Spatial Fano resonance and its implication for a glass microsphere

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ABSTRACT
Fano’s resonance is not obviously that of a system parameter depending on the state energy or frequency. The spatial distribution of light intensity may experience this resonance if the continuum of eigenmodes interferes with a resonant mode. We found this spatial Fano resonance in the exact solution of a diffraction problem: a hollow Bessel impinges on a dielectric sphere with optically substantial but not very large radius. Tuning the frequency, one may engineer a very sharp Fano minimum in free space. This point at which the electromagnetic field vanishes may serve an optical trap for molecules and atoms. In accordance with our calculations, such the trap has no analogues in the available literature in what concerns its parameters and its ultimate simplicity.

Keywords: Bessel beams, Mie coefficients, Fano’s resonance, optical trapping.

1. INTRODUCTION
After publication of seminal work [1] extensive studies of optical traps have been done, and a significant progress was achieved in trapping the nanoparticles, where simple and elegant technical solutions were found. Meanwhile, trapping the molecules and atoms is more demanding. The progress in this area is modest and is related either with expensive nanostructures [2] or with combinations of optical and static trapping that demands challenging setups [3]. Meanwhile, a new chapter of photonics promising for the atomic trapping arises now. It is engineering the sharp spatial variations of free-space light intensity in a submicron scale. This direction can be called creation of nanostructured light. Not obviously material nanostructures are required for it, some resonances of microparticles may be sufficient. Microparticles are much cheaper in fabrication and unlike nanostructures are available on the market. An example of nanostructured light obtained with a microparticle is the so-called photonic nanojet which results from the plane-wave diffraction by a dielectric sphere. The waist of the nanojet can be engineered with the effective width below 250 nm [4]. The optomechanical force in this area is suitable for trapping particles having dimensions of few nm [5]. However, for the molecular or atomic trapping one needs higher intensity gradients than those which can be offered by a nanojet. In this paper we explain how to create such the gradients using the same microsphere which would generates the nanojet if it is illuminated by a plane wave. When it is illuminated by a hollow collimated wave beam it generates the nanostructured light with much sharper contrast, which grant the needed gradient. This nanostructured light enables a robust and tiny atomic trap, and its spatial distribution is the spatial Fano resonance.
Fano’s resonances have been known in photonics where they arise as resonances of some optical systems such as absorption, emission, or extinction cross section, electric or magnetic polarizability, etc. Indeed, they hold in the frequency domain and describe the dispersion of the optical response [6]. Meanwhile, the key idea of the Fano resonance allows the coherent light to experience it in space. Really, the Fano resonance results from the interference of a continuum of states and a single resonant state. Its prerequisite is the nonzero phase shift of the continuum with respect to the resonance and the proper relation for the magnitudes of wave functions. However, both continuum and single state may refer not only to the energetic states of the system but to the electromagnetic field in the system and around it. Both requirements can be satisfied if this system is a simple glass microsphere. The continuum (quasi-continuum) of states may be the electric field of an infinite set of non-resonant spherical eigenmodes. The resonant state may be the field of the whispering gallery mode. It is possible to find the conditions when these two fields interfere so that to form a comparatively broad and high intensity maximum and a sharp minimum adjacent to it. We found that this situation occurs when the microsphere is illuminated by an axially symmetric Bessel beam with the small enough conical angle.

2. SOLUTION OF THE DIFFRACTION PROBLEM AND SPATIAL FANO RESONANCE
Azimuthally symmetric Bessel beam of 1st order propagating along the z-axis in the polar coordinate system has the $\phi$-component of the magnetic field $\mathbf{H}_\phi$ and two components of the electric field – radial $E_\rho$ and longitudinal $E_z$:

$$
\mathbf{H}_\phi^i = \frac{\varepsilon_0}{\eta} J_1(k\rho \sin \beta)e^{ikz \cos \beta}, \ E_\rho^i = E_0 \cos \beta J_1(k\rho \sin \beta)e^{ikz \cos \beta}, \ E_z^i = iE_0 \sin \beta J_0(k\rho \sin \beta)e^{ikz \cos \beta}.
$$

