Li, Xinze; Al-Tous, Hanan; Hajri, Salah Eddine; Tirkkonen, Olav

**Covariance Difference of Arrival based Fingerprinting Localization**

*Published in:*
2023 IEEE 97th Vehicular Technology Conference, VTC 2023-Spring - Proceedings

*DOI:*
10.1109/VTC2023-Spring57618.2023.10200236

Published: 01/01/2023

*Document Version*
Peer reviewed version

*Please cite the original version:*

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.
Covariance Difference of Arrival based Fingerprinting Localization

Xinze Li\textsuperscript{1}, Hanan Al-Tous\textsuperscript{3}, Salah Eddine Hajri\textsuperscript{2} and Olav Tirkkonen\textsuperscript{1}

\textsuperscript{1}Department of Information and Communications Engineering, Aalto University, Finland
\textsuperscript{2}Huawei Technologies CO. LTD, Lund Research Center, Sweden

Email: \{xinze.li, hanan.al-tous, olav.tirkkonen\}@aalto.fi, salah.eddine.hajri@huawei.com

Abstract—We define covariance difference of arrival (CDOA) features derived from channel state information that can be used for machine learning based fingerprinting localization in non-line of sight (NLoS) conditions, with minimal communication overhead. Taking advantage of the uniqueness of the multipath channel between the base station (BS) and user equipment (UE) at different locations in the geographical region of interest. UEs compute CDOA features, consisting of pair-wise distances between covariance matrices of received signals from multiple BSs. Measured features are fed back to the network, where fingerprinting localization is performed. We consider both $k$-nearest neighbour and neural network localization, and investigate the trade-off between localization performance and communication overhead. In simulations of a NLoS 5G NR factory scenario with eight-antenna BSs, CDOA features provide a localization error less than 0.91 m in 80\% of the cases, as compared to 0.78 m for a benchmark method where UEs feed back complete measured covariance matrices to the network, and 1.36 m for power difference of arrival features. Comparing to complete covariance feedback, CDOA features reduce communication overhead by 98\%.

Index Terms—Wireless localization, channel state information, non-line of sight, covariance matrix.

I. INTRODUCTION

Accurate localization is expected to be critical for future wireless systems to fully support industrial internet of things (IIoT), autonomous systems and extended reality services, among others. Positioning methods based on the fifth-generation (5G) network are envisioned to benefit from the high density of base stations (BSs), very high signal bandwidths, multi-antenna beamforming, and communications in higher frequency ranges, such as millimeter waves. Recent studies on 5G-based localization show positioning accuracy is below one meter by means of network-based time of arrival (ToA) and angle of arrival (AoA). However, these levels of accuracy are mainly achievable in line-of-sight (LoS) conditions, and are difficult to reach in the more challenging non-line-of-sight (NLoS) conditions [1]–[3].

A promising solution, to achieve improved accuracy in NLoS conditions and highly cluttered multipath environments, is machine learning (ML) based positioning using fingerprinting channel state information (CSI) based features. For example, $k$-nearest neighbours (KNN) fingerprinting is widely used for this purpose [4]–[6]. The localization accuracy of KNN based fingerprinting highly depends on the used CSI feature and the feature distance.

The simplest form of CSI feature based fingerprinting utilizes the received signal strength (RSS). In [7], a comprehensive study of distance and similarity measures for fingerprinting localization based on RSS was considered. Applying more generic CSI features for fingerprinting provides better localization performance than only relying on RSS [4].

The difference between the measurements from two BSs is a common approach with geometric localization approaches. The minimum localization network based on the received signal strength difference (RSSD) is analyzed through an algebraic model and compared to the performance of AoA based method in [8]. The time-difference-of-arrival (TDoA) approach is used to handle the synchronization problem between the UE and the BSs in [9]. The power difference of arrival (PDOA) is used for LoS localization based on a known path loss model and a geometric localization approach [10].

