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Practical folding meets measurable paper properties

Jarmo Kouko [a], Tomi Kankkunen [a,b], Miia Palmu [c], Jarkko Niiranen [b], Kirsi Peltonen [d,*]

[a] VTT Technical Research Centre of Finland Ltd, P.O. Box 1603, FI-40101 Jyväskylä, Finland
[b] Department of Civil Engineering, Aalto University, P.O. Box 11000, FI-00076 Aalto, Finland
[c] VTT Technical Research Centre of Finland Ltd, P.O. Box 1000, FI-02044 VTT, Finland
[d] Department of Mathematics and Systems Analysis, Aalto University, P.O. Box 11100, FI-00076 Aalto, Finland

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ABSTRACT

This paper was motivated by recent activity to replace synthetic, daily-life consumer package solutions with cellulose-based materials and tailored applications. This investigation included a collection of 26 commercial paperboards. The main objective was to create foundations and increase common understanding for further scientific research in a highly interdisciplinary topic. Currently, the Miura-ori family is widely used in applied origami, being the most studied origami tessellation pattern due to its simplicity and remarkable properties. The folded samples were evaluated using qualitative ranking of the folding performance developed in this study and measuring the tensile properties of the materials. Four qualitative folding parameters were defined, and a numerical ranking was given for them.

As a key result, the specific modulus of the materials seemed to correlate with the qualitative folding performance of the investigated materials. Therefore, the investigated Miura-ori folding can be assumed to be a "stiffness-driven" structure. Lastly, a finite element simulation was performed to assess whether the qualitative folding experience can be captured by numerical mechanics analysis by comparing two different boards. Simulations were able to identify that folding mechanics depend on the choice of orthotropic material orientation with respect to tessellation pattern, and therefore contributes to the qualitative folding experience.

1. Introduction

1.1. Folding of paper and board

Origami, considered as an art form of folding a square piece of paper into a figure in the 3D space, has expanded into processes with a wide range of applications, inspiration and active research in science, technology, architecture, and arts. Origami folding can be seen as a subfield of mathematics [1–3] with many connections to computer science via algorithms and computational aspects [4,5], or from the point of view of the physical products as a branch of design [6–7], engineering [8–11], or even biomedicine [12]. Some ideas about the broadness of the topic in different fields can be found in [13], reflecting the exponential growth of the community around International Meetings on Origami in Science, Mathematics and Education that have taken place around every fourth year since 1989.

Folding techniques are scale-independent, making them equally valid to nanoscale medical devices as large-scale engineering applications. Materials to be folded can vary accordingly from biomaterials to metals and synthetic fibres. Fabrication of folded objects is often made manually, and there are not too many solutions towards mass production yet. One such approach was developed in [14] for the continuous folding process, where sheet material is progressively folded in two dimensions, through a set of rollers, followed by a configured roller for the final folding in the third dimension. For smaller volume production, one could utilise synchronous methods, based on moulding structures mimicking the origami behaviour, which fold the whole sheet in one go [15]. Folds can also be weaved via jacquard techniques [16] familiar in the textile industry. We do not know if analogous weaving approaches have been developed to construct folds to other materials directly.

This paper is motivated by the scale of daily life packages, where, for example, ceramic tableware and glass items could be wrapped. We concentrate on cellulose-based materials that could replace traditional bubble wrap or other synthetic cushioning materials. Our long-term goal then is, naturally, a quest for the perfect material for different purposes and relevant production methods. An analogous challenge is met by all folders from origami hobbyists to dedicated artists and professionals. Some solutions can be found from the existing enormous selection of

* Corresponding author.
E-mail address: kirsi.peltonen@aalto.fi (K. Peltonen).

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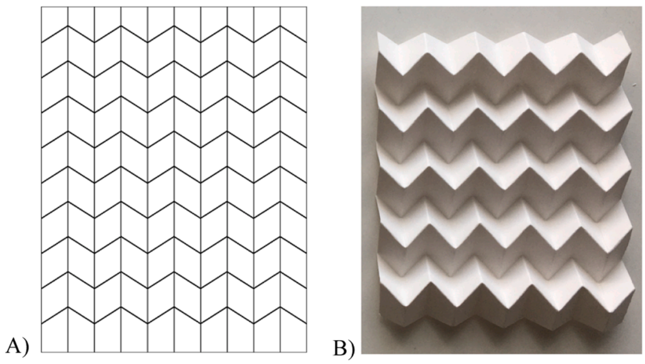

Fig. 1. A) a 2D Miura-ori tessellation pattern applied to the samples, B) a folded board sample with the Miura-ori tessellation (figures by Miia Palmu).

traditional, machine-made papers; but in the other extremity, there are techniques and recipes for customised hand paper making like [17] that are approaching perfection to individual needs. Besides the apparent visual appeal of origami structures, they can have excellent mechanical and energy absorption properties, and thus have inspired novel sandwich structures even in demanding applications [18]. In this article, we describe our first steps, starting from a collection of available commercial board and paper samples. To make the comparison between different materials and their suitability, we study how tensile tests for certain board samples are reflected in the actual folding activity. This will provide substantial knowledge about possible mechanical fabrication methods. Mechanical simulation tests will supplement the qualitative experiments.

Paper and board products contain folds in order to have designed shape and functionality [19]. Scoring, creasing, perforation, and kiss-cutting are widely used to enable exact folding lines for paper and board. Scoring and creasing causes minimal damage (no cut) to the material where a perforation and kiss-cut partly break the folding line. The creasing process is intended to delaminate paperboard layers in the fold and localise the fold [20]. According to Coffin and Nygårds [21], the folded edges of the board should not exhibit cracks on the outside surface, and the crease should offer minimal resistance to bending (not store energy) so that the box panels remain straight and form a parallelogram. The advantage of creasing is that it can be applied to paperboard packages that should have impermeable folds, such as corrugated boxes and liquid packages. Perforated lines consist of very small holes punched into the paper. Perforation reduces bending resistance at the line and significantly facilitates folding. The advantage of perforation is also that it enables parts of the sheet to be separated simply by folding and tearing along the dotted line. Kiss-cutting is a die cutting process that is applied to two-layered materials, and the top layer (e.g., sticker) is cut through, but the laminated backing paper is not.

