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## Coupling of Zero Sound to the Real Squashing Mode in Rotating $^3\text{He-B}$

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Rotation of superfluid  $^3\text{He-B}$  in a magnetic field enhances the coupling of the nonzero- $m_J$  substates of the real squashing collective mode to the zero sound, and the fivefold line splitting becomes observable even when  $\mathbf{H}$  is parallel to  $\boldsymbol{\Omega}$  and to the direction of sound propagation. Equilibrium vortex lattices and vortex-free states can be distinguished by their characteristic absorption spectra. The dependence of the sound attenuation on the angular velocity in magnetic fields up to 32 mT is reported; the data are qualitatively compared with theory.

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Experiments on rotating superfluid  $^3\text{He}$  have revealed a multitude of intriguing phenomena. The creation of the various types of quantized vortices in the liquid and their characteristics are of special interest, as well as the vortex-free flow states in which the superfluid component remains at rest while the normal component follows the rotation of the experimental cell. Several methods have been employed in studying the properties of rotating  $^3\text{He}$ , including NMR, persistent-current, and ion-mobility measurements (for a review, see Ref. 1).

In this Letter, the first zero-sound experiments in rotating  $^3\text{He}$  are reported. Ultrasound is a powerful probe of collective modes, arising from temporal oscillations of the  $3 \times 3$  complex order-parameter matrix, which characterizes the superfluid  $^3\text{He}$ .<sup>2</sup> The possible modes can be classified according to the total angular momentum quantum number  $J$ , which may have values 0, 1, or 2.<sup>3</sup> When the temperature-dependent frequency of a mode in the superfluid approaches that of the zero sound, the imaginary  $J=0$  mode, an attenuation peak is observed in the absorption spectrum. We have investigated the real  $J=2$  mode, also called the real squashing (rsq) mode.<sup>4</sup>

At rest, with no magnetic field or with  $H \neq 0$  aligned parallel to the direction of sound propagation, defined by the wave vector  $\mathbf{q}$ , the rsq mode gives rise to one intense attenuation peak only, located at  $\hbar\omega_0 = (8/5)^{1/2}\Delta$ ; here  $\Delta$  is the isotropic energy gap in  $^3\text{He-B}$  and  $f_0 = \omega_0/2\pi$  is the frequency of the zero sound. All five substates, with

$m_J = \pm 2, \pm 1, 0$ , can be observed when  $\mathbf{H}$  deviates from the direction of  $\mathbf{q}$ .<sup>5,6</sup>

We find that rotation of the  $^3\text{He-B}$  sample changes the coupling strengths of the substates: The  $m_J = \pm 2, \pm 1$  side peaks become visible even when  $\mathbf{H} \parallel \mathbf{q}$ . The intensities of the peaks vary with angular velocity. The zero sound also proved to be sensitive to different order-parameter distributions in  $^3\text{He-B}$ : When vortices were created in the sample, the ultrasonic absorption spectrum was radically different from that observed when a vortex-free flow state was formed instead.

In the experimental cell, we have two  $X$ -cut quartz-crystal transducers with the fundamental frequency of  $f_1 = 8.895$  MHz, 4 mm apart; the sound propagates along  $\boldsymbol{\Omega}$ , the axis of rotation. The cylindrical ultrasonic chamber, with a radius  $R = 3$  mm, has smooth quartz surfaces, and it is connected to the rest of the  $^3\text{He}$  sample only through small holes in its side wall. The transmitting crystal was excited by a pulse whose duration was 5 or 12  $\mu\text{sec}$ . A dual-line phase-sensitive detector decomposed the received signal into its  $0^\circ$  and  $90^\circ$  components, from which the attenuation,  $\Delta\alpha$ , and the phase of the signal were extracted.

During experiments the temperature of the superfluid was allowed to drift slowly, thereby bringing the various substate frequencies to resonance with the zero sound. Usually the fifth harmonic, at  $f_5 = 44.7$  MHz, was employed to measure the ultrasonic attenuation properties

of  $^3\text{He-B}$  at  $p=3.2$  bars; experiments were made also at 6.5 and 11.5 bars, with the higher harmonics  $f_7=62.6$  MHz or  $f_9=80.6$  MHz.

