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Equivalence of Angular Stability and Reflection Locality for Metasurfaces with Anomalous Reflection

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Abstract – **The so-called generalized reflection law defines the fields reflected from non-uniform boundaries in terms of the local reflection coefficient. Absolute majority of researchers utilize the approximation of so-called reflection locality in which the reflection coefficient at a given point (at the reference unit cell position) is assumed to be the same as if the metasurface were uniform, i.e., all the unit cells surrounding the reference one were identical. It is known that this locally periodic approximation is adequate for anomalous reflectors in case of small deviations from the usual reflection law. In this talk we will present a proof that this approximation is adequate also for strongly anomalous reflections if (and only if) the reflection coefficient of the corresponding uniform metasurfaces is independent from the incidence angle.**

I. Introduction

The development of metasurfaces (MSs) in the two last decades resulted in realizations of phase-gradient reflectors from the microwave to the optical range. These metasurfaces perform anomalous (non-specular) reflection to a prescribed direction. Surface currents obeying the linear reflection phase Φ_R coordinate dependence create only one plane wave in the desired direction. However, periodically modulated metasurfaces support infinitely many Floquet harmonics [1], and proper design of structures that effectively nullify the amplitudes of all unwanted propagating harmonics is a challenge. How to correctly engineer a periodically non-uniform metasurface (PNUMS)? The absolute majority of engineers use for it the locally periodical approximation. First, one designs a uniform MS with lumped loads in each unit cell. Varying these loads one obtains various reflection phases Φ_R covering the whole range $[-\pi, \pi]$ at needed frequencies. Once such periodical MS with variable loads (we call it generic MS) is designed, one simply assumes that in the PNUMS $\Phi_R(x)$ is locally the same as Φ_R of the generic MS (i.e. as if all the loads were the same as that of the unit cell centered at x) [2]. In other words, one assumes that $\Phi_R(x)$ does not feel the differences in the unit cells surrounding the reference one. This approximation that we call *locally periodical approximation* or *reflection locality* is so commonly adopted by researchers that some even formulate electromagnetic theorems based on it (see e.g. in [4]). However, this approximation cannot be adequate for arbitrary deviations from the usual reflection law, except the case of retroreflection [3]. Indeed, a PNUMS obeys the Floquet theory of arbitrary periodic gratings. In accordance to that, in the best case, when the anomalous reflection holds in the first diffraction order, we have $|\sin\theta_a - \sin\theta_i| = \lambda/D$. If this difference between the incidence angle θ_i and the anomalous reflection angle θ_a is large, e.g., exceeds $\pi/3$, the ratio λ/D is not much smaller than unity. Let $D = 2\lambda$ and $a = \lambda/6$ (here a is the unit cell size) $\Phi_R(x)$ differs from $\Phi_R(x \pm a)$ by $\pm\pi/6$. For this noticeable phase change the loads in two adjacent unit cells should be noticeably different. If the reference unit cell interacts with them, $\Phi_R(x)$ cannot be the same as that in the uniform MS. This approximation seems to be applicable only for small deviations when $D \gg \lambda$. Meanwhile, tunable MSs operating only with small deviations from the usual reflection law are impractical. Is it possible to achieve the reflection locality for deviation angles $|\theta_a - \theta_i|$ at least of the order of $\pi/3$?

Below we will see that it is possible to engineer PNUMSs for which the values $\Phi_R(x)$ are equal to those of the uniform generic MS for so large deviations. It is the case when the generic MS has the same Φ_R for different incidence angles for both TE and TM polarizations. This property is known for some reflecting MSs and is called *angular stability*. If it is respected for all loads needed for the PNUM the reflection locality and angular stability are equivalent concepts.

