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# A PMU Placement Framework in an Active Distribution Network Based on Voltage Profile Estimation Accuracy

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**Abstract**— Traditional distribution networks have been transformed into active distribution networks (ADNs) due to the growing penetration of flexible resources as well as renewable energy sources (RESs). However, conventional systems have low levels of automation and observability. Thus, it is necessary to install measurement devices like phasor measurement units (PMUs) in these networks to increase their observability. Motivated by the abovementioned facts, a precision criterion based on the voltage profile of the ADNs is introduced for PMU placement to enhance the accuracy of the distribution state estimation (DSE). This process is performed by specifying the optimal number and location of PMUs. Moreover, the incorrect input data in PMUs can affect the state estimation accuracy. Therefore, the effect of deliberately or non-deliberately erroneous input data in PMUs has been evaluated on the estimated voltage of all nodes. Finally, the developed methodology is implemented on the 77-bus-UK-test distribution network to analyze the effectiveness of the proposed approach in improving the accuracy of the estimated network voltage profile.

**Index Terms**— Active Distribution Networks, Network Observability, State Estimation Accuracy, Phasor Measurement Units, Voltage Profile.

## I. INTRODUCTION

Coordination and management of distributed energy resources (DERs), utilization of optimal energy for consumers, and expansion of controllable energy sources have increased the significance and application of modern distribution management systems (DMSs). From this perspective, the energy management system (EMS) is an advanced control function integrated into modern DMS to utilize and operate DERs, Energy Storage Systems, and responsive loads. In this regard, the measurement equipment, distribution state estimation (DSE), and EMS are critical elements to optimize the operation of the active distribution networks (ADNs) [1]–[4].

Nowadays, distribution networks are facing fundamental alterations in conventional operation and planning mechanisms with the introduction of DERs as well as increasing concerns on environmental matters. On the other hand, in the current situation, DMS controls and monitors the distribution network with low levels of observability which will not fulfill the challenges of ADNs. Therefore, DSE, as one of the pivotal aspects of DMS, should be used in an accurate and reliable manner to achieve appropriate levels of network observability. DSE has been extensively studied by considering the accuracy of the state estimation process (SEP), measurement allocation, network observability, and

investment cost issues [5]–[12]. In the conventional DSE process, traditional noisy measured data such as substation measurements along with historical network information (as pseudo-measurements) are used to estimate the distribution network conditions. However, these measurements are not precise enough and may be led to considerable inaccurate DSE results. Modern measurements such as phasor measurement units (PMUs), which have an appropriate reporting rate and accuracy, can be employed in these circumstances. On the other hand, it is not possible to utilize PMUs with a wide range in distribution systems due to economic constraints. Therefore, it is necessary to optimize the number and location of installed PMUs in ADNs [1], [2] considering the operational conditions of the system. In this regard, many precious contributions have been studied, considering the optimal allocation of PMUs to increase the accuracy of DSE while minimizing the designing costs [5], [6], [8], [10]–[14]. In [5], a framework for PMU and smart meter placement is investigated using the genetic algorithm (GA). Accordingly, designing cost and DSE accuracy indices are minimized as objective functions. Considering the economic constraints for implementation of measurement devices, the placement of a prespecified number of meters is studied in [10]. Accordingly, the voltage profile estimation error in the worst-case scenario of all possible scenarios has minimized. In [11], the optimal allocation of PMUs aiming to minimize the PMU-cost and line relays is introduced. In this regard, the network observability and the minimization of load loss with different configurations have been considered. Considering the fact that future distribution grids would be prone to some operational challenges such as grid congestion and power quality issues due to the ever-increasing penetration of the DERs, the authors in [15] have introduced a method to determine the optimal constellation of PMUs to enhance the state estimation accuracy considering the operational condition of ADNs. In this regard, a PMU placement framework is investigated to minimize the compensating cost of the accuracy of active power injection and load interruption owing to the inaccurate estimation of line-loadings. An integer linear programming framework is introduced in [12] to optimize measurement costs and network observability. Furthermore, the accuracy of DSE results in different network conditions, such as reconfiguration, alteration in load consumption, and measurement error degradation is addressed to assess the impact of the proposed placement. Considering the fact that any failure in measurement devices can affect state estimation

results, a PMU placement method has been investigated in [8] based on reliability and accuracy of state estimation results.

