\xi) , y(\xi), \lambda(\xi)) = c^T x(\xi) + q(\xi)^T y(\xi) + \lambda(\xi)^T x(\xi) + \rho \frac{1}{2} ||x(\xi) - x||_2^2\). The augmented Lagrangian relaxation (25) corresponds to the standard Lagrangian relaxation, as defined in Problem (21), of the non-anticipativity constraints through the terms \(\lambda(\xi)^T x(\xi)\), \(\forall \xi \in \Xi\), augmented by a quadratic term \(||x(\xi) - x||_2^2\). The augmented Lagrangian term is motivated by better convergence properties under more general conditions. Additionally, in the presence of duality gaps caused by non-convexity, the inclusion of the proximal term typically provides better bounds \(z^{LD} \leq z^{DP} < z\) (Bazaraa, Sherali, & Shetty, 2013). The respective augmented Lagrangian dual problem is given by
\[
z^{DP} = \max_{\xi \in \Xi} \left\{ \phi^\rho(\lambda) : \sum_{\xi \in \Xi} P(\xi) \lambda(\xi) = 0 \right\}.
\]
(26)

The issue, however, is that the inclusion of the proximal term prevents the sought-after separability in the set of scenarios \(\xi \in \Xi\). To achieve separability, one must decouple the solution of \(\phi^\rho(\lambda)\) in the dimensions \(x(\xi), \forall \xi \in \Xi, \), and \(x\). This leads to a method generally known as Alternating Direction Method of Multipliers (ADMM), of which PH is a special case. As the name suggests, by solving the problem for each “direction” \((x(\xi), \forall \xi \in \Xi, \) or \(x\) individually while keeping the other directions fixed, separability can be achieved. For instance, for a fixed \(x^k\) obtained at iteration \(k\) of the PH algorithm, we have
\[
\phi^\rho(\lambda) = \sum_{\xi \in \Xi} P(\xi) \phi^\rho_\xi(\lambda(\xi)),
\]
(27)
where, for each \(\xi \in \Xi\), \(\phi^\rho_\xi(\lambda(\xi)) = \min_{x(\xi), y(\xi)} \left\{ L^\rho(x(\xi) , y(\xi), x^k, \lambda(\xi)) : (x(\xi), y(\xi)) \in K(\xi) \right\}\). That is, having a fixed value for \(x^k\) provides a way to recover separability, as in Eq. (23). Interestingly, solving \(\phi^\rho(\lambda)\) for a fixed \(x(\xi)^k\) (and given Lagrangian multipliers \(\lambda^k\)) obtained at iteration \(k\) can be done in a closed form
\[
\arg \min_{x} L^\rho(x(\xi)^k, x, \lambda^k) = \sum_{\xi \in \Xi} P(\xi) x(\xi)^k,
\]
where effectively consists of calculating the probability-weighted average (i.e., expected value) of the scenario-dependent variable values \(x(\xi)\) at every iteration \(k\) of the PH algorithm.

The name Progressive Hedging comes from the gradual iterative enforcement of consensus among the variables \(x(\xi)^k\), \(\forall \xi \in \Xi\), by penalising their deviation from an expected value \(x^k\) using the proximal term. This expected value is then updated to \(x^{k+1}\) using new found \(x(\xi)^{k+1}\), which, are steered towards \(x^{k+1}\) by the updated penalty information encoded in the Lagrangian dual multipliers \(\lambda^k\). In effect, PH is aimed at solving the augmented Lagrangian problem (26). To do this, we employ the subgradient method, since subgradients of the form \(\rho(x^k - x, y(\xi))\), \(\forall \xi \in \Xi\), are immediately available after evaluating \(\phi^\rho(\lambda^k)\). Thus, dual ascent steps can be iteratively taken utilising these subgradients, eventually converging (under suitable conditions) to a maximising solution \(x^k\) for \(\phi^\rho(\lambda)\). The pseudocode for the Progressive Hedging algorithm is presented in Algorithm 1.

