Volovik, G.E.; Zubkov, M.A.

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Nambu sum rule and the relation between the masses of composite Higgs bosons

G. E. Volovik

Low Temperature Laboratory, School of Science and Technology, Aalto University, Finland, and L. D. Landau Institute for Theoretical Physics, Moscow, Russia

M. A. Zubkov

ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia

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We review the known results on the bosonic spectrum in various Nambu-Jona-Lasinio models both in condensed matter physics and in relativistic quantum field theory including ^{3}He-B, ^{3}He-A, the thin films of superfluid ^{3}He, and QCD (Hadronic phase and the color-flavor locking phase). Next, we calculate the bosonic spectrum in the relativistic model of top quark condensation suggested in [Phys. Lett. B **221**, 177 (1989)]. In all considered cases, the sum rule appears, which relates the masses (energy gaps) $M_{\rm boson}$ of the bosonic excitations in each channel with the mass (energy gap) of the condensed fermion M_f as $\sum M_{\rm boson}^2 = 4M_f^2$. Previously, this relation was established by Nambu for ^{3}He-B and for the s-wave superconductor. We generalize this relation to the wider class of models and call it the Nambu sum rule. We discuss the possibility to apply this sum rule to various models of top quark condensation. In some cases, this rule allows us to calculate the masses of extra Higgs bosons that are the Nambu partners of the 125 GeV Higgs.

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I. INTRODUCTION

It is difficult to overestimate the role of the Nambu-Jona-Lasinio (NJL) approximation in field theory (i.e., the approximation with the effective four-fermion interaction) [1]. It gives qualitative understanding of the formation of fermion condensates in a number of models that describe various physical problems from superconductivity and superfluidity [2] to top-quark condensation [3]. However, any NJL model is only a low-energy approximation to the microscopic theory. The NJL models are not renormalizable. Therefore, they are to be considered as the phenomenological models with the finite ultraviolet cutoff Λ. In most of the papers on the NJL models, the physical quantities are evaluated in one-loop approximation (i.e., in the leading order in $1/N$ expansion). It is worth mentioning that, formally, the contributions of higher loops to various physical quantities may be strong. For example, in Refs. [4,5], it has been shown that the next-to-leading (NTL) order approximation to the fermion mass M_T in the simplest model of top-quark condensation is weak compared to the one-loop approximation only if $M_T \sim \Lambda$. Actually, all dimensional parameters of the relativistic NJL models (calculated nonperturbatively or taking into account higher orders of the perturbation theory) are typically of the order of the cutoff unless their small values are protected by symmetry.

Nevertheless, there is another way to look at the NJL models. We can consider the one-loop approximation for the calculation of various quantities like fermion and boson masses (i.e., the mean field approximation or the leading order in $1/N$ expansion). The higher loops are simply disregarded. This is usually done in the NJL approximation to QCD [6] or technicolor [7], where all dimensional parameters are of the order of the ultraviolet cutoff so that the corrections to the leading-order $1/N$ approximation are not so large. (For the results of the nonperturbative numerical lattice investigation of the NJL model, see, for example, Ref. [8]). However, this is also done in many papers on the models of top-quark condensation (TC) [3,9], where the cutoff is assumed to be many orders of magnitude larger than the mass of the top quark. There were only a few papers on the next-to-leading-order approximation (see, for example, Refs. [4,5]). Besides, in the evaluation of the Standard Model fermion masses in the technicolor theory due to the extended technicolor (ETC) interactions [7], the effect of the ETC is taken into account through the effective four-fermion term. No loop contributions due to this term are considered. However, those loop contributions would give values of masses $\sim\Lambda_{\rm ETC}^2$, $\Lambda_{\rm ETC} \gg M_T$. The justification may be based on the assumption that we deal with the phenomenological model that is to be considered at the tree level (ETC), or in one loop (TC) without taking into account higher-loop contributions. However, the more rigorous explanation is that there exist the contributions of the microscopic theory due to the trans-Λ degrees of freedom that are not taken into account in the NJL approximation. Those contributions cancel the dominant higher-loop divergences. Therefore, the one-loop results (TC) and tree-level results (ETC) dominate. (See also the discussion in Sec. III A.) In this paper, we assume that this pattern takes place in the models of top-quark condensation. This means that there exist the contributions to the Higgs boson masses and to the quark masses that come from the energies larger than Λ and are not accounted for by the NJL model. They are assumed to cancel the quadratic divergent contributions to the (squared) Higgs boson masses and linear divergent contributions to the quark masses. The consideration of the possible mechanisms that may provide this are outside the scope of the present paper. We only mention that there exists the situation, when such a pattern is realized. Namely, in quantum hydrodynamics, there formally exist the divergent contributions to various physical quantities (for example, to the vacuum energy). Nevertheless, the hydrodynamics may be considered with these divergent contributions subtracted, and this is how classical hydrodynamics appears as a low-energy approximation to quantum theory. The origin of this cancellation is well-known [10]. It is provided by the thermodynamical stability of the vacuum. Recently, it was suggested that the similar mechanism is responsible for the cancellation of the ultraviolet divergences in vacuum energy (quantum gravity) and for the cancellation of the quadratically divergent contribution to the Higgs boson mass in the Weinberg-Salam model [11].