(1)
Here $k$ and $\eta$ are free-space wave number and impedance, respectively, $J_{0,1}$ are cylindrical Bessel functions, and $\beta$ is called conical angle. If $\beta \ll 1$ the radial field component dominates: $|E^i_\rho|^2 > |E^i_\theta|^2$. Such the Bessel beam is hollow — its electric intensity $I_i(\rho) = |E^i_\rho|^2 + |E^i_\theta|^2$ is minimal on the beam axis. It also refers to the electromagnetic intensity and power flux since on the beam axis both magnetic field and Poynting vector exactly vanish. In Fig.1(a) we depict $I_i(\rho)$ for the case when $a \approx \lambda/\pi \sin \beta$ ($a$ is the sphere radius). Notice, that the clamed effect holds for smaller values of $a$ than this one.

![Figure 1](image.png)

**Figure 1.** (a) A hollow Bessel beam illuminates a microsphere. Coordinates ($\rho, \phi, z$) are polar, coordinates ($r, \phi, \theta$) are spherical. (b) The normalized intensity distribution shown in the $x$-$z$ plane comprises the Fano-like maximum located inside the sphere and the Fano-like minimum at about $\lambda/4$ from its rear edge. This minimum (optical trap centre) and the potential barrier behind it, are marked by two red arrows. Here $\varepsilon = 3.1$, $x_0 = 9.36$, $\beta = 0.01$.

Diffraction of such the beam by a dielectric sphere is a classical problem, however, the expansion coefficients of the beam (1) onto spherical harmonics resulting in the Mie coefficients are so-called overlapping integrals, usually calculated numerically. We have done it analytically. Details of our closed-form solution can be found in our paper [7]. Since an axially symmetric beam with azimuthal magnetic field does not excite the TE-modes both types of Mie coefficients (internal $a_n$ and scattering $d_n$) ones, take the following form:

$$a_n = \frac{-\varepsilon j_n(R_0)|R_3|j_n(R_0)|r + j_n(R_0)|R_1 j_n(R_1)|'}{\varepsilon j_n(R_0)|R_0 h_n^{(1)}(R_0)|' - h_n^{(1)}(R_0)|R_0 j_n(R_1)|'}$$
$$d_n = \frac{\varepsilon j_n(R_1)|R_0 h_n^{(1)}(R_0)|' - h_n^{(1)}(R_0)|R_0 j_n(R_1)|'}{\varepsilon j_n(R_1)|R_0 h_n^{(1)}(R_0)|' - h_n^{(1)}(R_0)|R_0 j_n(R_1)|'}$$

(2)

Here it is denoted $R_0 = \kappa a$ and $R_0 = \varepsilon R_0$ ($N$ is the refractive index of the sphere material). Parameter $R_0$ is often called the size parameter and is denoted $x_0$ (below we use namely this notation), $\varepsilon = N^2$ is the relative permittivity. Spherical Bessel and spherical Hankel functions are denoted $j_n$ and $h_n^{(1)}$, respectively. Prime means the derivative. Formulas (2) are not more involved than the classical plane-wave Mie expressions. Moreover, our $a_n$, $d_n$ attenuate with $n$ more rapidly. This good convergence allowed us to accurately (with the error below 1%) calculate the scattered fields up to $x_0 = 20$ via standard Mie series without application of difficult or inaccurate high-frequency methods.

The spatial Fano resonance arises when the conditions $\lambda < a < \lambda/\pi \sin \beta$, and $\beta \ll 1$ are satisfied. These resonances form a gallery — they occur at discrete wavelengths starting from $n = 12$ where $n$ is the number of the resonant mode. In Fig. 1(b) we depict the calculation of the normalized intensity of the total electric field (in the log scale). In this map, we clearly see the Fano-like minimum in between the high Fano-like maximum and the spread rear maximum behind the minimum. Below we will see that this spread maximum will serve a potential barrier of the optical trap. In these calculations, we adopted $\varepsilon = 3.1$ (typical optical glass), $\beta = 0.01$ and $x_0 = 9.36$. For this size parameter the mode $n = 12$ is resonant, whereas the interference of all other modes forms the quasi-continuum with the pronounced phase linearly growing along $z$ from the centre of the sphere $r = 0$ in the rear direction ($z > 0$). This quasi-continuum is the field in which the evanescent component dominates. Therefore, behind the sphere its amplitude decays exponentially with $z$. The field of the resonant mode outside the sphere is more evanescent. Therefore, its phase weakly changes versus $z$ and its amplitude decays more rapidly. However, inside the sphere the resonant mode strongly prevails. As a result, we observe the interference of the resonant mode and quasi-continuum which is typical for the Fano resonance. Numerical analysis of the intensity calculated on the...
beam axis ($\rho=0$) versus $z$ shows that the famous Fano formula nicely fits to our plot and, moreover, the physical meaning of the Fano parameters entering this formula is respected. In the considered example, the light intensity distribution for $z>0$ can be well approximated as a following function of $z$:

$$I = I_{\text{max}} \left(1 - \frac{1}{\Omega^2 + 1}\right), \quad \Omega \equiv \left|\frac{z-z_{\text{min}}}{a-z_{\text{min}}^*}\right|. \quad (3)$$

Here $z_{\text{min}}$ is the coordinate of the Fano minimum, at which the phase shift between the quasi-continuum and the resonant mode approaches $\pi$. Formula (3) is the special case of the famous Fano formula:

$$F(\Omega) = F_{\text{max}} \left(\frac{1}{\Omega^2 + 1}\right), \quad q \equiv \cot \Delta$$

for the case when the continuum phase shift $\Delta$ in the middle point is equal $\pi/2$. It is clearly our case since at the maximum the resonant mode field and that of the quasi-continuum sum up in phase and in the minimum cancel out. In other words, the new resonance we have revealed not simply mimics the Fano resonance it has all its characteristic features and is namely the Fano resonance with $q=0$, not its approximate analogue.

In Fig. 2(a) we depict the evolution of the intensity distribution on the beam axis with the variation of the wavelength (the argument is the size parameter). In the whole range $x_0=9.98$-10.02 the magnitudes of the resonant mode $n=13$ and the quasi-continuum become equal behind the sphere but only when $x_0=10.075$ the phase shift $\Delta$ attains at the corresponding point $\pi$ and the intensity nullifies. More exactly, within the band $x_0=10.075\pm10^{-6}$ there is the value of the size parameter for which the quasi-continuum and the mode with $n=13$ have the same magnitudes and opposite phases at a certain point. In other words, both real and imaginary parts of the electric field nullify at the same point of the beam axis. It implies the zero electromagnetic temperature $T_e$, i.e. absolute darkness of this point if there is no parasitic light. The band $x_0=10.075\pm10^{-6}$ corresponds to the relative light bandwidth $10^{-7}$, that is typical for the spectral line of standard tuneable lasers with continuous operation. Within this band at the point ($\rho=0, z=z_{\text{min}}$) the electric field is as small as $E/E_0 < 10^{-6}$. This situation can be reproduced for an infinite set of harmonics (starting from the lowest resonant size parameter $x_0=9.36$ for which the resonant mode is $n=12$). If we change $\epsilon$ the same resonances will hold at other frequencies starting from certain $n$. To have this resonance for the modes $n<10$ one needs very high permittivity. Most important is that for any transparent material of the sphere, we may engineer the zero electromagnetic trapping potential.

Figure 2. (a) Normalized light intensity in the log scale on the beam axis behind the sphere when the size parameter varies from 9.98 to 10.02. The Fano resonance holds when $x_0=10.0075$ (not shown). Then the Fano minimum with zero intensity is located at $k_0\rho \approx 11.09$ ($k_0=\hbar$). (b) The colour map of the trapping potential (purple colour – $U>10$ mK, red colour – $U<0.1$ mK) for the case $x_0 = 9.36$. The centre of the optical trap is at $z=10.45/k$.