The basis weight of paperboards is usually higher compared to papers, and most of the paperboard are multi-ply products. Typically, the basis weight of paperboards is over 130–150 g/m^2, but there are exceptions to the rule. Scoring and perforation can be applied from very thin (15–20 g/m^2) to intermediate (60–90 g/m^2) basis weights and thicknesses. Creasing can be applied from intermediate to heavy papers and paperboard grades. Kiss-cut can be applied from relatively low thicknesses in the condition that paper is incompressible to high thicknesses, as well as to very thick corrugated board panels. In principle, all four methods can be applied to enhance origami-folding of paper and board. However, the suitable method, or combination of methods, depends on the application design and material properties of paperboards. Especially, the mechanical behaviour of the paperboard is nonlinear and depends on the loading direction (orthotropy) [22], which naturally poses challenges in the selection of suitable material and the manufacturing method.

Mathematical models of tessellation patterns and their properties deal with infinitely thin developable surfaces between the zero width creases. In practice, how the paper expands and contracts near the folds depends on the thickness and other properties. The folding action, likewise, influences the material, for example, as fibres break or loosen. The material itself can also be an enabler. For example, it can be shown that a well-known origami model, the pleated hyperbolic paraboloid, cannot be folded with the standard crease pattern in the standard mathematical model of ideal zero-thickness paper because positive Gaussian curvature is introduced at the vertices where three mountains/valleys meet [23]. However, it can easily be folded from standard copy paper or corresponding material according to the very same pattern due to resilience properties of real materials. Real shapes that can be achieved by folding different materials cannot be described by simple equations but can arise unexpected and turn out to be computationally impossible.

1.2. *Objectives*

The key objective of this study was to create foundations and increase common understanding for further scientific research in a highly interdisciplinary topic. People having background in design, engineering, mathematics, and paper physics need to agree about the terminology for the same phenomena in order to avoid misunderstandings and build a solid base for further research. Especially, the importance of material properties on successful folding needs to be understood. For these reasons this article is written in a sufficiently general level. Another objective was to study if few typical and relatively simple mechanical measurements of paper could define Miura-ori folding performance. Based on the results, the necessity of more complex and sophisticated measurements is assessed.

A material related objective of the investigation was to find a quantitative relation between Miura-ori foldability and mechanical properties of paperboards. The focus was on cardboards that are strong enough to be used to protect tableware and other fragile objects, and the goal was to try to identify a set of measurable quantities that could be stipulated a priori for materials to be used in an automated folding process. Quantification of easiness of folding by hand, together with the durability of the folded outcome, was the central theme. Lastly, a numerical mechanics simulation was performed in order to further analyse the deformation of the Miura-ori structure during the folding process. The simulation compared two different paperboards folded in different directions relative to orthotropic material axes – following the fold line orientation in actual Miura-ori tessellation pattern. In addition, the effect of pre-cut on folding response was determined. The main objective of mechanics simulations was to assess whether the numerical mechanics can capture the differences in qualitative folding experience.

2. Experimental

2.1. *Materials*

Commercial paperboard and speciality paper samples were used as test materials. The samples included white linerboard (four samples, Liner A-D), folding boxboard (FBB) (two samples, FBB A-B), laminated liquid packaging board (13 samples, LAM A-M), cup stock (four samples, Cup A-D) and sack paper (two samples, special A-B), grades that all are commonly known paper and board grades. A general introduction of the investigated paper and board grades are given in Appendix 1. Grammage and thickness of the investigated paper and board samples varied from 126 to 428 g/m^2 and from 135 to 547 μm, respectively. In addition to the paper and board samples, an EnDURO Ice 130 paper sample was used as a reference (IcePaper). Ice 130 paper is a paper-film-paper laminate polymer material with a grammage and thickness 120 g/m^2 and 220 μm, respectively.

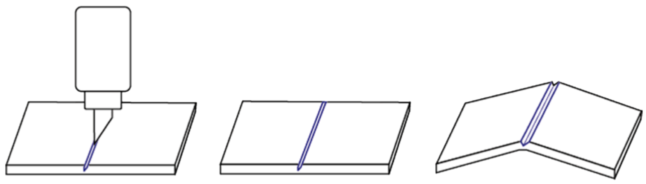

Fig. 2. Schematic presentation of kiss-cutting the Miura-ori tessellation pattern and folding of the samples.

Table 1
Qualitative folding parameters.

Durability	Foldability	Buckling	Bending	Performance
0: Unbroken	0: Very good	0: None	0: None	0–1: Good
1: Broken	1: Good	1: Moderate	1: Moderate	2–4: Moderate
	2: Moderate	2: Large	2: Large	5–7: Poor

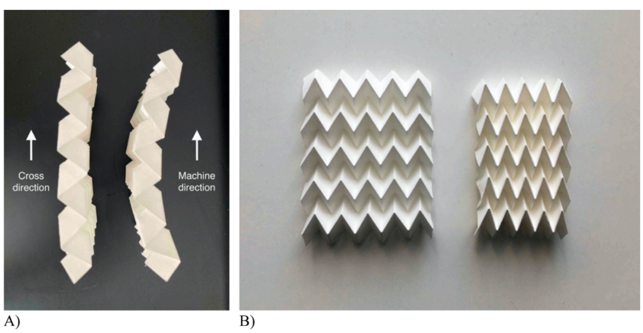

Fig. 3. A) Miura-ori folded perpendicular (CD) vs. parallel to machine direction (MD), B) Miura-ori in two different 200 g/m^2 materials (photos by Miia Palmu).

2.2. Methods

2.2.1. A pattern for comparison of materials

To make the comparison between material samples and their behaviour under folding, the simplest possible pattern was used in this work (shown in Fig. 1). The pattern belongs to the so called Miura-ori family of tessellations. The crease pattern is composed of parallelograms arranged in rows and columns separated by collinear lines in one direction and zigzag lines in the other direction.