Based on this assumption, we expect that in the relativistic models of top-quark condensation quantitative predictions of the one-loop NJL approximation may be as accurate as in the BCS models of superconductivity or superfluidity. It is worth mentioning that, in microscopic theories of top-quark condensation, there is no confinement (otherwise, the top quark would be confined into the regions of space smaller than 1 TeV^{-1}). In this aspect, these theories differ essentially from QCD, in which the absence of confinement in the NJL approximation does not allow us to use it widely for the consideration of low-energy physics. (For the attempts to include the description of confinement to the NJL approximation, see Ref. [12].)

More specifically, we investigate the particular case of the NJL model suggested in Ref. [9]. We calculate its bosonic spectrum and establish the relation between the masses of bosonic excitations and the fermion masses. This relation is similar to the relation found in ^{3}He-B and in the s-wave superconductors between the energy gaps of the scalar excitations and the fermion energy gap. This relation was first noticed in Ref. [2] by Nambu. In the form $M_1^2 + M_2^2 = 4\Delta^2$, it is valid in the effective NJL-like model of ^{3}He-B for the boson energy gaps $M_{1,2}$ existing at each value of $J = 0$, 1, 2, where J is the quantum number corresponding to the total angular momentum of the Cooper pair. It relates them to the constituent mass of the fermion excitation Δ existing due to the condensation. The similar relation was also discussed in the Nambu-Jona-Lasinio approximation [1] of QCD, where it relates the σ-meson mass and the constituent quark mass $M_\sigma \approx 2M_{\mathrm{quark}}$. (In the nonrelativistic BCS theory, the role of the masses of the fermionic and bosonic excitations is played by the energy gaps in the fermionic and bosonic spectrums, respectively.) Recent discussion of Higgs modes in condensed matter systems can be found in Refs. [13–17] and in references therein.

We introduce the notion of the Nambu sum rule that is the generalization of the above-mentioned relations to the theories with condensed fermions (such that there is the single fermion, for which the constituent mass M_f is essentially larger than the masses of the other fermions). This sum rule reads

$$\sum M_{H,i}^2 \approx 4M_f^2. \tag{1}$$

In the left-hand side of this equation, the sum is within the given channel over the composite scalar excitations such that the mentioned fermion with mass M_f contributes to their formation. We do not give here the general proof of this sum rule. Instead, we consider several models in which it holds. In addition to our results on the bosonic spectrum of the above-mentioned top-quark condensation model, we review several NJL models, for which the bosonic spectrum is already known.

Recall that the recent experimental results [18,19] on the 125 GeV Higgs exclude the appearance of the other Higgs bosons within the wide ranges of masses (approximately from 130 GeV to 550 GeV). However, this announced exclusion is related only to the particle with the same cross section as the only standard Higgs boson of the Standard Model. The particles that have the smaller cross sections are not excluded. To be more explicit, we refer to the recent data of CMS Collaboration [20]. In Fig. 4 of [20], the solid black curve separates the region where the scalar particles are excluded (above the curve) from the region where they are not excluded. For example, the particle with mass around 200 GeV and with the cross section about 1/3 of the Standard Model cross section is not excluded by these data. The similar exclusion curve was announced by ATLAS [21].