Algorithm 1 Progressive Hedging algorithm
1: initialise: \(\lambda(x)^0, \rho > 0, \epsilon > 0; k = 1\)
2: for \(\xi \in \Xi\) do
3: \((x(\xi)^0, y(\xi)^0) = \arg \min_{x(\xi), y(\xi)} \left\{ c^T x(\xi) + q(\xi)^T y(\xi) : (x(\xi), y(\xi)) \in K(\xi) \right\}\)
4: end for
5: \(x^0 = \sum_{\xi \in \Xi} P(\xi) x(\xi)^0\)
6: while \(\sum_{\xi \in \Xi} P(\xi) \left\| x(\xi)^k - x^k \right\|_2^2 > \epsilon\) do
7: for \(\xi \in \Xi\) do
8: \((x(\xi)^k, y(\xi)^k) = \arg \min_{x(\xi), y(\xi)} \left\{ L^\rho(x(\xi) , y(\xi), x^k) : (x(\xi), y(\xi)) \in K(\xi) \right\}\)
9: end for
10: \(x^k = \sum_{\xi \in \Xi} P(\xi) x(\xi)^k\)
11: \(\lambda(\xi)^k = \lambda(\xi)^{k-1} + \rho(x(\xi)^k - x^k)\)
12: \(k = k + 1\)
13: end while
14: \(x^k = x^k\)
15: for \(\xi \in \Xi\) do
16: \(z_\xi = \min_{x(\xi), y(\xi)} \left\{ q(\xi)^T y(\xi) : (x(\xi), y(\xi)) \in K(\xi) \right\}\)
17: \(\phi_\xi = \min_{x(\xi), y(\xi)} \left\{ L(x(\xi) , y(\xi), \lambda(\xi)^k) : (x(\xi), y(\xi)) \in K(\xi) \right\}\)
18: end for
19: \(z = c^T x^k + \sum_{\xi \in \Xi} P(\xi) z_\xi\)
20: \(\phi = \sum_{\xi \in \Xi} P(\xi) \phi_\xi\)
21: return \(x^k, z, \lambda, \phi\)

The PH algorithm is initialised in Lines 1–5, in which scenario-wise independent solutions \(x(\xi)^0\) are obtained and used for defining the initial non-anticipative solution \(x^0\). Notice that this corresponds to solving the augmented Lagrangian dual function.
(Eq. (25)) with \( \lambda(\xi) \) and \( \rho \) set to zero, meaning there is no linkage between the scenarios. The stopping criterion, Line 6, consists of the combined squared primal feasibility residuals \( P(\xi)||x(\xi)^k - x^k||_2^2 \) and squared dual feasibility residuals \( ||x^k - x^{k-1}||_2^2 \), given as

\[
\begin{align*}
\sum_{\xi \in \Xi} P(\xi)(||x(\xi)^k - x^k||_2^2 + ||x^k - x^{k-1}||_2^2) \\
= \sum_{\xi \in \Xi} P(\xi)||x(\xi)^k - x^{k-1}||_2^2.
\end{align*}
\]

We refer the reader to Boyd, Parikh, & Chu (2011) for a technical analysis of the primal and dual feasibility residuals and their convergence to zero in the course of the algorithm.

Line 8 represents the update of the scenario-dependent variables \( (x(\xi)^k, y(\xi)^k) \), which can be carried out separately (and in parallel). It is worth noticing that this step comprise the bulk of the computational effort in Algorithm 1. In Line 10, the non-anticipative solution \( x^k \) is calculated as the expected value among all scenarios, and in Line 11 the Lagrangian dual multipliers are updated accordingly.

Finally, due to the lack of convexity caused by the integrality requirement of the variables (cf. Eq. (17)), PH in this context can only be seen as a heuristic with no convergence guarantees. However, we can still extract an implementable (i.e., integer and non-anticipative) solution by employing rounding to obtain a final solution \( x^\ast \) (Line 14), which can then be evaluated to provide an upper bound (Lines 16 and 19). Furthermore, we can calculate a dual bound (for further details, please refer to Gade et al. (2016)) by evaluating the Lagrangian dual function (23) utilising the Lagrangian dual multipliers \( \lambda(\xi)^k \), \( \forall \xi \in \Xi \) (Lines 17 and 20). Combining the upper and lower bound yields an optimality gap for the solution obtained.

5. Algorithm performance

The proposed model and the decomposition method were implemented in Julia (version 1.6.2, with JuMP version 0.21.9) and solved using Gurobi version 9.2. All experiments were performed on an Intel Core i7 3.1 GHz Quad-Core with 16 GB RAM. The code with the implementation of the models and solution algorithm can be found at https://github.com/gamma-opt/2SSP-Platelet-Inventory.

To present the practical benefits of employing this solution algorithm, we ran the hospital-level full-scale deterministic equivalent model using Gurobi (Gurobi Optimization LLC, 2021) as the solver but without using the PH algorithm. One hospital (H1) could not be solved in a reasonable time frame; after 28 hours, the optimality gap was 8.85%. In contrast, PH requires approximately 45 minutes to obtain a solution for all eight hospitals. Thus, we can deduce that solving the model is not practical in a real-world setting without the PH algorithm (or another decomposition algorithm).