3. OPTICAL TRAP FOR MOLECULES AND ATOMS

An atom with Lorentzian polarizability $\alpha$ corresponding to its fundamental optical transition experiences in the monochromatic light with nonuniform intensity the gradient force $f_\rho = 0.5 Re(\alpha) V I$ [1]. In the case of the blue detuning (of the laser line centre from the fundamental excited state), $f_\rho$ is directed towards the intensity minimum. Our minimum is the Fano one and is, therefore, very sharp. Its effective size along $z$ is about 200 nm and that in the transverse plane is close to 400 nm. When cold atoms excited by the laser light are dragged by the gradient force into this trap, they remain cold. Their electromagnetic temperature $T_e$ (determined via the trapping potential $U$ and Boltzmann constant as $T_e = U/k_B$) will drop from dozens of mK to less than 1 mK (in the case of the laser flux of the order of 1 kW/cm$^2$ which is typical for optical trapping). Such the contrast is sufficient for a robust trapping i.e. when the lifetime of a single trapped atom exceeds 100 ms. The trapping potential is calculated in
In accordance with [2]: \( U = \mu^2 I/2k_0h\delta \), where \( \mu \) is dipole moment (matrix element) of the fundamental optical transition of trapped atoms and \( \delta \) is frequency detuning in Hz. For all alkaline atoms \( \mu \) is within \((2 - 3) \cdot 10^{-29}\) SI. The typical detuning is by 3 orders of magnitude larger than the damping frequency of the Lorentzian resonance describing the optical transition. On these conditions the spatial distribution of \( U \) is almost the same for all alkaline atoms. In Fig. 2(b) we present the colour map of \( U \) in the plane \( x-z \) calculated for an alkaline atom. Here the coordinates are dimensionless. Due to the problem symmetry the cartesian coordinate \( x \) can be treated as the cylindrical coordinate \( \rho \). This colour map corresponds to \( \omega_0 = 9.36 \) (resonance of the mode \( n=12 \)). We can see that our trap centered at the point \( U = 0 \) (\( \rho = 0, z = z_{\text{min}} \)) is almost surrounded by potential barriers. Green colour corresponds to \( U = 1 \) mK and the bounds of this region leave the opportunity to a moving atom to escape from the trap. We estimated the lifetime of a single Cs atom in such the trap as few seconds. So long trapping times have been reported for dipole traps formed in the focus of a powerful laser beam [3]. Heating was suppressed due to a sophisticated pulsation regime that demanded a specially engineered modulator. Besides of the optical trapping such traps comprise the static part (coils for static magnetic field etc.) and represent cumbersome and expensive setups. However, the size of the atomic trap obtained in this setup is much larger (several microns) than our optical trap. Submicron traps offer fine location of the atoms and molecules. They have been previously reported only for plasmonic nanostructures and for nanotips of near-field optical microscopes and atomic force ones. However, these nanostructures do not grant so long lifetime (see e.g. in [2]). Indeed, the combination of a long trapping time and deeply submicron size makes the suggested optical trap unique and deserving an experimental check. To build this setup one need only commercially available equipment producing a hollow Bessel beam from a usual laser light, a simple glass microsphere in a vacuum camera and the source of atoms, ions, or molecules.

4. DISCUSSION AND CONCLUSIONS

In this work, we have reviewed the novel concept of the spatial Fano resonance which holds for the light intensity distribution if a dielectric microsphere is illuminated by a hollow Bessel beam. This resonance implies the nanostructured light in free space having a so sharp and deep minimum that it may serve a deeply submicron but very robust optical trap that does not demand to be reinforced by a static trap and may trap the separate atoms, ions, or molecules. The idea of the Fano resonance is the interference of the quasi-continuum of non-resonant spherical eigenmodes with the resonant one. It implies that the sphere should be larger than the wavelength but not very large so that to keep high-order Mie resonances (whispering gallery resonances) pronounced and well separated from one another over the frequency axis. This requirement is easier to respect with a microcylinder where the 2D geometry grants a better separation of these resonances. In the case of the cylinder, the hollow beam may be simply an antisymmetric pair of plane waves propagating with the small angle between the wave vectors. These waves have identical amplitudes and opposite phases, and this pair of waves is called cosine beam. In work [8], we considered the incidence of cosine beams on glass microcylinders and found the spatial Fano resonances in the 2D case. The optical trap in the 2D case is a cylindrical region of submicron thickness parallel to the glass cylinder and located at a substantial distance \( d \) from it \((d \) may exceed \( \lambda/2 \)). The exact vanishing of the optical potential on the axis of this optical trap can be also engineered.

So, the results obtained in our works [7, 8] promise a technical breakthrough in the molecular and atopic optical trapping and may stimulate the development of a new chapter of photonics – photonics of nanostructured light.

REFERENCES