This is the analogy with the superconductivity and superfluidity that prompts that the Higgs boson may be composite. (See Refs. [22–24] for the foundation of the Higgs mechanism in quantum field theory.) In our opinion, the models of the top-quark condensation [3,4,7,9,25–29] are of special interest as they relate the Higgs boson to the only known Fermi particle (top quark) with the mass of the same order as the Higgs boson mass. Therefore, we have in mind the pattern of top-quark condensation dealing with the Nambu sum rule, and, in the right-hand side (rhs) of Eq. (1), the top-quark mass stands. For the review of the conventional technicolor, we refer to Refs. [7,30–32]. The so-called top-color assisted technicolor that combines both technicolor and top-color ingredients was considered, for example, in Refs. [33–37]. For the related models based on the extended color sector, see Refs. [38,39] and references therein. The top seesaw mechanism was considered in Ref. [40]. We also mention the attempts to consider the recently found 125 GeV resonance as a top pion [29].

An interesting consequence of the Nambu sum rule with the top-quark mass is that, if there are only two states in the given channel, then the partner of the 125 GeV Higgs should have the mass around 325 GeV. It is worth mentioning that in 2011 the CDF Collaboration [41] announced the preliminary results on the excess of events in the $ZZ \to ll\bar{l}\bar{l}$ channel at the invariant mass ≈ 325 GeV. The CMS Collaboration also reported a small excess in this region [42]. Although the particle with the cross sections of the Standard Model Higgs is excluded at this mass, this exclusion does not work for the particles with smaller cross sections. Originally, the mentioned excess of events was treated as a statistical fluctuation. However, in Refs. [43,44], it was argued that it may point out the possible existence of a new scalar particle with mass $M_{H2} \approx 325$ GeV.

The paper is organized as follows. In Sec. II, we consider the condensed-matter NJL models of ^{3}He. In Sec. II A, we review the hydrodynamic action for ^{3}He. Next, in Sec. II B, we consider the bosonic spectrum in the NJL model of ^{3}He-B. (This model was considered originally in this respect by Nambu). We present the simple method for the calculation of the bosonic spectrum in this model. In principle, this method with some modifications can be applied to the other models of this section, although we do not present the corresponding calculations. In Sec. II C, we consider the 3D A phase of the superfluid ^{3}He. In this case, the fermions are gapless. However, the Nambu sum rule Eq. (1) works if, in its rhs, the average of the angle-dependent energy gap is substituted. In Sec. II D, we consider two-dimensional (2D) thin films of ^{3}He. There are two main phases (a and b), in which the Nambu sum rule works within the effective 2D four-fermion model similar to that of ^{3}He-B.

Section III is devoted to the relativistic NJL models. In Sec. III A, we describe the top-quark condensation model of Ref. [9] and its particular case considered in this paper. In Sec. III B, we calculate the bosonic spectrum of the model. In Sec. III C, we present the other example of the relativistic model, in which the Nambu sum rule holds, i.e., the NJL model of the color superconductor in the so-called color-flavor locking (CFL) phase. In the corresponding four-fermion effective model, there are two different fermionic energy gaps. Both of them are related to the bosonic masses by the relation similar to the ordinary relation between the constituent quark mass and the mass of the sigma meson. In Sec. III D, we consider the analogy with the Veltman identity for the vanishing of the quadratic loop divergencies in the scalar boson masses.

The model of Ref. [9] considered in Sec. III A and III B suffers from various problems, and a lot of physics is to be added in order to make it realistic. However, this is the first example in which the Nambu sum rule in the nontrivial form appears in the relativistic model. There may appear the other nontrivial (and more realistic) models of the top-quark condensation (and the other technicolorlike models) in which there are several composite Higgs bosons, for which the masses are related by the Nambu sum rule. We suggest looking for such schemes based on the analogy with superfluid ^{3}He (we refer to Ref. [45] and to the references therein).

