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Published in:
SYMMETRY

DOI:
10.3390/sym15050991

Published: 01/05/2023

symmetry

MDPI

Article

Planck Constants in the Symmetry Breaking Quantum Gravity

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Abstract: We consider the theory of quantum gravity in which gravity emerges as a result of the symmetry-breaking transition in the quantum vacuum. The gravitational tetrads, which play the role of the order parameter in this transition, are represented by the bilinear combinations of the fermionic fields. In this quantum gravity scenario the interval ds in the emergent general relativity is dimensionless. Several other approaches to quantum gravity, including the model of superplastic vacuum and BF theories of gravity support this suggestion. The important consequence of such metric dimension is that all the diffeomorphism invariant quantities are dimensionless for any dimension of spacetime. These include the action S, cosmological constant Λ, scalar curvature R, scalar field Φ, wave function ψ, etc. The composite fermion approach to quantum gravity suggests that the Planck constant $\hbar$ can be the parameter of the Minkowski metric. Here, we extend this suggestion by introducing two Planck constants, bar $\hbar$ and slash $\hslash$, which are the parameters of the correspondingly time component and space component of the Minkowski metric, $g^{\mu\nu}_{\rm Mink} = {\rm diag}(-\hbar^2, \hslash^2, \hslash^2, \hslash^2)$. The parameters bar $\hbar$ and slash $\hslash$ are invariant only under $SO(3)$ transformations, and, thus, they are not diffeomorphism invariant. As a result they have non-zero dimensions—the dimension of time for $\hbar$ and dimension of length for $\hslash$. Then, according to the Weinberg criterion, these parameters are not fundamental and may vary. In particular, they may depend on the Hubble parameter in the expanding Universe. They also change sign at the topological domain walls resulting from the symmetry breaking.

Keywords: Planck constant; quantum gravity; emergent tetrads; dimensionless interval

Citation: Volovik, G.E. Planck Constants in the Symmetry Breaking Quantum Gravity. *Symmetry* **2023**, *15*, 991. https://doi.org/10.3390/sym15050991

Academic Editors: Stefano Profumo and Sergei D. Odintsov

Received: 28 March 2023
Revised: 24 April 2023
Accepted: 25 April 2023
Published: 27 April 2023

1. Introduction

It is becoming clear that quantum gravity cannot be obtained by quantization of the classical gravity. Gravity can arise as an emergent low-energy phenomenon, which comes from underlying quantum fields of the quantum vacuum. The typical example is provided by condensed matter, where the effective gravity emerges in the topological Weyl and Dirac materials, semimetals, superfluids, and superconductors. Gravitational tetrads emerge there in the vicinity of the conical points in the spectrum of fermionic quasiparticles [1], see also recent papers [2,3]. Another condensed matter example of effective gravity is provided by the B-phase of superfluid ^{3}He, where vielbein emerge as bilinear combinations of the fermionic fields [4]. Similar mechanism of the formation of the composite tetrads in the low-energy physics has been suggested in the relativistic quantum field theories [5–9]. The emergent tetrads give rise to the effective metric (the four fermions object), to the interval, and finally to the effective action for the gravitational field. The important consequence of this mechanism is that all the diffeomorphism invariant physical quantities are dimensionless. Here, we discuss one more consequence of such dimensionless physics, which is related to the Planck constant [10]. Actually there are two Planck constants, bar $\hbar$, and slash $\hslash$. Both are the elements of the metric and tetrads in Minkowski vacuum. The bar $\hbar$ is the time component of the tetrad and has dimension of time, while the slash $\hslash$ enters the space components of tetrads and has dimension of length.

2. Composite Tetrads from Relative Symmetry Breaking

The gravitational tetrads may appear as composite objects made of the more fundamental fields, the quantum fermionic fields [5–9,11]:

$$\hat{E}^a_\mu = \frac{1}{2}\left(\Psi^\dagger \gamma^a \partial_\mu \Psi - \Psi^\dagger \overleftarrow{\partial_\mu} \gamma^a \Psi\right) . \tag{1}$$

The original action does not depend on tetrads and metric and is described solely in terms of differential forms:

$$S = \frac{1}{24} e^{\alpha\beta\mu\nu} e_{abcd} \int d^4x \, \hat{E}^a_\alpha \hat{E}^b_\beta \hat{E}^c_\mu \hat{E}^d_\nu . \tag{2}$$

This action, which is the operator analog of the cosmological term, has high symmetry. It is symmetric under coordinate transformations $x^\mu \to \tilde{x}^\mu(x)$, and, thus, is also scale invariant. In addition, the action is symmetric under spin rotations, or under the corresponding gauge transformations when the spin connection is added to the gradients. The action may also contain the operator analog of the Einstein–Hilbert–Cartan term [6], $e^{\alpha\beta\mu\nu} e_{abcd} \int d^4x \, \hat{E}^a_\alpha \hat{E}^b_\beta F^{cd}_{\mu\nu}$, where $F^{cd}_{\mu\nu}$ is the Cartan curvature two-form. Additionally, the four-form field can be included, $e^{\alpha\beta\mu\nu} e_{abcd} \int d^4x \, F^{abcd}_{\alpha\beta\mu\nu}$, which is also related to the problem of the vacuum energy and cosmological constant [12–18].

