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# Leakage current of a superconductor–normal metal tunnel junction connected to a high-temperature environment

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We consider a voltage-biased normal metal-insulator-superconductor (NIS) tunnel junction, connected to a high-temperature external electromagnetic environment. This model system features the commonly observed subgap leakage current in NIS junctions through photon-assisted tunneling which is detrimental for applications. We first consider a NIS junction directly coupled to the environment and analyze the subgap leakage current both analytically and numerically; we discuss the link with the phenomenological Dynes parameter. Then, we focus on a circuit where a low-temperature lossy transmission line is inserted between the NIS junction and the environment. We show that the amplitude of the transmitted frequencies relevant for the photon-assisted tunneling is exponentially suppressed as the length  $\ell$  and the resistance per unit length  $R_0$  of the line are increased. Consequently, the subgap current is reduced exponentially as well. This property can not be obtained by means of lumped circuit elements. We finally discuss our results in view of the performance of NIS junctions in applications.

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## I. INTRODUCTION

The peculiar nature of single-particle electronic transport through a normal metal-insulator-superconductor (NIS) junction is at the origin of several interesting applications. Such junctions are widely used in experiments of mesoscopic physics as a spectroscopic tool,<sup>1,2</sup> as a very sensitive thermometer,<sup>3–5</sup> and as a key element in nanorefrigeration.<sup>3,6,7</sup> Furthermore, NIS junctions are currently investigated in view of achieving a high accuracy when controlling the current through a single-electron SINIS turnstile. Such a device is one of the interesting candidates for the completion of the so-called quantum metrological triangle, i.e., it can be used to obtain a precise realization of current.<sup>8,9</sup> These applications are all based on the existence of the Bardeen-Cooper-Schrieffer (BCS) energy gap  $\Delta$  in the density of states (DOS) of the superconductor.<sup>10</sup> Ideally, one would expect no single-electron current to flow through a NIS junction at low temperature as long as the bias voltage  $V$  satisfies the inequality  $-\Delta < eV < \Delta$ .

In practice, the subgap current is different from zero. This is a central problem which limits the performance of applications based on energy-selective single-particle transport in NIS junctions. The presence of unwanted accessible states in the subgap region manifests itself as a smearing of the junction's current-voltage ( $I$ - $V$ ) characteristic as well as of its differential conductance. Giaever was the first to experimentally study the NIS junction. He noticed that this deviation from the ideal behavior was present even if the junction was kept at a temperature much lower than the critical one  $T_c$  of the superconductor.<sup>11</sup> A possible source of subgap leakage currents is the occurrence of many-electron tunneling processes, such as Andreev reflection.<sup>12–14</sup> However, these many-electron processes are strongly suppressed if the tunnel resistance  $R_T$  of the junction is chosen high enough and do not account for the observed residual subgap transport either.

Dynes modified the BCS superconducting DOS introducing a single phenomenological dimensionless parameter  $\gamma_{\text{Dynes}}$  in order to fit the behavior of the subgap quasiparticle tunneling current through a Josephson junction.<sup>15</sup> The modified DOS, normalized to the corresponding normal-state DOS at the Fermi energy, is given by

$$N_S^{\text{Dynes}}(E) = \left| \text{Re} \left[ \frac{E/\Delta + i\gamma_{\text{Dynes}}}{\sqrt{(E/\Delta + i\gamma_{\text{Dynes}})^2 - 1}} \right] \right|. \quad (1)$$

It can be seen that  $\gamma_{\text{Dynes}}$  indeed accounts for the broadening of the DOS around  $\Delta$  and the occurrence of states within the gap. This expression is frequently used in both numerical and analytical calculations,<sup>16</sup> but concerning the microscopic origin of the Dynes parameter  $\gamma_{\text{Dynes}}$ , for temperatures far below  $T_c$ , relatively little is known. In general, the smearing of the DOS can be energy dependent.

Recently, it was realized that the exchange of energy between the NIS junction and its surrounding electromagnetic environment may be one of the causes of the smearing of the BCS DOS.<sup>17,18</sup> Indeed, under certain conditions, energy absorption from such an environment enables the crossing of the tunnel barrier by single electrons even for  $|V|$  much less than  $\Delta/e$ . Within this framework, an analytical expression for  $\gamma_{\text{Dynes}}$  has been obtained in terms of the parameters characterizing the NIS junction's environment.<sup>17</sup> In this particular case, the Dynes parameter found describes the smearing at all energies. Following the idea of photon-assisted tunneling demonstrated in Ref. 17, we generalize the approach here for an external circuit characterized by an arbitrary impedance  $Z(\omega)$ , kept at a temperature  $T_{\text{env}}$  that is not necessarily the temperature  $T_{\text{jun}}$  of the NIS junction [see Fig. 1(a)]. We obtain expressions for the subgap leakage current and the subgap Dynes parameter  $\gamma_{\text{Dynes}}^{\text{sub}}$ , valid for energies smaller than the gap  $\Delta$ . Then, we turn our attention to the circuit depicted in Fig. 1(b), where we study the effects of the insertion of a

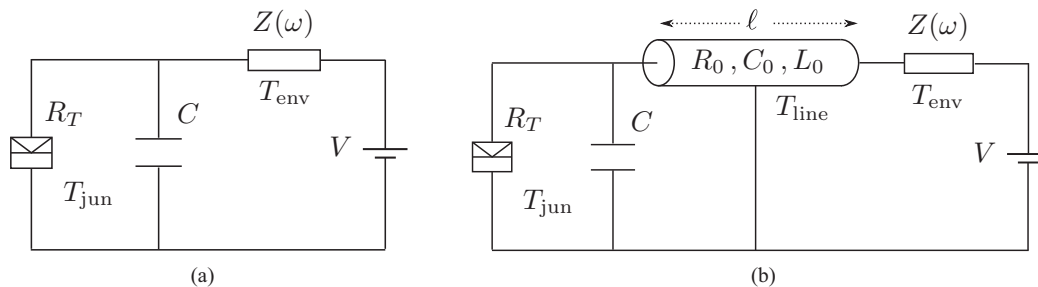


FIG. 1. Circuit representation of the two configurations studied in this paper. (a) A NIS junction at temperature  $T_{\text{jun}}$  is connected in parallel to its capacitance  $C$  and to an impedance  $Z(\omega)$  which represents the high-temperature environment at temperature  $T_{\text{env}} \gg T_{\text{jun}}$ . The whole circuit is biased by the constant voltage  $V$ . (b) A transmission line of length  $\ell$  is inserted between the junction and the impedance  $Z(\omega)$  of circuit (a). It is described by the parameters  $R_0$ ,  $C_0$ , and  $L_0$ , the resistance, the capacitance, and the inductance per unit length, respectively, as well as by its temperature  $T_{\text{line}}$  which is assumed equal to  $T_{\text{jun}}$ .

lossy transmission line, meant to act as a frequency-dependent filter, between the cold junction and the high-temperature external impedance  $Z(\omega)$ . In particular, we use our results to understand under which conditions the transmission line will behave as a filter capable of reducing the photon-assisted tunneling induced by the high-temperature external impedance and thus reducing  $\gamma_{\text{Dynes}}^{\text{sub}}$  to values that are compatible with the accuracy requirements for applications such as the SINIS turnstile.

## II. NIS JUNCTION COUPLED TO A HIGH-TEMPERATURE ENVIRONMENT

### A. Single-particle current

We start by considering the basic circuit illustrated in Fig. 1(a) where a NIS junction is connected in series to an effective high-temperature impedance  $Z(\omega)$ . The junction itself is characterized by a tunnel resistance  $R_T$  in parallel with a capacitance  $C$ . The entire circuit is voltage biased. This constitutes a minimal model for a junction embedded in an external electromagnetic environment at temperature  $T_{\text{env}}$ , which can be much higher than the temperature  $T_{\text{jun}}$  of the junction.