The advantage of the PH algorithm is that it can be parallelised (specifically, the for-loops in Lines 2–4, 7–9, and 15–18 in Algorithm 1). This enables the computations to be sped up by more efficiently utilising the available threads. Table 2 displays the percentage change from one to eight threads in the computational time needed to solve each of the eight hospitals with the platelet shelf life set to seven days. In general, there is a 50–72% reduction in computing time if eight threads are in use. In reality, utilising one and eight threads solves all eight hospitals in approximately 45 and 20 minutes, respectively. We highlight that further speedups could likely be achieved by improving our parallel implementation (e.g., eliminating latency due to shared memory access and utilising a distributed approach instead of thread-based parallelisation). However, this approach is left as a suggested direction for future development.

Table 2 also displays the optimality gaps for each hospital. The gaps for all hospitals except H4 are below 2.5%. Upon analysing the demand scenarios generated for hospital 4 (H4), we notice that an outlying peak value appears in one of the scenarios. This compromises the ability of the PH algorithm to find a solution that performs well over all scenarios, leading to a larger optimality gap.

6. Practical results and policy recommendations

In this section, we present optimised platelet inventory management policies assuming a five- or seven-day shelf life. Firstly, the results for the five-day shelf life will serve as a method of validation of the model presented in Section 3.2 and a baseline upon which to examine extending the shelf life. Secondly, the results of extending the shelf life of platelets to seven days will be analysed. Notice that this implies that the usable shelf lives of the platelets increase from 3 to 5 days. In this, we assume that the FRCBS bears the change in the cost of manufacturing the platelets due to the extension methods, i.e., the ordering costs for the hospitals remain the same (recall that our ordering cost is given by the total number of units ordered times the unit cost of the platelets, which changes according to the shelf-live extension method employed). Nevertheless, we also present results on the effect of sharing the added cost with the hospitals. Lastly, policy implications of these results for the FRCBS and hospitals will be discussed.

6.1. Results for five-day shelf life

The results for the five-day shelf life are presented in Table 3. In particular, the table shows the average daily number of platelet units ordered, expired, and in shortage, together with the order-up-to level of the decision-making units (H1–H8, FRCBS). Regarding the two-stage stochastic programming model, the order-up-to level is the first-stage decision, and the remaining results are the second-stage decisions (see Fig. 2). This means that the optimal policy recommendation for each decision-making unit is the order-up-to inventory size. For instance, the optimal policy for ordering platelets with a shelf life of five days at hospital 1 (H1) is to have an order-up-to level of 29 units, meaning placing an order whenever the inventory falls below this level. These results were shared with FRCBS experts, and they confirmed that the order-up-to policy is realistic in the Finnish blood supply chain.

The order-up-to inventory level and daily orders of each decision-making unit in Table 3 act as simple yet effective validation tools. The order-up-to level (variable \( s \) in the optimisation model) can indicate the hospital’s busyness and, as a proxy, its

| Table 2 |
| Solution times from 1 to 8 threads and optimality gaps. |
|----------------|----------------|----------------|
| DMU           | Change in solution time | Optimality gap (%) |
| H1            | -72%                | 0.4%            |
| H2            | -60%                | 0.4%            |
| H3            | -58%                | 2.4%            |
| H4            | -50%                | 6.5%            |
| H5            | -52%                | 0.4%            |
| H6            | -56%                | 0.5%            |
| H7            | -55%                | 0.4%            |
| H8            | -52%                | 1.8%            |

DMU: Decision-making unit, H1–H8: Hospital 1–Hospital
size. For example, hospital 2 (H2) has an order-up-to level of 38 units, whereas hospital 3 (H3) has 21 units. In our model, both are classified as large hospitals, but in real life, hospital 2 is indeed larger and sees more patients than hospital 3. Additionally, using the original FRCBS data, we have compared the histograms of observed orders for each hospital with those generated by our model, and performed a two-way Kolmogorov-Smirnov test for discrete data to investigate whether the ordering quantities come from the same distribution (see the Supplementary material). In summary, for most hospitals, these histograms are indeed very similar. The only peculiar cases are hospital 4 (H4) and hospital 8 (H8). For H4, the discrepancy between the distribution of original and model-based ordering quantities is likely related to the optimality gap we observed for this hospital, as discussed in Section 5. For H8 (which is in fact a collection of small hospitals), this may be due to highly sporadic demand: the most frequent number of ordered units is 0 (i.e., not placing an order), and the highest number of ordered units in any given day is eight.