The tetrads e^a_μ appear as the vacuum expectation values of the bilinear fermionic 1-form $\hat{E}^a_\mu$ as a result of the spontaneous symmetry breaking:

$$e^a_\mu = < \hat{E}^a_\mu > . \tag{3}$$

This order parameter breaks the separate symmetries under orbital and spin transformations, but remains invariant under the combined rotations. On the level of the Lorentz symmetries the symmetry breaking scheme is $L_L \times L_S \to L_J$. Here, L_L is the group of Lorentz transformations in the coordinates space, L_S is the group of Lorentz transformations in the spin space, and L_J is the symmetry group of the order parameter, which is invariant under the combined Lorentz transformations L_J.

Similar symmetry breaking mechanism of emergent gravity is known in condensed matter physics, where the effective gravitational vielbein also emerges as the bilinear fermionic 1-form [4]. This scenario takes place in the p-wave spin-triplet superfluid ^{3}He-B, where the corresponding relative symmetry breaking [19] occurs between the spin and orbital rotations, $SO(3)_L \times SO(3)_S \to SO(3)_J$. This means that the symmetry under the relative rotations in spin and orbital spaces is broken, while the properties of ^{3}He-B are isotropic.

3. Dimensionful Metric and Dimensionless Interval

The metric field is the bilinear combination of the tetrad fields

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu , \tag{4}$$

and, thus, in this quantum gravity the metric is the fermionic quartet (in principle the signature can be the dynamical variable O_{ab}, [20,21] and η_{ab} may also emerge as the vacuum expectation value of the corresponding symmetry breaking phase transition, $\eta_{ab} = < O_{ab} >$.

It is important that in this quantum gravity, the fermionic fields Ψ are dimensionless, since they are normalized by the Berezin integral [8]. Thus, the tetrads in Equation (3) have the dimensions of the inverse time and inverse length, $[e^a_0] = 1/[t]$ and $[e^a_i] = 1/[L]$, while the metric elements in Equation (4) have dimensions $1/[t]^2$, $1/[L]^2$ and $1/[t][L]$. Due to these dimensions of tetrads and metric, the interval is dimensionless:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \; , \; [s^2] = [1] . \tag{5}$$

The reason for that is that the interval is the diffeomorphism invariant, while in this approach to quantum gravity all the diffeomorphism-invariant quantities are dimensionless [8].

The same takes place for the other diffeomorphism invariant quantities: the action S (an example is in Equation (2)); scalar curvature R; scalar field Φ; the wave function ψ; masses M; cosmological constant Λ; etc. [22,23]. This is valid for the arbitrary dimension of spacetime, and, thus, is universal, which is one of the most important consequences of the composite tetrads.

Note that the original action (2) does not contain the Planck constant $\hbar$. One can show that this is the property of any action, if it is written in the diffeomorphism invariant form, see Section 12.

4. Action, Mass and Scalar Field Are Dimensionless

Let us consider the simplest example of the dimensionless action—the action describing interaction of a charged point particle with the $U(1)$ gauge field.

$$S = q \int dx^\mu A_\mu \,. \tag{6}$$

As the original action (2), this action does not depend on the metric field and is described solely in terms of differential forms, now in terms of the one-form $U(1)$ gauge field A_μ. The $U(1)$ field is the geometric quantity, which comes from the gauging of the global $U(1)$ field. The field A_μ comes from the gauging of gradient of the phase field, and, thus, has dimension of the gradient of phase, with $[A_0] = 1/[t]$ and $[A_i] = 1/[L]$. The charge q here is dimensionless—it is the integer (or fractional) geometric charge of the fermionic or bosonic field. As a result the action (6) is naturally dimensionless, $[S] = 1$.

Such an action can be extended to the objects of higher dimensions, which interact with the corresponding gauge fields: $1+1$ strings interacting with a two-form gauge field, $2+1$ branes interacting with the three-form field, and also $3+1$ medium interacting with the four-form field.

Now, let us consider the action describing the classical dynamics of a point particle. This action requires the metric field, since it is expressed in terms of the interval:

$$S = M \int ds \;,\; ds^2 = -g_{\mu\nu} dx^\mu dx^\nu \,. \tag{7}$$

Since both the interval ds and the action S are dimensionless, from Equation (7) it follows that the particle mass M is also dimensionless, $[M] = [S] = [s] = [1]$.

Let us consider the quadratic terms in the action for the classical scalar field Φ:

$$S = \int d^4x \, \sqrt{-g} \left(g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi + M^2 |\Phi|^2 \right) . \tag{8}$$

Comparing the gradient and the mass terms, and using the dimension of the metric, one again obtains that the mass M is dimensionless, $[M] = [1]$. Then, since the action S and volume element $d^4x \sqrt{-g}$ are dimensionless, it follows that the scalar field is also dimensionless, $[\Phi]^2 = [M] = [S] = [1]$.