II. NAMBU SUM RULES IN HELIUM-3 SUPERFLUID

A. "Hydrodynamic action" in ^{3}He

According to Ref. [46], ^{3}He may be described by the effective theory with the action

$$\begin{aligned} S = &\int dt d^3x \bar{\chi}_s \left\{ i\partial_t + \mu + \frac{1}{2m}\Delta \right\} \chi_s \\ &- \frac{1}{2}\int dt \int d^3x \int d^3y u(x-y) \\ &\times \sum_{s,s'} \bar{\chi}_s(x,t)\chi_s(x,t)\bar{\chi}_{s'}(y,t)\chi_{s'}(y,t). \end{aligned} \tag{2}$$

Here, χ is an anticommuting spinor variable, $s = \pm$, μ is the chemical potential, and $u(x)$ is the interatomic potential. Then, the integration over the "fast" Fermi fields (i.e., those with sufficiently large values of momenta) gives the effective action for the modes living near the Fermi surface. Assuming imaginary time and the spin-triplet p-wave pairing (i.e., the Cooper pairing in the state with orbital angular momentum $L = 1$ and spin angular momentum $S = 1$), in the first approximation, this effective action can be written as

$$S_{\rm low} = \sum_{p,s} \bar{a}_s(p)\epsilon(p)a_s(p) - \frac{g}{\beta V}\sum_{p,i=1,2,3} \bar{J}_i(p)J_i(p), \tag{3}$$

where

$$\begin{aligned} &p = (\omega, k), \qquad \hat{k} = \frac{k}{|k|}, \qquad \epsilon(p) = i\omega - v_F(|k| - k_F) \\ &J_i(p) = \frac{1}{2}\sum_{p_1+p_2=p} (\hat{k}_1 - \hat{k}_2)a_\alpha(p_2)\sigma_i a_\beta(p_1)\epsilon^{\alpha\beta}. \end{aligned} \tag{4}$$

Here, $a_\pm(p)$ is the fermion variable in momentum space, v_F is Fermi velocity, k_F is Fermi momentum, and g is the effective coupling constant. The authors of Ref. [46] proceed with the bosonization using the following trick. The unity is substituted into the functional integral that is represented as $1 \sim \int D\bar{c}Dc \exp(\frac{1}{g}\sum_{p,i,\alpha}\bar{c}_{i,\alpha}(p)c_{i,\alpha}(p))$, where $c_{i,\alpha}$, $(i,\ \alpha = 1, 2, 3)$ are bosonic variables. These variables may be considered further as the field of the Cooper pairs, which serves as the analog of the Higgs field in relativistic theories. The shift of the integrand in $D\bar{c}Dc$ removes the four-fermion term. Therefore, the fermionic integral can be taken. As a result, we arrive at the hydrodynamic action for the Higgs field c:

$$S_{\rm eff} = \frac{1}{g}\sum_{p,i,\alpha} \bar{c}_{i,\alpha}(p)c_{i,\alpha}(p) + \frac{1}{2}\log {\rm Det}M(\bar{c}, c), \tag{5}$$

where

$$M(\bar{c},c) = \begin{pmatrix} (i\omega - v_F(|k|-k_F))\delta_{p_1p_2} & \frac{1}{(\beta V)^{1/2}}[(\hat{k}_1-\hat{k}_2)c_\alpha(p_1+p_2)]\sigma_\alpha \\ -\frac{1}{(\beta V)^{1/2}}[(\hat{k}_1-\hat{k}_2)c_\alpha(p_1+p_2)]\sigma_\alpha & -(i\omega - v_F(|k|-k_F))\delta_{p_1p_2} \end{pmatrix}. \quad (6)$$

B. Nambu sum rules in ^{3}He-B

In the B phase of ^{3}He, the condensate is formed in the state with $J=0$, where $\mathbf{J}=\mathbf{L}+\mathbf{S}$ is the total angular momentum of Cooper pair [45]

$$c^{(0)}_{i\alpha}(p) = (\beta V)^{1/2} C\delta_{p0}\delta_{i\alpha}. \quad (7)$$