Folding performance of the materials was tested using 118 mm x 150 mm pieces that were cut from each sample. The characteristic dimensions for a parallelogram in the Miura-ori pattern were chosen to be 14 mm for the major fold, 15.5 mm for the minor fold and roughly 60° for the acute angle. These measurements give 10 × 10-unit pieces that are 'customary', providing deployable structure where units do neither overlap nor stay far apart in the fully folded form. These extremal phenomena would take place if the acute angle would be closer to 90° or less than 30° respectively. The chosen parameters also give a pattern that is relatively convenient to fold by hand, after the folds are kiss-cut by a vinyl cutter. The dimensions of the used vinyl cutter also influenced these choices in practice.

2.2.2. Folding method

The materials were tested using hand-folded samples with a Miura-ori pattern. The folding pattern was first cut on the surface with a vinyl cutter, and then sheets were folded manually in order to determine folding endurance. The pattern was cut on the material by using a vinyl cutter Roland Camm-1 GS24 that cuts with a selected pressure/speed ratio. Used operation parameters of the vinyl cutter were as follows: Cutting force 515 mN (cutting force 60 gf and pen force −0.5 gf), moving speed of the blade 150 mm/s, blade offset depth 0.25 mm. The parameters were defined by testing several value combinations and by evaluating the cutting result and foldability. Depth of the actual cut was not measured from the samples.

Instead of cutting through the material, only cuts on the surface of the material were made, and thus a kind of gentle kiss-cut was created (see Fig. 2). These gentle kiss-cuts were made only on one side of the material, leaving the other side intact. The pattern used was a Miura-ori, with a 60-degree angle between the diagonal (mountain or valley) fold and the upright vertical fold. The samples were folded manually by gently pushing each mountain and valley fold in place first and then pressing the folds tightly together according to the pattern's vertical direction. Based on hand-folding experience, delamination of folding lines in samples was considered disadvantageous and therefore creasing was not applied.

2.2.3. Qualitative ranking of board folding performance

Folded samples were evaluated using qualitative ranking of folding performance and measuring the dimensional changes by the folding. The qualitative folding parameters were defined as follows: Durability described cracking of folding, foldability described easiness of folding, buckling described crumpling of individual faces, and bending described curving of the whole folded structure. Numerical ranking for the qualitative folding parameters is presented in Table 1. The Performance parameter was used for ranking the general behaviour of the samples. The smaller the sum (performance), the better the material is suitable to the considered folding purposes. Some reference pictures for rating durability and buckling are provided in Appendix 3 (Fig. 9). Samples were considered bending moderately if the distortion was less than 5 mm. The foldability categories were based on the folders hands-on experience. Folding performance was evaluated using linear regression between the mechanical properties and quantitative folding parameters. The regression was used in this study as an indicative tool and not for ranking the materials. The buckling defect defined in this study was considered not to originate from moisture [24,25], but compressive forces [26]. Dias reviewed buckling of thin sheets in nature and applications and explained the origins of thin elastic sheet using mathematical models [27].

In addition, dimensional changes by the folding were measured in length and width direction of the samples. The direction of the folds varied in relation to the machine direction (MD) and cross-machine direction (CD) with the investigated the samples. The direction of straight folding was along the MD for the 13 laminated (LAM A-M) samples having grammage 190–390 g/m^2. The direction of straight folding was along the CD for the cup stock, liner board, FBB, sack paper and "Icepaper" samples having grammage 120–270 g/m^2.

2.2.4. Mechanical tests of the materials

The grammage of the board samples was determined according to ISO 536:1995. The thickness was determined according to ISO 534:1998, and the density was determined based on the measured values of the grammage and thickness. The tensile strength and the strain at the break were determined with a Lloyd tensile tester, in accordance with ISO 5270:1998. Paper samples were conditioned, and mechanical testing of the samples took place at a temperature of 23 °C and at 50% relative humidity. Tensile tests of board samples were conducted in the machine direction (MD) and cross-machine direction (CD).

3. Results

3.1. Folding performance of boards

The machine direction (MD, in which statistically most of the pulp fibres are orientated in paper) was chosen for most of the samples to be aligned with the minor folds. This had a slightly distinguishable consequence that some thicker materials seemed to admit the major folds

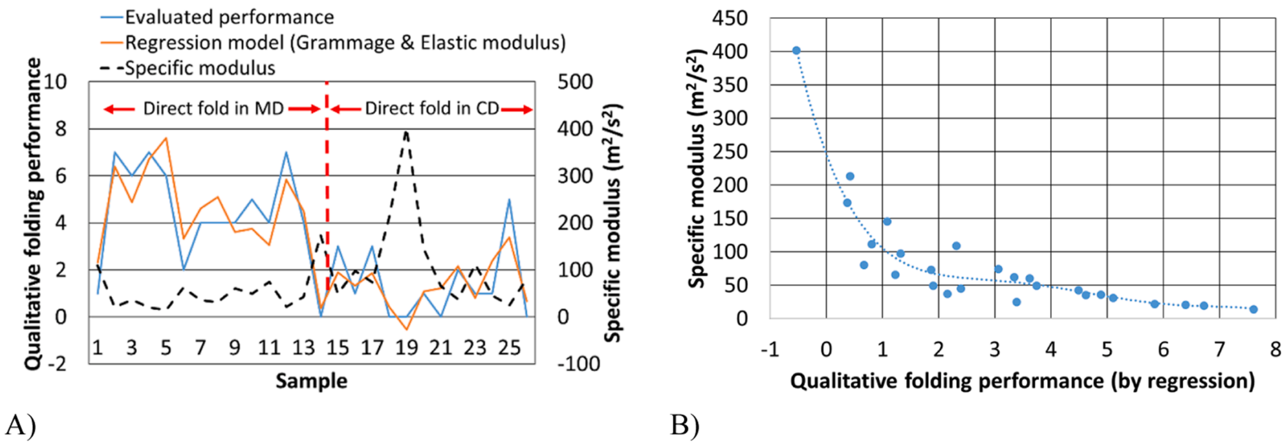

Fig. 4. A) Qualitative folding performance of the studied samples. Evaluated performance and linear regression for performance (applying direction of straight folding, grammage and elastic modulus in the direction of straight folding of the samples). B) Specific modulus as a function of qualitative folding performance given by the linear regression model.