On the other hand, in the current situation, the data reporting rate by smart meters is inappropriate for real-time monitoring of future distribution networks. Thus, considering the limited number of voltage magnitude meters and PMUs, a sub-modular saturation algorithm has studied in [13]. The developed algorithm could provide a robust meter placement to perform the DSE while achieving network observability. The authors in [14], have performed a multi-objective optimization method to optimize location and number of measurement devices for an accurate state estimation. In the proposed model, three objective functions including the total cost of PMUs, smart meter devices, and the average relative percentage of state estimation errors are considered. Finally, a multi-objective pareto-based PSO-Krill Herd algorithm is utilized to calculate the optimal answer.

The obtained results from the DSE have a pivotal role in the operation of distribution networks as shown in Fig. 1.

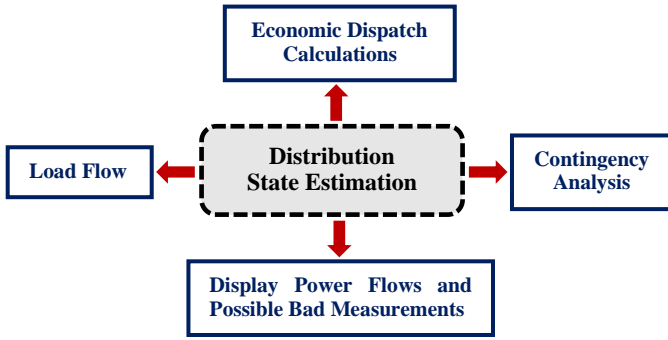


Fig. 1. The DSE role in distribution networks

Therefore, accurate estimation of network state variables – which are typically voltage magnitude and phase angle of network buses- is quite crucial throughout the network. Motivated by the mentioned points, a new precision criterion based on minimizing the difference between actual and estimated bus voltages is investigated in this article for optimal PMU placement. Furthermore, the developed procedure ensures the acceptable voltage magnitude for all network buses. Note that the minimum and maximum acceptable range of the bus voltage magnitudes are considered between 0.95 to 1.05 per-unit (p.u.), respectively. Also, intentional or unintentional incorrect input data in measurement devices can cause severe effects on the DSE results. From this perspective, the impact of this condition will be explored on the precision criterion and estimated voltage profile of the network. Additionally, to take into account the measurement uncertainty, several scenarios are generated using the K-means clustering process.

The rest of this paper is structured as follows. Section II describes the distribution state estimation algorithm, the proposed framework, and the optimization of measurement placement. Section III provides information on the test system and presents the simulation results. Finally, Section IV concludes the paper.

## II. DISTRIBUTION STATE ESTIMATION, METHODOLOGY, AND OPTIMIZATION OF MEASUREMENT ALLOCATION

### A. State Estimation Process

The desired goal of electric power systems is to deliver electric energy with high reliability to all customers. This process would be accomplished by knowing the state of power system, e.g., its voltage profile throughout the network. In this context, state estimation is a computational process that aims to evaluate the states of the power system considering limited redundant measurements subjected to noise and measurement errors [16]–[18].

In this paper, the weighted least square (WLS) based formulation is used for the DSE [5], [8], [16]–[18]. The WLS-based DSE formulation minimizes the sum of the squares of weighted deviations between actual and estimated measurements. If there are  $n$  numbers of state variables and  $m$  measurements, then the mathematical relationship between the set of actual measurements ( $z$ ) could be modelled as follow:

$$z = f(x) + e \quad (1)$$

Where,  $f^T(x) = [f_1(x), f_2(x), \dots, f_m(x)]$ ;  $f_i(x)$  is the  $i^{\text{th}}$  measurement function;  $x^T = [x_1, x_2, \dots, x_n]$  and  $e^T = [e_1, e_2, \dots, e_m]$  are the system state vector and the vector of measurements uncertainty (i.e., measurements errors), respectively. Moreover, the system state vector consists of bus voltage magnitudes and phase angles. Generally, in the state estimation process [5]–[10], [12], the optimization problem is formulated with the following objective function ( $J(x)$ ):

$$\text{Min}_x J(x) = [z^{meas} - f(x)]^T [W]^{-1} [z^{meas} - f(x)] \quad (2)$$

where,  $z^{meas}$  is the vector of measured quantities and  $W = \text{diag}\{\sigma_1^2, \sigma_1^2, \dots, \sigma_m^2\}$  is the covariance matrix of measurement errors.

As mentioned, traditional measurement devices through the network have inappropriate reporting rates and accuracy. Therefore, accurate synchro-phasors data obtained from PMUs can be utilized to enhance the accuracy of the SEP results in ADNs. In this respect, the measurement vector is considered as follows:

$$Z^{meas} = [V_{mag}, V_{ang}, I_{mag}, I_{ang}, P_{flow}, Q_{flow}, P_{inj}, Q_{inj}]^T$$

where,  $V_{mag}$  and  $V_{ang}$  are sub-vectors that include the voltage magnitude and phase angle measurements in the substation bus (slack bus) and buses with PMUs, respectively. Moreover, the line flow current magnitude and phase angle measurements for the lines connected to the buses with PMUs are represented in  $I_{mag}$  and  $I_{ang}$  sub-vectors, respectively. Additionally,  $P_{flow}$  and  $Q_{flow}$  consist the active and reactive power flow measurements; while the active and reactive power injection measurements (pseudo-measurements) are considered as  $P_{inj}$  and  $Q_{inj}$ .

Finally, the state variable  $x$  can be calculated according to:

$$x_{j+1} - x_j = [G(x_j)]^{-1} [H(x_j)]^T [W]^{-1} [z^{meas} - f(x_j)] \quad (3)$$

where,  $x_j$  is the estimated state vector from the  $j^{\text{th}}$  iteration of  $x$ ,  $G(x) = [H]^T [W]^{-1} [H]$  is the gain matrix, and  $H(x)$  represents the Jacobian matrix.

### B. Mathematical Modeling of Performance Index

Increasing the level of observability and monitoring in the distribution network to improve the DSE accuracy will raise the design cost of the state estimation system. Therefore, focusing on the error minimization function in the DMS, the installation cost of accurate measurement devices is optimized with respect to the desired accuracy. In this regard, a precision criterion is performed to enhance the estimated voltage of all network buses by optimal placement of PMUs. This is done by minimizing the root mean square of the difference between the real and imaginary parts of the actual and estimated voltage of each network bus, which is presented in (4). Then, based on the determined precision index, the placement of PMUs has been performed with the aim of proper accuracy for the DSE process and minimizing the number of allocated PMUs.

Precision =

$$\frac{1}{N_{scn}} \sum_{sc=1}^{scenarios} \sum_{i=1}^N \text{sqr} \left( \left| (V_{real}^{base})_{sc,i} - (V_{real}^{est})_{sc,i} \right|^2 + \left| (V_{imag}^{base})_{sc,i} - (V_{imag}^{est})_{sc,i} \right|^2 \right) \quad (4)$$

where,  $V_{real}^{base}$  and  $V_{imag}^{base}$  are the actual real and imaginary parts of a bus voltage, respectively. Also,  $V_{real}^{est}$  and  $V_{imag}^{est}$  are the estimated real and imaginary parts of a bus voltage, respectively. The number of network buses and scenarios are also presented with  $N$  and  $N_{scn}$ , respectively. In (4), first, the precision criterion is calculated for all network buses (taking into account the localized PMUs) in each of the generated scenarios. Finally, by averaging all the scenarios, the precision value is calculated for the mentioned PMU configuration.