This corresponds to the symmetry-breaking scheme $G\to H$ with the symmetry of physical laws $G = SO_L(3)\times SO_S(3)\times U(1)$ and the symmetry of the degenerate vacuum states $H = SO_J(3)$. The parameter C satisfies the gap equation

$$0 = \frac{3}{g} - \frac{4}{\beta V}\sum_p (\omega^2 + v_f^2(|k|-k_F)^2 + 4C^2)^{-1}. \quad (8)$$

The value $\Delta = 2C$ is the constituent mass of the fermion excitation. There are 18 modes of the fluctuations $\delta c_{i\alpha} = c_{i\alpha} - c^{(0)}_{i\alpha}$ around this condensate. Tensor $\delta c_{i\alpha}$ realizes the reducible representation of the $SO_J(3)$ symmetry group of the vacuum (acting on both spin and orbital indices). The mentioned modes are classified by the total angular momentum quantum number $J = 0, 1, 2$.

According to Refs. [47,48], the quadratic part of the effective action for the fluctuations around the condensate has the form

$$S^{(1)}_{\rm eff} = \frac{1}{g}(u,v)[1-g\Pi]\begin{pmatrix} u \\ v\end{pmatrix}, \quad (9)$$

where $\delta c_{i\alpha}(p) = u_{pi\alpha} + iv_{pi\alpha}$, and the polarization operator at $k=0$ is given by

$$\Pi = \begin{pmatrix} \Pi^{uu} & 0 \\ 0 & \Pi^{vv}\end{pmatrix}. \quad (10)$$

At each value of $J = 0, 1, 2$, the modes u and v are orthogonal to each other and correspond to different values of the bosonic energy gaps.

At $k=0$, the polarization operator can be represented as

$$\Pi(\omega) = \int_0^\infty dz \frac{\rho(z)}{z+\omega^2}, \quad (11)$$

where the spectral function $\rho \sim \sum |F_{Q\to ff}|^2$, and $|F_{Q\to ff}|^2$ is the probability that the given mode Q (in the case of ^{3}He-B, the quantum number $Q=J$) decays into two fermions.

At $J=0$, the v bosonic mode is gapless and can easily be obtained using the gap equation. Also, this follows from the fact that this is the Goldstone mode, which comes from the broken $U(1)$ symmetry. Next, for any J, we have ($\sqrt{t} = \epsilon_+ + \epsilon_-$; $k_+ = -k_-$; $\epsilon_\pm^2 - v_f^2(|k|-k_F)^2 - \Delta^2 = 0$):

$$\begin{aligned} \rho^u(t) &\sim \theta(t-4\Delta^2)\sqrt{1-\frac{4\Delta^2}{t}}\,{\rm Sp}G^{-1}(\epsilon_+,k_+)O^{(J)}_u G^{-1}(-\epsilon_-,k_-)O^{(J)}_u \\ &\sim \sqrt{1-\frac{4\Delta^2}{t}}[(t/2-\Delta^2) - \eta^{(J)}\Delta^2]\theta(t-4\Delta^2) \\ \rho^v(t) &\sim \theta(t-4\Delta^2)\sqrt{1-\frac{4\Delta^2}{t}}\,{\rm Sp}G^{-1}(\epsilon_+,k_+)O^{(J)}_v G^{-1}(-\epsilon_-,k_-)O^{(J)}_v \\ &\sim \sqrt{1-\frac{4\Delta^2}{t}}[(t/2-\Delta^2) + \eta^{(J)}\Delta^2]\theta(t-4\Delta^2). \end{aligned} \quad (12)$$

Here,

$$G^{-1}(\epsilon,k) = \begin{pmatrix} (\epsilon - v_F(|k|-k_F)) & 2C(\hat{k}\sigma) \\ -2C(\hat{k}\sigma) & (\epsilon + v_F(|k|-k_F))\end{pmatrix}, \qquad O^{ij}_{u,v} = \begin{pmatrix} 0 & \hat{k}^i_+\sigma^j \\ \mp\hat{k}^i_+\sigma^j & 0\end{pmatrix}, \qquad O^{(0)} = \frac{1}{\sqrt{D}}O^{ii},$$
$$[O^{(1)}]^{ij} = \frac{1}{\sqrt{D(D-1)/2}}O^{[ij]}, \qquad [O^{(2)}]^{ij} = \frac{1}{\sqrt{D(D+1)/2-1}}\left[O^{\{ij\}} - \frac{1}{D}O^{kk}\delta^{ij}\right], \quad (13)$$