Most fingerprinting-based localization methods are network based. The location is estimated at the network side, while the fingerprints can be estimated either at the network or the user equipment (UE) side. Utilizing the communication signal at the UE and using it for localization is attractive, as no additional measurements need to be conducted and no specific resources need to be reserved. In 5G NR, the available reference signals in the downlink for CSI measurements, i.e., channel state information reference signal (CSI-RS) and synchronization signal/PBCH block (SSB), can be used for localization and sensing. The CSI from multi-antenna BSs could provide better localization accuracy, as the multi-path components (MPCs) in the radio environment can be well captured (resolved in the angle domain). With multiple antennas and/or panels at the UE and/or gNB, the length of CSI-based features increases. The length of the CSI feature plays a key role in communication overhead between the UEs and the network. Therefore, devising a fingerprint with reduced length and able to capture the radio environment is important.

In this paper, we extend the concept of PDOA to distances between CSI features. We concentrate on Covariance Distance of Arrival (CDOA) features, i.e., distances between covariance CSI features measured at a UE from multiple BSs, because of its robustness to small scale fading. We investigate several linear algebraic distances to measure the distance between covariance CSI features from several BSs, and create the fingerprint based on the pairwise CSI distances. The length
of the fingerprint, depends only on the number of pairwise distances (i.e., the number of BSs) and does not scale with the number of antennas, at either the UE or BS. We devise a local topology preservation measure to evaluate the performance of the proposed feature and compare it to the performance of selected benchmark features. Additionally, we evaluate the localize performance using the root-mean-squared error (RMSE) based on WKNN and deep neural network (DNN) approaches. The remainder of this paper is organized as follows: In Section II, the system model is presented. In Section III, CSI features and distances are introduced. In Section IV, the localization framework is presented. In Section V, the ML algorithms are presented. In Section VI, the computational complexity and communication overhead are addressed. Simulation results are presented and discussed in Section VII. Finally, conclusions are drawn in Section VIII.

II. SYSTEM MODEL

We consider a communication system with \( B \) BSs, each BS is equipped with a uniform linear array (ULA) of \( M \) antennas. We also consider a UE with a single omnidirectional antenna. A cyclic prefixed orthogonal frequency-division multiplexing (CP-OFDM) waveform is considered, assuming that the cyclic prefix length is larger that the maximum delay spread. The channel matrix models the path loss as well as large scale and small scale multipath fading effects. The channel matrix between UE \( u \) and BS \( b \) over subcarrier \( n \) at time-sample \( s \) is \( \mathbf{R}_{u,b,n,s} \in \mathbb{C}^{M \times 1} \).

III. CSI FEATURES AND DISTANCES

For fingerprinting based localization, several CSI features have been considered [4], [5]. The selection of the CSI feature highly affects the resulting performance and the required computational complexity. Aiming to reduce the communication overhead between the UE and network, we devise a CSI feature that is not affected by the number of antennas at the BS. This feature is computed at the UE side and transmitted to the network. The UE applies matrix operation as well as vector and matrix norms to create this feature.

A. Covariance Matrix

The covariance matrix of the channel between UE \( u \) and BS \( b \), is denoted as \( \mathbf{R}_{u,b} \) and is given by,

\[
\mathbf{R}_{u,b} = \frac{1}{SN} \sum_{s=0}^{S-1} \sum_{n=0}^{N-1} \mathbf{r}_{u,b,n,s} \mathbf{r}_{u,b,n,s}^H,
\]

where \( S \) is the number of time samples and \( \mathbf{R}_{u,b} \in \mathbb{C}^{M \times M} \). In \( \mathbf{R}_{u,b} \), the effect of small scale fading is averaged.

B. CSI Distances

Multiple natural metrics can be derived from vector and matrix norms. The simplest distance between two matrices \( \mathbf{R} \) and \( \mathbf{R}' \) is the Euclidean distance

\[
d_{\text{Euc}}(\mathbf{R}, \mathbf{R}') = \left\| \mathbf{R} - \mathbf{R}' \right\|_F,
\]

which is invariant under unitary transformation including permutations.