Spectra of the type that we denote by V, shown in Fig. 1, with the  $m_J=0$  and  $m_J=+1$  resonances merged into a prominent peak at the center and the side peaks barely separating, were invariably and exclusively encountered when the rotation was started before the sample had been cooled to the B phase. This procedure is believed to create an equilibrium vortex lattice, and we thus identify the V spectrum with a vortex state. The reproducibility of the peak heights and their separations was better than 5% from one cooldown to the next.

When rotation was started well below  $T_c$ , another equilibrium state emerged, with equal stability (changes  $< 5\%$ ); the received signal of this type, VF, is shown in Fig. 1. The side peaks now couple more strongly to the zero sound than in the V state, while the intensities of the center peaks decrease. The separation of the clearly visible side peaks in the VF spectrum is increased by 10% with respect to that in the V state. The VF spectrum, being distinctly different from the V spectrum, is interpreted as representing an equilibrium vortex-free state. The VF spectra were always obtained when a very slow acceleration ( $0.004$  rad/sec $^2$ ) was used. At higher values ( $0.008$  or  $0.15$  rad/sec $^2$ ), intermediate spectra (denoted VVF) were sometimes observed, more often at the fastest acceleration rate, representing a combination of counterflow and a nonequilibrium number of vortices. The peak intensities and separations varied up to 10% even within one experiment. These intermediate spectra

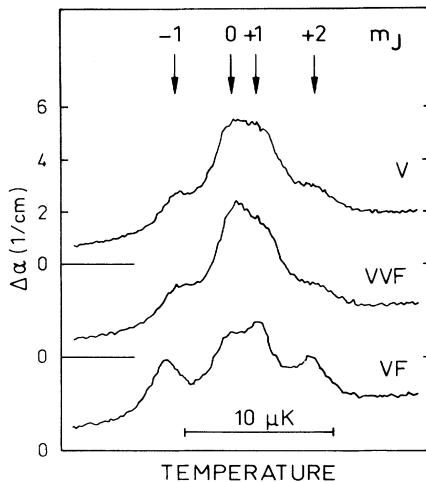


FIG. 1. rsq-mode absorption spectra of  $^3\text{He-B}$ , rotating at  $\Omega=0.87$  rad/sec;  $p=3.2$  bars and  $f_5=44.7$  MHz. The magnetic field  $H=25$  mT was aligned parallel to  $\Omega$  and  $\hat{q}$ . Type-V spectrum represents a vortex state, whereas type VF was obtained in a vortex-free flow state. The VVF spectrum indicates an intermediate nonequilibrium texture. The attenuation  $\Delta\alpha = \alpha(T) - \alpha(T_c)$ ;  $\alpha(T_c)$  is measured just above  $T_c$ .

were resolved at  $p=3.2$  bars and  $H=25$  mT only; in lower fields the Zeeman-split substrate peaks are already quite close to each other, while at  $H=32$  mT only VF spectra were observed. For angular velocities  $\Omega > 0.9$  rad/sec, the VVF spectra did not appear, nor did they at elevated pressures.

The resonance frequencies of substrates in the rsq mode are given in the limit of high  $H$  ( $\geq 50$  mT) by the formula $^7$

$$[\omega(r)]^2 = (\omega_0 + \gamma m_J H)^2 + c_1^2 q^2 + c_2^2 q^2 (\hat{q} \cdot \hat{h})^2, \quad (1)$$