6.2. Results for seven-day shelf life

Table 4 shows the results when the shelf life of platelets is extended from five to seven days using either the bacteria culture method or the pathogen inactivation method. As illustrated by the right-most column of this table, the extension decreases the total operating costs of the hospitals from 2% (H2 and H7) to up to 43% (H4), all while keeping the inventory order-up-to levels approximately the same and reducing the average daily expiry of units and unmet demand. Such cost reduction is to be expected since the FRCBS assumes the additional production costs involved in the extension. In both extension methods, the inventory order-up-to level and average daily orders for the FRCBS are decreased by approximately ten units (Table 4) compared to the five-day shelf life (Table 3). The average daily expiry remains at zero; this is vital as there cannot be expiry at the start of the chain for an efficient supply chain. The average daily unmet demand at the FRCBS increases slightly for both extension methods; however, the level remains below half a unit per day. The main effect in extending the shelf life is the change in total costs for the FRCBS. Using the bacteria culture method, the total costs are increased by 2%. However, using the pathogen inactivation method decreases the total costs by 24%.

6.3. Comparison of cost-sharing policies

As the FRCBS holds a monopoly on the supply, it stands to reason that they have the power to pass on the additional costs due to extending the platelet shelf lives to the hospitals. However, the FRCBS is a not-for-profit organisation and thus, cannot make a profit from the additional costs to the hospitals. This motivates us to consider alternative cost-sharing policies, so we could understand to what extent the costs borne by the FRCBS could be compensated by means of operational savings incurred at the hospitals, i.e., passing on a percentage of the extra costs to the hospitals for the manufacturing of the platelet units. Although this can be idealised, in practice, finding an optimal cost-sharing policy that is both efficient and fair comprises a whole research question in itself, and thus is left outside the scope of our study.

Based on discussions with the management of FRCBS, we chose to evaluate two fixed policies for distributing the additional cost between the FRCBS and the hospitals. The first of these policies is the equal split of additional costs from the method, herein referred to as the 50:50 policy. The second is the 25:75 policy, wherein the hospitals carry 25% of the additional costs and the FRCBS the remaining 75%. The baseline policy is that of the results shown in Section 6.2; the FRCBS bears the total additional costs (i.e., 100%) due to the extension method used. Changing the price of the platelet units only affects the total costs, that is, the order-up-

Table 3
Key figures representing the optimal policies for five days shelf life in platelet units.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Order-up-to level (s)</th>
<th>Avg. daily orders (q)</th>
<th>Avg. daily expiry (e)</th>
<th>Avg. daily unmet demand (f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>29</td>
<td>15.93</td>
<td>1.17</td>
<td>0.3</td>
</tr>
<tr>
<td>H2</td>
<td>38</td>
<td>23.65</td>
<td>0.17</td>
<td>0.48</td>
</tr>
<tr>
<td>H3</td>
<td>21</td>
<td>8.44</td>
<td>2.07</td>
<td>0.05</td>
</tr>
<tr>
<td>H4</td>
<td>14</td>
<td>4.95</td>
<td>3.26</td>
<td>0.00</td>
</tr>
<tr>
<td>H5</td>
<td>27</td>
<td>12.53</td>
<td>1.90</td>
<td>0.14</td>
</tr>
<tr>
<td>H6</td>
<td>23</td>
<td>10.59</td>
<td>1.21</td>
<td>0.15</td>
</tr>
<tr>
<td>H7</td>
<td>33</td>
<td>19.77</td>
<td>0.15</td>
<td>0.49</td>
</tr>
<tr>
<td>H8</td>
<td>9</td>
<td>3.22</td>
<td>1.76</td>
<td>0.01</td>
</tr>
<tr>
<td>FRCBS</td>
<td>131</td>
<td>100.47</td>
<td>0.00</td>
<td>0.19</td>
</tr>
</tbody>
</table>

DMU: Decision-making unit, H1–H8: Hospital 1–Hospital 8, FRCBS: Finnish Red Cross Blood Service

Table 4
Key figures representing the optimal policies for seven days shelf life in platelet units.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Order-up-to level (s)</th>
<th>Avg. daily orders (q)</th>
<th>Avg. daily expiry (e)</th>
<th>Avg. daily unmet demand (f)</th>
<th>Total cost change from 5 to 7 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>31</td>
<td>14.9</td>
<td>0.02</td>
<td>0.18</td>
<td>-14%</td>
</tr>
<tr>
<td>H2</td>
<td>41</td>
<td>23.71</td>
<td>0.00</td>
<td>0.25</td>
<td>-2%</td>
</tr>
<tr>
<td>H3</td>
<td>21</td>
<td>6.66</td>
<td>0.29</td>
<td>0.05</td>
<td>-33%</td>
</tr>
<tr>
<td>H4</td>
<td>14</td>
<td>3.17</td>
<td>1.48</td>
<td>0.00</td>
<td>-43%</td>
</tr>
<tr>
<td>H5</td>
<td>27</td>
<td>10.70</td>
<td>0.07</td>
<td>0.14</td>
<td>-25%</td>
</tr>
<tr>
<td>H6</td>
<td>24</td>
<td>9.45</td>
<td>0.03</td>
<td>0.11</td>
<td>-20%</td>
</tr>
<tr>
<td>H7</td>
<td>36</td>
<td>19.83</td>
<td>0.00</td>
<td>0.28</td>
<td>-2%</td>
</tr>
<tr>
<td>H8</td>
<td>9</td>
<td>2.21</td>
<td>0.75</td>
<td>0.01</td>
<td>-40%</td>
</tr>
<tr>
<td>FRCBS Bacteria</td>
<td>122</td>
<td>91.29</td>
<td>0.00</td>
<td>0.20</td>
<td>+2%</td>
</tr>
<tr>
<td>FRCBS Pathogen</td>
<td>121</td>
<td>91.15</td>
<td>0.00</td>
<td>0.33</td>
<td>+24%</td>
</tr>
</tbody>
</table>