Table 2
Correlation (R^2) of linear regression between the qualitative folding performance and measured mechanical properties of samples.

Direction of material property	Grammage	Thickness	Tensile strength	Tensile stiffness	Elastic modulus	Energy to break
CD	0.82	0.81	0.64	0.65	0.15	0.50
MD	0.82	0.81	0.72	0.50	0.10	0.19
As straight fold	0.82	0.81	0.71	0.66	0.13	0.53

Table 3
Correlation (R^2) of linear regression between grammage and qualitative folding parameters of the samples.

Direction of straight fold	Durability	Foldability	Buckling	Bending
CD and MD data combined	0.48	0.71	0.68	0.42

transversal to the machine direction (cross-machine direction, CD) more easily. Accordingly, changing the minor folds to the CD made the straight lines a bit easier to fold for the same materials. As the orientation turned out to be insignificant for the Miura-ori pattern applied to the thicknesses relevant for our purposes, the comparison was based on this chosen orientation. However, one phenomenon was observed that might be related to fibre orientation or to the multi-layer structures. Namely, some samples were slightly bending along the minor folds

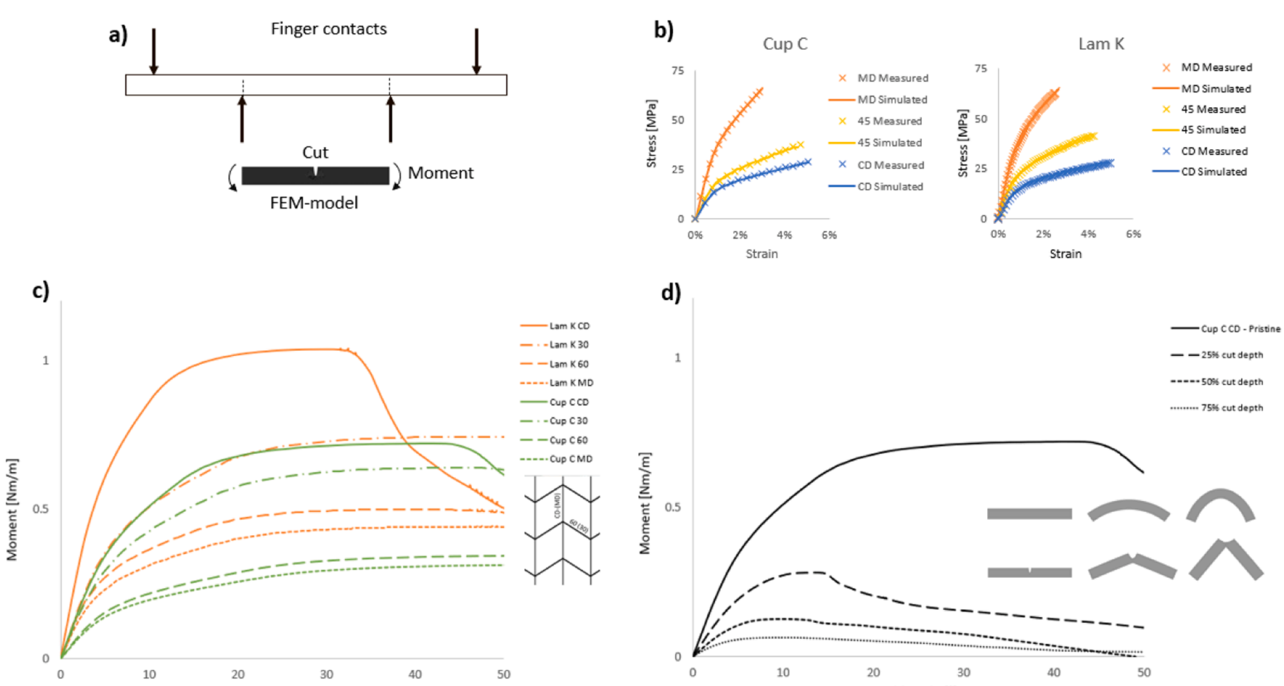

Fig. 5. A) Simulation setup; B) Material model calibration; C) Simulated response (moment versus fold angle) for fold lines with different material orientations; D) Effect of pre-cut depth.

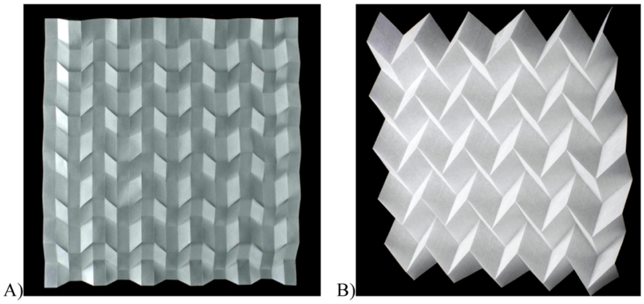

Fig. 6. Miura-ori variations (figures by Anne Kinnunen).

Fig. 7. Semi-folded Miura-ori prototype patterns by Miia Palmu.

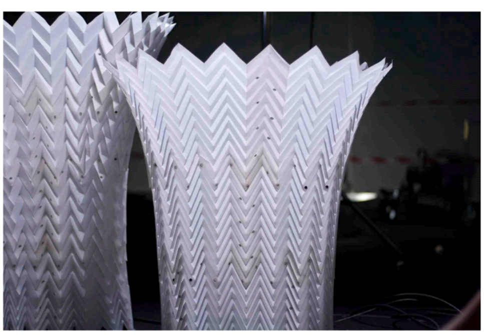

Fig. 8. EnDURO Ice 130 Miura-ori construction, Photo: Kilian Kottmeier.

parallel to the MD in the folded state. Especially, those materials that had an observable coating on one side only were bending more to the other direction than those with identical lining on both sides. Fig. 3A show the difference in bending for one of the 200 g/m^2 samples. As bending also took place in some homogeneous samples, it can be assumed, that this can also have to do with other fabrication details of paperboards that were beyond our studies [28]. The folded state of the samples varied greatly, depending not only on the thickness of the paper. Fig. 3B presents two different 200 g/m^2 samples: the material on the right-hand side keeps its folded state sharper.