According to the so-called  $P(E)$  theory,<sup>19</sup> the single-particle tunneling current through a NIS junction coupled to an external environment is given by

$$I_{\text{NS}}(V) = \frac{1}{eR_T} \int dE \int dE' N_S(E') [1 - f(E')] \times \{f(E - eV) - f(E + eV)\} P(E - E'). \quad (2)$$

Here, the energy  $E$  refers to the electrons of the normal metal,  $E'$  is the energy of the superconductor quasiparticles,  $N_S(E')$  is the BCS density of states of the superconducting wire divided by the normal-metal DOS at the Fermi level, and  $f(E) = [e^{\beta_{\text{jun}} E} + 1]^{-1}$  is the Fermi-Dirac distribution with  $\beta_{\text{jun}} = 1/k_B T_{\text{jun}}$  the inverse temperature of the junction. Expression (2) does not take into account the higher-order processes in tunneling which will be ignored throughout this paper. The validity of this assumption will be discussed in Sec. IV.

The function  $P(E)$  in Eq. (2) is the probability density that the tunneling electron exchanges an amount of energy  $E$  with the environment. This process takes place through the emission

or absorption of photons. It is defined as

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt e^{iEt/\hbar} e^{J(t)}, \quad (3)$$

i.e., it is the Fourier transform of the exponential of the correlation function

$$J(t) = 2 \int_0^{+\infty} \frac{d\omega}{\omega} \frac{\text{Re}[Z_{\text{tot}}(\omega)]}{R_K} \times \left\{ \coth\left(\frac{1}{2}\beta_{\text{env}}\hbar\omega\right) [\cos(\omega t) - 1] - i \sin(\omega t) \right\}. \quad (4)$$

Here,  $Z_{\text{tot}}(\omega)$  is the total impedance seen by the junction, resulting from the connection in parallel of  $C$  and  $Z(\omega)$ ,  $R_K = h/e^2$  is the quantum resistance, and  $\beta_{\text{env}} = 1/k_B T_{\text{env}}$ .

The function  $J(t)$  determines the strength of the coupling between the NIS junction and the environment. Indeed, if  $J(t) = 0$ , the probability density  $P(E)$  is equal to a Dirac delta  $\delta(E)$  and the single-particle tunneling current is elastic. Expression (2) then reduces to the standard expression for single-particle tunneling in NIS junctions valid in the absence of environment. The environment-induced inelastic tunneling processes occur only when  $J(t) \neq 0$ . In general, the time intervals where the inelastic effects are important are related to the energy ranges where  $P(E) \neq 0$ . The order of magnitude of  $J(t)$  sets the number of photons responsible for the single-particle tunneling. Depending on this number, the coupling between the NIS junction and the multimode environment can be considered weak or strong. Throughout this paper we will treat both regimes of weak and strong coupling in more detail.

In order to analyze the smearing of the NIS junction's  $I$ - $V$  characteristic due to the presence of the high-temperature environment, we will ignore the thermal smearing induced by finite temperature of the N and S electrodes. This is an adequate approximation under standard experimental conditions where  $T_{\text{jun}} \ll \Delta/k_B$ . Hereafter, we will set the temperature of the junction  $T_{\text{jun}}$  to zero. Under this assumption, the single-particle current (2) becomes

$$I_{\text{NS}}(V) \simeq \frac{1}{eR_T} \int_{-eV}^{+eV} dE \int_{\Delta}^{+\infty} dE' N_S(E') P(E - E'). \quad (5)$$

We furthermore will focus on the subgap region of the  $I$ - $V$  curve considering  $|eV| \ll \Delta$ . As a result, the integration variables  $|E| \ll E'$  in Eq. (5), and we can approximate

$P(E - E') \approx P(-E')$ . The resulting integral over  $E$  can be performed immediately to yield

$$I_{\text{NS}}^{\text{sub}}(V) \simeq \gamma_{\text{env}} \frac{V}{R_T}, \quad (6)$$

where the factor  $\gamma_{\text{env}}$  is given by the integral

$$\gamma_{\text{env}} = 2 \int_{-\Delta}^{+\infty} dE' N_S(E') P(-E'). \quad (7)$$

We see that for the parameter  $\gamma_{\text{env}}$  [Eq. (7)] and hence the subgap current given by Eq. (6) to be nonzero, the function  $P(E)$  should be nonzero for energies  $E \leq -\Delta$ . This reflects the fact that under subgap conditions  $eV, k_B T_{\text{jun}} \ll \Delta$ , a nonzero single-particle current occurs only if the tunneling electrons absorb an energy  $\gtrsim \Delta$  from the environment. For instance,  $\gamma_{\text{env}} = 0$  for elastic tunneling in the absence of an environment, when  $P(E) = \delta(E)$ . We also expect  $\gamma_{\text{env}}$  to vanish when the temperature of the environment  $k_B T_{\text{env}}$  is much less than the energy gap  $\Delta$ . Indeed, due to detailed balance,<sup>19</sup>  $P(-E) = e^{-E/k_B T_{\text{env}}} P(E)$ , the function  $P(E)$  is strongly suppressed for negative energies  $E < -k_B T_{\text{env}}$ . This means that the integral in Eq. (7) will vanish unless the environment is sufficiently hot,  $k_B T_{\text{env}} \gtrsim \Delta$ .

In order to make a connection with the aforementioned approach due to Dynes, we linearize the usual expression for elastic single-particle tunneling in a NIS junction, using the Dynes DOS (1) to characterize the superconducting electrode. One obtains the linear subgap current-voltage relationship

$$I_{\text{NS}}^{\text{sub}}(V) \simeq \sqrt{\frac{\gamma_{\text{Dynes}}^2}{\gamma_{\text{Dynes}}^2 + 1}} \frac{V}{R_T}.$$

Comparing this result with Eq. (6) above, we conclude that, in the linear regime,  $\gamma_{\text{env}}$  can be related to the Dynes parameter in the subgap region  $\gamma_{\text{Dynes}}^{\text{sub}}$  according to  $\gamma_{\text{env}} = \sqrt{\gamma_{\text{Dynes}}^{\text{sub}2} / (\gamma_{\text{Dynes}}^{\text{sub}2} + 1)}$ . We see in particular that the two parameters coincide  $\gamma_{\text{Dynes}}^{\text{sub}} = \gamma_{\text{env}}$  whenever  $\gamma_{\text{env}}, \gamma_{\text{Dynes}}^{\text{sub}} \ll 1$ . This shows that fluctuations of a high-temperature electromagnetic environment constitute a possible microscopic source of the phenomenological Dynes parameter, at least under subgap conditions  $eV, k_B T_{\text{jun}} \ll \Delta$ .

### B. Weak and strong coupling

As we have seen above, the strength of the coupling between the NIS junction and the environment is determined by the function  $J(t)$ . Let us assume that this function is small, in a sense to be detailed in the following. Expanding the exponential function  $\exp[J(t)]$  up to the first order in  $J(t)$ , Eq. (3) becomes

$$P(E) \simeq \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt e^{iEt/\hbar} [1 + J(t)]. \quad (8)$$

The evaluation of the integral over time in Eq. (8) gives

$$P(E) \simeq \delta(E) + \frac{1}{\hbar} \int_0^{+\infty} \frac{d\omega}{\omega} \frac{\text{Re}[Z_{\text{tot}}(\omega)]}{R_K} \times \left\{ \left[ \coth\left(\frac{1}{2}\beta_{\text{env}}\hbar\omega\right) - 1 \right] \delta\left(\frac{E}{\hbar} + \omega\right) \right.$$

$$\left. + \left[ \coth\left(\frac{1}{2}\beta_{\text{env}}\hbar\omega\right) + 1 \right] \delta\left(\frac{E}{\hbar} - \omega\right) - 2\hbar \coth\left(\frac{1}{2}\beta_{\text{env}}\hbar\omega\right) \delta(E) \right\}. \quad (9)$$

We see that the function  $P(E)$  has an elastic contribution and an inelastic one involving the exchange of exactly one photon between the junction and the environment. In fact, the first and the fourth terms represent the elastic tunneling involving zero and one virtual photon, respectively. The second and third terms are related to the process of absorption and emission of one real photon, respectively. We define this one-photon regime as weak coupling. On the other hand, the coupling becomes strong whenever the single-photon exchange between the junction and the environment is no longer the dominant effect. In this case, the higher-order terms can not be neglected in the series expansion of  $\exp[J(t)]$ , indicating that multiphoton processes have to be taken into account.