DMU: Decision-making unit, H1–H8: Hospital 1–Hospital 8, FRCBS: Finnish Red Cross Blood Service, Bacteria: Bacteria culture extension method, Pathogen: Pathogen inactivation extension method
to level, average daily orders, expiry, and unmet demand do not change.

Fig. 3 displays the percentage change in total costs from a shelf life of five days to the extended seven days for each hospital (3a) and the FRCBS (3b) using different extension methods and cost sharing policies. Fig. 3a shows that for both extension methods, cost reductions for all hospitals except H2 and H7 remain significant even if costs of the extension are shared between the hospitals and the FRCBS. The reason for the cost increase at H2 and H7 when some of the costs are borne by the hospitals is that the average daily expiry at H2 and H7 in the case of a five-day shelf life is close to zero (cf. Table 3). Thus, these hospitals stand to gain only a little from the extension. The average cost savings over all hospitals from five to seven days shelf life using pathogen inactivation are 17.7% for the 25:75 policy and 13.1% for the 50:50 policy. In the case of the bacterial cultures extension method, the average cost reductions for the 25:75 and 50:50 policies are 21.0% and 19.6%, respectively. As expected, sharing the costs of extension with hospitals reduces the total costs for the FRCBS compared to the baseline policy (Fig. 3b). In particular, using the 50:50 cost sharing policy with bacterial cultures extension method actually results in a 2% cost saving for the FRCBS compared with the total costs of the five-day shelf life. In contrast, using the 25:75 bacteria culture method results in no total cost changes for the FRCBS compared with the five-day shelf life.

6.4. Inventory control policy recommendations

For Finland’s national hospitals, our modelling suggests that extending the shelf life of platelets would be advantageous in cost savings. In addition, Tables 3 and 4 provide inventory control policies at each hospital for both shelf lives in the form of optimal order-up-to levels. These tables also show the average daily orders placed by each hospital for both shelf lives. For instance, with a five-day shelf life, hospital 1 (H1) is likely to place an order of around 16 units per day (Table 3). If daily orders consistently fall above or below this average, updating the demand scenarios to include the most recent data and re-running the model could be beneficial. This would provide an updated inventory control policy.

Inventory control policies are also provided for the Finnish Red Cross Blood Service. An optimal policy for their current situation is presented in Table 3. It states that an order-up-to level of 130 units is sufficient to satisfy national platelet demand while ensuring that expiry and unmet demand are kept to a minimum. Currently, the FRCBS has an average daily order-up-to level of 149 units with a minimum level of 100 units. Our results suggest that the daily

![Graph showing percentage change in total costs from 5 to 7 days shelf life](image)
order-up-to level can be significantly reduced while keeping expiry and unmet demand to a minimum. Table 4 provides optimal inventory policies if platelet shelf lives were extended using either a bacteria culture or pathogen inactivation method. The bacteria culture method has been identified as the most cost-efficient method, with only a 2% increase in total costs for the FRCBS in the case in which it assumes all costs involved in the extension. If the FRCBS shares the additional costs due to the bacteria method equally with the hospitals, its operational costs can in fact be reduced by 2%. However, it needs to be made clear that our model only considers costs associated with the platelet supply chain and does not include any investigation of parameters associated with the effectiveness and safety of either extension method.

7. Conclusion

This paper presents a two-stage stochastic programming model that utilises a Progressive Hedging algorithm to solve our real-world platelet inventory problem in an acceptable time. With this, we can accommodate sizeable real-world scenario sets to assess the inventory control and cost implications of extending the shelf life of platelets and reducing the risk of infection. The novelty of our work comes in the use of a decentralised two-level approach, the combination of the model and the solution algorithm approach, and the inventory policy recommendations and assessment of extending the shelf life.