### C. Optimization of Measurement Placement and GA

Based on the aforementioned performance criterion in the previous section, the objective function is proposed for the DSE accuracy improvement as follows:

$$\text{Min} \begin{cases} \text{obj}_1 = \text{PMU}^{cost} \\ \text{obj}_2 = \text{Precision} \end{cases} \quad (5a)$$

Subject to:

$$\text{Precision}_j < \text{Precision}^{base} \quad (5b)$$

$$\left| V_i^{\min} \right| < \left| V_i \right| < \left| V_i^{\max} \right| \quad (5c)$$

where,  $\text{Precision}_j$  is the calculated voltage accuracy index after placement of the  $j^{\text{th}}$  PMU configuration, and  $\text{Precision}^{base}$  is the base voltage accuracy index in the network without any PMU. Note that the minimum and maximum acceptable range of the bus voltage magnitudes are considered between 0.95 and 1.05 per-unit (p.u.),

respectively.  $\text{PMU}^{cost}$  is also the total cost of optimally located PMUs in the network which is calculated using the following equation.

$$\text{PMU}^{cost} = \sum_{i=1}^N x_i \quad (6)$$

Where,  $x$  is a binary variable. For a given bus,  $x$  is equal to one if a PMU exists at that bus otherwise it would be zero. Also,  $N$  is the total number of network buses. Furthermore, according to (5b) and (5c), it is considered that the DSE process should perform with appropriate accuracy and the voltage magnitudes of buses would be in the allowable range, respectively. Otherwise, inappropriate responses, which would not satisfy the constraints, will be penalized by adding a large number to their cost functions in the multi-objective optimization.

Since PMU placement is a non-deterministic problem, GA is used as an optimization tool to find the optimal location and number of PMUs, considering objective functions and restrictions [5], [19], [20]. Therefore, in this article, considering the multi-objective nature of optimization problem, a multi-objective genetic algorithm (MOGA) is utilized for the optimal PMU placement.

Implementing a MOGA has the following steps:

- **Input data:** distribution network topology, bus data, distributed generations data, measurements uncertainty, and objective function related to the DSE accuracy and PMU placement.
- **Generating a set of population:** a set of individuals which are the possible PMU configuration is generated.
- **Individuals' assessment:** the DSE process and MOGA are run simultaneously, then the fitness value of each individual is calculated based on the objective function illustrated in (5). Finally, individuals with good fitness values are selected as parents for the next generation.
- **Reproduction:** if the MOGA termination criteria aren't satisfied after the assessment of all individuals, new individuals must be generated by mutation and crossover functions. Although, individuals with enough good fitness value remain unaltered for the next generation. This procedure is repeated until a termination condition is satisfied.
- **Termination:** after assessing the fitness value of all individuals in the population, the optimization process is terminated. In this regard, if the stopping criteria of MOGA are respected, the best PMU configuration in the network would be reported. Stopping criteria include: an acceptable accuracy based on (5), exceeding the maximum iteration number, and no more improvement in the optimization process.

## III. CASE STUDY

### A. Test System and Simulation Conditions

The modified version of the generic 77-bus UK distribution system is chosen as a balanced three-phase ADN to evaluate the performance of the proposed method on the DSE accuracy and meter placement. As illustrated in Fig. 2, the selected test system has the radial network topology, 7

DGs, and 75 load points. Information relevant to the bus data, DG outputs, and line parameters can be found in [5], [21].