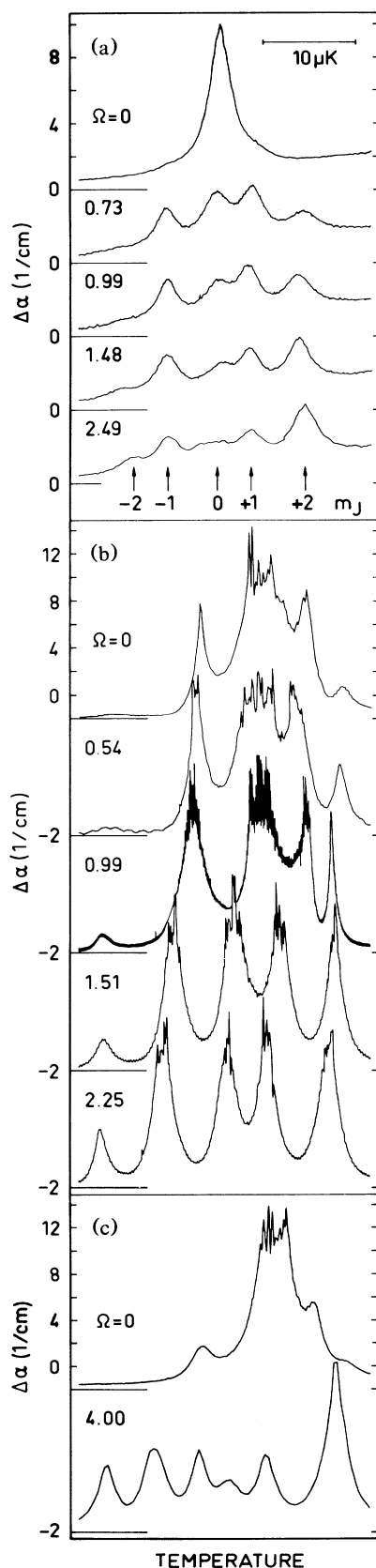
where  $\omega_0$  is the angular frequency given earlier,  $\gamma m_J H$  defines the Zeeman splitting, and  $\hat{q} = \mathbf{q}/q$  is a unit vector in the direction of the sound propagation. The vector  $\hat{h}_i = R_{ai} H_a / H$  defines the effect of the magnetic field on  $R_{ai}$ , the three-dimensional rotation matrix which specifies the order parameter in  $^3\text{He-B}$ . $^2$  Coefficients  $c_1$  and  $c_2$  (on the order of the Fermi velocity  $v_F$ ) are slightly different for each substrate. $^8$  The coordinate dependence is contained in the term with  $\hat{q} \cdot \hat{h}$ .

In the local oscillator approximation, $^{7,8}$  the intensity of a resonance peak is proportional to the spectral density

$$P(\omega') = \frac{1}{\pi R^2} \int r dr \delta[\omega' - \omega(r)] \\ = \frac{2r(\omega')}{R^2} \left| \frac{\partial \omega(r)}{\partial r} \right|_{r=r(\omega')}^{-1} \quad (2)$$

at the applied angular frequency  $\omega'$ . For Eq. (2) to be valid, the wavelength of the zero sound,  $\lambda = 2\pi/q$ , must be significantly shorter than the magnetic healing length  $\xi_H$ , over which the direction of  $\hat{h}$  can change appreciably. This condition is fulfilled in our case:  $\lambda \approx 5$   $\mu\text{m}$  while  $\xi_H \approx 1$  mm, at  $p=3.2$  bars and  $f_5=44.7$  MHz. The intensity of a resonance peak is given by the product of  $P(\omega')$  and the coupling strength of the appropriate mode in  $^3\text{He-B}$  to the zero sound. For the rsq mode, the coupling has been proposed to arise from a small particle-hole asymmetry, $^9$  which leads to a factor  $8[(\Delta/2\epsilon_F) \times \ln(0.1\epsilon_F/\Delta)]^2$ , where  $\epsilon_F$  is the Fermi energy. This quantity has to be multiplied by the angle-dependent Legendre polynomial  $[P_{2m_J}((\hat{q} \cdot \hat{h})^2)]^2$ , $^{5,10}$  which has its maximum value for  $m_J=0$  when  $\hat{h} \parallel \hat{q}$ ; the angular factors of all  $m_J \neq 0$  substrates are zero for this orientation. For a tilted angle between  $\hat{h}$  and  $\hat{q}$ , the Legendre polynomials become nonzero for all five substrates.