We proceed by determining the time interval where the expansion (8) holds. Given the fact that  $J(t=0) = 0$ , we expect this to be the short-time interval.<sup>19</sup> We set  $Z(\omega) = R$  for simplicity and introduce the dimensionless time  $\tau = t/R_K C$  as well as the ratio  $\rho = R/R_K$ . The quantity  $\exp[\text{Re}\{J(\tau, \rho)\}]$  decays monotonically with increasing time  $\tau$ , starting from unity at  $\tau = 0$ . The rate at which it decays depends on  $\rho$ : the larger  $\rho$ , the faster it decays, in agreement with Ref. 19. We determine the relevant short-time interval by determining the characteristic time  $\tau_{10\%}$ , at which the quantity  $\exp[\text{Re}\{J(\tau, \rho)\}]$  dropped by 10%. Figure 2(a) shows  $\tau_{10\%}$  as a function of the parameter  $\rho$ , keeping  $T_{\text{env}}$  and  $C$  fixed. The line  $\tau_{10\%}(\rho)$  separates the weak coupling regime found at short times from the strong coupling regimes reached for longer times. As expected,<sup>19</sup> with increasing  $\rho$ , the separatrix  $\tau_{10\%}(\rho)$  decreases as  $1/\rho$ , and then saturates at a value  $\tau_S \sim \sqrt{\hbar/k_B T_{\text{env}} R_K C}$  for  $\rho > \rho_{\text{th}} \sim \tau_S$ . As shown in Fig. 2(b), the curve  $\tau_{10\%}(\rho)$  shifts up when decreasing the temperature of the environment  $T_{\text{env}}$ , thereby increasing the time interval where the expansion (8) holds.

We now return to the inelastic tunneling of single electrons through the NIS junction. Under subgap conditions  $k_B T_{\text{jun}}, eV \ll \Delta$ , the energy  $E$  relevant for the photon-assisted tunneling processes is in the interval  $\Delta \lesssim E \lesssim k_B T_{\text{env}}$ . The upper bound corresponds to the largest energy the junction can absorb from the environment. In time domain, we thus have to consider the interval  $\tau_e < \tau < \tau_\Delta$  where  $\tau_\Delta = \hbar/\Delta R_K C$  and  $\tau_e = \hbar/k_B T_{\text{env}} R_K C$ . This interval is represented by the colored strip in Fig. 2(a). Note that on the logarithmic scale used here, the lower bound  $\tau_e$  almost coincides with the value  $\tau_S$  at which the separatrix saturates for large values of  $\rho$ . The intersection between  $\tau_\Delta$  and the 10% curve  $\tau_{10\%}(\rho)$  defines the characteristic resistance  $\rho_\Delta$  separating the weak and strong coupling regimes. When  $\rho < \rho_\Delta$ , coupling is weak and only single-photon absorption processes occur (green area); if  $\rho \sim \rho_\Delta$ , both single-photon and multiphoton processes occur during single-electron tunneling (yellow-orange area); as soon as  $\rho \gg \rho_\Delta$ , multiphoton processes become dominant (red area). In particular, the two limiting cases  $\rho \ll \rho_\Delta, \rho_{\text{th}}$  and  $\rho \gg \rho_\Delta, \rho_{\text{th}}$  are equivalent to the conditions  $R/R_K \ll \Delta/k_B T_{\text{env}}$  and  $R/R_K \gg \Delta/k_B T_{\text{env}}$ , respectively.

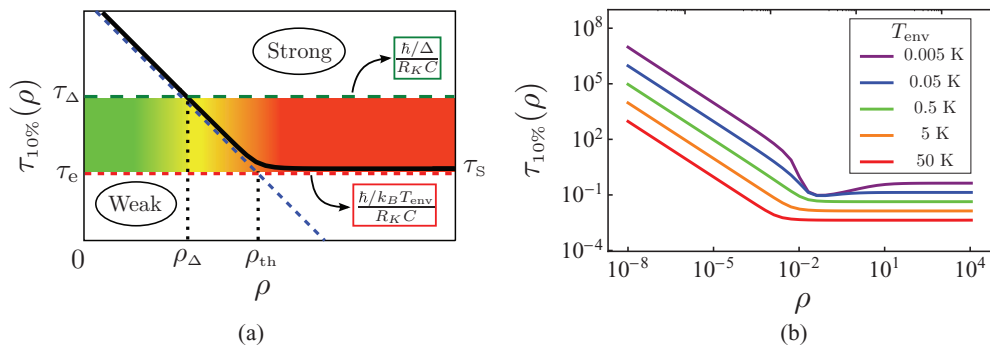


FIG. 2. (Color online) Plot of the separatrix  $\tau_{10\%}(\rho)$  as a function of the dimensionless resistance  $\rho = R/R_K$ , defined as the solution of the equation  $\exp[\text{Re}[J(\tau_{10\%}, \rho)]] = 0.9$  with  $C$  fixed. Both plots are in double-logarithmic scale. (a) For a fixed value of  $T_{\text{env}}$ ,  $\tau_{10\%}(\rho)$  separates the weak and strong coupling regions (black thick line). The colored strip indicates the time interval bound by  $\tau_\Delta = \hbar/\Delta R_K C$  (dark green dashed line) and  $\tau_e = \hbar/k_B T_{\text{env}} R_K C$  (red dashed line). The intersection between  $\tau_\Delta$  and the separatrix defines the resistance  $\rho_\Delta$ . The asymptotic expression for  $\tau_{10\%}(\rho)$  valid for  $\rho \rightarrow 0$  and proportional to  $1/\rho$  is also shown (blue dashed line). Its intersection with the line corresponding to  $\tau_S \sim \sqrt{\hbar/k_B T_{\text{env}} R_K C}$  defines the threshold resistance  $\rho_{\text{th}}$ . On the logarithmic scale used here,  $\tau_S$  almost coincides with  $\tau_e$ . (b) As the temperature of the environment  $T_{\text{env}}$  is decreased, the curve  $\tau_{10\%}(\rho)$  moves up, thereby increasing the weak coupling region.

### C. Subgap leakage current: Weak coupling

We start by dealing with the weak coupling case. Since we are interested in the subgap region of the  $I$ - $V$  characteristic  $k_B T_{\text{jun}}, eV \ll \Delta$ , the behavior of the function  $P(E)$  at energies  $E > -\Delta$  is irrelevant. Therefore, we can ignore the elastic contributions in Eq. (9). Evaluating the integral over frequencies in Eq. (9), the relevant contribution to the function  $P(E)$  for energies  $E \neq 0$  reads as

$$P(E) \simeq 2 \frac{\text{Re}[Z_{\text{tot}}(E/\hbar)]}{R_K} \left( \frac{1 + n(E)}{E} \right). \quad (10)$$

Here,  $n(E) = [e^{\beta_{\text{env}} E} - 1]^{-1}$  is the Bose-Einstein distribution of the photons of the environment.