5. Schrödinger Equation in Minkowski Spacetime and Two Planck Constants

Expanding the Klein–Gordon equation for scalar Φ in Equation (8) over $1/M$ one obtains the non-relativistic Schrödinger action. In Minkowski spacetime, introducing the Schrödinger wave function ψ

$$\Phi(\mathbf{r}, t) = \frac{1}{\sqrt{M}} \exp\left(iMt/\sqrt{-g^{00}} \right) \psi(\mathbf{r}, t) \,, \tag{9}$$

one obtains the Schrödinger-type action in the form

$$S_{\rm Schr} = \int d^3x dt \sqrt{-g}\, \mathcal{L}_{\rm q}\,, \tag{10}$$

$$2\mathcal{L}_{\rm q} = i\sqrt{-g^{00}}(\psi\partial_t\psi^* - \psi^*\partial_t\psi) + \frac{g^{ik}}{M}\nabla_i\psi^*\nabla_k\psi + 2U|\psi|^2\,. \tag{11}$$

Here, we added the potential term with $U = \sqrt{-g^{00}}qA_0$, where A_0 of the electromagnetic gauge field and q is the geometric charge of the scalar field.

Equation (11) suggests that the metric element $\sqrt{-g^{00}}$ of the Minkowski vacuum plays the role of the Planck constant $\hbar$. This connection between g_{00} and $\hbar$ was also suggested in Ref. [24], where it was noticed that if $\hbar$ is absorbed into Minkowski metric it does not enter equations written in the covariant form. Since in the Akama-Diakonov (AD) approach to quantum gravity the interval is dimensionless, the Planck constant has dimension of time, $[\hbar] = [t]$. However, the term with the space gradients suggests that spatial elements of the Minkowski metric play the roles of another Planck constant, which we denote as slash $\hbar\!\!/$:

$$-g^{00}_{\rm Mink} \equiv \hbar^2\;,\; g^{ik}_{\rm Mink} \equiv \hbar\!\!/^2\delta^{ik}\,. \tag{12}$$

These Planck constants, $\hbar$ and $\hbar\!\!/$, enter correspondingly the time derivative and space derivative terms in the Schrödinger equation:

$$i\hbar\partial_t\psi = -\frac{\hbar\!\!/^2}{2M}\nabla^2\psi + U\psi\,. \tag{13}$$

$\hbar$ and $\hbar\!\!/$ have different dimensions

$$[\hbar] = [t]\;,\; [\hbar\!\!/] = [L]\,, \tag{14}$$

and their ratio $\hbar\!\!/ / \hbar$ determines the speed of light c in Minkowski vacuum. Equation (14) suggests that the Planck constants represent the units of space and time, rather than the units of the phase space.

All the terms in the Schrödinger Equation (13) are dimensionless, including the potential energy U. This can be checked for the Coulomb potential for electron with the geometric charge $q = -1$ in the field of nucleus with the geometric charge $q = Z > 0$. This potential has the conventional form $U(r) = -Ze^2/r$, where e is the "physical charge", which can be expressed in terms of the fine structure constant. The fine structure constant is diffeomorphism invariant and thus is dimensionless. That is why, from

$$\alpha = \frac{e^2}{\hbar\!\!/}\,, \tag{15}$$

it follows that $e^2 = \hbar\!\!/\alpha$ has dimension of length,

$$[e^2] = [\hbar\!\!/] = [L]\,, \tag{16}$$

and, thus, the potential $U(r) = -Ze^2/r$ is dimensionless, $[U] = [1]$. Then one has

$$i\hbar\partial_t\psi = \mathcal{H}\psi\;,\; \mathcal{H} = -\frac{\hbar\!\!/^2}{2M}\nabla^2 - \alpha Z\frac{\hbar\!\!/}{r}\,. \tag{17}$$

The other possible potential terms are also dimensionless. For example, the dipole term $U_{\rm dip} \sim d^2/r^3$ is dimensionless, since $[d^2] = [e^2][L^2] = [L]^3$. The Pauli term for electron $U_{\rm P} = -\frac{\hbar\!\!/^2}{M_e}\mathbf{B}\cdot\sigma$ is dimensionless, since $[\hbar\!\!/] = [L]$, the dimension of geometric magnetic field $[B] = 1/[L]^2$, and the dimension of electron mass $[M_e] = 1$.

Note that the Hamiltonian $\mathcal{H}$, which enters the Schrödinger equation, comes from the variation of the dimensionless action over the dimensionless ψ^* and, thus, is dimensionless, $[\mathcal{H}] = [1]$. On the other hand, the Hamiltonian, which comes from the action as $H = dS/dt$, has dimension of frequency, $[H] = 1/[t]$. The relation between the energy and frequency will be discussed in Section 7.

Schrödinger equation contains two Planck constants with different dimensions, while the other parameters, such as M and α, are dimensionless.