Concerning extending the shelf life of platelets in Finland, our main findings are as follows: (i) The extension is advantageous for the national hospitals as the order-up-to levels, the number of expired units and unmet demand decrease together with a reduction in total costs. (ii) The extension decreases the order-up-to level for the Finnish Red Cross Blood Service (FRCBS) while ensuring the expiry and unmet demand is low. (iii) The cost change of the extension for the FRCBS compared to the current situation ranges between -2% and 24%, depending on the extension method and cost sharing policy. (iv) By using the bacteria culture extension method and passing 25% of the additional cost to the hospitals, the costs for the FRCBS stay the same while for the hospitals they either decrease or remain the same.

A strength of our work is that it results in implementable policy recommendations for Finnish national hospitals and the FRCBS. An optimal order-up-to level policy for each hospital and the FRCBS is presented for the current five-day and prospective seven-day shelf life settings. An order-up-to level is straightforward to implement and does not require any additional resources or personnel. Thus, the FRCBS and the hospitals can implement these recommendations with minimal effort and potentially considerable cost savings. Another advantage to our approach is solving the model in parallel, which enables the computational burden to remain reasonable. Incorporating a decomposition solution algorithm into the stochastic programming model reduces the computational solution time dramatically. In fact, without a decomposition method, our problem setting could not be solved and consequently renders the model not fit for purpose.

A limitation of our case study is the lack of RhD-type differentiation. However, as previously discussed, since different RhD types do not incur differentiated ordering or holding costs, the conclusions related to changes in overall costs from our study are reliable. On the other hand, it would be interesting to investigate whether the fact that RhD-positivity is predominant in the Finnish population would pose additional challenges to the management of the platelets, but we defer this to a future study.

Product differentiation is another appealing avenue for further developments. The explicit differentiation between apheresis (disregarded in our study) and pooled platelets may be of interest in other blood inventory systems. In some cases, apheresis platelets can be used to satisfy pooled demand, which helps minimising its wastage. In addition, some agencies may impose ABO types to be matched, which would require the model to keep track of the different platelet types. In any case, the proposed model could be straightforwardly adapted to consider type differentiation if we had that data available. Previous work from two of the authors (Dillon et al., 2017) has considered multiple blood types and substitution, and we are confident that a similar modelling approach could be employed. There would likely be an increased computational requirement admittedly, however, the proposed decomposition method would most certainly aid in that.

A limitation of our model formulation is that we have a deterministic age on arrival parameter, and, although we confirmed our assumption with practitioners, it does not truly reflect reality. In the future, a stochastic parameter should be utilised to capture real life better. Another drawback to our approach is that despite the model being programmed in an open-source programming language, we employ a commercial optimisation solver. Not all companies have or can afford access to commercial solvers; therefore, the usability of our model could be limited in practice. It should, however, be noted that without the use of a commercial solver, the problem setting size would have to be reduced, and even then, a solution may not be reached.

Finally, the use of representative small- and medium-sized hospitals (H7 and H8) do not allow for policy recommendations for the individual Finnish hospitals of these sizes. Hence, our results are not directly implementable for these hospitals. We decided to use such representative hospitals as a means of limiting the number of decision-making units in the analysis, so we could more clearly communicate our main findings. Nevertheless, combining smaller units still provides a useful estimate to the FRCBS for their composite production planning even if the demand for distinct smaller units is not provided. Moreover, in practice, these small hospitals do not hold platelet inventories (they either issue orders as needed or, in emergency cases, transfer patients to the nearest hospital that does have platelets stored). We highlight that there is nothing that prohibits the framework to be applied while considering each hospital individually.

Various future research directions seem encouraging at this point. One interesting direction to be explored is the inclusion of clinical effectiveness in assessing methods used to extend the shelf life of platelets. At present, our model only presents cost and inventory control implications for the shelf life extension methods, and thus separate research into the effectiveness of these methods must be carried out. Another direction is developing a robust demand prediction model that can be utilised to create future demand scenarios. Linking the prediction and optimisation models could provide more real-time policy recommendations. Experiments employing alternative solution algorithms to PH and implementing a Branch-and-Bound algorithm to close the optimality gaps presented in Section 4 is another interesting direction to pursue. Furthermore, extending the hospital model to allow for transshipments between nearby hospitals could reduce expiry in smaller, more remote hospitals, as they could pool their inventories to better deal with sparse demand scenarios. Lastly, due to the large geographical area of Finland, the use of satellite inventory stocks could be beneficial. Developing a location-allocation model for optimising the setup and management of inventory in such a setting offers interesting topics for future research.

