Lamberg, Joel; Tamminen, Aleksi; Taylor, Zachary; Ala-Laurinaho, Juha

Wavefront Modified Spherical Vector Beams for THz Cornea Imaging

Published in:
2023 48th International Conference on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz)

DOI:
10.1109/IRMMW-THz57677.2023.10299240

Published: 31/10/2023

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Lamberg, J., Tamminen, A., Taylor, Z., & Ala-Laurinaho, J. (2023). Wavefront Modified Spherical Vector Beams for THz Cornea Imaging. In *2023 48th International Conference on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz)* (International Conference on Infrared, Millimeter, and Terahertz Waves). IEEE. https://doi.org/10.1109/IRMMW-THz57677.2023.10299240

Wavefront-modified spherical vector beams for THz cornea imaging

Joel Lamberg[1], Faezeh Zarrinkhat[2], Aleksi Tamminen[1], Juha Ala-Laurinaho[1] and Zachary Taylor[1]
[1] Department of Electronics and Nanoengineering, MilliLab, Aalto University, 02150 Espoo, Finland
[2]TeraView Ltd, 1, Enterprise Cambridge Research Park, Cambridge, CB25 9PD, UK

Ocular diseases can be detected in early-stage with non-destructive terahertz imaging by analyzing cornea thickness and water content changes. We propose wavefront-modified spherical vector beam imaging, which reduces nominal 4-5 % super-confocally focused Gaussian beam imaging errors to less than 0.1 %.

I. INTRODUCTION

TERAHERTZ imaging is a promising method to diagnose ocular diseases in the early stage by analyzing changes in cornea thickness and water content (CTWC) [1]. The cornea is typically imaged by Gaussian beams, and the scattering is compared to the stratified medium theory (SMT) model [2]. However, the mismatch between the Gaussian beams' wavefront and the cornea's spherical surface creates a 4-5 % error in this analysis due to the mismatch in the coupling between the incident and scattered fields [3], see Fig. 1.

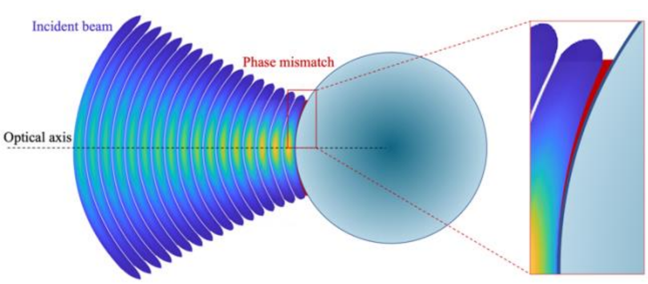

Fig. 1. Incident Gaussian beam with the mismatch between the wavefront and spherical surface.

This research investigates cornea imaging with wavefront-modified spherical vector beams (SVB), whose wavefronts are modified to match the spherical surface of the cornea, reducing the original analysis error down to less than 0.1 %. The imaging properties are evaluated with the coupling coefficient (K), describing spatial matching between the incident and scattered fields [4], and it is defined as

$$K = \frac{\iint [\boldsymbol{E}_{inc} \cdot \boldsymbol{E}_{scat}] dS}{\iint [\boldsymbol{E}_{inc} \cdot E_{inc}^{*}] dS} \tag{1}$$

where $\boldsymbol{E}_{inc}$ and $\boldsymbol{E}_{scat}$ are the incident and scattered fields, respectively.

Cornea possesses gradually increasing water content from the anterior surface to the posterior surface. This radial permittivity gradient can be modeled as multilayered spherical structure, whose electromagnetic scattering is obtained using Mie theory. However, wavefront modified SVBs do not have unequivocal and symmetric focal planes and must be synthesized using the 3D angular spectrum method (3D ASM) combined with the vector spherical harmonic (VSH) expansion [5]. Scattered fields are mapped from the incident fields by extended boundary condition method (EBCM) [6].

To maximize the CTWC analysis properties of electromagnetic beams, the scattering behavior from layered spheres should approach plane wave scattering from planar SMT models. The scattering behavior of wavefront modified SVBs for cornea imaging is studied by modeling the corneal radial permittivity gradient as 50-layer spherical structure and the results are compared to the reference plane wave scattering from the SMT model and to super-confocally focused Gaussian beam scattering [7,8].