Table 4 in Appendix 2 presents grammage, thickness and qualitative folding performance of the board samples. Table 5 in Appendix 2 presents measured tensile strength, energy to the break, strain at the break and tensile stiffness in the CD and MD of the investigated samples. Furthermore, Table 5 in Appendix 2 presents the estimated Poisson's ratio and shear stiffness of the samples. The direction of the straight folding was MD for the Lam A-M (LBP) samples and CD for all other samples. In this study, strength properties of materials were reported in consistent with practises commonly applied in paper engineering [29]. However, they are not following traditional mechanics of materials due to the difficulty to measure the thickness of fibre network materials.

The direction of straight folding seemed to correlate with the quality of folding (see Fig. 4). The quality was better when the direction was CD. However, it remained unclear as to whether this was due to the orientation direction or some structural properties (e.g., laminate layer) of the board samples. The grammage was clearly higher in the samples with a straight fold in MD, which may also have influenced the result. The linear regression presented in Fig. 4 was based on three parameters: the direction of straight folding (values either 0 or 1), grammage and elastic modulus in the direction of the straight folding of the samples. With the chosen parameters, the linear regression gave the best estimate for the evaluated performance. As the linear regression was not predicting a physical quantity, it was allowed also to have negative values indicating in this case very good performance.

The correlation between the qualitative folding performance with grammage was 0.82 (see Table 2) and almost the same with thickness, which were the best observed correlations. Taking into account the other parameters did improve the correlation between the qualitative folding performance and mechanical properties of the samples only marginally. However, in addition to grammage, elastic modulus was observed as a useful parameter for the regression shown in Fig. 4A. Interestingly, application of elastic modulus, instead of tensile stiffness, led to a significantly lower correlation, which may indirectly relate to the absence of thickness in tensile stiffness compared to elastic modulus. On the other hand, the elastic modulus alone had poor correlation with the qualitative folding performance. The application of LARS (Least-angle regression) did not improve correlations compared to the linear regression.

As the ratio of elastic modulus and grammage was evaluated using the measured values, a higher ratio seemed to indicate better qualitative folding performance. Furthermore, the ratio of elastic modulus and density, known also as specific modulus, indicated for qualitative folding performance (as shown in Fig. 4B). By independently increasing either the elastic modulus or decreasing density (or grammage), the ratio can be increased, and the qualitative folding performance is expected to improve.

Foldability and buckling are the folding parameters that have the highest correlations of linear regression with grammage (see Table 3). The durability of the folds is most probably related to the fibre composition, material structure and performance of the surface layer, and especially corners are critical locations for fractures [30]. Bending of the whole structure may be related to in-plane tensions and their release of the multilayer structures [28]. Beex and Peerlings sums up nicely according to literature that the quality of the folds depends on two converting processes: the manufacture of fold lines (creasing) and the subsequent folding [31]. A good crease contains some delamination, initiated during creasing, to reduce the bending stiffness and to prevent the board from breaking during folding. They showed also by simulations that the top layer of high grammage board is most likely to fail at the creasing zone. Between foldability (easiness of folding) and grammage (thickness), inverse relation can be considered obvious. According to Coffin and Nygårds [21], thickness is of prime importance for crease severity, but they also point out the lack of experimental data on different paperboard grades related to creasing and folding. In our study creasing was not performed, but instead kiss-cutting on the one side. However, thickness and local elongation were most likely important for the folding performance. Bending is related to material parameters, layered structure of the board and naturally linked to dimensions and dimensional ratios of the tessellation [28]. Buckling of the individual unit faces is, in this study, a parameter that needs to be explained in more detail, which is presented in the next section.

3.2. Mechanics simulation

The aim of the mechanics simulations was to gain further insight on (1) how orthotropic paper folds in different directions and (2) how the

Table 4
Grammage, thickness and qualitative folding performance of the samples.

Sample	Grammage, g/m^2	Thickness, µm	Direction of straight fold	Durability	Foldability	Buckling	Bending	Qualitative folding performance
Lam A	185	250	MD	0	0	1	0	1
Lam B	372	477	MD	1	2	2	2	7
Lam C	300	368	MD	1	2	2	1	6
Lam D	387	473	MD	1	2	2	2	7
Lam E	428	547	MD	1	2	2	1	6
Lam F	231	321	MD	0	1	0	1	2
Lam G	285	386	MD	1	2	1	0	4
Lam H	310	411	MD	1	2	1	0	4
Lam I	241	301	MD	1	2	1	0	4
Lam J	245	344	MD	1	1	1	2	5
Lam K	218	291	MD	1	1	1	1	4
Lam L	342	480	MD	1	2	2	2	7
Lam M	284	355	MD	0	2	1	1	4
Cup A	203	275	CD	0	0	1	0	1
Cup B	226	311	CD	1	1	1	0	3
Cup C	164	246	CD	0	0	1	0	1
Cup D	229	333	CD	0	0	1	0	1
FBB A	207	355	CD	1	1	1	0	3
FBB B	273	436	CD	1	2	2	0	5
Liner A	157	205	CD	0	0	0	0	0
Liner B	175	180	CD	0	0	0	0	0
Liner C	135	135	CD	0	0	0	0	0
Liner D	200	210	CD	0	0	1	0	1
Special A	150	195	CD	0	0	0	0	0
Special B	196	258	CD	0	1	0	1	2
IcePaper	126	220	CD	0	0	0	0	0

Table 5
Some measured and estimated mechanical properties of the samples.

