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Anti-Hermitian Optical Media with Gain and Loss

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Abstract – **Distributing loss and gain in different effective parameters of optical media suggests an alternative branch in the domain of non-Hermitian optics. In this conference talk, we contemplate an extreme scenario in which those optical media are *anti-Hermitian* and investigate various aspects of wave interactions with such media. Particularly, we show that it is possible to *fully* convert evanescent source fields to transmitted propagating waves and vice versa or to provide lasing effect creating both evanescent and propagating plane waves. We hope that this talk persuades the non-Hermitian optics community to consider anti-Hermiticity as an alternative feature for shaping light-matter interactions in unprecedented ways.**

I. Introduction

Recently, non-Hermitian systems have been at the core of attention in quantum and classical physics including optics [1]. Separating gain and loss in non-Hermitian optical systems allows for providing parity-time reversal symmetry [2] which results in attaining novel effects and functionalities [3]. Focusing on conventional optical media, parity-time symmetry is realized by proper engineering of the permittivity such that its real and imaginary parts are even and odd functions with respect to space, respectively [4]. However, one alluring possibility is to separate the gain and loss in different effective material parameters rather than in space. For that, we need to employ non-Hermitian media having both electric and magnetic responses such as magnetodielectric ones [5]. More enticingly, one scenario is if the corresponding permittivity representing the electric response and the permeability being responsible for the magnetic response have zero real parts [6, 7]. In this case, the magnetodielectric medium is anti-Hermitian which is in fact the generalized duality transformation of Tellegen nihility [7]. In this work, we thoroughly scrutinize such anti-Hermitian media by studying the wave scattering under different types of excitation, polarization, and geometry. Here, due to the limited space, we focus only on the interface and grounded slab problems (with specific excitation and polarization), while in the oral presentation, our whole comprehensive study will be shown, including scattering from spherical particles made of anti-Hermitian magnetodielectric materials.

II. Transformation from evanescent to propagating waves

We consider an interface between vacuum and an anti-Hermitian magnetodielectric half-space, whose effective material parameters are described by

$$\epsilon = \epsilon_0(-\mathrm{j}\chi), \qquad \mu = \mu_0(+\mathrm{j}\chi), \tag{1}$$

see Fig. 1(a). Here, χ is a real-valued number, ϵ_0 and μ_0 represent the free-space permittivity and permeability, respectively, and "j" denotes the imaginary unit. In this study, the source field in free half-space is assumed to be a homogeneous or inhomogeneous plane wave that we call the incident wave. In the present case, this wave is evanescent with the decay in the x-direction, and its polarization is transversely electric (TE). Therefore, the electric field has one component along the y-axis, and there are two components of the magnetic field – the z- and x-ones. On the other hand, we suppose that the transmitted wave in the anti-Hermitian half-space is a propagating plane wave (it has been shown that the wave number for an anti-Hermitian magnetodielectric medium with gain and loss compensation is real allowing wave propagation inside the medium while the time-averaged Poynting

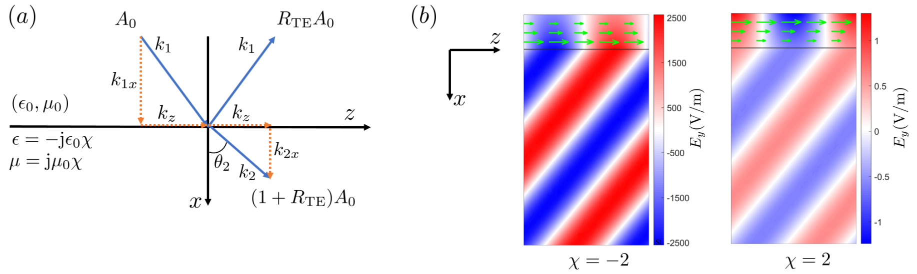

Fig. 1: (a)–Interface between vacuum and anti-Hermitian medium. (b)–Lasing effect (left) and conversion of incident evanescent wave to transmitted propagating plane wave without reflection (right). The green arrows show the time-averaged Poynting vector.

vector is zero). By writing the fields in the vacuum and anti-Hermitian half-spaces, and by imposing the boundary conditions, we finally find the reflection coefficient as

$$R_{\text{TE}} = \frac{\chi|k_{1x}| - k_{2x}}{\chi|k_{1x}| + k_{2x}}, \tag{2}$$

in which $|\cdot|$ determines the absolute value, and k_{ix} is the wave-vector normal component in vacuum ($i = 1$) and in the medium ($i = 2$). This component is purely imaginary in free space and purely real in the anti-Hermitian half-space. At a glance, Eq. (2) indicates that there is a zero at $k_{2x} = \chi|k_{1x}|$ for positive real-valued χ. Technically, this means that the incident evanescent wave is fully converted to the transmitted plane wave with zero reflection, and it propagates toward infinity without carrying power. It is difficult to find this feature by using usual dielectric materials due to the complete mismatch between vacuum and dielectric half-space for such an incident wave.