6. From Quantum Vacuum to Classical Physics via Symmetry Breaking

From Equation (12) it follows that in the Minkowski spacetime the tetrads have the following values:

$$e_a^\mu = \mathrm{diag}(-\hbar, \hslash, \hslash, \hslash)\ ,\ e^a_\mu = \mathrm{diag}(-\frac{1}{\hbar}, \frac{1}{\hslash}, \frac{1}{\hslash}, \frac{1}{\hslash})\,, \tag{18}$$

Here, e_a^μ are tetrads that are inverse to e^a_μ. They have dimension of length, $[e^i_a] = [L]$, and time $[e^0_a] = [t]$. Their determinant (which is inverse to the determinant e of e^a_μ) has dimension of the four-volume. As a result, the vacuum expectation value of the original action (2) serves as the number of the elementary four-volumes.

From Equation (3) it follows that the tetrads e^a_μ represent the order parameter of the symmetry breaking phase transition with $e^a_\mu = 0$ in the symmetric phase. This suggests that, in the AD approach to quantum gravity, the symmetric phase of the vacuum corresponds to $\hbar = \infty$ and $\hslash = \infty$. It is the pure quantum vacuum, with quantum correlations at the infinitely long distances due to scale invariance. The scale invariance is broken by the finite values of $\hbar$ and $\hslash$ in the broken symmetry phases, and this, finally, gives rise to the classical physics for large masses. The classical physics emerges only in the broken symmetry states. The $\hbar$ expansion in the classical limit [25] is opposite to the $1/\hbar$ expansion in the quantum limit.

In this respect the Planck constant $\hslash$ has analogy with the coherence length ξ in the second order phase transitions in superconductors and superfluids. The scale of ξ is intermediate between the microscopic length scale a, the interatomic distance, and the macroscopic scale l of superfluid hydrodynamics, $a \ll \xi \ll l$. In microscopic physics we use a/ξ as small parameter (quantum limit), while in macroscopic physics the small parameter is ξ/l (classical limit).

The action for the massive Dirac particles is

$$S = \int d^4x\, e\, (ie^\mu_a \bar{\Psi}\gamma^a \nabla_\mu \Psi - M\bar{\Psi}\Psi)\,. \tag{19}$$

The Dirac field is dimensionless, $[\Psi] = [1]$, as well as the four-volume element, $[d^3x dt\, e] = [1]$. In the limit of large wavelength, $\lambda \gg \hslash/M$, one obtains the Schrödinger equation for the non-relativistic fermions in Equation (13), and from that equation—the classical physics of massive particle at large M.

Since the Minkowski metric in Equation (12) is quadratic in $1/\hbar$ and $1/\hslash$, these Planck parameters may have negative signs, which corresponds to the different signs of the tetrad elements in Minkowski vacuum in Equation (18). In principle, there can be the topological objects related to the symmetry breaking, such as the cosmological domain walls between the Minkowski vacua with positive and negative signs of $1/\hbar$ and/or $1/\hslash$ [24,26]. Inside the domain wall the symmetric quantum vacuum with $1/\hbar = 1/\hslash = 0$ is restored, or partially restored if only one of the Planck constants changes sign. Example of such walls can be found in Ref. [27]. The same takes place in the cores of the other topological objects, such as torsion strings [26] and topological instantons [28,29]. Analytic extension of $1/\hbar$ and $1/\hslash$ across the Big Bang is also possible, which is similar to the analytic extension of metric in Refs. [30,31].

7. Energy and Frequency

Since the action is dimensionless, it may serve as the phase of the wave function in path integral presentation or in the path integrals over the quantum fields. For the point particle, one has

$$e^{iS} = e^{iM\int ds} \,. \tag{20}$$

Let us consider the particle at rest in the Minkowski vacuum:

$$e^{iS} = e^{i\int \mathcal{L}(t)dt} = e^{iMt\sqrt{-g_{00}}} \,. \tag{21}$$

This function is the periodic in time with period

$$\mathcal{T} = \frac{2\pi}{M\sqrt{-g_{00}}} \,, \tag{22}$$

which corresponds to the frequency of oscillations,

$$\omega = M\sqrt{-g_{00}} \,. \tag{23}$$

The quantum mechanical relation between energy of stationary particle and frequency, $M = \hbar\omega$, demonstrates again that the Planck constant $\hbar$ can be considered as the element of Minkowski metric:

$$\hbar = \frac{1}{\sqrt{-g_{00}^{\rm Mink}}} = \sqrt{-g^{00}_{\rm Mink}} \,. \tag{24}$$

Note that the Planck constant was introduced by Planck as a quantum of action. However, now, since the action is dimensionless, the quantum of action is also dimensionless, $\Delta S = 2\pi$ (or $\Delta S = \pi$ for fermions). Nevertheless, the main property of the Planck constant remains valid: the $\hbar$ enters the relation between the energy and frequency, $M = \hbar\omega$. However, now it has dimension of time, $[\hbar] = 1/[t]$.

