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Anisotropic hysteresis representation of steel sheets based on a vectorization technique applied to Jiles-Atherton model

F. Martin¹, R. Chen¹,², J. Taurines¹,³, and A. Belahcen¹, Senior, IEEE

¹Department of Electrical Engineering and Automation, Aalto University, FI-00076 Espoo, Finland
²State Key Laboratory of EERI, School of Electrical Engineering, Hebei University of Technology, Tianjin 300130, China
³Electrical Engineering Unit, Tampere University, FI-33014 Tampere, Finland

Hysteresis model of soft magnetic material enables an accurate estimation of the losses in electrical application. In rotating machine, the flux distribution can magnetize many directions of the iron core. The combination of the rotational and the alternating flux density appears between the slot and the yoke region. Even if the alternating flux density appears in the middle of each tooth, the magnetic properties varies with the magnetized direction with respect to the rolling direction. Whereas the losses strictly increases with the amplitude of an alternating flux density, the hysteresis loss presents a maximum under rotational flux density condition. Although this phenomenon is properly described by the coherent rotation of the domains at the microscopic scale, the macroscopic representation of the material should represent this specific feature. In this paper, a general vectorization technique is applied to the Jiles-Atherton model of hysteresis. The anisotropic anhysteresis behavior is derived from the analysis of a cubic crystal. This anhysteresis model drives the coherent rotation of the hysteresis model. Even if the model parameters are identified with alternating flux condition, the rotational flux density can be predicted properly.

Index Terms—Ferromagnetic materials, hysteresis, magnetic anisotropy.

I. INTRODUCTION

VECTOR model of hysteresis could be decomposed into two main categories. The first one consists in applying a vectorization technique to a scalar hysteresis model [1]. The second method consists in vectorizing the hysteresis operator [2]. Both vectorization techniques can be employed to represent accurately many symmetric cycles of hysteresis [3]. However, the consistent rotation of the magnetic domains at high field intensity requires a modification of the hysteron distribution with respect to the flux density amplitude [5], [6], [7], [8]. Similarly the vectorization technique requires an additional term to consider the consistent rotation at high field intensity [9]. However, the model parameters are identified from both alternating and rotational measurements. Recently, Chen et al. [10] introduces an energy-based hysteresis model which considers the coherent rotation of the domain orientations in both the anhysteresis anisotropic model and the energy-based hysteresis model. In this article, we analyze the same anisotropic vectorization technique as developed in [10]. The vectorization consists in defining an effective field from the definition of the single crystal energy. Whereas the energy based hysteresis model decomposes the applied magnetic field into a reversible and an irreversible part, we implement the vectorization into the Jiles-Atherton model [11] which decomposes the magnetization into a reversible and an irreversible part. However, the definition of the reversible magnetization needs to be extended for considering the coherent rotation of the magnetic domains. First, the model parameters are identified under alternating flux density condition with 4 different flux orientations. Then, the model is validated under rotating flux density condition. The physical description of the hysteresis phenomena can predict properly the consistent rotation.

II. VECTOR ANISOTROPIC HYSTERESIS MODEL

A. Macroscopic anisotropic energy

The magnetocrystalline anisotropy energy density \( W_a \) of a single crystal with cubic symmetry is generally expressed by:

\[
W_a = K_1 \gamma_1^2 \gamma_2^2 + K_2 \gamma_2^2 \gamma_3^2 + K_3 \gamma_3^2 \gamma_1^2 + K_2 \gamma_1^2 \gamma_2^2 \gamma_3^2. \tag{1}
\]

The component \( \gamma_i \) of the magnetization direction \( \gamma \) are expressed in the single crystal coordinate system. Three edges of the cube defined the orthogonal unit system of this crystal frame. The magnetization direction of the magnetic domains can be expressed in the macroscopic coordinate system where the rolling, the transverse, and the normal direction of the sheet forms an orthogonal unit system. The magnetization direction \( m \) is deduced from rotating the crystal frame to match with the macroscopic frame so that \( \tilde{m} = R \cdot \gamma \). In the case of in plane magnetization, the resulting anisotropic energy of a magnetic domain can be expressed with only 7 terms

\[
W_a(\tilde{m}) = K_{40} \tilde{m}_1^4 + K_{64} \tilde{m}_2^4 + K_{22} \tilde{m}_1^2 \tilde{m}_2^2 + K_{60} \tilde{m}_1^6 + K_{42} \tilde{m}_1^4 \tilde{m}_2^2 + K_{24} \tilde{m}_1^2 \tilde{m}_2^4 + K_{06} \tilde{m}_2^6. \tag{2}
\]