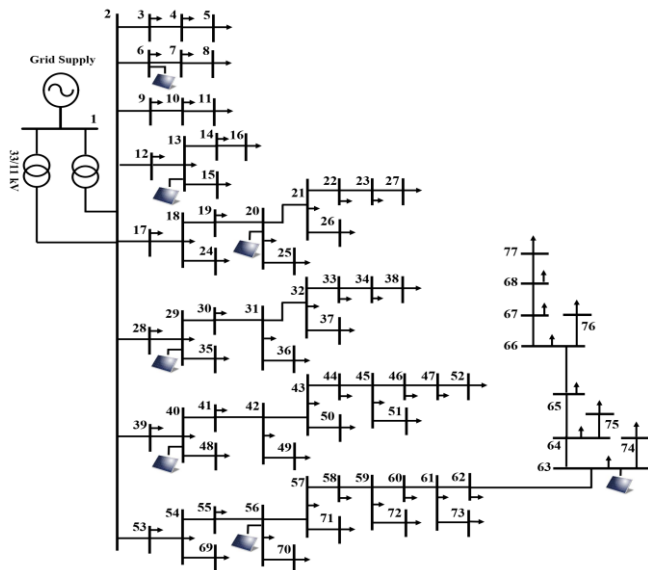


Fig. 2. UK 77-bus distribution test system

The power flow results from the test system are considered to model the actual value of measurements. Measurement deviations also obeyed the normal distribution. For the SEP, four different types of measurements are assumed to achieve full observability of the network [5], [22], [23]: conventional voltage measurement for the main substation bus (with 1% maximum error), power flow measurements at the sending side of the primary feeders (with 3% maximum error), all load buses with/without DGs in the form of active and reactive power injections as pseudo-measurements (with 50% maximum error), and measured quantities of located PMUs (with 0.7% and 0.7 centiradian errors for magnitude and phase angle of bus voltages). Eventually, the K-means clustering trials are employed to take into account different scenarios for measurement deviations [22]. In this regard, in the first place, 10000 scenarios were produced for different measurement data using the normal distribution. For the aforementioned purpose, the power flow results and measurements' errors were assumed as the mean values and standard deviations, respectively. Finally, utilizing the K-means clustering process, 1000 final scenarios were selected for the simulation study. This process is illustrated in the flowchart of Fig. 3. Note that these scenarios can reflect real operating points since they stem from power flow outputs.

### B. Results and Analysis

Implementing the MOGA for the aforementioned optimization problem, different optimal numbers and locations of PMUs are demonstrated in Table I to improve the DSE accuracy and voltage profile of the studied test network. As can be traced in Table I, the DSE accuracy is improved by locating PMUs and increasing their numbers. Also, the relevant total cost of PMUs is considered equal to their numbers, considering that the price of one PMU is one p.u. As mentioned before, the precision criterion shows the difference between the actual and the estimated value of all bus voltages, and it can be rendered as the extent of the inaccuracy between the actual and estimated voltage value of all network buses.

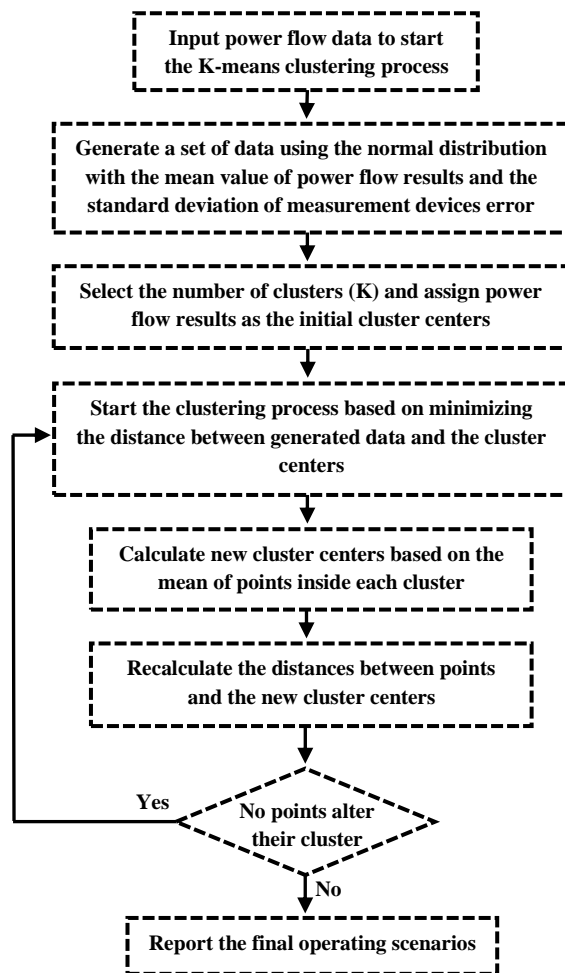


Fig. 3. Generate different operating scenarios using K-means clustering method

TABLE I. OPTIMAL PMU CONFIGURATIONS WITH MINIMUM COSTS AND RELATED PERFORMANCE INDICES FOR EACH CONFIGURATION.