II. No reflection locality in the absence of angular stability

Let us assume that a generic periodic MS does not possess angular stability. Suppose that there are N lumped loads for which the generic MS offers the needed Φ_R for the normal incidence: $Z_1,\ Z_2, \dots Z_N$, whereas $\Phi_R^{(m)}(\theta_i = 0)$ differ from $\Phi_R^{(m\pm1)}(\theta_i = 0)$ by $\pm 2\pi/N$, where $m = 2, \dots N-1$. No angular stability means that the values $\Phi_R^{(m)}$ for a substantial incidence angle, e.g. $\theta_i = \pi/3$, for the same values Z_m are noticeably different from $\Phi_R^{(m)}(0)$. For example, we may have $\Phi_R^{(m)}(\pi/3) - \Phi_R^{(m\pm1)}(\pi/3) = \pm 4\pi/N$. Postulating the reflection locality for the PNUMS (as many researchers do), we obtain the period of the reflection phase $D = 2\lambda$ for the normal incidence and $D = \lambda$ for the incidence under the angle $\theta_i = \pi/3$ on the same PNUMS. In the first case, the normally incident wave deflects under the angle $\theta_a = \pi/3$. In the second case, the wave incident under the angle $\theta_i = \pi/3$ is not deflected at all, since the necessary condition of the anomalous reflection $D > \lambda$ is not respected. Now let us assume that $|\Phi_R^{(m)}(\pi/3) - \Phi_R^{(m\pm1)}(\pi/3)|$ is not larger but smaller than $2\pi/N$, e.g. equals π/N. In this case, the values $\Phi_R^{(m)}(\pi/3)$ give the same period $D = 2\lambda$, but vary from 0 to π. So, in both cases of the angular instability (larger values of $\Phi_R(\pi/3)$ than $\Phi_R(0)$ or smaller), the transmittance from the channel $\theta = 0$ to the channel $\pi/3$ and the transmittance from the channel $\pi/3$ to the channel $\theta = 0$ are essentially different. This violation of reciprocity results from the assumed reflection locality that cannot hold for a PNUMS if the generic MS has no angular stability.

III. Reflection locality holds in the presence of angular stability

Meanwhile, if we postulate the angular stability for a uniform MS, consisting of a periodic planar grid of metal elements with lumped loads located on a metal-backed dielectric, the reflection locality holds for it. To prove it, one can use the equivalence of such MS to an array of uniaxial bianisotropic particles [5], see Fig. 1. Generalizing the approach of [5] to oblique incidence one can express the collective polarizabilities of the bianisotropic array via the sheet reactance of the grid $Z_{sg} = jX_g$ (in the lossless case), the electromagnetic thickness kh and the incidence angle θ_i. One can apply the theory of bianisotropic grids (see e.g. in [5, 6, 7]) and imposing the angular stability requirement for the generic MS formulate the conditions to which the bianisotropic array should satisfy.

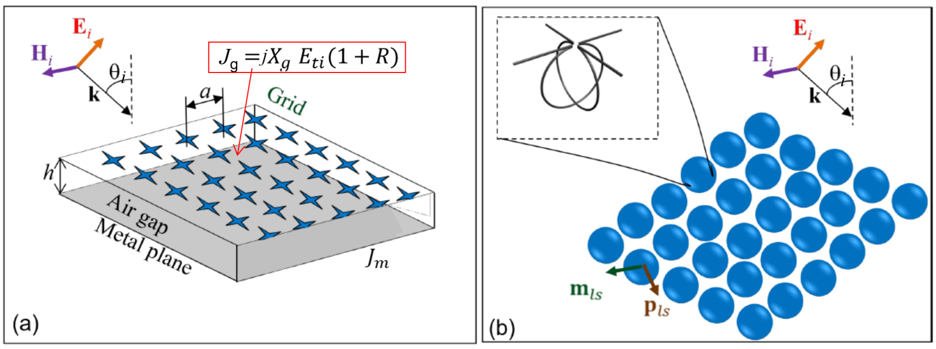

Fig. 1: (a) A generic MS is formed by a planar grid of metal elements (loaded by lumped loads) located over a metal plane with the gap h. J_g and J_m are the surface currents in the homogenized grid and on the metal plane, respectively. (b) An array of uniaxial bianisotropic particles illuminated by a plane wave is impenetrable in the resonance band and can be equivalent to the generic MS.