According to Eqs. (1) and (2), the maximum zero-sound absorption is expected in those locations of the experimental cell where  $\hat{q} \cdot \hat{h}$  is constant and, consequently, the derivative  $|\partial\omega/\partial r|^{-1}$  makes  $P(\omega')$  infinite; the coupling factor at that angle must simultaneously be nonzero for the mode in question. At rest, with  $H \neq 0$  parallel to  $\hat{q}$ ,  $\hat{h}$  tends to align along  $\mathbf{H}$  (and  $\hat{q}$ ) in the bulk liquid; a prominent  $m_J=0$  peak is observed, while the coupling factors of the other substrates are small. A weak response for the  $m_J \neq 0$  modes is expected from the



liquid near the cylindrical side wall of the experimental cell, where  $\hat{h}$  prefers alignment perpendicular to the wall and, therefore, turns perpendicular to  $\hat{q}$  as well.

Figure 2 illustrates the effect of increased rotation on the attenuation of zero sound by the rsq mode. Only the  $m_J=0$  peak was observed at  $p=3.2$  bars when  $\Omega=0$ ; the rsq mode crossing occurs at  $T=0.81T_c=1.06$  mK, using the Greywall temperature scale,<sup>11</sup> with  $f_5=44.7$  MHz. Rotation changes the situation entirely. The  $m_J \neq 0$  side peaks appear, but only in the presence of a nonzero  $H$ . The splitting is  $150 \mu\text{K}/T$  for  $\Delta m_J=1$ , in good agreement with earlier stationary measurements.<sup>5</sup> At  $p=11.5$  bars and  $H=25$  mT, however, where  $f_9=80.6$  MHz was employed and the temperature of the crossing is  $\approx 0.6T_c$ , the ratio of the Zeeman splitting to the natural width of the resonance lines<sup>9</sup> is larger than at 3.2 bars, and low-intensity side peaks became visible. At 6.5 bars additional textural splitting<sup>6,7</sup> of the  $m_J=0$  peak was observed at high rotation speeds.

If we assume that coupling to zero sound originates from the small particle-hole asymmetry in  $^3\text{He-B}$ ,<sup>9</sup> the only effect of rotation is to create a difference between the normal and superfluid velocities,  $\mathbf{v}_n - \mathbf{v}_s$ , which acts as a new orienting axis, perpendicular to  $\hat{q}$ . Consequently,  $\mathbf{v}_n - \mathbf{v}_s$  tends to turn  $\hat{h}$  away from the direction determined by  $\mathbf{H}$  alone; the spatial distribution of  $\hat{q} \cdot \hat{h}$  in Eq. (1) thus changes. The plateaus in this parameter appear at different locations in the experimental cell, thus altering the spectral density in Eq. (2). The coupling factors also change, as the angle between  $\hat{h}$  and  $\hat{q}$  at the plateaus varies with  $\Omega$ .

Another mechanism has also been proposed for the enhancement of the side peaks in Fig. 2: The superflow is predicted to affect the coupling strengths directly.<sup>12</sup> This scheme leads to line intensities proportional to  $[p_F(v_n - v_s)/\Delta]^2$ , where  $p_F$  is the Fermi momentum; it exhibits a complex dependence on the angles between  $\hat{q}$ ,  $\mathbf{v}_n - \mathbf{v}_s$ , and  $\hat{h}$ . In our lower-pressure data, no clear parabolic  $\Omega$  dependence of the  $m_J \neq 0$  peak heights was seen. At 11.5 bars, however, nearly  $\Omega^2$  behavior was observed for the  $m_J = \pm 2$  states; these results are illustrated in Fig. 3.

Our data cannot be qualitatively understood using the high- $H$  approximation, Eq. (1); the asymmetry of the

FIG. 2. The effect of rotation on the rsq-mode attenuation, when  $\mathbf{H} \parallel \hat{q} \parallel \boldsymbol{\Omega}$ . (a) rsq spectra at  $p=3.2$  bars and  $H=32$  mT; the horizontal alignment of the curves is arbitrary, because determination of the absolute temperature to an accuracy better than  $10 \mu\text{K}$  was not possible. Only VF spectra are displayed. (b) Spectra at  $p=11.5$  bars and  $H=25$  mT; the temperature scale is arbitrary for each curve, because of insufficient heat conduction between the sound cell and the thermometer. (c) Two spectra at  $p=6.5$  bars and  $H=25$  mT demonstrating the textural splitting of the  $m_J=0$  peak. We can resolve attenuations up to about  $10 \text{ cm}^{-1}$  only, and therefore peaks with higher intensity are truncated off to this level.