8. de Sitter Spacetime and Planck Constants

The same parameters $\hbar$ and $\not{h}$ exist for any $D+1$ Minkowski spacetime. However, these parameters are not diffeomorphism invariant. Being the element of the Minkowski metric they are invariant only under space rotations. As a result the Planck constants are not dimensionless. Then, according to the Weinberg criterion [32], they cannot be the fundamental constants (see also Refs. [33–36] on fundamental constants).

Let us consider the possible variation of the Planck constants on example of the de Sitter (dS) spacetime. The dS spacetime can be obtained from the 4 + 1 Minkowski spacetime:

$$-\frac{1}{\hbar^2}dt^2 + \frac{1}{\not{h}^2}\sum_1^4 X^i X^i = \alpha^2 \,. \tag{25}$$

It contains one more parameter, the dimensionless constant α, the radius of the 4 + 1 sphere. The corresponding Hubble parameter has dimension of frequency:

$$H = \frac{1}{\hbar\alpha} \,,\ [H] = \frac{1}{[t]} \,. \tag{26}$$

In the Paineve–Gullstrand form, the interval in dS spacetime contains three parameters, $\hbar$, $\not{h}$, and H,

$$ds^2 = -\frac{1}{\hbar^2}dt^2 + \frac{1}{\not{h}^2}\left((dr - Hrdt)^2 + r^2 d\Omega^2\right) . \tag{27}$$

At $r = 0$ the metric is Minkowski. However, since the Planck constants are not fundamental, it is not excluded that in the dS Universe they may deviate from their values in Minkowski vacuum and depend on H. The phonon analog of the metric emerging in liquids suggests the following corrections to the Planck constants [37]:

$$\frac{\Delta \not{h}}{\not{h}} \sim \frac{\Delta \hbar}{\hbar} \sim \hbar^2 H^2 = \frac{\not{h}^2}{r_c^2} \ll 1\,. \tag{28}$$

Here, r_c is the radius of the cosmological horizon.

Note that the main cosmological constant problem is not affected by this dependence of $\hbar$. In the q-theory of the quantum vacuum [15], the vacuum energy is self-tuned to zero in the full equilibrium, and this does not depend on the value of $\hbar$.

9. Black Hole and Planck Constants

Let us consider the possible variation of the Planck constants coming from the black hole. The black hole metric in the Paineve–Gullstrand form is

$$ds^2 = -\frac{1}{\hbar^2}dt^2 + \frac{1}{\not{h}^2}\left((dr - vdt)^2 + r^2 d\Omega^2\right), \tag{29}$$

where $v(r)$ is the corresponding shift function. The radius of the black hole event horizon is

$$r_h = 2MG\,, \tag{30}$$

where M is the dimensionless mass of the black hole and G is the Newton constant, which has dimension of length, see Section 11. So the metric also contains three parameters, $\hbar$, $\not{h}$, and the parameter MG. The metric becomes Minkowski at $r \to \infty$. However, near the horizon it may deviate from the vacuum values. The comparison with Equation (28), where the corrections to the Planck constants are inverse proportional to the square of the event horizon radius, suggests the following corrections:

$$\frac{\Delta \not{h}}{\not{h}} \sim \frac{\Delta \hbar}{\hbar} \sim \frac{\not{h}^2}{r_h^2} \ll 1\,. \tag{31}$$

10. Planck Constants and Tolman Law

In the dS spacetime, the probability of Hawking radiation of particle with mass M detected by observer at $r = 0$ is determined by parameters $\hbar$ and H:

$$w \propto \exp\left(-\frac{2\pi M}{H\sqrt{-g^{00}_{\rm Mink}}}\right) = \exp\left(-\frac{2\pi M}{\hbar H}\right) = \tag{32}$$

$$= \exp\left(-\frac{M}{T_H}\right). \tag{33}$$

Here, T_H is the Gibbons–Hawking temperature measured at $r = 0$, where the metric is Minkowski:

$$T_H = T(r=0) = \frac{\sqrt{-g^{00}_{\rm Mink}}H}{2\pi} = \frac{\hbar H}{2\pi}\,. \tag{34}$$

This temperature is dimensionless due to the time dimension of the Planck constant: $[T_H] = [\hbar][H] = [t]/[t] = [1]$.

On the other hand, the parameter $H/2\pi$ plays the role of Tolman temperature, which enters the Tolman law

$$T(r) = \frac{T_{\rm Tolman}}{\sqrt{-g_{00}(r)}}\,,\ T_{\rm Tolman} = \frac{H}{2\pi}\,, \tag{35}$$

has dimension of inverse time, $[T_{\rm Tolman}] = [H] = 1/[t]$, see also Refs. [22,23].

The Hawking temperature of black hole, which is measured at the asymptotic Minkowski vacuum, is

$$T_H = T(r = \infty) = \frac{\not\hbar}{4\pi r_h}\,, \tag{36}$$

where r_h is the position of the black hole horizon. As the Gibbons–Hawking temperature in Equation (34), the Hawking temperature (36) is also dimensionless, now due to the length dimension of the second Planck constant, $[T_H] = [\not\hbar]/[r_h] = [L]/[L] = [1]$.