These conditions are as follows:

$$\hat{\alpha}_{ee}^{TE,TM} = \hat{\alpha}_{ee}^{(0)} \cos^{\pm1}\theta_i, \qquad \hat{\alpha}_{mm}^{TE,TM} = \hat{\alpha}_{mm}^{(0)} \cos^{\mp1}\theta_i, \quad \hat{\alpha}_{me}^{TE,TM} = \hat{\alpha}_{me}^{(0)}, \tag{1}$$

where index (0) corresponds to the normal incidence, and the upper and lower signs correspond to TE and TM polarizations, respectively. Here, scalar collective polarizabilities are defined for electric and magnetic dipoles of the reference particle as follows:

$$p = \hat{\alpha}_{ee} E_{ti} + \hat{\alpha}_{em} H_{ti}, \qquad m = \hat{\alpha}_{me} E_{ti} + \hat{\alpha}_{mm} H_{ti}, \quad \hat{\alpha}_{me} = \hat{\alpha}_{em}, \tag{2}$$

where index t means tangential components of the incident fields. Of course, equations (1) cannot be satisfied for all angles, however, one may design single-unit (local) polarizabilities, for which these conditions can be respected approximately in the range $\theta_i = [0, \pi/3]$ (for uniaxial omega particles formed by two non-identical capacitively coupled metal elements). However, we need these conditions for collective polarizabilities of particles in the array. Collective polarizabilities of a uniaxial binaisotropic array for both TE and TM cases were expressed through the individual ones in work [6]. Inspecting these relations we see that conditions (1) can be satisfied (approximately) if and only if we can neglect the real parts of the so-called interaction factors $\beta_{ee,mm,me}$. These parameters are defined through the interaction fields (tangential components) $E_{t,\rm int}$ and $H_{t,\rm int}$: the fields created by all the other particles (with moments $p_{ls},\ m_{ls}$) at the center of the reference particle:

$$E_{t,\rm int} = \beta_{ee}p + \beta_{em}m, \qquad H_{t,\rm int} = \beta_{mm}m + \beta_{me}p, \quad \beta_{me} = \beta_{em}. \tag{3}$$

The imaginary parts of $\beta_{ee,mm,me}$ cannot be neglected, because they are linked with the reflection coefficient. However, in the expressions for the collective polarizabilities, these imaginary parts exactly cancel out [7]. The real parts of the interaction factors describe near-field interactions between the array elements. They very strongly depend on θ_i, and if their contribution into $\hat{\alpha}^{TE,TM}_{me,mm}$ is important, the interaction effects destroy the angular stability. That is, to ensure the angular stability we need to minimize the near-field interactions in the array. For a homogenized array it means that the quasi-static electromagnetic field taken at the reference point A is practically determined by the electric and magnetic surface polarization inside a circle of small radius $\rho < a$. The quasi-static field of the electromagnetic polarization sheet with a removed circle (i.e. the reactive field of adjacent particles) brings a negligible contribution into the reflected field calculated at A. This is what we call the reflection locality and this is what is required for the angular stability. Is it physically possible? The analysis of the contribution of $\mathrm{Re}(\beta_{ee,mm,me})$ into $\hat{\alpha}_{ee,mm,me}$ (with the use of aforementioned relations between $\hat{\alpha}_{ee,mm,me}$ and the parameters of the generic MS) shows that it is possible: both reflection locality and angular stability are achievable in the magnetic (parallel) resonance range of the MS where $X_g \approx -kh$. Near the magnetic resonance of the MS $\hat{\alpha}_{me,ee,mm}$ are resonant, and a small deviation of the resonant frequency with respect to the operation one brings a large deviation to Φ_R. We see that the model of angular stability demanding the reflection locality is consistent with the requirement of the full reflection phase control. Since the angular stability is necessary for reflection locality and vice versa, these concepts, though both being approximate, are mathematically equivalent.

IV. Conclusion

In summary, we have shown that the angular stability is a necessary and sufficient condition for the reflection locality of generic metasurfaces, and these concepts are equivalent. As to the reconfigurable surfaces, the reflection locality allows conventional design based on the generic locally periodical MS with variable loads. If the generic MS with angular stability transforms to a non-uniform one by varying the unit cell loads along the surface, its reflection response evidently remains local, and the standard design procedure results in its proper operation for arbitrary incident and deviation angles within the validity of the angular stability assumption.

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