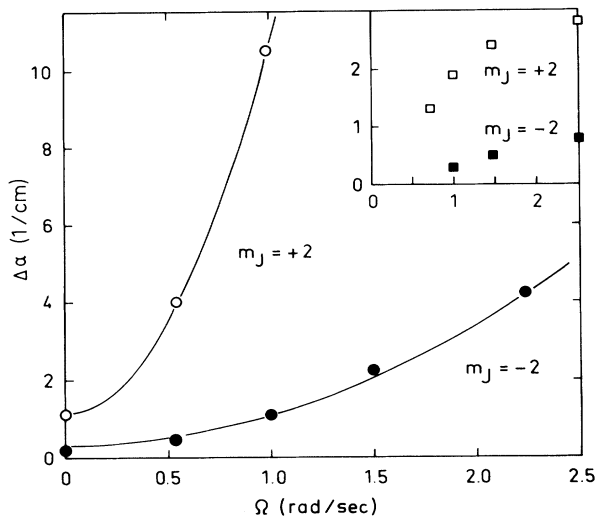


FIG. 3. Dependence of the  $m_J = \pm 2$  attenuation peak heights on the angular velocity  $\Omega$  (see Fig. 2) at  $p = 11.5$  bars,  $f_9 = 80.6$  MHz, and  $H = 25$  mT. Inset: Data points at  $p = 3.2$  bars,  $f_5 = 44.7$  MHz, and  $H = 32$  mT. The curves in the main graph display parabolas fitted to the experimental points:  $\Delta\alpha = [9.6 \text{ cm}^{-1} (\text{rad/sec})^{-2}] \Omega^2 + 1.1 \text{ cm}^{-1}$  for  $m_J = +2$ , and  $\Delta\alpha = [0.77 \text{ cm}^{-1} (\text{rad/sec})^{-2}] \Omega^2 + 0.32 \text{ cm}^{-1}$  for  $m_J = -2$ .

$m_J < 0$  and  $m_J > 0$  substate intensities, in particular, cannot be accounted for in this limit. Salomaa and Volovik<sup>13</sup> have quite recently performed phenomenological calculations for the rsq spectra and line intensities in homogeneous textures over the whole experimentally relevant range of  $H$ , including both interaction mechanisms. In the stationary state,  $\hat{q} \parallel \hat{h}$  holds practically everywhere in the experimental cell, while at high rotation speeds ( $\Omega \approx 5$  rad/sec),  $\hat{q} \perp \hat{h}$  is approached; the latter situation is met at lower velocities when the pressure is reduced, since the dipolar velocity becomes smaller. Nearly parabolic behavior can be expected at  $p = 11.5$  bars with  $\Omega = 0-2.5$  rad/sec, while at  $p = 3.2$  bars,  $\hat{q} \perp \hat{h}$  is reached at low velocities and  $\Delta\alpha$  saturates.<sup>13</sup> For the extrema, the theory of the homogeneous state accounts well for the mutual ordering of the side peak intensities, even when only the particle-hole-asymmetry factor is considered; for small angles between  $\hat{q}$  and  $\hat{h}$ , the intensities of the  $m_J$  substates decrease as  $0 \rightarrow +1 \rightarrow -1 \rightarrow +2 \rightarrow -2$ , while for large angles ( $> 60^\circ$ ) the  $m_J = +2$  component dominates. In the sta-

tionary state, the decreasing separation of substates with increasing  $m_J$  also agrees with the new theory<sup>13</sup> and a fully microscopic treatment.<sup>14</sup> At intermediate velocities, however, extensive numerical calculations of the textures, created in the equilibrium counterflow and vortex states, are a prerequisite to explain quantitatively the observed intensities and substate frequencies.

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