Desired incident fields are computed using 3D ASM method by integrating VSH expanded source points at the spherical surface as

$$\boldsymbol{E}_{inc}(\boldsymbol{r}) = \frac{1}{4\pi^2} \iint_{\Omega} E_0(\theta,\phi)\, \boldsymbol{E}_t(r;\theta,\phi) sin\theta d\theta d\phi, \tag{2}$$

where VSH expanded source points are defined as

$$\boldsymbol{E}_t(r;\theta,\phi) = \sum_m \sum_n [a_e \boldsymbol{M}_e^1(k\boldsymbol{r}) + a_o \boldsymbol{M}_o^1(k\boldsymbol{r}) + b_e \boldsymbol{N}_e^1(k\boldsymbol{r}) + b_o \boldsymbol{N}_o^1(k\boldsymbol{r})], \tag{3}$$

where $\boldsymbol{M}_e^1, \boldsymbol{M}_o^1, \boldsymbol{N}_e^1$ and $\boldsymbol{N}_o^1$ are the VSH of the first kind and a_e, a_o, b_e and b_o are the beam shape coefficients (BSCs). Scattered fields are presented similarly by changing VSH to third kind and mapping scattered field BSCs with EBCM as [9]

$$[f_e\ f_o\ g_e\ g_o]^T = T[a_e\ a_o\ b_e\ b_o]^T \tag{4}$$

where T is T-matrix, and $f_e, f_o, g_e,$ and g_o are the scattered field BSCs.

II. RESULTS

Desired wavefront modified SVBs are synthesized by Gaussian distributed electric fields with equal phases positioned on the surface of the cornea. This approach enables inverse beam design to compute incident field distribution on the location of optical element (P1) for creating the goal field on the cornea´s surface. Also, the scattered field and beam coupling K can be computed at P1 from the multilayered cornea model, see Fig. 2.

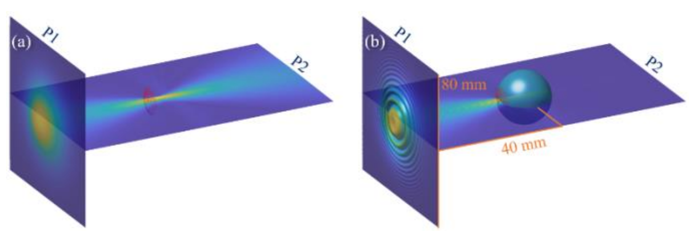

Fig. 2. Simulation arrangement: a) the incident SVB field and b) incident + scattered SVB field from the 50-layer cornea model.

Spherical cornea model is defined as a 580-μm or 680-μm thick shell in a 7.8-mm radius sphere with a lossy water core. Cornea´s gradient permittivity is modeled as a 50-layer structure, in which permittivity of the layers are obtained with the effective medium theory via the Bruggeman model [10]. The lossy core permittivity is calculated by the double-Debye model [11]. Also, the cornea anterior water content (AWC) is fixed at 40 %, while the posterior water content (PWC) is changed between 70 % and 90 %.

The total fields of SVB are simulated on the propagation plane to illustrate the beam behavior and the equal phase wavefront on the spherical surface, see Fig. 3.

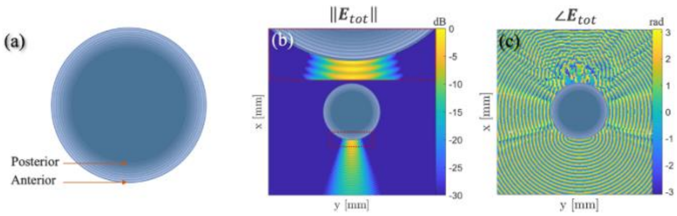

Fig. 3. a) Cornea model with radially changing permittivity and b) amplitude and c) phase of the total electric fields of SVB outside the 50-layer cornea model. Top part in b) shows a zoom-in plot in the vicinity of cornea marked with the rectangle.

The K from cornea model is computed on the evaluation plane P1 in the 200 – 400 GHz frequency range with SVB and super-confocally focused Gaussian beam [8]. Obtained results are compared to the reference SMT model with the same layer thickness and permittivity values, see Fig. 4. SVBs show almost perfect match with the SMT theory, which increases cornea imaging properties compared to the Gaussian beam imaging.