The general 3D model can also be derived by factorizing the terms in \( m_i^nm_j^p \) after applying the rotation in equation (1). Whereas the macroscopic behavior is the sum of the contributions of each magnetic domains in every grain, we assume a single equivalent grain can represent the anisotropic energy at the macroscopic level. However, the presence of the magnetic domains involves that the magnetization remains along the easy axis in the absence of external magnetic field. It results in a vanishing anisotropic energy term in every domain \( (W_a = 0) \). While increasing the amplitude of the field, the rotation of the magnetization orientation can be computed by the minimization of the single crystal energy for every domain. Although this tedious operation properly considers the
coherent rotation of the $N_{\text{dom}}$ magnetic domains, we model this phenomena by modulating the anisotropic energy $W_a$ with a function $\mathcal{F}$ which depends on the level of saturation of the material.

$$W_{\text{an}} = \sum_{n=1}^{N_{\text{dom}}} W_a(\mathbf{m}_n) \approx \mathcal{F} \left( \frac{M_{\text{an}}}{M_s} \right) W_a(\mathbf{m})$$

(3)

where $M_s$ is the saturation magnetization of the material, $M_{\text{an}}$ is the amplitude of the anhysteresis magnetization. We have selected arbitrarily the function $\mathcal{F}(x) = (1-\eta^3)x/(1-(\eta x)^3)$ as in [10]. Whereas, $W_{\text{an}}$ can be computed for every magnetic domain with their respective orientation $\gamma$, our model computes $W_{\text{an}}$ with the resulting magnetization direction $\mathbf{m}$ of a grain, ie. the vectorial sum of the domain orientations.

B. Anhysteris magnetization

The crystal energy density can be expressed from the Gibbs free energy in the macroscopic frame by

$$W = -\mu_0 M_s h_e \cdot \mathbf{m} + W_a$$

(4)

The effective field $h_{\text{eff}}$ can be deduced from differentiatation with respect to the magnetization

$$h_{\text{eff}} = -\frac{1}{\mu_0 M_s} \partial W \partial \mathbf{m}$$

(5)

At the macroscopic scale, this effective field becomes

$$h_{\text{eff}} = h_e - \frac{1}{\mu_0 M_s} \mathcal{F} \left( \frac{M_{\text{an}}}{M_s} \right) \partial W_a \partial \mathbf{m}$$

(6)

Since the micromagnetism equilibrium involves no torque, ie. $h_{\text{eff}} \times \mathbf{m} = 0$, the effective field is colinear with the magnetization direction $\mathbf{m}$. In ferromagnetic material, the direction of the effective field and the magnetization are the same, $h_{\text{eff}} = H_{\text{eff}} \mathbf{m}$.

The anhysteretic magnetization $M_{\text{an}}$ is modeled with a strictly monotonous spline interpolation $\mathcal{L}(H_{\text{eff}})$ with polynomial of order 3. The interpolant can be fitted from the magnetic measurement until the measured maximum field $H_{\text{max}}$. Since the function $\mathcal{F}$ requires the saturation of the anhysteris characteristic, a law to approach the saturation is employed for the extrapolation. The anhysteris magnetization is computed from the amplitude of the effective field with

$$M_{\text{an}}(H_{\text{eff}}) = \begin{cases} \mathcal{L}(H_{\text{eff}}) & \text{for } H_{\text{eff}} \leq H_{\text{max}} \\ M_s (1 - a_s H_{\text{eff}}^{-2}) & \text{for } H_{\text{eff}} > H_{\text{max}} \end{cases}$$

(7)

where the parameters $M_s$, and $a_s$ are deduced from the spline interpolant $\mathcal{L}$ at $H_{\text{max}}$ to ensure the differential class $C^1$.

The implicit equation (6) is solved by minimizing the residual $r_{\text{an}}$ with a Newton method. The residual and the Jacobian $\mathbf{J}_{\text{an}}$ are given in equation (8) and (9)

$$r_{\text{an}} = h_e - \frac{1}{\mu_0 M_s} \mathcal{F} \left( \frac{M_{\text{an}}}{M_s} \right) \partial W_a \partial \mathbf{m} - h_{\text{eff}}$$

(8)

$$-\mu_0 M_s \mathbf{J}_{\text{an}} = \frac{\partial \mathcal{F}}{\partial M_{\text{an}}} \frac{\partial \mathcal{L}}{\partial H_{\text{eff}}} h_{\text{eff}} \partial W_a \partial \mathbf{m} + \mathcal{F} \partial^2 W_a \partial m^2 - \mathbf{I}$$

(9)

where $\mathbf{I}$ is the unit matrix and $\otimes$ is the dyadic product.