The probability density (10) can be used to get a limiting expression for  $\gamma_{\text{env}}$ :

$$\gamma_{\text{env}} = 4 \int_{\Delta}^{+\infty} dE N_S(E) \frac{\text{Re}[Z_{\text{tot}}(E/\hbar)]}{R_K} \frac{n(E)}{E}. \quad (11)$$

Let us apply this result to the example of a purely resistive external environment. This model has been used before to study devices based on tunnel junctions in connection with experiments.<sup>17,20,21</sup> Replacing the external impedance  $Z(\omega)$  of the circuit of Fig. 1(a) by a pure resistance  $R$ , the real part of the total impedance is

$$\text{Re}[Z_{\text{tot}}(\omega)] = \frac{R}{1 + (\omega RC)^2}. \quad (12)$$

Numerical integration of Eq. (11) using Eq. (12) is straightforward. Results for  $R_K \gamma_{\text{env}}/R$  as a function of  $k_B T_{\text{env}}/\Delta$  are shown in Fig. 3 for various values of the parameter  $\Delta RC/\hbar$ . We see that  $\gamma_{\text{env}}$  increases monotonically with temperature. Also shown is the asymptotic linear temperature dependence of  $\gamma_{\text{env}}$  reached for temperatures  $k_B T_{\text{env}} \gg \Delta$ :

$$\gamma_{\text{env}} \simeq 2\pi \frac{R}{R_K} \frac{k_B T_{\text{env}}}{\Delta} \left[ 1 - \frac{\Delta RC/\hbar}{\sqrt{1 + (\Delta RC/\hbar)^2}} \right]. \quad (13)$$

This high-temperature expression is correct up to a constant shift  $\approx \Delta/k_B$  along the temperature axis [see Fig. 3].

From Fig. 3 we see that as the parameter  $\Delta RC/\hbar$  is increased, the slope characterizing the limiting dependence decreases: photon-assisted inelastic tunneling is effectively reduced by increasing the junction capacitance. Note that in the limit  $\Delta RC/\hbar \ll 1$ , the result (13) tends to  $\gamma_{\text{env}}^{\text{D}} = 2\pi(R/R_K)(k_B T_{\text{env}}/\Delta)$ . This formula has been already obtained in Ref. 17 using  $P(E)$  theory under similar conditions, but for any bias voltage, using a high-temperature expansion for the environment [see also Ref. 22].

Consequently, the subgap parameter  $\gamma_{\text{env}}^{\text{D}}$  coincides with the Dynes parameter  $\gamma_{\text{Dynes}}$ . In other words, within this limiting case, the thermal energy  $k_B T_{\text{env}}$  determines the NIS junction's  $I$ - $V$  characteristic even around the superconducting gap  $e|V| \sim \Delta$ .

### D. Subgap leakage current: Strong coupling

We do not aim to present a general analysis in the strong coupling limit. In the particular case where  $\text{Re}[Z_{\text{tot}}(\omega)]$  is strongly peaked around  $\omega = 0$ , the probability density  $P(E)$  can be calculated explicitly<sup>19</sup> and results for the parameter  $\gamma_{\text{env}}$  obtained. Let us illustrate this by considering a purely resistive environment. When the resistance is big,  $R \gg R_K \Delta/k_B T_{\text{env}}$

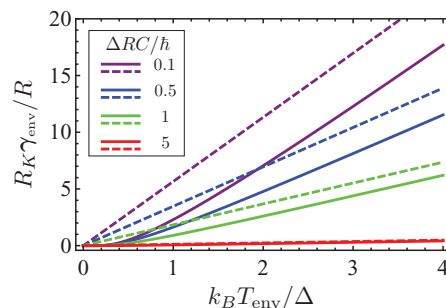


FIG. 3. (Color online) Plot of the rescaled parameter  $R_K \gamma_{\text{env}}/R$  as a function of  $k_B T_{\text{env}}/\Delta$  for different values of the ratio  $\Delta RC/\hbar$ . Solid lines are obtained by a numerical integration of Eq. (11) using Eq. (12). Dashed lines refer to the asymptotic  $\gamma_{\text{env}}$  given by Eq. (13).



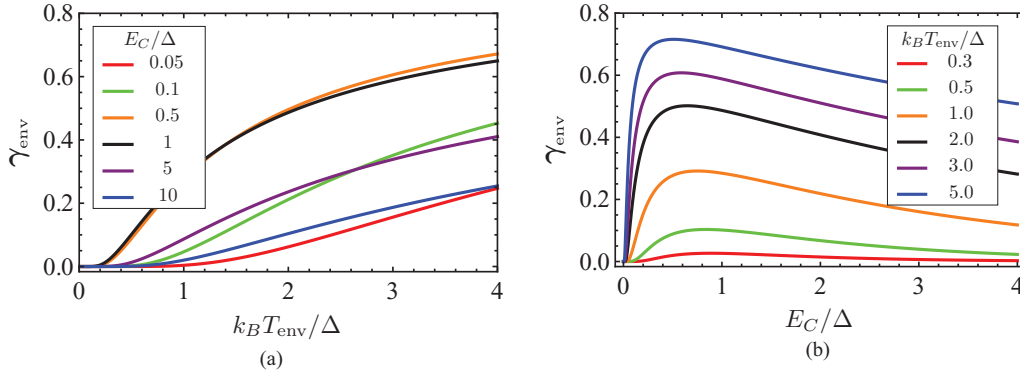


FIG. 4. (Color online) (a) Plot of the parameter  $\gamma_{\text{env}}$  as a function of  $k_B T_{\text{env}}/\Delta$  obtained considering the numerical integration of Eq. (16). Each curve refers to a certain fixed value of the ratio  $E_C/\Delta$  (see legend). (b) Numerical plot of the same quantity [Eq. (16)] as a function of  $E_C/\Delta$  for different values of the ratio  $k_B T_{\text{env}}/\Delta$ , as indicated.

(see Sec. II B), the impedance (12) becomes

$$\text{Re}[Z_{\text{tot}}(\omega)] \simeq \left(\frac{\pi}{C}\right) \delta(\omega). \quad (14)$$

As a result, the function  $P(E)$  is given by

$$P(E) \simeq \frac{1}{\sqrt{4\pi k_B T_{\text{env}} E_C}} \exp\left[-\frac{(E - E_C)^2}{4k_B T_{\text{env}} E_C}\right]. \quad (15)$$

Here, we defined the charging energy  $E_C = e^2/2C$ . Inserting the function (15) in Eq. (7), we find

$$\gamma_{\text{env}} = \frac{1}{\sqrt{\pi E_C k_B T_{\text{env}}}} \int_{\Delta}^{+\infty} dE N_S(E) \exp\left[-\frac{(E + E_C)^2}{4E_C k_B T_{\text{env}}}\right]. \quad (16)$$

Note that this result depends on  $R$  implicitly only, through the requirement  $R \gg R_K \Delta/k_B T_{\text{env}}$ . Direct numerical integration of (16) yields  $\gamma_{\text{env}}$  as a function of  $k_B T_{\text{env}}/\Delta$  and  $E_C/\Delta$ , as shown in Figs. 4(a) and 4(b). Some remarks are in order at this point. First of all, for  $E_C \ll \Delta$ , the integral in Eq. (16) can be evaluated approximately,  $\gamma_{\text{env}} \simeq e^{-\Delta^2/k_B T_{\text{env}} E_C}$ . As in the weak coupling regime, large values of the capacitance lead to a reduction of the parameter  $\gamma_{\text{env}}$ . Upon increasing the ratio  $E_C/\Delta$ ,  $\gamma_{\text{env}}$  will first increase, then it decreases again when  $E_C/\Delta > 1$ , which is a manifestation of the Coulomb blockade. As a function of temperature,  $\gamma_{\text{env}}$  increases monotonically, similarly to the weak coupling limit. However, rather than reaching an asymptotic linear dependence,  $\gamma_{\text{env}}$  saturates at  $\gamma_{\text{env}} = 1$  for temperatures  $k_B T_{\text{env}} E_C \gg \Delta^2$ : the noise is so strong that features of the order of the gap  $\Delta$  are washed out.