| Config. number | PMU Locations (BUS NUMBER)  | PMU <sup>Cost</sup> (p.u.) | Precision (p.u.) | VMD (p.u.) | VAD (p.u.) |
|----------------|-----------------------------|----------------------------|------------------|------------|------------|
| 1              | [-]                         | 0                          | 0.7337           | 0.6509     | 0.2220     |
| 2              | [28,47]                     | 2                          | 0.3247           | 0.1992     | 0.2219     |
| 3              | [39,47,67]                  | 3                          | 0.2918           | 0.1397     | 0.2236     |
| 4              | [23,28,47,67]               | 4                          | 0.2757           | 0.1221     | 0.2232     |
| 5              | [20,23,28,47,67]            | 5                          | 0.2719           | 0.1179     | 0.2232     |
| 6              | [20,23,28,34,47,67]         | 6                          | 0.2683           | 0.1130     | 0.2231     |
| 7              | [20,23,28,34,47,53,67]      | 7                          | 0.2655           | 0.1092     | 0.2230     |
| 8              | [20,23,28,34,39,47,53,67]   | 8                          | 0.2632           | 0.1061     | 0.2227     |
| 9              | [7,20,23,28,34,39,47,53,67] | 9                          | 0.2611           | 0.1035     | 0.2227     |

Therefore, as can be seen from the results of the precision criterion, without any PMU there is almost 1% error between the actual and estimated values of every bus voltage in the network since the precision percentage for all network buses is 73.37% in scheme 1. But, as expected, placing PMUs has increased the accuracy of the measurements and consequently, the DSE accuracy. As a result, the error in the network voltage profile has reached from 1% to below 0.5% in scheme 2, and 0.4% in the last scheme. Moreover, for

smoothing the network voltage profile, the locations of PMUs are mainly at the beginning and end of the feeders.

On the other hand, for further analysis of the estimated bus voltages, two indexes for voltage magnitude deviation (VMD) and voltage angle deviation (VAD) are introduced as follows:

$$VMD = \frac{1}{N_{scn}} \sum_{sc=1}^{scenarios} \sum_{i=1}^N \left| |V_{i,sc}^{base}| - |V_{i,sc}^{est}| \right| \quad (7)$$

$$VAD = \frac{1}{N_{scn}} \sum_{sc=1}^{scenarios} \sum_{i=1}^N \left| \angle V_{i,sc}^{base} - \angle V_{i,sc}^{est} \right| \quad (8)$$

where,  $|V_{i,sc}^{base}|$  and  $|V_{i,sc}^{est}|$  are the actual and estimated voltage magnitude of each bus, respectively. Furthermore,  $\angle V_{i,sc}^{base}$  and  $\angle V_{i,sc}^{est}$  are also demonstrate the actual and estimated voltage angle of each bus, respectively. It can be seen from the results of the VMD and VAD indexes that PMU placement significantly affects the magnitude of bus voltages. In this respect, the VMD of all network buses remarkably decreases by increasing the number of PMUs. In comparison, PMU placement has a light and even negative impact on the voltage angles. This might be due to the proximity of bus voltage angles to each other in a distribution network.