The anhysteretic magnetization, $\mathbf{m}_{\text{an}} = M_{\text{an}} \mathbf{m}$, can be modeled with 18 parameters for in-plane magnetization:

- 7 parameters for the anisotropic energy, $K_{ij}$,
- 1 parameter for the coherent rotation, $\eta$,
- 10 parameters for the anhysteris spline interpolant $\mathcal{L}$.

The general 3D case should include 8 more parameters in the anisotropic energy. The spline interpolant could also contains more than 10 control points.

C. Vectorization technique for hysteresis

Whereas the anhysteris behavior involves an homogeneous cubic crystal pattern in every grain, various defects tends to pin some domain walls. These heterogeneous distortions of the crystal lattice hinder the magnetization process by retaining the domain volume fraction. Generally, these defects can be represented with point force or dislocation line from a micro-mechanical analysis. The stress field which is localized in the vicinity of the defect, vanishes with the distance to the defect core. Whereas the domain wall is pinned to these defects, the domain orientation is also influenced only in the vicinity of a defect. In soft magnetic material with low coercive field, the defect density remains low. Hence, we can assume that the magnetic domain orientation is not influenced by the defect heterogeneity. Although the domain orientation can be free to rotate even with a pinned domain wall, the macroscopic magnetization contains the contribution of every domains.

The hysteretic magnetization can be generally expressed by $\mathbf{m}_{\text{h}} = M_s \mathbf{e}_h$. The resulting magnetization orientation $\mathbf{e}_h$ is expressed by

$$\mathbf{e}_h = \mathcal{F} \left( \frac{M_{\text{an}}}{M_s} \right) \mathbf{e}_{\text{rev}} + \left( 1 - \mathcal{F} \left( \frac{M_{\text{an}}}{M_s} \right) \right) \mathbf{e}_{\text{irr}}$$

(10)

where $\mathbf{e}_{\text{rev}}$ is the direction of a reversible magnetization and $\mathbf{e}_{\text{irr}}$ is the direction of the irreversible magnetization. The function $\mathcal{F}$ plays the phenomenologial role of the coherent rotation of the magnetic domain. When the magnetization approaches the saturation, the domains should align toward the reversible magnetization as the density of domain walls drops. However, it aligns mainly toward the irreversible magnetization at lower magnetization amplitude. Since, the domain rotation phenomenon is already representated in the anhysteris behavior, the vectorization technique does not require any additional parameters.

D. Jiles-Atherton model

The Jiles-Atherton model developed in [13], [14] consists in solving the following differential equations

$$\frac{d\mathbf{m}_{\text{irr}}}{||\mathbf{m}_{\text{irr}}||} = \frac{\mathbf{e}_{\text{irr}}}{||\mathbf{e}_{\text{irr}}||} \cdot \mathbf{d}h_{\text{eff}}$$

$$\mathbf{e}_{\text{irr}} = k^{-1} \cdot (\mathbf{m}_{\text{an}} - \mathbf{m}_{\text{irr}})$$

(11)

where the norm is denoted by $||\mathbf{x}|| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$ and the ReLu function $[x]^+ = x$ for positive $x$ and $[x]^+ = 0$ for strictly negative $x$. In practice, the differential equation is resolved
with a Crank-Nicholson technique with a Newton method for the implicit equation.

Because the domain wall moves mainly along the easy direction of the crystal, the irreversible energy should present a similar anisotropy as the anhysteretic behavior. By analogy of the energy term $k \cdot \xi_{irr} \cdot (m_{an} - m_{irr})$ and the anisotropic energy in equation 2, the matrix $k$ is deduced

$$k = k_{iso}I + k_{ani} \left( \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} - W_0 I \right)$$  \hspace{1cm} (12)$$

with $k_{11} = K_{20}m_1^4 + K_{60}m_2^4, k_{22} = K_{04}m_2^4 + K_{06}m_2^4,$ and $k_{12} = K_{22}m_1m_2 + K_{42}m_2^3m_2 + K_{24}m_1m_2^3.$ Only the parameters $k_{iso}$ and $k_{ani}$ describes the anisotropic irreversible pinning force tensor $k$. Since the anisotropic energy can be defined at a given constant, the term $W_0$ is defined as the minimum of the anisotropic energy. This term ensures the definite positive property of the pinning force tensor for any set of the anisotropic parameters $K_{ij}$.

The hysteretic magnetization amplitude $M_h$ and the direction $e_{irr}$ are computed with

$$M_h = |(c \cdot m_{an} + (I - c) \cdot m_{irr})|$$ $$e_{irr} = \frac{c \cdot m_{an} + (I - c) \cdot m_{irr}}{M_h}$$  \hspace{1cm} (13)$$

The matrix $c$ relates to the initial susceptibility of the first magnetization of the material. It is defined by analogy to the pinning force tensor $k$ with two parameters $c_{iso}$ and $c_{ani}$.