### III. NIS JUNCTION COUPLED TO A HIGH-TEMPERATURE ENVIRONMENT BY MEANS OF A TRANSMISSION LINE

In the previous section, we have studied the subgap leakage current in a NIS junction which is directly coupled to the external environment  $Z(\omega)$ . We have seen that a reduction of the subgap leakage current is possible when the capacitance of the junction  $C$  is increased and/or the resistance of the environment  $R$  is decreased. Unfortunately, in real experiments  $R$ , and in particular  $C$ , can not be chosen arbitrarily and one needs other means to achieve the

accuracy requirements for the aforementioned NIS junction's applications. We therefore consider the circuit of Fig. 1(b) where the junction is indirectly coupled to the external environment via a cold, lossy transmission line acting as a frequency-dependent filter.

#### A. Voltage fluctuations in the presence of a transmission line

In order to find the correlation function  $J(t)$  in the presence of the transmission line, we follow the method developed in Ref. 23 to solve the intermediate problem of the propagation of the noise generated by the high-temperature environment with impedance  $Z(\omega)$  through the line towards the junction, as shown in Fig. 5. The line has a length  $\ell$  and is described by the parameters  $R_0$ ,  $C_0$ , and  $L_0$ , the resistance, the capacitance, and the inductance per unit length, respectively. We ignore the thermal noise produced by the impedance  $Z_J(\omega)$  and by the line, assuming both components at zero temperature. The high-temperature element produces current noise  $\delta I$  which in turn induces voltage noise  $\delta V$ .

To understand how the potential drop  $\delta V_J$  across  $Z_J(\omega)$  is connected to  $\delta V = Z(\omega) \delta I$ , we start considering the potential  $V(x)$  and the current  $I(x)$  at a given point  $x$  along the transmission line. They satisfy the two partial differential equations

$$\frac{\partial V(x)}{\partial x} = -I(x)[R_0 - i\omega L_0], \quad \frac{\partial I(x)}{\partial x} = i\omega C_0 V(x).$$

Combining them, one obtains the wave equation

$$\frac{\partial^2 V(x)}{\partial x^2} = -K^2(\omega) V(x), \quad (17)$$

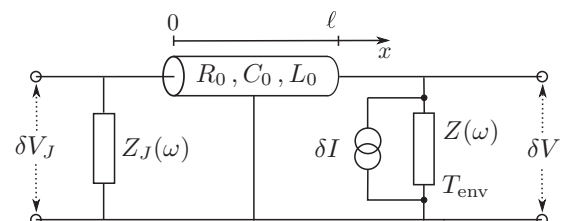


FIG. 5. Sketch of the circuit discussed in Sec. III A.

where  $K^2(\omega) = \omega^2 L_0 C_0 + i\omega R_0 C_0$  is the wave vector squared of the signal which propagates along the line. A general solution of Eq. (17) is given by

$$V(x) = A e^{iK(\omega)x} + B e^{-iK(\omega)x}. \quad (18)$$

Consequently, the current along the line is

$$I(x) = \frac{1}{Z_\infty(\omega)} [A e^{iK(\omega)x} - B e^{-iK(\omega)x}], \quad (19)$$

with  $Z_\infty(\omega) = i(R_0 - i\omega L_0)/K(\omega)$ . The parameters  $A$  and  $B$  can be determined by means of the boundary conditions

$$\begin{aligned} V(\ell) &= Z(\omega)[I(\ell) + \delta I] = Z(\omega) I(\ell) + \delta V, \\ V(0) &= -Z_J(\omega) I(0). \end{aligned}$$

As a result, the potential drop  $\delta V_J = V(0) = A + B$  across the impedance  $Z_J(\omega)$  depends on the noise  $\delta V$  according to the relation

$$\delta V_J = T(\omega) \delta V. \quad (20)$$

In this last equation, we introduced  $T(\omega)$ , the transmission function

$$\begin{aligned} T(\omega) &= \frac{2 Z_\infty(\omega) Z_J(\omega)}{[Z_\infty(\omega) + Z(\omega)][Z_\infty(\omega) + Z_J(\omega)]} \\ &\times \frac{1}{e^{-iK(\omega)\ell} - \lambda_1(\omega) \lambda_2(\omega) e^{iK(\omega)\ell}}, \end{aligned} \quad (21)$$

where

$$\lambda_1(\omega) = \frac{Z_\infty(\omega) - Z(\omega)}{Z_\infty(\omega) + Z(\omega)}, \quad \lambda_2(\omega) = \frac{Z_\infty(\omega) - Z_J(\omega)}{Z_\infty(\omega) + Z_J(\omega)}$$

are the reflection coefficients. Assuming that the potential  $\delta V$  satisfies the quantum fluctuation-dissipation theorem

$$\langle \delta V(t) \delta V(0) \rangle_\omega = 2\hbar\omega \frac{\text{Re}[Z(\omega)]}{1 - e^{-\beta_{\text{env}}\hbar\omega}},$$

the spectral density function of the potential (20) is

$$\langle \delta V_J(t) \delta V_J(0) \rangle_\omega = |T(\omega)|^2 2\hbar\omega \frac{\text{Re}[Z(\omega)]}{1 - e^{-\beta_{\text{env}}\hbar\omega}}. \quad (22)$$

This expression describes the propagation of the noise from  $Z(\omega)$  to the noiseless impedance  $Z_J(\omega)$  through the noiseless transmission line. The voltage-voltage correlation function (22) is in agreement with the general formula given in Ref. 23.

### B. Correlation function for the transmission line circuit

We use Eq. (22) to calculate the modified correlation function  $J_T(t)$  which appears in Eq. (3). According to Ref. 19,  $J(t)$  is defined as the correlation function

$$J(t) \equiv \langle \varphi_J(t) \varphi_J(0) - \varphi_J(0) \varphi_J(t) \rangle, \quad (23)$$

where the phase  $\varphi_J(t)$  is the time integral of the potential  $\delta V_J(t)$  across the NIS junction

$$\varphi_J(t) \equiv \frac{e}{\hbar} \int_{-\infty}^t \delta V_J(\tau) d\tau.$$

In other words,

$$\langle \varphi_J(t) \varphi_J(0) \rangle_\omega = \left( \frac{e}{\hbar} \right)^2 \frac{1}{\omega^2} \langle \delta V_J(t) \delta V_J(0) \rangle_\omega. \quad (24)$$

Using the fluctuation-dissipation relation (22) in Eq. (24), we rewrite Eq. (23) as a function of  $T(\omega)$ ,  $Z(\omega)$ , and  $T_{\text{env}}$ . Taking the impedance  $Z_J(\omega)$  to be the one of a capacitance  $C$ , the modified function  $J_T(t)$  reads as

$$\begin{aligned} J_T(t) &= 2 \int_0^{+\infty} \frac{d\omega}{\omega} |T_C(\omega)|^2 \frac{\text{Re}[Z(\omega)]}{R_K} \\ &\times \left\{ \coth\left(\frac{1}{2}\beta_{\text{env}}\hbar\omega\right) [\cos(\omega t) - 1] - i \sin(\omega t) \right\}. \end{aligned} \quad (25)$$

Here,  $T_C(\omega)$  is the function  $T(\omega)$  [Eq. (21)] with  $Z_J(\omega) = Z_C(\omega) = -1/i\omega C$ . Since the transmission line is considered noiseless, its temperature  $T_{\text{line}}$  should be low,  $T_{\text{line}} \ll \Delta/k_B$ . In what follows, we set  $T_{\text{line}} = T_{\text{jun}} = 0$ .

### C. Transmission function

In order to understand the effect of the insertion of the transmission line in the circuit of Fig. 1(a), a discussion about the general behavior of  $T_C(\omega)$  is necessary. In general, the modulus squared of the transmission function (21) is characterized by a series of resonance peaks, whose properties depend on  $\ell$ ,  $R_0$ ,  $C_0$ , and  $L_0$  as well as on the external impedance  $Z(\omega)$ . To have an idea of the behavior of  $|T_C(\omega)|^2$ , let us consider the case of a purely resistive environment  $Z(\omega) = R$ .