It is not excluded that the Tolman temperature is the parameter of the equilibrium system, which may influence the Planck constants. Equations (28) and (31) suggest the following corrections to the Planck constants:

$$\frac{\Delta \not\hbar}{\not\hbar} \sim \frac{\Delta\hbar}{\hbar} \sim \hbar^2 T^2_{\rm Tolman} \ll 1\,. \tag{37}$$

11. Length Dimension of Newton Constant and Planck Length

In the AD composite fermion gravity, the gravitational potential $U(r) = -GM_1M_2/r$ is dimensionless and contains masses M_1 and M_2, which are also dimensionless. As a result the Newton constant has the dimension of length, $[G] = [L]$. This suggests that G is not diffeomorphism invariant, and, thus, cannot be the fundamental constant. That is why in the gravitational action it must be compensated by $\not\hbar$, which also has dimension of length, $[G] = [\not\hbar] = [L]$:

$$S = \frac{1}{16\pi}\frac{\not\hbar}{G}\int d^4x\,\sqrt{-g}\,R\,. \tag{38}$$

Since the scalar curvature R is dimensionless, the Einstein–Hilbert action (38) is dimensionless. It can be written via diffeomorphism invariant quantities if we introduce the Planck mass $M_{\rm P} = \sqrt{\not\hbar/G}$, which is dimensionless as all the masses M in the composite tetrad approach, $[M_{\rm P}]^2 = [\not\hbar]/[G] = [L][L]^{-1} = [1]$:

$$S = \frac{M^2_{\rm P}}{16\pi}\int d^4x\,\sqrt{-g}\,R\,. \tag{39}$$

The Planck length scale has the conventional form $l^2_{\rm P} = \not\hbar G$, with $[l_{\rm P}]^2 = [\not\hbar][G] = [L][L] = [L]^2$. The slash Planck constant $\not\hbar$ has the same dimension as the Planck length, $[\not\hbar] = [l_{\rm P}] = [L]$. Whether this "Planck constant length" is related to the "Planck length scale", is an open question [38]. This question was considered on the example of the acoustic gravity, where the analog of the trans-Planckian phyics—atomic physics—is known [37]. It was demonstrated that the acoustic analog of $\not\hbar$ is on the order of the interatomic distance. This suggests that in AD quantum gravity the Planck constant $\not\hbar$ is on the order of Planck length $l_{\rm P}$, i.e., the Planck mass is on the order of unity, $M_{\rm P} = \sqrt{\not\hbar/G} \sim 1$.

12. No $\hbar$ and $\not\hbar$ in Diffeomorphism Invariant Equations

Let us consider the diffeomorphism invariant equations on example of the statistical entropy, which is dimensionless in any units. The Gibbons–Hawking entropy of the de Sitter cosmological horizon is

$$S_H = \frac{M^2_{\rm P}}{4\pi T^2_H}\,, \tag{40}$$

where T_H is the Gibbons-Hawking temperature in Equation (34). The Bekenstein–Hawking entropy of the black hole is

$$S_H = \frac{4\pi M^2}{M^2_{\rm P}} = \frac{M}{2T_H}\,, \tag{41}$$

where M is the black hole mass and T_H is the Hawking temperature of black hole radiation.

All quantities in Equations (40) and (41) are dimensionless, $[S_H] = [M_P] = [T_H] = [M] = [1]$. Both equations do not contain Planck constants. This demonstrates the general property of diffeomorphism invariant equations: they do not contain $\hbar$ and $\not{h}$, because the Planck constants are not diffeomorphism invariant and have dimensions of time and length correspondingly.

13. Conclusions

The important consequence of the Akama-Diakonov composite tetrads approach to quantum gravity is the "dimensionless physics"; all the diffeomorphism invariant quantities are dimensionless for any dimension of spacetime. These include the action S, interval s, cosmological constant Λ, Hawking temperature T_H, scalar curvature R, scalar field Φ, Planck mass M_P, masses M of particles and fields, etc.

Another consequence of this approach to quantum gravity, is that there are two Planck constants, bar $\hbar$ and slash $\not{h}$, which are the elements of the Minkowski metric. As the elements of the Minkowski metric, $\hbar$ and $\not{h}$ are invariant only under $SO(3)$ space rotations, and, thus, they are not diffeomorphism invariant. As a result, the Planck constants are not dimensionless, with bar $\hbar$ having dimension of time, $[\hbar] = [t]$, and slash $\not{h}$ having dimension of length $[\not{h}] = [L]$, and they do not enter the diffeomorphism invariant equations.

Since the Planck constants are not dimensionless, then according to Weinberg criterion they cannot be the fundamental constants, and thus may vary with space and time. The possible corrections to the Planck constants in the de Sitter Universe and near the event horizon of black hole are in Equations (28) and (31).