Furthermore, we can determine the best PMU configuration in Table I regarding DSE accuracy and PMU installation cost by conducting a sensitivity analysis on the precision criterion and cost of PMUs. The formulation of sensitivity analysis is as follows:

$$S_{PMU\ cost}^{pr} = \frac{\Delta precision}{\Delta PMU\ cost} \quad (9)$$

where,  $\Delta precision$  and  $\Delta PMU\ cost$  are the difference between precision value and PMU installation cost of configurations  $i, i+1$ , respectively. The results of the sensitivity analysis are summarized in Table II. According to the results, schemes 2 and 3 have the most enhancement rate in the DSE accuracy per PMU. So, these two schemes are the most optimal PMU constellation in the viewpoint of accuracy and installation cost. Also, a comparison among network voltages profile for three different cases is demonstrated in Fig. 4 and Fig. 5: actual voltage profile (based on power flow results), estimated voltage profile without any PMU, and estimated voltage profile with the presence of PMUs (scheme 3 in Table I). These two Fig. clearly shows the significant impact of accurate measurement placement on the DSE results.

Additionally, the influence of deliberately or non-deliberately erroneous input data in PMUs has been evaluated on the DSE accuracy and precision criterion. In this regard, two cases were considered to demonstrate this effect. In the first case, the impact of 15% and 30% error percentages in the input of one of the installed PMUs is evaluated on the precision, VMD, and VAD criteria. While the second one analyses the effects of incorrect data in the input of two installed PMUs on the predefined criteria. The results of this assessment are summarized in Table III and IV.

TABLE II. SENSITIVITY ANALYSIS ON THE PRECISION CRITERION AND PMU INSTALLATION COST.

| Config. number | PMU Locations (BUS NUMBER)  | $S_{PMU\ cost}^{pr}$ (%) |
|----------------|-----------------------------|--------------------------|
| 1              | [-]                         | -                        |
| 2              | [28,47]                     | 20.45                    |
| 3              | [39,47,67]                  | 3.29                     |
| 4              | [23,28,47,67]               | 1.61                     |
| 5              | [20,23,28,47,67]            | 0.38                     |
| 6              | [20,23,28,34,47,67]         | 0.36                     |
| 7              | [20,23,28,34,47,53,67]      | 0.28                     |
| 8              | [20,23,28,34,39,47,53,67]   | 0.23                     |
| 9              | [7,20,23,28,34,39,47,53,67] | 0.21                     |

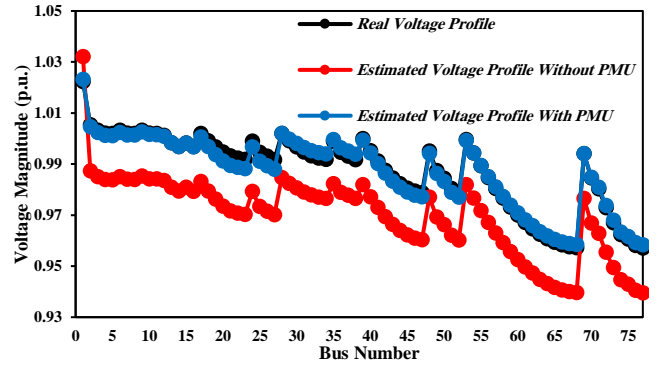


Fig. 4. Comparison of actual and estimated bus voltage magnitudes.

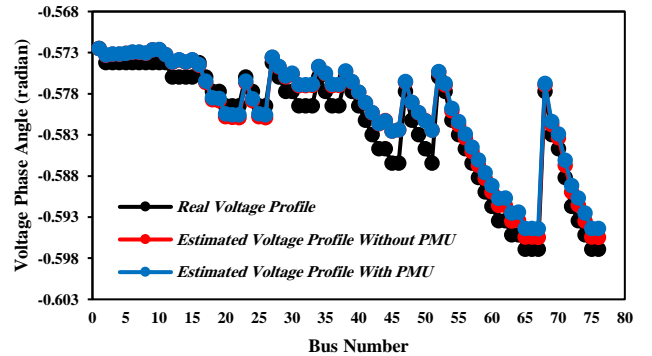


Fig. 5. Comparison of actual and estimated bus voltage angles.