Sample	Tensile strength, N/m		Energy to break, J/m^2		Strain at break, %		Tensile stiffness, kN/m		Poisson's ratio *		Shear stiffness, kN/m *
	CD	MD	CD	MD	CD	MD	CD	MD	CD	MD	CD/MD
Lam A	7.5	15.6	377	328	7.2	3.3	596	1260	0.041	0.338	263
Lam B	16.1	33.8	818	946	7.1	4.6	1040	1770	0.040	0.339	318
Lam C	11.2	23.7	578	578	6.9	3.9	789	1480	0.098	0.340	156
Lam D	15.5	30.1	790	824	7	4.5	1060	1720	0.040	0.330	305
Lam E	16.6	33.2	756	956	6.4	4.7	1090	1840	0.046	0.333	367
Lam F	10.2	21.5	520	489	7	3.6	743	1490	0.054	0.339	151
Lam G	12.9	22.4	624	517	6.6	3.7	925	1500	0.033	0.317	267
Lam H	12.4	23.4	604	554	6.7	3.7	899	1620	0.041	0.327	335
Lam I	10.2	18.5	585	413	7.8	3.5	706	1310	0.105	0.322	209
Lam J	10.5	20.3	525	431	6.7	3.4	772	1440	0.043	0.330	410
Lam K	9	19.2	473	400	7.1	3.3	661	1370	0.041	0.340	368
Lam L	11.3	29.1	482	678	5.9	3.9	813	1710	0.046	0.360	372
Lam M	12.8	25.3	646	614	7.2	3.9	843	1520	0.035	0.332	400
Cup A	8.8	20.5	417	432	6.3	3.3	749	1500	0.043	0.350	407
Cup B	8	20.6	366	389	6.1	3	664	1610	0.049	0.360	335
Cup C	5.1	11.8	222	181	5.9	2.4	429	1110	0.046	0.349	408
Cup D	7.4	12.4	218	190	4.1	2.4	657	1140	0.048	0.313	438
FBB A	6	13.5	177	211	4	2.4	581	1290	0.039	0.346	335
FBB B	7.3	15.3	253	273	4.7	2.7	645	1310	0.046	0.338	418
Liner A	5.2	12.8	188	303	4.7	3.4	542	1150	0.053	0.356	455
Liner B	7.5	15	325	294	6.4	3.1	558	1210	0.048	0.333	467
Liner C	5.8	11.2	259	186	6.6	2.6	467	988	0.042	0.329	355
Liner D	9.9	15.5	422	304	6.4	3.1	698	1290	0.053	0.305	522
Special A	5.3	11.4	420	1090	10.7	18.5	431	377	0.050	0.341	525
Special B	7.9	13.9	564	1210	9.9	16.2	599	490	0.041	0.319	456
IcePaper	4.5	6.7	312	128	8.3	3	309	492	0.051	0.299	548

* *Eqs* $G_{12} = \sqrt{E_1 E_2}/2(1+\sqrt{\nu_{12}\nu_{21}})$ *and* $\sqrt{\nu_{12}\nu_{21}} = 0.293$ *from [22] used, where subscripts '1' and '2' refer to MD and CD, respectively.*

cut depth affects the folding response. Due to orthotropy, the folding behaviour in Miura-ori tessellation depends on the orientation of the fold. Since accurate prescription of the cut depth in the fold lines is not a trivial task, the mechanics simulations were performed to gain further insight on the sensitivity of the cut depth on the folding response. In addition, it was studied whether the numerical simulations can explain some of the qualitative differences in folding experience.

The FEM-simulation setup was based on the idea how a single fold line can be seen folded by using fingers. This corresponds to the so-called four-point beam bending experiment leading to pure bending near the fold region (Fig. 5A). The direction of the fold (MD, 30, 60, CD) was varied based on an example Miura-ori pattern, where '30′ and '60′ denote angular deviation from CD in degrees. The cut was modelled by using an ellipsoidal shape, and to vary the cut depth this shape was translated in the thickness direction of the cross section of the paper. The inherit anisotropic behaviour of paper was modelled in a simplified

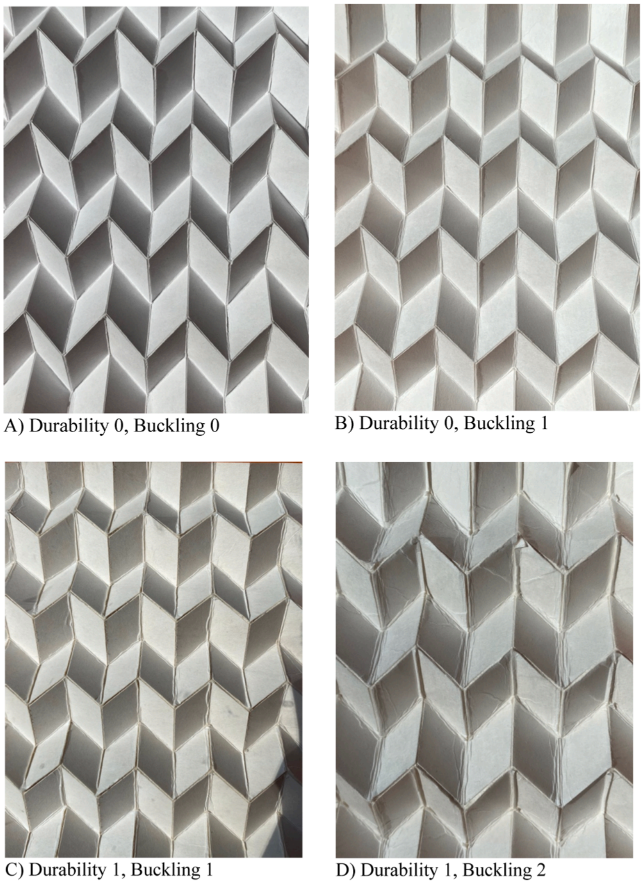

Fig. 9. Reference pictures for Durability (0/1) and Buckling (0/1/2).

manner by adopting a linearly elastic orthotropic material model [32] extended with the quadratic Hill's yield criterion, nonlinear isotropic hardening, and associated flow rule – following the source [33] without tension compression separation.

The boards Cup C (folding performance 1) and Lam K (folding performance 4) were chosen for further inspection, with Lam K having a slightly larger thickness. Cup C and Lam K were folded in such a way that the straight folds in Miura-pattern align with CD- and MD-orientation of the sheet, respectively (see Table 4). This indicates that the most interesting fold directions for Cup C are 'CD' and '60′, and for Lam K 'MD' and '30′. These boards were calibrated for the aforementioned material model (Fig. 5B) according to extension experiments in different material directions (MD, 45, CD).