According to Vladimirov and Diakonov [8], "the unconventional dimensions of the fields ... are natural and adequate for a microscopic theory of quantum gravity". The similar "dimensionless physics" appears also in several other approaches to quantum gravity. It appears, in particular, in the BF theories, where the composite metric is formed by the triplet of the two-form fields (Schönberg-Urbantke metric) [39–44]. It appears also in the model of the superplastic vacuum [45], which is described in terms of the so-called elasticity tetrads [46–52], and in acoustic gravity [37]. All this suggests that the physics with two Planck constants, bar $\hbar$ and slash $\not{h}$, can be reasonable.

Funding: This work has been supported by Academy of Finland (grant 332964).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: I thank Y.N. Obukhov for discussion and criticism.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Volovik, G.E. *The Universe in a Helium Droplet*; Clarendon Press: Oxford, UK, 2003.
2. Sabsovich, D.; Wunderlich, P.; Fleurov, V.; Pikulin, D.I.; Ilan, R.; Meng, T. Hawking fragmentation and Hawking attenuation in Weyl semimetals. *Phys. Rev. Res.* **2022**, *4*, 013055. [CrossRef]
3. Könye, V.; Morice, C.; Chernyavsky, D.; Moghaddam, A.G.; van Den Brink, J.; van Wezel, J. Horizon physics of quasi-one-dimensional tilted Weyl cones on a lattice. *Phys. Rev. Res.* **2022**, *4*, 033237. [CrossRef]
4. Volovik, G.E. Superfluid ^{3}He-B and gravity. *Phys. B* **1990**, *162*, 222–230. [CrossRef]
5. Akama, K. An Attempt at Pregeometry: Gravity with Composite Metric. *Prog. Theor. Phys.* **1978**, *60*, 1900–1909. [CrossRef]
6. Diakonov, D. Towards lattice-regularized Quantum Gravity. *arXiv* **2011**, arXiv:1109.0091.
7. Vladimirov, A.A.; Diakonov, D. Diffeomorphism-invariant lattice actions. *Phys. Part. Nucl.* **2014**, *45*, 800. [CrossRef]
8. Vladimirov, A.A.; Diakonov, D. Phase transitions in spinor quantum gravity on a lattice. *Phys. Rev. D* **2012**, *86*, 104019. [CrossRef]
9. Obukhov, Y.N.; Hehl, F.W. Extended Einstein—Cartan theory a la Diakonov: The field equations. *Phys. Lett. B* **2012**, *713*, 321–325. [CrossRef]
10. Volovik, G.E. Dimensionless physics: Planck constant as an element of Minkowski metric. *JETP Lett.* **2023**, *117*, 240–244. [CrossRef]
11. Maiezza, A.; Nesti, F. Parity from gauge symmetry. *Eur. Phys. J. C* **2022**, *82*, 491. [CrossRef]
12. Hawking, S.W. The cosmological constant is probably zero. *Phys. Lett. B* **1984**, *134*, 403. [CrossRef]