As can be seen from the results, incorrect input data in the measurements has a terrible influence on the DSE results and all precision criteria. Also, increasing the error percentage in the PMUs (and generally all measurement devices) dramatically increases the estimation error of the state variables and concluded DSE outputs in the network. However, as can be conducted from the results, when the number of installed PMUs increases, this unfavorable effect becomes slighter. This is because increasing the number of accurate measurements enables the rest to cover the measuring error of one/two affected measurement devices. Also, a comparison between the results presented in Tables III and IV demonstrates that applying errors in the input of more measurement devices has more destructive effects on the output accuracy of the SEP.



TABLE III. IMPACT OF ERRONEOUS DATA IN THE INPUT OF ONE PMU ON THE PRECISION, VMD, AND VAD CRITERIA.

| Config. number | PMU <sup>Cost</sup> (P.U.) | 15%       |        |        | 30%       |         |        |
|----------------|----------------------------|-----------|--------|--------|-----------|---------|--------|
|                |                            | precision | VMD    | VAD    | precision | VMD     | VAD    |
| 1              | 0                          | -         | -      | -      | -         | -       | -      |
| 2              | 2                          | 5.7112    | 5.6984 | 0.2720 | 11.3893   | 11.3802 | 0.3223 |
| 3              | 3                          | 3.7415    | 3.7238 | 0.2487 | 7.4233    | 7.4123  | 0.2752 |
| 4              | 4                          | 2.8278    | 2.8055 | 0.2430 | 5.6054    | 5.5916  | 0.2659 |
| 5              | 5                          | 2.2689    | 2.2422 | 0.2391 | 4.4780    | 4.4617  | 0.2579 |
| 6              | 6                          | 1.9009    | 1.8600 | 0.2365 | 3.7430    | 3.7243  | 0.2527 |
| 7              | 7                          | 1.6377    | 1.6032 | 0.2334 | 3.2121    | 3.1918  | 0.2452 |
| 8              | 8                          | 1.4423    | 1.4040 | 0.2318 | 2.8161    | 2.7936  | 0.2417 |
| 9              | 9                          | 1.4625    | 1.4279 | 0.2323 | 2.8562    | 2.8359  | 0.2424 |

TABLE IV. IMPACT OF ERRONEOUS DATA IN THE INPUT OF TWO PMUS ON THE PRECISION, VMD, AND VAD CRITERIA.

| Config. number | PMU <sup>Cost</sup> (P.U.) | 15%       |         |        | 30%       |         |        |
|----------------|----------------------------|-----------|---------|--------|-----------|---------|--------|
|                |                            | precision | VMD     | VAD    | precision | VMD     | VAD    |
| 1              | 0                          | -         | -       | -      | -         | -       | -      |
| 2              | 2                          | 11.4174   | 11.4082 | 0.3242 | 22.8025   | 22.7949 | 0.4079 |
| 3              | 3                          | 7.4685    | 7.4574  | 0.2810 | 14.8824   | 14.8743 | 0.3391 |
| 4              | 4                          | 5.6336    | 5.6201  | 0.2665 | 11.2217   | 11.2123 | 0.3143 |
| 5              | 5                          | 4.5062    | 4.4904  | 0.2573 | 8.9596    | 8.9489  | 0.2971 |
| 6              | 6                          | 3.7639    | 3.7458  | 0.2515 | 7.4791    | 7.4672  | 0.2859 |
| 7              | 7                          | 3.2307    | 3.2107  | 0.2456 | 6.4105    | 6.3979  | 0.2720 |
| 8              | 8                          | 2.8323    | 2.8101  | 0.2423 | 5.6112    | 5.5975  | 0.2645 |
| 9              | 9                          | 2.8743    | 2.8540  | 0.2442 | 5.6933    | 5.6807  | 0.2667 |

#### IV. CONCLUSION

In this paper, based on the estimated voltage profile accuracy of the network, a framework is presented for PMU placement. In this regard, a precision criterion is investigated for determining the optimal number and location of PMUs in the network based on the maximization of the DSE results' accuracy. To evaluate the efficiency of the proposed method, the developed framework is implemented on the modified 77-bus UK distribution test network. The results show that applying the proposed scheme leads to a PMU configuration that improve the DSE accuracy and network voltage profile estimation.

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