Acknowledgments

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Appendix A. Table of notation

<table>
<thead>
<tr>
<th>Category</th>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sets</td>
<td>$\xi \in \mathbb{Z}$</td>
<td>Scenarios</td>
<td>Varies by HOS and FRCBS</td>
<td>FRCBS data</td>
</tr>
<tr>
<td></td>
<td>$t \in T$</td>
<td>Time periods</td>
<td>$T = 14$</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$m \in \mathcal{M}$</td>
<td>Remaining shelf life</td>
<td>$M = 5$ or $7$ for HOS, $M = 2$ for FRCBS</td>
<td>Assumption</td>
</tr>
<tr>
<td>Parameters</td>
<td>$B_m$</td>
<td>Initial inventory of units with remaining shelf life $m$</td>
<td>Varies by HOS and FRCBS</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$L$</td>
<td>Lead time</td>
<td>$1/2$ equal probability</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$P(\xi)$</td>
<td>Probability of scenario $\xi$</td>
<td>Varies by HOS, for FRCBS is sum of all HOS quantity ordered</td>
<td>FRCBS data</td>
</tr>
<tr>
<td></td>
<td>$D(\xi)_t$</td>
<td>Demand of units in period $t$ in scenario $\xi$</td>
<td>$\in$ 822 for HOS, $\in$ 10,000 for FRCBS</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$G_m$</td>
<td>Shortage cost per unit in HOS and FRCBS</td>
<td>$\in$ 1 for HOS and FRCBS</td>
<td>FRCBS</td>
</tr>
<tr>
<td></td>
<td>$O$</td>
<td>Order cost per unit in HOS or production cost in FRCBS</td>
<td>$\in$ 411 for HOS, $\in$ 300 for FRCBS</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$Oe$</td>
<td>Extra order cost for weekend and emergency orders</td>
<td>$\in$ 1413 for HOS only</td>
<td>FRCBS</td>
</tr>
<tr>
<td></td>
<td>$Or$</td>
<td>Transportation cost</td>
<td>Same as order/production cost</td>
<td>FRCBS</td>
</tr>
<tr>
<td></td>
<td>$E$</td>
<td>Expiry cost per unit in HOS and FRCBS</td>
<td>90%</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$A$</td>
<td>Minimum service level for units</td>
<td>60 for HOS, 250 for FRCBS</td>
<td>Assumption</td>
</tr>
<tr>
<td></td>
<td>$S$</td>
<td>Maximum (a 'sufficiently large number') inventory capacity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$q(\xi)_t$$ HOS order quantity or FRCBS supply quantity at period $t$ in scenario $\xi$

$$i(\xi)_{h,m}$$ Inventory level of units with shelf life $m$ at the start of period $t$ in scenario $\xi$

$$i(\xi)_{h,m}$$ Inventory level of units with shelf life $m$ at the end of period $t$ in scenario $\xi$

$$v(\xi)_{t}$$ Total inventory at the end of period $t$ in scenario $\xi$

$$e(\xi)_{h}$$ Quantity of expired units at period $t$ in scenario $\xi$

$$d(\xi)_{h,m}$$ Quantity of units with shelf life $m$ used to fulfill demand in period $t$ in scenario $\xi$

$$f(\xi)_{h}$$ Unmet demand in period $t$ in scenario $\xi$

$$s$$ Total order-up-to level

$$b(\xi)_{h}$$ Binary variable needed for linearising the ordering policy constraints

Appendix B. Formulation of alternative ordering policies

The formulation of New-Inventory-to-S (NIS) and Two Demand (2D) alternative ordering policies (Civelek et al., 2015) require additional indices and parameters. Hence, here we present the full amended hospital model with notation and ordering policies.

Sets and indices:

$\xi \in \mathbb{Z}$ Scenarios

$t \in T := \{1, 2, \ldots, T\}$ Time periods

$[1, 2, \ldots, M]$ Remaining shelf life, i.e., age groups

Parameters:

$B_m$ Initial inventory of units with remaining shelf life $m$

$M$ Maximum shelf life

$L$ Lead time

$P(\xi)$ Probability of scenario $\xi$

$D(\xi)_t$ Demand of units with shelf life $m$ in period $t$ in scenario $\xi$

$G_m$ Shortage cost per unit for platelets with shelf life $m$

$O$ Holding cost per unit

$E$ Expiry cost per unit

$X$ Substitution penalty

$Q_{m,k}$ Substitution priority index for substituting units with shelf life $k$ with units with shelf life $m$

$\alpha_m$ Minimum service level for units with shelf life $m$

$\beta$ Minimum exact match rate for "Young" units

$\delta$ Maximum inventory capacity, a 'sufficiently large number' needed to linearise constraints containing binary variables

Variables:

$q(\xi)_t$ Order quantity at period $t$ in scenario $\xi$

$i(\xi)_{h,m}$ Inventory level of units with shelf life $m$ at the start of period $t$ in scenario $\xi$