The Euclidean distance, while simple, is not a natural metric on the space of covariance matrices, since covariance matrices live on a non-Euclidean space. Several covariance distances have been considered [11], [12]. The log-Euclidean metric is geodesic, rotation invariant, scale invariant, and inversion invariant [13], [14]. The log-Euclidean distance between two covariance matrices is computed as

\[
d_{\text{LogEuc}}(\mathbf{R}, \mathbf{R}') = \left\| \log(\mathbf{R}) - \log(\mathbf{R}') \right\|_F.
\]

The log indicates matrix logarithm. Generally, matrix logarithm of positive semidefinite matrix \( \mathbf{R} \) is calculated by performing a singular value decomposition (SVD), i.e., \( \mathbf{R} = \mathbf{U} \Sigma \mathbf{V}^H \), \( \Sigma \) is a diagonal matrix and \( \log(\mathbf{R}) = \mathbf{U} \log(\sigma_1, \ldots, \sigma_M) \mathbf{V}^H \) and \( \sigma_m > 0 \) for \( m = 1, \ldots, M' \) and \( M' \leq M \).

The Jensen-Bregman LogDet Divergence (JBLD) enjoys several desirable theoretical properties, at the same time is computationally less demanding compared to log-Euclidean [15]. The JBLD between two matrices is computed as

\[
d_{\text{JBLD}}(\mathbf{R}, \mathbf{R}') = \log \left[ \frac{1}{2} \left( \mathbf{R} + \mathbf{R}' \right) \right] - \frac{1}{2} \log \left\| \mathbf{R} \mathbf{R}'^H \right\|_F,
\]

where \( \left\| \cdot \right\| \) denotes the determinant. The square-root of JBLD is a metric.

IV. LOCALIZATION FRAMEWORK

It is worth mentioning that the focus is on localization in NLoS condition, since such an environment is more challenging. We consider fingerprint-based localization. Figure 1 shows the localization framework, the CSI feature is measured at the UE side and the location is estimated at the network side based on an ML approach. A direct ML-based approach employs a model to predict UE position from a set of features/fingerprints.
A data set consisting of $U$ fingerprints $\{f_u\}$ is created on the online phase, with the corresponding physical positions $\{p_u\}$. The created data set will be used on the online phase to estimate the position $p_{u'}$ of UE $u' \notin \{1, \ldots, U\}$ using $f_{u'}$ and an ML algorithm.

### A. Benchmark Fingerprints

We consider three types of raw features to be the benchmark features, namely, covariance matrices from $B$ BSs, received powers and pairwise PDOA from $B$ BSs. We consider both Euclidean and log-Euclidean distances for the benchmarks.

The covariance raw feature $f_{u}^{\text{Cov}}$ of UE $u$ is created from $B$ covariance matrices. As the channel covariance matrix is hermitian, the covariance raw feature consists of $BM^2$ real values. Similarly, the log covariance raw feature $f_{u}^{\text{Cov Log}}$ of UE $u$ is created form $B$ covariance matrices, applying matrix logarithm.

The received power raw feature $f_{u}^{\text{Pow}}$ of UE $u$ is created from $B$ received powers, this feature is of $B$ non-negative values. The received power raw feature $f_{u}^{\text{Pow Log}}$ of UE $u$ is created from $B$ received powers in dB scale.

The pairwise PDOA feature from $B$ BSs is

$$f_{u}^{\text{PDOA}} = \left[p_1 - p_2, \ldots, p_1 - p_j, \ldots, p_B - p_B\right]^T.$$  

The pairwise PDOA feature from $B$ BSs in the log scale is

$$f_{u}^{\text{PDOA Log}} = \left[\log(p_1) - \log(p_2), \ldots, \log(p_{B-1}) - \log(p_B)\right]^T.$$  

### B. Fingerprint based on Covariance Difference of Arrival

We create the fingerprint of UE $u$ from pair-wise distance from $B$ BSs. The CSI feature of UE $u$ is created based on all combinations of pair-wise CSI distances, i.e.,

$$f_{u}^{\text{CDOA}} = \left[d(R_{u,1}, R_{u,2}), \ldots, d(R_{u,B-1}, R_{u,B})\right]^T,$$

where $f_{u}^{\text{CDOA}} \in \mathbb{R}^{D \times 1}$ and $D$ is the feature dimension, which can be upper bounded by the binomial coefficients of choosing a subset of 2 elements from a set of $B$ elements. Any of the distances presented in the previous subsection (i.e., Euclidean, log Euclidean and JBLD) can be considered to create the CDOA feature.