Figure 6 illustrates the behavior of  $|T_C(\omega)|^2$  as a function of  $\omega RC$  for different values of the dimensionless parameters  $z_0 = \sqrt{L_0/C_0}/R$ ,  $c_0 = \ell C_0/C$ , and  $r_0 = \ell R_0/R$ . Also shown is the Lorentzian result

$$|T_C(\omega)|^2 = 1/[1 + (\omega RC)^2] \quad (26)$$

found for  $\ell = 0$ , i.e., in the absence of the transmission line. In other words, Eq. (26) describes the spectrum of the transmitted signal through a lumped  $RC$  low-pass filter. In order for the line to be an efficient filter, we require  $|T_C(\omega)|^2$  to be below this Lorentzian curve in the relevant frequency ranges. We see that both the position and the width of the resonance peaks are proportional to  $\pi/2c_0z_0$ : the longer the transmission line, the denser around zero and the sharper are the peaks. Their height decreases rapidly as the dimensionless frequency  $\omega RC$  is increased. This can be seen in particular when the line has no losses,  $r_0 = 0$  [see Figs. 6(a)–6(d)]. Although the Lorentzian curve is approached for lossless lines when  $c_0$  or  $z_0$  is reduced, we observe no real reduction below it.

A significant reduction of the height of the peaks is possible if the line which connects the NIS junction and the environment is lossy,  $r_0 > 0$ . Indeed, we see from Figs. 6(e) and 6(f) that the bigger is  $r_0$  the smaller are the local maxima of  $|T_C(\omega)|^2$ . Moreover, the transmission function is even much smaller than  $1/[1 + (\omega RC)^2]$  when the condition  $r_0 \gg z_0$  is satisfied, as is seen in Figs. 6(f) and 7. Therefore, within this particular limit, the insertion of a resistive transmission line may be convenient.

### D. Subgap leakage current: Weak coupling

We expect that the single-photon and multiphoton regimes, weak and strong coupling, respectively, are strongly related to the resistance per unit length  $R_0$ . Let us analyze the situation proceeding as in Sec. II B. We consider the function  $\tau_{10\%}(\rho)$  for

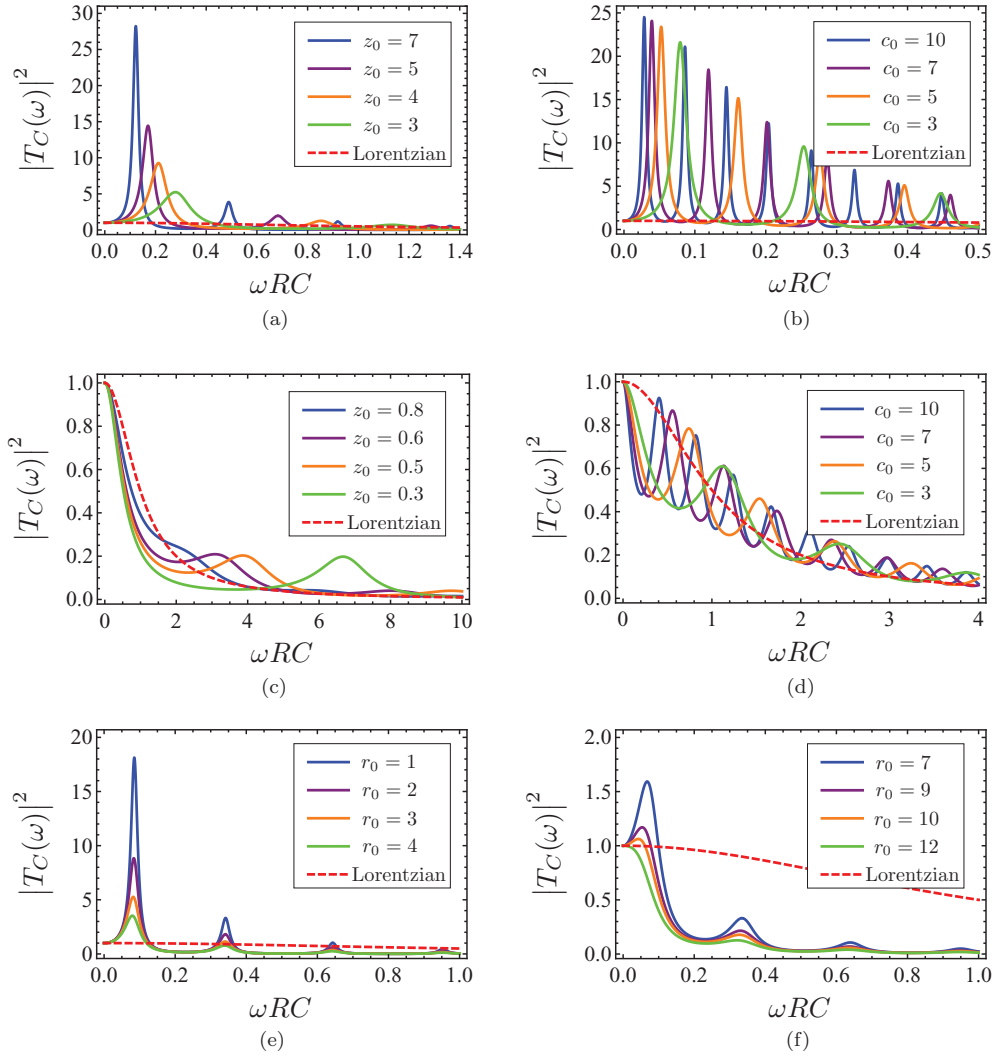


FIG. 6. (Color online) Plots of the transmission function  $|T_C(\omega)|^2$  as a function of the dimensionless variable  $\omega RC$ . Each panel corresponds to a different set of the parameters  $z_0$ ,  $c_0$ , and  $r_0$ : (a)  $r_0 = 0$ ,  $c_0 = 1$ ,  $z_0 = (7, 5, 4, 3)$ ; (b)  $r_0 = 0$ ,  $z_0 = 5$ ,  $c_0 = (10, 7, 5, 3)$ ; (c)  $r_0 = 0$ ,  $c_0 = 1$ ,  $z_0 = (0.8, 0.6, 0.5, 0.3)$ ; (d)  $r_0 = 0$ ,  $z_0 = 0.7$ ,  $c_0 = (10, 7, 5, 3)$ ; (e)  $z_0 = 10$ ,  $c_0 = 1$ ,  $r_0 = (1, 2, 3, 4)$ ; (f)  $z_0 = 10$ ,  $c_0 = 1$ ,  $r_0 = (7, 9, 10, 12)$ .

a purely resistive environment. In Fig. 8, we plot  $\tau_{10\%}(\rho)$  as a function of the dimensionless resistance  $\rho$  for different values of  $R_0$ . We see that the lossier the transmission line is, the more

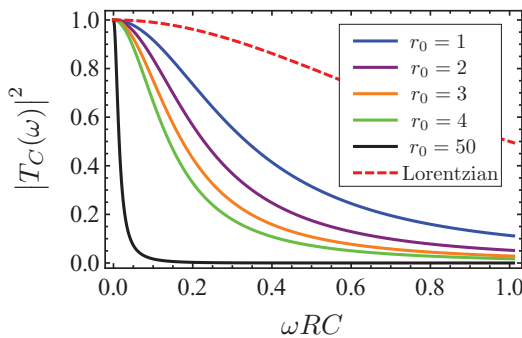


FIG. 7. (Color online) Plot of the transmission function  $|T_C(\omega)|^2$  as a function of the dimensionless variable  $\omega RC$  for different values of the parameter  $r_0$ . The other parameters are  $c_0 = 1$ ,  $z_0 = 0.7$ . Also shown is the Lorentzian corresponding to the function  $|T_C(\omega)|^2$  in the limit  $\ell \rightarrow 0$ , given by Eq. (26).

the weak coupling region spreads out. The resistance  $\rho_\Delta$ , given by the intersection between  $\tau_{10\%}(\rho)$  and the line corresponding to the dimensionless time  $\tau_\Delta = \hbar/\Delta R_K C$ , significantly shifts towards higher values of  $\rho$  as  $R_0$  is increased; the lossy line indeed protects the junction from the high-temperature external environment. Hereafter, we will therefore focus on a highly resistive transmission line and only the weak coupling regime will be treated.