$i(\xi)_{h,m}$ Inventory level of units with shelf life $m$ at the end of period $t$ in scenario $\xi$

$v(\xi)_{t}$ Total inventory at the end of period $t$ in scenario $\xi$

$e(\xi)_{h}$ Quantity of expired units at period $t$ in scenario $\xi$

$d(\xi)_{h,m}$ Quantity of units with shelf life $m$ used to fulfill demand in period $t$ in scenario $\xi$

$f(\xi)_{h}$ Unmet demand in period $t$ in scenario $\xi$

$s$ Total order-up-to level

$b(\xi)_{h}$ Binary variable needed for linearising the ordering policy constraints

Mathematical model:

$$\min_{\xi \in \mathbb{Z}} z = \sum_{\xi \in \mathbb{Z}} P(\xi) \left[ \sum_{t \in T} \left( O q(\xi)_t + H v(\xi)_t + E e(\xi)_t + \sum_{m \in M} G_m f(\xi)_{t,m} + X \sum_{k \in M} Q_{m,k} a(\xi)_{t,m,k} \right) \right]$$

s.t. $i(\xi)_{1,m} = B_m, \quad \forall \xi, m$
\[ is(\xi)_{t,m} = \sum_{k \in M} a(\xi)_{t,m,k} + ie(\xi)_{t,m}, \quad \forall \xi, t, m \quad \text{(B.3)} \]

\[ \sum_{k \in M} a(\xi)_{t,m,k} + f(\xi)_{t,m} = D(\xi)_{t,m}, \quad \forall \xi, t, m \quad \text{(B.4)} \]

\[ \nu(\xi) = \sum_{m \in M \setminus \{1\}} ie(\xi)_{t,m}, \quad \forall \xi, t \quad \text{(B.5)} \]

\[ e(\xi) = ie(\xi)_{t,1} - \nu(\xi), \quad \forall \xi, t \quad \text{(B.6)} \]

\[ \nu(\xi)_{t+1,m} = \nu(\xi)_{t+1,m} - \nu(\xi)_{t}, \quad \forall \xi, t \in T \setminus \{T\}, \quad m \in M \setminus \{M\} \quad \text{(B.7)} \]

\[ is(\xi)_{t+1,M} = q(\xi)_{t}, \quad \forall \xi, t \leq T - L \quad \text{(B.8)} \]

\[ \sum_{t \in T} a(\xi)_{t,m,k} \geq \alpha_{m} \sum_{t \in T} D(\xi)_{t,m}, \quad \forall \xi \in \Xi, \quad m \in M \quad \text{(B.9)} \]

\[ \sum_{t \in T} a(\xi)_{t,m,M} \geq \beta \sum_{t \in T} a(\xi)_{t,m,k}, \quad \forall \xi \in \Xi \quad \text{(B.10)} \]

\[ \text{Sta.} \text{.} \text{S.} \text{.} \text{M.} \in \mathbb{Z}_{>0}. \quad \text{(B.11)} \]

### Ordering policies

If using the NIS policy, the inventory levels of the youngest units are kept constant, so an amount equal to \( S_{m} \) is ordered each day, as stated by constraint (B.12).

\[ q(\xi)_{t} = S_{m}, \quad \forall \xi, t \quad \text{(B.12)} \]

The 2D policy is a combination of NIS and 1D (presented in the main text) policies. The order amount is defined as in 1D case, but it must be greater than or equal to the order-up-to level of the youngest units, \( S_{m} \). This condition is presented in constraint (B.13).

\[ q(\xi)_{t} = \max\{S_{m}, S_{m} - \sum_{m \in L} is(\xi)_{t,m}\}, \quad \forall \xi, t \quad \text{(B.13)} \]

Again, it can be replaced using four linear inequality constraints (B.14)-(B.17). The binary variable will now be equal to one if the order amount defined by the 1D policy is greater than \( S_{m} \) and zero otherwise.

\[ q(\xi)_{t} \geq S_{eq} - \sum_{m \in L} is(\xi)_{t,m}, \quad \forall \xi, t, m > L \quad \text{(B.14)} \]

\[ q(\xi)_{t} \geq S_{m}, \quad \forall \xi, t \quad \text{(B.15)} \]

\[ q(\xi)_{t} \leq S_{eq} - \sum_{m \in L} is(\xi)_{t,m} + 5(1 - b(\xi)_{t}), \quad \forall \xi, t, m > L \quad \text{(B.16)} \]

\[ q(\xi)_{t} \leq S_{m} + 5b(\xi)_{t}, \quad \forall \xi, t \quad \text{(B.17)} \]

### Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi: 10.1016/j.ejor.2023.03.007.

### References


Heidari-Fathian, H., & Pasandideh, S. H. R. (2018). Green-blood supply chain net-