Apart from zero, concerning Eq. (2), the most intriguing case is likely to uncover the pole which occurs at $k_{2x} = -\chi|k_{1x}|$ for negative values of χ (now, the loss is due to magnetic properties, and the gain shows itself through permittivity). For this pole, the reflection magnitude becomes ideally infinite. Since the transverse component of the wave vector is conserved (no spatial variation along the z-axis), such a reflected field is certainly associated with an evanescent wave whose magnitude attenuates exponentially as we move far from the interface, and it does not carry power in the normal direction. However, it carries a nonzero time-averaged power density that is directed along the z-axis. Consequently, a strong surface wave is generated, which is the amplified version of a weak localized incident wave. This is clear in Fig. 1(b). Finally, it is worth mentioning that one can redo these calculations for the TM polarization to see that the transformation and amplification effects interestingly occur contrariwise to the same values of χ.

III. Anti-Hermitian slab backed by a PEC mirror

Next, we study a grounded anti-Hermitian slab of thickness d which is infinitely extended in the y- and z-directions. The volume $x < -d$ is filled by vacuum, and there is a perfect electric conductor (PEC) boundary at $x = 0$ (see Fig. 2(a)). Let a transverse-magnetic (TM) polarized plane wave with a magnetic field purely along the y-axis be incident on the slab. The magnetic field in the half-space above the slab and inside the slab can be written as a superposition of forward and backward propagating waves. From Maxwell's curl equation $\nabla \times \mathbf{H} = \text{j}\omega\epsilon\mathbf{E}$ we can calculate the corresponding electric field, which has components along x- and z-directions. By imposing continuity of the transverse field components at the interface $x = -d$ we find the amplitude reflection factor

$$R_{\text{TM}} = \frac{\chi k_{1x} + k_{2x}\tan(k_{2x}d)}{\chi k_{1x} - k_{2x}\tan(k_{2x}d)}. \tag{3}$$

If the incident wave is evanescent in x-direction (k_{1x} is imaginary), the reflection factor always has magnitude one, which can be easily verified from Eq. (3). If the incident wave is propagating, the reflection factor can in principle diverge (lasing) or go to zero (perfect absorption) for both propagating and evanescent waves in the slab. Figure 2(b) shows the absolute value of the reflection factor for TM polarization for $0 \le k_z/k_0 \le 1$, $-2 \le \chi \le 2$, and the normalized slab thickness $k_0d = 2$. One intriguing result is the fact that for TM polarization lasing can be

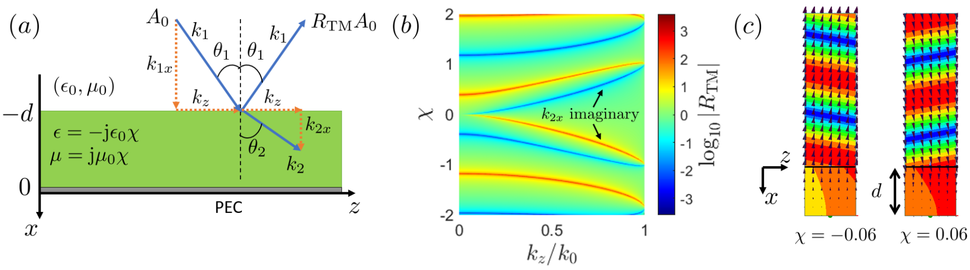

Fig. 2: (a)–Slab of thickness d filled by an anti-Hermitian medium (see Eq. (1)) and terminated by a PEC boundary. (b)–Reflection factor of the slab for TM polarization for $k_0d = 2$. (c)–Simulation of fields for $k_0d = 2$, TM polarization and incidence angle $\theta_1 = 10.64°$. The magnetic field is color coded and the black arrows show the time-averaged Poynting vector. For $\chi = -0.06$ the reflection factor diverges, for $\chi = 0.06$ it is zero.

achieved for an arbitrarily small value of χ at normal incidence, as long as χ is negative. This branch in Fig. 2(b) corresponds to evanescent waves inside the slab, as $|\chi| < k_z/k_0$.

Further, we observe that for $k_z \to k_0$ (grazing incidence), there are special points in the reflection factor for specific values of χ, where the infinite and zero reflection curves intersect. One can show that for TM polarization this happens for $k_0d\sqrt{\chi^2 - 1} = m\pi$, $m \in \mathbb{Z}$. For $m = 0$ we find $\chi = 1$ and for $m = 1$ we find $\chi \approx 1.86$, which agrees with Fig. 2(b). Simulation results for a TM polarized plane wave incident under the angle of $\theta_1 = 10.64°$ on the slab for $k_0d = 2$ and $\chi = \pm 0.06$ are shown in Fig. 2(c). For $\chi = -0.06$, the slab acts as a laser, for $\chi = 0.06$ the slab acts as a perfect absorber. The numerical values for the reflection factor in simulations agree with the theoretical results calculated from Eq. (3). To conclude this discussion, the qualitative behavior of the slab under TE illumination is almost identical to the TM polarization case, merely with different values of χ and k_z resulting in zero or infinite reflection. The major difference is that the reflection factor does not feature the two infinite and zero reflection curves emerging from the origin in Fig. 2(b).

IV. Conclusion

In this summary, we have shown that anti-Hermitian optical media with gain and loss allow us to efficiently convert evanescent waves to propagating plane waves. Also, we have theoretically predicted surface-wave lasing: The presence of an evanescent wave at an interface with vacuum results in generation of a surface wave, ideally with an infinite amplitude, which carries power only along the interface. Also, considering a slab backed by a PEC mirror, we have found that small amounts of gain and loss give rise to the lasing effect for propagating plane waves. Finally, we have indicated that there is a particular point where the lasing and absorbing effects coalesce.

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