With the help of Eq. (25), the function  $P(E)$  for the circuit of Fig. 1(b) can be obtained in the weak coupling regime. Proceeding as in Sec. II C, we find

$$P(E) \simeq 2 |T_C(E/\hbar)|^2 \frac{\text{Re}[Z(E/\hbar)]}{R_K} \left( \frac{1+n(E)}{E} \right). \quad (27)$$

Evaluating the relation (27) for negative energies and inserting the result into Eq. (7), the parameter  $\gamma_{\text{env}}$  can be written as

$$\gamma_{\text{env}} = 4 \int_{\Delta}^{+\infty} dE N_S(E) |T_C(E/\hbar)|^2 \frac{\text{Re}[Z(E/\hbar)]}{R_K} \frac{n(E)}{E}. \quad (28)$$



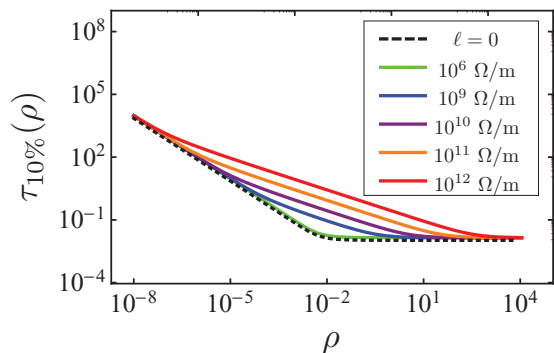


FIG. 8. (Color online) Plot of the dimensionless time  $\tau_{10\%}(\rho)$  as a function of the dimensionless resistance  $\rho = R/R_K$  for different values of the resistance per unit length  $R_0$  of the transmission line (see legend). Also shown is the curve  $\tau_{10\%}(\rho)$ , valid for the circuit of Fig. 1(a) (black dashed line). The values of the other parameters are  $\Delta \simeq 200 \mu\text{eV}$  (energy gap of aluminum),  $T_{\text{env}} = 5 \text{ K}$ ,  $C = 10 \text{ fF}$ ,  $C_0 = 6 \varepsilon_0$ ,  $L_0 = \mu_0$ ,  $\ell = 10 \mu\text{m}$ .

We next specialize to the case of large resistance per unit length  $R_0$ . In order to obtain a limiting expression for  $|T_C(\omega)|^2$  for  $R_0 \rightarrow \infty$ , let us assume that the inductive properties of the line are negligible compared to  $R_0$ . Since the relevant frequency scale is given by  $\Delta/\hbar$ , this means that the condition  $R_0 \gg L_0 \Delta/\hbar$  should hold. Within this  $RC$  limit, we find that the wave vector  $K(\omega)$  of the signal propagating through the transmission line has an imaginary part equal to  $\sqrt{\omega R_0 C_0}/2$ . As a result, the amplitude of the noise is exponentially attenuated along the line [see Eqs. (18) and (19)] being proportional to  $\exp[-\ell \sqrt{2\omega R_0 C_0}]$ . We see that the bigger  $\ell$  and  $R_0$  are, the smaller is the voltage noise which reaches the junction. In particular, an exponential suppression of the propagating signal is achieved when the inequality  $\ell \sqrt{2\Delta R_0 C_0}/\hbar \gg 1$  is valid as well. This additional condition allows us to write the equation

$$|e^{-iK(\omega)\ell} - \lambda_1(\omega) \lambda_2(\omega) e^{iK(\omega)\ell}|^2 \simeq 4 e^{\ell \sqrt{2\omega R_0 C_0}}.$$

Then, the modulus squared of the transmission function  $T_C(\omega)$  becomes

$$|T_C(\omega)|^2 \simeq \left| \frac{Z_\infty(\omega) Z_C(\omega) e^{-\ell \sqrt{2\omega R_0 C_0}/2}}{[Z_\infty(\omega) + Z(\omega)][Z_\infty(\omega) + Z_C(\omega)]} \right|^2, \quad (29)$$

where  $Z_\infty(\omega) \simeq (1+i)\sqrt{R_0/2\omega C_0}$  for a line in the  $RC$  limit. Combining the two conditions used so far, we find that the approximated function (29) holds when the resistance of the transmission line  $\ell R_0$  is much bigger than its characteristic impedance  $Z_\infty = \sqrt{L_0/C_0}$ .

Increasing the resistance per unit length  $R_0$ , one also expects that interference effects become negligible. Indeed, when  $R_0$  is very big, the amplitude of the signal across the junction is much smaller than its starting value and its reflected counterpart vanishes rapidly before reaching the noise source again. In terms of our description of the transmission line given in Sec. III A, this happens when the reflection coefficients  $\lambda_1(\omega)$  and  $\lambda_2(\omega)$  tend to 1. In fact, in this limit, the potential drop (18) tends to 0 across the junction and to  $\delta V$  across the impedance  $Z(\omega)$ . For a purely resistive environment, this regime is reached when  $R_0$  is such that the two inequalities

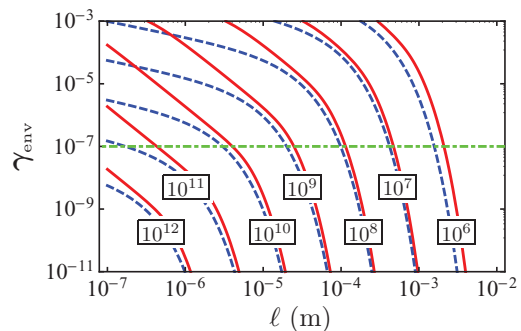


FIG. 9. (Color online) Plot of the parameter  $\gamma_{\text{env}}$  [Eq. (28)] as a function of the length of the transmission line  $\ell$ . The red solid line is obtained by means of the numerical integration of Eq. (28) for a purely resistive environment. The blue dashed line is the plot of the asymptotic parameter given by Eq. (31). These two curves are plotted for different values of the resistance per unit length  $R_0$  ( $\Omega/\text{m}$ ) (as indicated in the graph). All the plots are obtained considering the gap parameter of the aluminum,  $\Delta \simeq 200 \mu\text{eV}$ . The other parameters are  $T_{\text{env}} = 5 \text{ K}$ ,  $C = 10 \text{ fF}$ ,  $R = 10 \Omega$ ,  $C_0 = 6 \varepsilon_0$ ,  $L_0 = \mu_0$ .

$R^2 \ll \hbar R_0/2C_0\Delta$  and  $\Delta R_0 C^2/\hbar C_0 \gg 2$  hold, in other words, the resistance of the environment  $R$  is much smaller than  $R_0 C/2C_0$ . Equation (29) then reduces to the asymptotic expression

$$|T_C(\omega)|^2 \simeq \frac{e^{-\ell \sqrt{2\omega R_0 C_0}}}{1 + \omega R_0 C^2/\hbar C_0}. \quad (30)$$

Unlike the lumped  $RC$  low-pass filter described by the  $1/\omega$ -decaying Eq. (26), in this case we see that the amplitude of the transmitted frequencies relevant for the photon-assisted tunneling is exponentially suppressed as the length  $\ell$  and the resistance per unit length  $R_0$  of the line are increased. By means of Eq. (30), the integral in Eq. (28) can be evaluated approximately with the result

$$\gamma_{\text{env}} \simeq 4 \frac{R}{R_K} \frac{1}{e^{\Delta/k_B T_{\text{env}}} - 1} \times \sqrt{\frac{\pi}{\ell \sqrt{2\Delta R_0 C_0}/\hbar}} \frac{e^{-\ell \sqrt{2\Delta R_0 C_0}/\hbar}}{1 + \Delta R_0 C^2/\hbar C_0}. \quad (31)$$