with $D = 3$ and

$$\eta^{(J)} = \frac{\mathrm{Sp} V O^{(J)} V O^{(J)}}{\mathrm{Sp} O^{(J)} O^{(J)}}, \tag{14}$$

with

$$V = \begin{pmatrix} 0 & \hat{k}_+ \sigma \\ -\hat{k}_+ \sigma & 0 \end{pmatrix}. \tag{15}$$

In the v channel at $J = 0$, the energy gap is equal to zero, which leads to the condition

$$\mathrm{const} \int_{4\Delta^2}^{\Lambda^2} \sqrt{1 - \frac{4\Delta^2}{t}} dt = \frac{3}{g}, \tag{16}$$

where Λ is the ultraviolet cutoff. The bosonic energy gaps $E_{u,v}^{(J)}$ are defined by the equation

$$\mathrm{const} \int_{4\Delta^2}^{\Lambda^2} \sqrt{1 - \frac{4\Delta^2}{t}} \frac{t - 2\Delta^2(1 \pm \eta^{(J)})}{t - [E_{u,v}^{(J)}]^2} dt = \frac{3}{g}, \tag{17}$$

with the same constant as in Eq. (16). Comparing these two equations, we come to the following Lemma.

Lemma II.1.—The energy gaps are given by

$$E_{u,v}^{(J)} = \sqrt{2\Delta^2(1 \pm \eta^{(J)})}, \tag{18}$$

which proves the Nambu sum rule for ^{3}He-B:

$$[E_u^{(J)}]^2 + [E_v^{(J)}]^2 = 4\Delta^2. \tag{19}$$

Explicit calculation of Eq. (14) gives $\eta^{J=0} = \eta^{J=1} = 1$, and $\eta^{J=2} = \frac{1}{5}$. Thus, we get immediately the result obtained in Ref. [48] via the direct solution of the equation $\mathrm{Det}(g\Pi(iE) - 1) = 0$:

(1) $J = 0$: For $J = 0$, there is one pair of the Nambu partners (the gapless Goldstone sound mode and the so-called pair-breaking mode with the energy gap $E = 2\Delta$):

$$E_1^{(0)} = 0, \qquad E_2^{(0)} = 2\Delta. \tag{20}$$

(2) $J = 1$: For $J = 1$, there are three pairs of Nambu partners (three gapless Goldstone modes—spin waves and three corresponding pair-breaking modes with the energy gap $E = 2\Delta$):

$$E_1^{(1)} = 0, \qquad E_2^{(1)} = 2\Delta. \tag{21}$$

(3) $J = 2$: For $J = 2$, there exist five pairs—five so-called real squashing modes with the energy gap $E = \sqrt{2/5}(2\Delta)$ and, correspondingly, five imaginary squashing modes with the energy gap $E = \sqrt{3/5}(2\Delta)$:

$$E_1^{(2)} = \sqrt{2/5}(2\Delta), \qquad E_2^{(2)} = \sqrt{3/5}(2\Delta). \tag{22}$$

(Zeeman splitting of the imaginary squashing mode in a magnetic field has been observed in Ref. [49]; for the latest experiments, see Ref. [50].)

C. Nambu sum rules in ^{3}He-A

In the A phase of ^{3}He, the condensate is formed in the state with $S_z = 0$ and $L_z = 1$ [45]. In the orbital sector, the symmetry breaking in ^{3}He-A is similar to that in the electroweak theory: $U(1) \otimes SO_L(3) \to U_Q(1)$, where the quantum number Q plays the role of the electric charge (see, e.g., Ref. [51]), while in the spin sector, one has $SO_S(3) \to SO_S(2)$. According to Ref. [52], one has

$$\begin{aligned} c_{i\alpha}^{(0)}(p) &= (\beta V)^{1/2} C \delta_{p0} (\delta_{i1} + i\delta_{i2}) \delta_{\alpha 3} \\ &= (\beta V)^{1/2} C \delta_{p0} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{23}$$

where C satisfies the gap equation

$$0 = \frac{1}{g} - \frac{2}{\beta V} \sum_p \frac{1 - \hat{k}_3^2}{\omega^2 + v_f^2(|k| - k_F)^2 + 4C^2(1 - \hat{k}_3^2)}. \tag{24}$$