### C. CSI Feature Evaluation

To shed light on the possible performance gain of the proposed CDOA feature, we devise a simple performance measure that can be used to evaluate the CSI feature before testing the localization accuracy. We call it the average neighbourhood preservation ratio (NPR).

For a given data set of $U$ UE positions and the corresponding CSI features. We use a distance measure, for example the Euclidean distance to compute the distance between two CSI features. For each data point in the data set, we find the set of $k$-nearest neighbours based on the feature distance and the set of $k$-nearest neighbours based on physical distance; the physical distance is also measured by the Euclidean distance. For UE $u$, the set of $k$-nearest neighbours based on feature distance is called $A_k^u$ and the set of $k$-nearest neighbours based on physical distance is called $B_k^u$. The average NPR is computed by considering the average number of common points in $A_k^u \cap B_k^u$ over all UEs in the data set, i.e., the average NPR of $k$-nearest neighbours is

$$\eta_k = \frac{\sum_u |A_k^u \cap B_k^u|}{U_k},$$

where $|$ denotes the cardinality, and $\eta_k \in [0, 1]$. The CSI feature with a larger $\eta_k$ is the better in neighbourhood preservation.

We consider a simple scenario with four BSs located at the corner of an area with dimensions $20 \times 20$ m, and each BS is equipped with a ULA of eight antennas. The UEs are on a grid with 0.4 m distances. The CSI for indoor NLoS condition is generated using the Quasi Deterministic Radio Channel Generator (QuaDRiGa) emulator [16]. Figure 2 shows the average NPR performance of different CSI features, including the covariance feature in the linear and log scales, the power in the linear and log scales, the PDOA in the linear and log scales, and the CDOA using Euclidean and Log-Euclidean distance. Compared with the linear scale features, the logarithm scale features have better NPR performances. The value of the average NPR of the covariance feature $f_{u}^{\text{Cov Log}}$ indicates that this feature has the richest neighbourhood preservation information. The CDOA can preserve half of the neighbors in physical space with small $k$. Among CDOA distances, JBLD and log-Euclidean show similar NPR performance and outperforms the Euclidean distance. Since, the logarithm scale shows better performance compared to the linear scale for the power, covariance and PDOA features, from now on only the features in logarithm scale are considered for localization.

### D. Illustrative Example

To illustrate that the pair-wise CSI distance from $B$ BSs can be used as CSI feature for localization, we consider a simple scenario where the UE is moving in a straight line as shown in Fig. 2.
in at the left top corner of Figure 3. Four BSs are considered, each with eight antennas. The CDOA feature with log Euclidean distance is considered. The feature distance is measured by the Euclidean distance with respect to the feature of the first location on the mobility path. Figure 3 shows that the feature distance increases with the physical distance. Note that the dependence is not one-to-one. The feature distance depends on changes in the radio channel, which sometimes conspire to reduce the feature distance while physical distance increases.

E. Feature Estimation based on 5G NR Signals

It is important to consider the effect of channel estimation error on localization performance. Most works in the literature assume that the CSI is correctly estimated and the effect of estimation error is ignored. We are interested in estimating the channel covariance matrix at the UE side. 5G NR considers several down-link (DL) reference signals such as CSI-RS, positioning reference signal (PRS) and SSB. According to 3GPP TS 38.214, 5G NR DL-CSI-RS is configurable from 5 MHz to 100 MHz and DL-PRS can be configured from 25 MHz to 100 MHz [17]. We estimate the CSI using the least squares (LS) algorithm based on DL-CSI-RS. 3GPP 38.211, Table 7.4.1.5.3-1, row 6 is used to map the reference signal (random sequence) to resource elements.

V. ML Algorithms

A. k-Nearest-Neighbours

KNN is the simplest ML algorithm for regression problems. To predict the position of a new sample using KNN, first the distance between a new feature vector and all other feature vectors in the training data set is calculated. Then the k-nearest neighbors are selected and the new sample’s position is determined by averaging over the k-nearest neighbors’ position value. We consider the Euclidean distance to compute the feature distance, other distances will be investigated in a future work.