According to the results (Fig. 5C), one can conclude that the fold line orientation has a significant effect on the response (moment versus fold angle). In general, it is noted that Cup C curves are below Lam K curves which naturally can contribute to the easier foldability of Cup C (0 vs 1). To compare the folding responses of Cup C and Lam K considering the fold line orientation in Miura-pattern, one can follow curves 'Lam K 30′, 'Lam K MD', 'Cup C CD' and 'Cup C 60′. In particular, the difference is most obvious when comparing the folding moments over the zigzag fold lines, where 'Lam K 30′ has clearly higher folding moment compared to 'Cup C 60′. On the contrary, along the straight lines, 'Lam K MD' has significantly lower folding moment compared to 'Cup C CD'. To summarize above, when straight fold lines are orientated in CD, zigzag folds need less folding moment compared to straight folds – And opposite when the straight fold lines are orientated in MD. This might not necessary itself explain why folding is qualitatively easier in CD orientation, but clearly shows the difference in folding mechanics of Miura-pattern formed in different directions with respect to paper orthotropy. In addition, this difference can lead to different forms of parallelogram shape distortion in the folding process and might even contribute to the global curvature formation ('bending').

By varying the cut depth (Fig. 5D), it was observed that even a small cut (25%) reduces the required folding moment significantly, around 60%. In addition, adding a cut causes the maximum moment to occur with relatively small, under 20° folding angle. Increasing the cut depth further did similarly drop the maximum moment and corresponding fold angle but the effect was slightly milder. To extend above discussion, it is noted that the folding behaviour will be essentially different if the folding is performed in opposite direction (closing the cut).

4. Discussion

The alternating mountain and valley zigzags of Miura-ori family support a certain functionality that is present in nature, for example, in the leaves of some plants as well as in fashion, in early sixteenth century ruffs [34]. It is widely known amongst fashion designers under the name V-pleat, due to its millennia-long history related to pleating techniques and their derivatives. A modern systematic treatment about traditional pleat patterns up to contemporary design modifications can be found in [7]. The shape was also well known in architecture via Josef Albers and his Bauhaus students in the 1920s [35]. However, the name Miura-ori, coined after the work of Japanese astrophysicist Koryo Miura in 1970, was introduced originally as 'the developable double corrugation surface' [36–37]. The Miura map fold, herringbone or chevron patterns are also names for this structure. amongst origami tessellation patterns, it can be considered to belong to the class of corrugations, in a sense that it has no triple or more layers, leaving the entire original surface of the paper visible also in the folded state.

Today, the Miura-ori family is widely used in applied origami, and it is the most studied origami tessellation pattern due to its simplicity and remarkable properties. For example, this pattern can be deformed between the folded and unfolded states without bending any of the faces between the folds. This particular property is called rigid foldability, being suitable to a wide range of materials, like aluminium, copper, stainless steel, composites, or plastics. In the case of a perfectly stiff material, the entire pattern could fold and unfold in a single motion. This feature means that the pattern has one single degree of freedom. A theoretically interesting problem, that also gives insight into the practical challenges of folding, is related to the number of ways to assign mountains and valleys to the given crease pattern so that it folds flat. For Miura-ori, partial results and connections to colouring problems have been found [38]. Different variations of Miura-ori, like the one in Fig. 6, are potential in sandwich structures, for example, in aircraft engineering [39,40], where strength and lightness are the main criteria. A thorough treatment of Miura-ori can be found in Lang's excellent book [41] and references therein. Following Lang, in Fig. 1A, we call the zigzag folds of Miura-ori the major folds of the pattern, and the alternating-parity folds, formed from the vertical straight lines in the crease pattern, are the minor folds. This doubly periodic pattern is completely fixed after the choice of the length of the minor and major folds and the acute angle between them in a parallelogram.

Despite, there are some great examples of practical folded outcomes applying Miura-ori family, there is not a single large volume industrial production line producing the pattern for different applications. At the same time these patterns can be lightweight, flat foldable, highly transformable, and flexible. There are clearly industrial challenges and material limitations that may give directions to the design process. On the other hand, commercial potential of the industrial production is estimated to be high due to flexibility of the artefacts. Specifically, new type of cushioning and wrapping structures to be produced using a roll-to-roll process would be commercially interesting. This study intended to increase scientific understanding of paper physics and mechanical interactions involved in Miura-ori tessellations, that have strong connection to design and mathematics. Moreover, the folding and mechanical behaviour of the novel designs was addressed experimentally.

This study thus aimed to increase basic knowledge by combining experimental results and demonstrative functional 3D examples and structures that could be used in developing processes made from cellulose based materials.

The use of Miura-ori in our comparison tests was also justified since, via surprisingly few modifications, this pattern yields flexible families for different purposes. For example, any intrinsic (Gaussian) curvature of a surface can be approximated via Miura-ori generalisations that use quadrilateral unit cells, which are not necessarily congruent, but vary slowly in shape across the tessellation along minor folds [42]. It is not necessary for these patterns to be flat foldable in general, but in some cases, it is possible for them to be further approximated with periodic Miura-ori patterns [43]. Varying the spacing between minor folds regularly or unevenly leads to the major fold repetitions deviating from the horizontal direction. This will cause less overlapping in the flat-folded state and produce endless visual variations in the surface of a semi-folded pattern [7]. Some prototype patterns are shown in Fig. 7.

Under the folding operation of this study, the samples (paperboard) were subjected to high mechanical stress that was tolerated in varying ways. Excessive stretching or compression of the sheet led, in the worst case, to cracks. In the case of coated materials, the outer coating was bursting out due to kiss-cut. Thickness of the coating layer was greatly enhancing the coating cracking. In general, the paperboard orientation direction had a strong influence on the folds. Folding in the CD cracked the fibres in most of the studied cases. However, for some materials, such as Special A, the difference was almost indistinguishable, as the folds were equally even, clean and flat in both directions (MD and CD). Instead, a rather dramatic bending effect of the whole sheet took place (Fig. 3A) when the minor folds were made parallel to the machine direction.