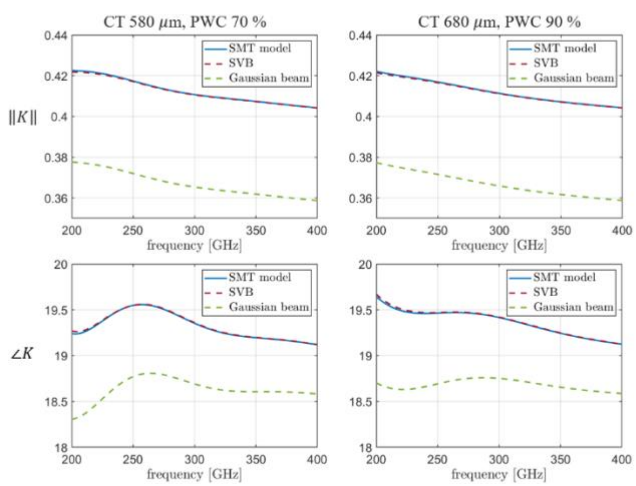

Fig. 4. The amplitude and phase of the coupling coefficient from the SMT model, SVB, and focused Gaussian beam at 200 to 400 GHz frequency range. Cornea is modeled with 580 and 680 μm thicknesses (CT) along with 70 and 90 % PWC.

III. Summary

Wavefront-modified spherical vector beams have clear advantage in cornea imaging compared to the nominal super-confocally focused Gaussian beams. The scattering profile of SVB is in a perfect match with the planar SMT model increasing the cornea imaging accuracy by reducing analysis error from 4-5 % to less than 0.1 %.

References

[1] Z. D. Taylor, J. Garritano, S. Sung, N. Bajwa, D. B. Bennett, B. Nowroozi, P. Tewari, J. Sayre, J. P. Hubschman, S. Deng, E. R. Brown, and W. S. Grundfest, "THz and mm-wave sensing of corneal tissue water content: Electromagnetic modeling and analysis," IEEE Transactions on Terahertz Sci. Technol. 5, 170–183 (2015).

[2] P. Yeh, A. Yariv, and C.-S. Hong, "Electromagnetic propagation in periodic stratified media. i. general theory∗," J.Opt. Soc. Am. 67, 423–438 (1977).

[3] A. Tamminen, S. V. Pälli, J. Ala-Laurinaho, M. Salkola, A. V. Räisänen, and Z. D. Taylor, "Quasioptical system for corneal sensing at 220–330 GHz: Design, evaluation, and ex vivo cornea parameter extraction," IEEE Transactions on Terahertz Sci. Technol. 11, 135–149 (2021).

[4] A. Tamminen, M. Baggio, I. Nefedova, Q. Sun, S. Presnyakov, J. Ala-Laurinaho, E. Brown, V. Wallace, E. Macpherson, T. Maloney, N. Kravchenko, M. Salkola, S. Deng, and Z. Taylor, "Extraction of thickness and water content gradients in hydrogel-based, water-backed corneal phantoms via submillimeter wave reflectometry," IEEE Transactions on Terahertz Sci. Technol. pp. 1–1 (2021).

[5] J. Lamberg, F. Zarrinkhat, A. Tamminen, J. Ala-Laurinaho, E. E. M. Khaled, J. Rius, J. Romeu, and Z. Taylor, "Simulation of scattering from layered spheres with known surface electric field distributions using Mie theory and modified angular spectrum method", Optics express, in peer-review process, 2023, https://doi.org/10.48550/arXiv.2209.03043

[6] U. P. O. Peña, "Scattering of electromagnetic radiation by a multilayered sphere," Computer Physics Communications, vol. 180, pp. 2348-2354, 2009

[7] Zarrinkhat F, Lamberg J, Tamminen A, Baggio M, Nefedova I, Ala-Laurinaho J, Khaled EEM, Rius J, Romeu J, Taylor Z. Vector spherical harmonic analysis and experimental validation of spherical shells illuminated with broadband, millimeter wave Gaussian beams: applications to corneal sensing. Biomed Opt Express. 2022 Jun 2;13(7):3699-3722. doi: 10.1364/BOE.456613.

[8] F. Zarrinkhat *et al*., "Fourier Analysis of Submillimeter-Wave Scattering from the Human Cornea," 2021 15th European Conference on Antennas and Propagation (EuCAP), Dusseldorf, Germany, 2021, pp. 1-5, doi: 10.23919/EuCAP51087.2021.9411287.

[9] E. E. M. Khaled, S. C. Hill, and P. W. Barber, "Scattered and internal intensity of a sphere illuminated with a gaussian beam," IEEE Transactions on Antennas Propag. 41, 295–303 (1993).

[10] E. R. Brown, O. B. McMahon, T. J. Murphy, G. G. Hogan, G. D. Daniels, and G. Hover, "Wide-band radiometry for remote sensing of oil films on water," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 46, no. 12, pp. 1989-1996, Dec. 1998, doi: 10.1109/22.739271.

[11] J. T. Kindt and C. A. Schmuttenmaer, "Far-infrared dielectric properties of polar liquids probed by femtosecond terahertz pulse spectroscopy," The J. Phys. Chem. 100, 10373–10379 (1996).