We notice that also the asymptotic parameter  $\gamma_{\text{env}}$  decreases exponentially in terms of  $\ell$  and  $R_0$ ; the dependence on the junction capacitance  $C$  is rather weak. The insertion of a highly resistive and noiseless transmission line between the NIS junction and the high-temperature environment indeed helps to suppress the subgap leakage current. The plot of Fig. 9 shows the exponential decay for a set of values of  $R_0$  and  $\ell$  that can be used in real experiments. Particularly interesting is the region where  $10^8 \Omega/\text{m} \lesssim R_0 \lesssim 10^{10} \Omega/\text{m}$  and  $10 \mu\text{m} \lesssim \ell \lesssim 10^2 \mu\text{m}$ . A transmission line with these values of  $R_0$  and  $\ell$  allows one to go far below  $\gamma_{\text{env}} \simeq \gamma_{\text{Dynes}}^{\text{sub}} \sim 10^{-7}$ , i.e., a value of  $\gamma_{\text{env}}$  which guarantees the achievement of the accuracy requirements for the superconducting gap-based technological applications of the NIS junction.<sup>9</sup>

#### IV. MULTIPARTICLE TUNNELING

Our analysis focuses on the single-particle subgap current through the NIS junction. We ignore the contribution due

to higher-order processes in tunneling, such as Andreev reflection.<sup>12–14</sup> Hence, in order to establish the validity of our single-particle tunneling assumption, one has to compare the parameter  $\gamma_{\text{env}}$  characterizing the leakage current with the dimensionless Andreev subgap conductance  $g_A = G_A R_T$ . In ballistic junctions, second-order perturbation theory yields the standard two-particle subgap conductance

$$G_A \simeq R_K / [R_T^2 (k_F^2 S)], \quad (32)$$

where  $k_F^2 S$  is the number of conduction channels in the tunnel barrier. Two-electron tunneling can be ignored as long as  $\gamma_{\text{env}} > R_K / R_T k_F^2 S$ . Typical estimates<sup>14</sup> yield  $R_K / R_T k_F^2 S \sim 10^{-7}$ .

On the other hand, in the diffusive case, the electrons reflected by the barrier are backscattered by the impurities randomly situated close to the barrier in the normal metal. Interference between the electrons in a region characterized by the coherence length  $\xi_N = \sqrt{\hbar D} / \max\{eV, k_B T_{\text{jun}}\}$ , where  $D$  is the diffusion coefficient, affects the two-particle tunneling probability.<sup>24</sup> As a result,  $G_A$  is given by

$$G_A \simeq R_N / R_T^2, \quad (33)$$

where  $R_N$  is the resistance of the diffusive normal metal over a length  $\xi_N$ . General estimates are hard to give in this situation since the result is strongly geometry dependent; the condition  $\gamma_{\text{env}} > R_N / R_T$  will be more stringent than the one for the ballistic case, especially under subgap conditions where  $\xi_N$  and hence  $R_N$  can be large.

Should Andreev reflection become dominant, one can always suppress it efficiently using the Coulomb blockade feature<sup>14</sup> that suppresses two-particle tunneling more strongly than single-particle tunneling.

## V. CONCLUSIONS

In conclusion, we studied the single-particle tunneling current through a voltage-biased NIS junction. Due to the presence of the superconducting energy gap  $\Delta$  in the BCS density of states, when the junction is kept at the temperature  $T_{\text{jun}} \ll \Delta / k_B$ , no current is expected to flow within the subgap region  $-\Delta < eV < \Delta$ . Actually, even if the higher-order tunneling processes are suppressed, a small subgap current is still measured experimentally. This leakage current limits the accuracy in applications involving NIS junctions. The origin of the leakage current is the exchange of energy exceeding the gap  $\Delta$  between the junction and the external high-temperature environment in which it is embedded. We studied this mechanism analytically and numerically. In particular, we found that a cold and lossy transmission line inserted between the junction and the environment reduces exponentially the subgap leakage current acting as a frequency-dependent filter. This indirect configuration helps to achieve the required suppression of noise.

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<sup>1</sup>H. Pothier, S. Guéron, Norman O. Birge, D. Esteve, and M. H. Devoret, *Phys. Rev. Lett.* **79**, 3490 (1997).

<sup>2</sup>H. Courtois, M. Meschke, J. T. Peltonen, and J. P. Pekola, *Phys. Rev. Lett.* **101**, 067002 (2008).

<sup>3</sup>F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, *Rev. Mod. Phys.* **78**, 217 (2006).

<sup>4</sup>J. M. Rowell and D. C. Tsui, *Phys. Rev. B* **14**, 2456 (1976).

<sup>5</sup>D. R. Schmidt, C. S. Yung, and A. N. Cleland, *Appl. Phys. Lett.* **83**, 1002 (2003).

<sup>6</sup>N. Nahum, T. M. Eiles, and John M. Martinis, *Appl. Phys. Lett.* **65**, 3123 (1994).

<sup>7</sup>M. M. Leivo, J. P. Pekola, and D. V. Averin, *Appl. Phys. Lett.* **68**, 1996 (1996).

<sup>8</sup>A. Kemppinen, M. Meschke, M. Möttönen, D. V. Averin, and J. P. Pekola, *Eur. Phys. J. Special Topics* **172**, 311 (2009).

<sup>9</sup>J. P. Pekola, O.-P. Saira, V. F. Maisi, A. Kemppinen, M. Möttönen, Yu. A. Pashkin, D. V. Averin, [Rev. Mod. Phys. (to be published)].

<sup>10</sup>J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).

<sup>11</sup>I. Giaever, H. R. Hart Jr., and K. Megerle, *Phys. Rev.* **126**, 941 (1962).

<sup>12</sup>A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys.–JETP **19**, 1228 (1964)].

<sup>13</sup>G. E. Blonder, M. Tinkham, and T. M. Klapwijk, *Phys. Rev. B* **25**, 4515 (1982).

<sup>14</sup>V. F. Maisi, O.-P. Saira, Yu. A. Pashkin, J. S. Tsai, D. V. Averin, and J. P. Pekola, *Phys. Rev. Lett.* **106**, 217003 (2011).

<sup>15</sup>R. C. Dynes, V. Narayanamurti, and J. P. Garno, *Phys. Rev. Lett.* **41**, 1509 (1978).

<sup>16</sup>For a recent reference, see T. Greibe, M. P. V. Stenberg, C. M. Wilson, T. Bauch, V. S. Shumeiko, and P. Delsing, *Phys. Rev. Lett.* **106**, 097001 (2011).

<sup>17</sup>J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Yu. A. Pashkin, O.-P. Saira, M. Möttönen, and J. S. Tsai, *Phys. Rev. Lett.* **105**, 026803 (2010).

<sup>18</sup>O.-P. Saira, M. Möttönen, V. F. Maisi, and J. P. Pekola, *Phys. Rev. B* **82**, 155443 (2010).

<sup>19</sup>G. L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret NATO ASI Series B (Plenum, New York, 1992), Vol. 294, pp. 21–107.

<sup>20</sup>J. M. Martinis and M. Nahum, *Phys. Rev. B* **48**, 18316 (1993).

<sup>21</sup>J. M. Hergenrother, J. G. Lu, M. T. Tuominen, D. C. Ralph, and M. Tinkham, *Phys. Rev. B* **51**, 9407 (1995).

<sup>22</sup>H. Grabert, G.-L. Ingold, and B. Paul, *Europhys. Lett.* **44**, 360 (1998).

<sup>23</sup>J. Siewert, Y. V. Nazarov, and G. Falci, *Europhys. Lett.* **38**, 365 (1997).

<sup>24</sup>F. W. J. Hekking and Yu. V. Nazarov, *Phys. Rev. Lett.* **71**, 1625 (1993); *Phys. Rev. B* **49**, 6847 (1994).