The numerical mechanics simulations focused on the moment-fold angle- response of single folds in Miura-ori-pattern taking into account the fact that folding mechanics depend on the direction of fold line in an orthotropic paper sheet. It is recognized that paper folding response has a complex physical nature [8], including large permanent strains and damage formation (cracks) with time-dependant relaxation behaviour. Especially, modelling the response of pre-cutted fold depends on whether the cut will close or open during the folding process. In addition, the Miura-ori pattern unit has four fold lines meeting at one intersection point. These intersection points are susceptible for fracture initiation, and their accurate simulation would require an advanced 3-dimensional nonlinear analysis by using experimentally calibrated parameters. However, the apparent orthotropy of paper is a driving property on the moment-fold angle- response [21], indicating that at least the early stage of the folding process was captured with a reasonable accuracy for comparison purposes.

All the samples were cut from one side only, which probably had some influence on the results. This approach was chosen mainly due to practical reasons as aligning cuts only to mountains would have required more sophisticated machinery. On the other hand, it is also meaningful to study cutting on one side only as cutting different coatings or colours of cardboards can have an essential impact on design and aesthetics [44]. The cut mountain folds were resisting the treatment best in general. In the best cases (Cup A, … Liner A-D, Special A, B) the valley folds created kind of clean hinges on the cut side, but in many cases (LAM B-H) the cutting bulge was uneven and bursting. Additionally, a lamination layer started to peel off from the faces bound by cut edges in samples (LAM B-D). In those materials, the cut mountains split into layers, where the inner material was emerging between sharp edges connected to the faces. The uncut valley on the other side also showed up imprecisely. In the valleys, the cut became completely invisible, leaving the fold unsharp. However, the uncut valley on the other side showed up in these cases remained even and beautiful.

For the optimal result in most of the cases, scoring should have been made on both sides, adjusting onto the mountain creases only. However, there were cases (LAM C-E, LAM L-M) where pre-cut creases that were folded into mountains on one side turned into sloppy and bubbling valleys on the other side. Some materials (Liner B) folded ideally for Miura-ori, but did not tolerate twisting at all. The same material (Liner B) started to break from the creases after some time, possibly due to interaction with air, humidity, or some other reason. The observations related to bending and dimensional changes clearly need further investigation.

As the ratio of elastic modulus and density (or grammage) was evaluated using the measured values, a higher ratio seemed to indicate better qualitative folding performance (as shown in Fig. 4). A high specific modulus is known to be beneficial for applications in which minimal structural weight is required. Those applications are also known as “stiffness-driven” structures. Physical SI units of the ratio of elastic modulus and density is length squared per time squared. The observed relation between specific modulus and qualitative folding performance may not be a coincidence and it may be worth of further investigation.

5. Conclusions

The main objective of this investigation was to find a quantitative relation between Miura-ori foldability and the mechanical properties of paperboards. Twenty-six different commercial paperboard and speciality paper samples were used as the test materials. The folded samples were evaluated using qualitative ranking of folding performance and measuring the tensile properties of the materials. Four qualitative folding parameters were defined and a numerical ranking was given for them. The qualitative folding parameters were namely the following: durability, foldability, buckling and bending. The performance (sum) parameter was used for ranking the general behaviour of the samples.

As a key result of this investigation, the specific modulus of the materials seems to correlate with the qualitative folding performance of the investigated materials. Therefore, the investigated Miura-ori foldings can be assumed to be “stiffness-driven” structures. The mechanics simulation compared two different paperboards folded in different directions relative to orthotropic material axes – following the fold line orientation in actual Miura-ori tessellation pattern. Simulations were able to identify that folding mechanics depend on the choice of orthotropic material orientation with respect to tessellation pattern, and therefore contributes to the qualitative folding experience. More advanced simulations are recommended to capture the other physical processes including the fold intersection point effect, parallelogram deformation and varying properties through the thickness of board. This helps to optimise the whole folding process and choose the best materials and manufacturing methods.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix 1

Linerboard: Linerboards are usually used for sides of corrugated fibreboard. The grammage range for linerboard is from 125 to 350 g/m^2. Bleached chemical pulp is used in the top ply for white linerboard. In this case, good formation is targeted; hardwood pulp is typically the main component of the top ply of white linerboard. Filler is often used in the top ply to improve opacity and, therefore, the appearance of the top side. The base ply of linerboard is typically unbleached chemical softwood pulp.

Folding boxboard (FBB): FBB is the most common paperboard grade, and it is used for demanding packaging. FBB can bend without fracture and typically consists of multiple layers of chemical and mechanical pulp. The grammage range for folding boxboard is relatively large, from 160 to 450 g/m^2. Top and back plies typically are bleached chemical pulp, whereas middle ply or plies may be mechanical or chemi-thermomechanical pulp (CTMP) and machine broke. The top side is usually pigment coated and the back side can also be coated.

Liquid packaging board (LPB) and **cup stock** are materials for meal and liquid packaging of trays, bowls, cups and plates made up of multiple chemical and mechanical pulp and polymer barrier layers. LPB is typically two-sided, low-density, polyethylene-coated paperboards. LPB can be made of unbleached kraft pulp, bleached chemical pulp or CTMP, and purity and cleanliness are very important for the materials.

Sack paper is a speciality paper grade, porous packaging material with high elasticity and high tear strength made of single-layer chemical kraft pulp. The grammage range for sack paper is from 80 to 180 g/m^2. A more detailed description for the common paper and board grades can be found, e.g., [45].

Ice paper can be used as a dust cover for documents, reference books, travel guides and encyclopaedias. It works particularly well in prototyping origami tessellations, as it is highly tear-resistant, safe, high-quality translucent material and has great dimensional stability, as well as excellent durability. It folds easily after scoring the creases with a vinyl or laser cutter. This material has been used in several student projects and exhibitions, like in the context of an exhibition, Sensual Mathematics [46] at Heureka, the Finnish Science Centre in 2017 (Fig. 8).

Appendix 2

Appendix 3

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