Weighted-KNN (WKNN) uses a weight function such that the largest weight is given to the data point nearest the point of estimation and the smallest weight to the data point that is furthest away from the k-nearest neighbors. The use of the weights can improve the performance of KNN, however, the selection of the weight function may affect the performance. The most common weight functions are the exponential function and the Gaussian function with parameter $\tau$, the parameter of these functions needs to be tuned carefully.

B. Deep Neural Network

For the positioning problem, we train a DNN to infer the location of a UE from the CSI feature. The DNN takes in a CSI feature and passes it through several fully connected layers until the output layer. The structure of the DNN for different CSI features is summarized in Table I. The structure that gives the best performance in terms of the loss function is reported. In the table, the number under the hidden layers column represents the number of neurons in that layer. In all layers, the rectified linear unit (ReLU) is used as the activation function, except the last layer where linear activation is used to generate the location of the UE. All the hidden layers are fully connected and weights are initialized using the He algorithm [18]. Batch normalization (BN) is configured between hidden layers.

In the training phase, the loss function of the predicted value against the ground truth value is computed. The mean squared error (MSE) is considered as the loss function. The trainable parameters are then updated by back-propagation. The training of the DNN takes place during an off-line phase with Adam optimizer. We assume that a data set containing the CSI feature and corresponding ground truth location is available.

VI. Computational Complexity and Communication Overhead

We will discuss the computational complexity in terms of feature generation and the location estimation on the online phase. The communication overhead is discussed in terms of the feature length. The complexity of channel estimation based on LS is not discussed as it is common for both the devised CSI distance based feature and covariance CSI feature.

A. Feature Generation

In $n$-digit computation, the complexity of an addition operation and multiplication operation is $\mathcal{O}(n)$ and $\mathcal{O}(n^2)$, respectively. We neglect additions for simplicity.

The log of the matrix needs SVD operation whose computation complexity is $C_{\text{SVD}} = \mathcal{O}(4M^3)$ in the worst case [19].

We consider Euclidean, log-Euclidean and JBLD to generate CDOA feature. The cost of Frobenius norm is equivalent to the trace of multiple matrices. Considering the multiplexing times of

<table>
<thead>
<tr>
<th>Feature</th>
<th>Input Layer</th>
<th>Hidden Layer</th>
<th>Output Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cov Log</td>
<td>256</td>
<td>[256, 128, 64]</td>
<td>2</td>
</tr>
<tr>
<td>Pow Log</td>
<td>4</td>
<td>[16, 16, 8]</td>
<td>2</td>
</tr>
<tr>
<td>Pow DOA</td>
<td>6</td>
<td>[64, 32, 16]</td>
<td>2</td>
</tr>
<tr>
<td>Pow CDOA</td>
<td>6</td>
<td>[128, 64, 32, 16]</td>
<td>2</td>
</tr>
</tbody>
</table>

Fig. 3. A simple scenario showing the locations of 4 BSs and a UE in a mobility path at the left top corner of the figure. The feature distance based on the pair-wise log-Euclidean distance versus the physical distance. The distances are computed with respect to the first location on the mobility path.
two conjugate complex numbers is 2, the trace operation mainly focuses on the diagonal values with cost $C_{T}\approx O(2M^2)$. Therefore, the cost of CDOA feature with Euclidean distance is [15]:

$$C_{\text{CDOA-Euc}} = \left( \frac{B}{2} \right) C_{T}. \quad (9)$$

The cost of CDOA feature with log-Euclidean distance is

$$C_{\text{CDOA-LogEuc}} = B C_{\text{SVD}} + \left( \frac{B}{2} \right) C_{T}. \quad (10)$$

The cost of CDOA feature with JLDB is [15]

$$C_{\text{CDOA-JLDB}} = \left( \frac{B}{2} \right) C \left( \frac{1}{3} M^4 \right). \quad (11)$$

**B. Complexity of Position Estimation**

For WKNN, we consider the worst situation of complexity with $U$ data points in the data set and $k$ neighbors. The brute-force computational complexity is

$$C_{\text{WKNN}} = U C_d + C_w, \quad (12)$$

where $C_d$ is cost of distance calculation and $C_w$ is the cost of weights calculation. In the case of Euclidean distance and uniform weights, $C_d \approx O(l)$ and $C_w \approx O(2)$, where $l$ is the feature length.