E. Reversible magnetization

In order to complete the vectorization technique, the direction of a reversible magnetization $e_{rev}$ is defined within the effective field definition. This reversible magnetization should represent the coherent rotation of the domain. At saturation, the single domain should be aligned with the reversible magnetization. In order to preserve the amplitude of the magnetization $M_h$ during the hysteresis process, the reversible orientation is looking for a reversible field amplitude $H_{rev}$ within the effective field subset. It is expressed with the following implicit equation.

$$H_{rev} = \frac{h_e}{|h_e|} - \frac{1}{\mu_0M_s} \nabla \left( \frac{M_h}{M_s} \right) \frac{\partial W_a}{\partial m} = L^{-1}(M_h)e_{rev}.$$  \hspace{1cm} (14)$$

The reversible magnetic field shares the same direction as the external field. Besides, the anisotropic energy $W_a$ is computed with the direction of the reversible field $e_{rev}$.

In practice, this implicit equation requires to determine $H_{rev}$ and $e_{rev}$. It is solved with a Newton method with a Langrange multiplier to ensure $|e_{rev}| = 1$.

III. APPLICATION

The parameters of the proposed vector hysteresis model are identified on the magnetic measurement of a M350-50A steel sheet reported in [9], [15]. Alternating and rotating flux density measurement are performed on the round rotational single sheet tester realized by Gorrican et al., 2001 [15].

The alternating measurement are performed at 20 Hz and the rotating measurement at 50 Hz. The proposed model only requires the alternating flux density measurement along different direction in order to identify the parameters. Then, the model validation is performed on rotating measurement. Due to the different frequencies in the measurement set, the eddy current and the excess field needs to be retrieve from the measured magnetic field. This extraction of the hysteresis term in the magnetic field is based on the Bertotti loss decomposition $p = k_h f B^2 + k_{ex} (f B)^{1.5} + \pi^2 \sigma d^2/6 (f B)^2$, where $f$ and $B$ are the frequency and the flux density amplitude, $\sigma$ and $d$ are the electrical conductivity and the thickness of the sheet. The coefficient $k_h$ and $k_{ex}$ are evaluated from the measured losses under alternating flux density condition with amplitude measured from 0.2T to 1.8T and three available frequencies 20 Hz, 50 Hz, and 100 Hz. The validity of the eddy current loss term should be correct since the skin depth equals half the sheet thickness for a frequency near 200 Hz. However, the hysteresis term could present some discrepancies at saturation where the losses also saturates with the flux density amplitude. Nevertheless, the excess loss term corresponds to 10 % and 30 % of the total losses at 20 Hz and 50 Hz respectively. Since the hysteresis relates to a static phenomenon, the results are presented in term of energy per cycle.

1) Identification of the model parameter

The model parameters are identified from the measurements under alternating flux density at 20 Hz along the rolling direction (0 deg), the transverse direction (90 deg) and two flux density directions at 30 deg and 60 deg. The parameters are identified from the first magnetization curve and the hysteresis energy at 1.65 T and 1.8 T. In Fig. 1, the first magnetization curve can be accurately evaluated with a relative error below 8%. In Fig. 2, the hysteresis energy per loop is computed with less than 10 % of relative error with respect to the measured hysteresis energy.

2) Validation of the model

The vector hysteresis model is first validated under alternating flux density condition with different orientations. In Fig. 3, the model can predict properly the hysteresis energy loss with a relative error lower than 40 %. The model mainly overestimates the hysteresis energy per cycle at low amplitude.
of the flux density. The anisotropic trend can be properly predicted by the proposed model for different orientations. Then, the model is validated under rotating flux density condition measured at 50 Hz. In Fig. 4, the rotational hysteresis energy can be accurately predicted by the vector model with a relative error lower than 22%. Although some discrepancies can be noticed at the peak of hysteresis energy, the drop of the energy caused by the consistent rotation of the domains is properly represented by the model.

**IV. Conclusion**

In this paper, we propose an anisotropic vectorization technique for magnetic hysteresis in electrical steel sheets. Although, the vector hysteresis is developed with the Jiles-Atherton model, it can be deployed to many other hysteresis models by following the proposed method. The model parameters require an identification from few static magnetic measurements under alternating conditions in at least three different orientations. Thanks to the physical relation to the cubic crystal symmetry, the model can predict the hysteresis energy in alternating conditions but also in rotating flux density conditions. Since the measurements were not performed in the quasi-static case, some notable discrepancies in the model validation and identification arises due to the extraction of the hysteresis magnetic field from the measurements. Besides, the model accuracy can also increase by considering more than one equivalent single crystal. In the future, the model should be employed to evaluate the magnetic properties of grain oriented materials.

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