13. Duff, M.J. The cosmological constant is possibly zero, but the proof is probably wrong. *Phys. Lett. B* **1989**, *226*, 36. [CrossRef]
14. Wu, Z.C. The cosmological constant is probably zero, and a proof is possibly right. *Phys. Lett. B* **2008**, *659*, 891. [CrossRef]
15. Klinkhamer, F.R.; Volovik, G.E. Dynamic vacuum variable and equilibrium approach in cosmology. *Phys. Rev. D* **2008**, *78*, 063528. [CrossRef]
16. Klinkhamer, F.R.; Volovik, G.E. Dark matter from dark energy in q-theory. *JETP Lett.* **2017**, *105*, 74–77. [CrossRef]
17. Nitta, M.; Yokokura, R. Higher derivative three-form gauge theories and their supersymmetric extension. *J. High Energy Phys.* **2018**, *10*, 146. [CrossRef]
18. Kaloper, N. Hidden variables of gravity and geometry and the cosmological constant problem. *Phys. Rev. D* **2022**, *106*, 065009. [CrossRef]
19. Leggett, A.J. NMR lineshifts and spontaneously broken spin-orbit symmetry. I. General concepts. *J. Phys. C* **1973**, *6*, 3187. [CrossRef]
20. Bondarenko, S.; Zubkov, M.A. Riemann–Cartan Gravity with Dynamical Signature. *JETP Lett.* **2022**, *116*, 54–60. [CrossRef]
21. Bondarenko, S. Dynamical Signature: Complex Manifolds, Gauge Fields and Non-Flat Tangent Space. *Universe* **2022**, *8*, 497. [CrossRef]
22. Volovik, G.E. Dimensionless physics. *JETP* **2021**, *132*, 727–733. [CrossRef]
23. Volovik, G.E. Dimensionless physics: Continuation. *JETP* **2022**, *135*, 663–670. [CrossRef]
24. Volovik, G.E. $\hbar$ as parameter of Minkowski metric in effective theory. *JETP Lett.* **2009**, *90*, 697–704. [CrossRef]
25. Brodsky, S.J.; Hoyer, P. The $\hbar$ expansion in quantum field theory. *Phys. Rev. D* **2011**, *83*, 045026. [CrossRef]
26. Volovik, G.E. Combined Lorentz symmetry: Lessons from superfluid ^{3}He. *J. Low Temp. Phys.* **2022**, *206*, 1–15. [CrossRef]
27. Vergeles, S.N. Domain wall between the Dirac sea and the 'anti-Dirac sea'. *Class. Quantum Grav.* **2022**, *39*, 038001. [CrossRef]
28. Hanson, A.J.; Regge, T. Torsion and quantum gravity. In Proceedings of the Integrative Conference on Group Theory and Mathematical Physics, Austin, TX, USA, 11–16 September 1978.
29. d'Auria, R.; Regge, T. Gravity theories with asymptotically flat instantons. *Nucl. Phys. B* **1982**, *195*, 308. [CrossRef]
30. Boyle, L.; Finn, K.; Turok, N. The Big Bang, CPT, and neutrino dark matter. *Ann. Phys.* **2022**, *438*, 168767. [CrossRef]
31. Boyle, L.; Teuscher, M.; Turok, N. The Big Bang as a Mirror: A Solution of the Strong CP Problem. *arXiv* **2022**, arXiv:2208.10396.
32. Weinberg, S.; Taylor, J.G. Overview of theoretical prospects for understanding the values of fundamental constants. *Philos. Trans. R. Soc. Lond. A* **1983**, *310*, 249–252.
33. Duff, M.J. Trialogue on the number of fundamental constants (III): A party political broadcast on behalf of the Zero Constants Party. *J. High Energy Phys.* **2002**, *2002*, 023. [CrossRef]
34. Okun, L.B. Cube or hypercube of natural units, In *Multiple Facets of Quantization and Supersymmetry Michael Marinov Memorial Volume*; Olshanetsky, M., Vainshtein, A., Eds.; World Scientific: Singapore, 2002.
35. Okun, L.B. On the Article of Gamow, G.; Ivanenko, D.; Landau, L. World Constants and Limiting Transition. *Phys. At. Nucl.* **2002**, *65*, 1370–1372. [CrossRef]
36. Gamow, G.; Ivanenko, D.; Landau, L. World constants and limiting transition. *Phys. At. Nucl.* **2002**, *65*, 1373–1375. [CrossRef]
37. Volovik, G.E. Acoustic metric and Planck constants. *JETP Lett.* **2023**, *117*, 550–555. [CrossRef]
38. Carlip, S. Spacetime foam: A review. *arXiv* **2022**, arXiv:2209.14282
39. Schönberg, M. Electromagnetism and Gravitation. *Riv. Bras. Fis.* **1971**, *1*, 91–122.
40. Urbantke, H. On integrability properties of SU (2) Yang-Mills fields. I. Infinitesimal part. *J. Math. Phys.* **1984**, *25*, 2321. [CrossRef]
41. Capovilla, R.; Dell, J.; Jacobson, T.; Mason, L. Self-dual 2-forms and gravity. *Class. Quantum Grav.* **1991**, *8*, 41–57. [CrossRef]
42. Obukhov, Y.N.; Tertychniy, S.I. Vacuum Einstein Equations in Terms of Curvature Forms. *Class. Quantum Grav.* **1996**, *13*, 1623–1640. [CrossRef]
43. Hehl, F.W.; Obukhov, Y.N. *Foundations of Classical Electrodynamics*; Birkhauser: Boston, MA, USA, 2003.
44. Friedel, L.; Speziale, S. On the relations between gravity and BF theories? *SIGMA* **2012**, *8*, 032. [CrossRef]
45. Klinkhamer, F.R.; Volovik, G.E. Tetrads and q-theory. *JETP Lett.* **2019**, *109*, 364–367. [CrossRef]
46. Dzyaloshinskii, I.E.; Volovick, G.E. Poisson brackets in condensed matter physics. *Ann. Phys.* **1980**, *125*, 67–97. [CrossRef]
47. Nissinen, J.; Volovik, G.E. Elasticity tetrads, mixed axial-gravitational anomalies, and (3+1)-d quantum Hall effect. *Phys. Rev. Res.* **2019**, *1*, 023007. [CrossRef]
48. Nissinen, J. Field theory of higher-order topological crystalline response, generalized global symmetries and elasticity tetrads. *Ann. Phys.* **2022**, *447*, 169139. [CrossRef]
49. Nissinen, J. Emergent spacetime and gravitational Nieh-Yan anomaly in chiral $p + ip$ Weyl superfluids and superconductors. *Phys. Rev. Lett.* **2020**, *124*, 117002. [CrossRef]
50. Laurila, S.; Nissinen, J. Torsional Landau levels and geometric anomalies in condensed matter Weyl systems. *Phys. Rev. B* **2020**, *102*, 235163. [CrossRef]
51. Gioia, L.; Wang, C.; Burkov, A.A. Unquantized anomalies in topological semimetals. *Phys. Rev. Res.* **2021**, *3*, 043067. [CrossRef]
52. Yi, J.; Ying, X.; Gioia, L.; Burkov, A.A. Topological order in interacting semimetals. *Phys. Rev. B* **2023**, *107*, 115147. [CrossRef]