For DNN, the computation depends on the number of layers and neurons. Let $N_i$ be the number of neurons in the $i$-th layer, and $I$ be the number of layers. The number of multiplications is

$$C_{\text{DNN}} = O \left( \sum_{i=1}^{I-1} N_i N_{i+1} \right). \quad (13)$$

**C. Communication Overhead**

The communication overhead is caused by the CSI feature length. For the covariance feature, the communication length in term of real values is $L_{\text{Cov}} = BM^2$, for the received power feature, the communication length is $L_{\text{Pow}} = B$. For the PDOA feature and the CDOA feature, the communication length is

$L_{\text{PDOA}} = L_{\text{CDOA}} = \left( \frac{B}{2} \right).$

**VII. SIMULATION**

The data set is split into 80% and 20% for training and testing, respectively. For DNN, 10% of the training is used for validation. We study the CDOA feature with the Log-Euclidean distance. We consider the received power feature in the log scale, the covariance in the log scale from four BSs, and PDOA in the log-scale as the benchmark features. We consider the following positioning approaches: WKNN with exponential weights, and DNN.

First, we consider the statistics of the distance error, i.e., the difference of the predicted and ground truth locations in the test data set. Figure 4 shows the cumulative distribution function (CDF) of distance error obtained for different features and ML algorithms using true CSI. Table III summarizes the accuracy of these CSI features and approaches. The localization based on covariance feature using DNN outperforms the power, PDOA and CDOA based features using both WKNN and DNN approaches. The covariance feature with DNN has 0.29 m RMSE which is approximately 70% of the distance between two nearest samples in the data set. The 80% percentile distance error performance for the CDOA feature is less than 1 m. Simulation results, show that using WKNN for the power feature outperforms DNN based approach. This may due that the feature is not rich enough to learn the radio environment, i.e., the signal angles at BS are not considered. The performance of the CDOA feature outperforms the power and PDOA features.

Figure 5 shows the CDF of the distance error based on DNN.
The power, covariance, PDOA, and CDOA feature are considered. DNN and WKNN (with exponential weight, \( \tau = 0.1 \) and \( k = 5 \)) are applied for localization.

The power, covariance, PDOA, and CDOA feature are considered based CSI feature for four BSs, each with 8 antennas. The received power based fingerprint is 0.13 m in terms of RMSE. The approach is 0.14 m in terms of RMSE. The gain compared to localization accuracy loss compared to covariance based positioning. Simulation results showed that covariance distance-based features proved to be quite beneficial for fingerprinting positioning. Using the CSI estimated based on CSI-RS. Clearly, the estimation of the CSI degrades the localization performance. The power, PDOA, and CDOA features are robust to channel estimation compared to the covariance feature.

**VIII. CONCLUSIONS**

In this paper, we have analyzed CSI-based feature design for fingerprinting based positioning. Considering the use case of interest, namely indoor localization, a low overhead CSI-based feature was determined and its impact on direct ML-based positioning was analyzed. Owing to its capability to capture, the distance in the spatial domain between two channel realizations, covariance distance-based features proved to be quite beneficial for fingerprinting positioning. Simulation results showed that the localization accuracy loss compared to covariance based approach is 0.14 m in terms of RMSE. The gain compared to received power based fingerprint is 0.13 m in terms of RMSE. The overhead is reduced by 98% compared to the covariance based CSI feature for four BSs, each with 8 antennas. The proposed covariance difference of arrival feature is robust to CSI estimation error. The localization based on CSI estimation results in 0.01 m RMSE for the covariance difference of arrival feature. The localization based on covariance estimation results in 0.14 m RMSE loss compared to localization based on true CSI.

**ACKNOWLEDGMENT**

This work was funded in part by Huawei Technologies Co., Ltd. We acknowledge the computational resources provided by the Aalto Science-IT project.

**REFERENCES**