The A phase is anisotropic. The special direction in the orbital space appears, which is identified with the direction of the spontaneous orbital angular momentum of Cooper pairs, which is chosen here along the z axis. In this phase, fermions are gapless. However, the value $\Delta(\theta) = 2C\sqrt{1 - \hat{k}_3^2} = \Delta_0 \sin\theta$ may be considered as the technical gap depending on the direction in space that enters the expressions to be considered below. (Here, θ is the angle between the anisotropy axis and the direction of the momentum k.)

In the BCS theory of ^{3}He-A, all bosonic modes are triply degenerate. This is the consequence of the hidden symmetry of the BCS theory applied to ^{3}He-A, which, in particular, gives rise to nine gapless Goldstone modes instead of five modes required by symmetry breaking [53,54]. On the language of $c_{i\alpha}$, this hidden symmetry leads to the representation of the one-loop effective action as the sum of the three terms. Each of those terms depends on $c_{i\alpha}$ with a definite value of $\alpha = 1, 2, 3$. The term with c_{i3} is transformed into the term with c_{i2} via the substitution $c_{i3} \to ic_{i2}$. The term with c_{i2} is transformed into the term with c_{i1} via the substitution $c_{i2} \to c_{i1}$. Among five Goldstone bosons corresponding to the breakdown pattern $U(1) \otimes SO_L(3) \otimes SO_S(3) \to U_Q(1) \otimes SO_S(2)$, there are $u_{11} + v_{21}$, $u_{12} + v_{22}$, $u_{23} - v_{13}$ that are transformed to each other by the above-mentioned transformation. Also, there are the Goldstone modes u_{33}, v_{33}. The latter modes may be transformed by this transformation to u_{31}, u_{32}, v_{31}, v_{32}. Therefore, four additional gapless modes appear in weak coupling limit. Recall that, in the strong coupling regime, these four modes become gapped.

The values of the energy gaps are given by the solutions of the equation $\mathrm{Det}(g\Pi(iE) - 1) = 0$. Exact solutions of the given equations are presented in Ref. [52]. The energy gaps are complex-valued; that means that the states are not stable. (The decay into the massless fermions is possible.) However, the real parts of the energy gaps can be evaluated in the approximation when the effective action at $k = 0$ is represented as the sum of the two terms: the first term corresponds to $\omega = 0$, while the second term is proportional to ω^2. Such a calculation gives the mass term for the modes of the field $c_{i\alpha}$ with the contribution due to the terms depending on higher powers of ω disregarded. This procedure gives six unpaired gapless Goldstone modes and two pairs of modes (triply degenerated) that satisfy a version of the Nambu sum rule. In this case, the role of the square of the fermion mass is played by the angle average of the square of the anisotropic gap:

$$\bar{\Delta}^2 \equiv \langle \Delta^2(\theta)\rangle = \frac{2}{3}\Delta_0^2. \tag{25}$$

The Nambu pairs are the following:

(1) One (triply degenerated) pair of bosons (the phase and amplitude collective modes in Nambu terminology) is formed by the "electrically neutral" ($Q = 0$) massless Goldstone mode and the "Higgs boson" also with $Q = 0$:

$$E_1^{(Q=0)} = 0, \qquad E_2^{(Q=0)} = 2\bar{\Delta} = \sqrt{8/3}\Delta_0. \tag{26}$$

(2) The other (triply degenerated) pair represents the analog of the charged Higgs bosons in ^{3}He-A with $Q = \pm 2$ (see, e.g., Ref. [54]). These are the so-called clapping modes for which the energies are

$$E_1^{(Q=2)} = E_2^{(Q=-2)} = \sqrt{2}\bar{\Delta} = \sqrt{4/3}\Delta_0. \tag{27}$$

Lemma II.2.—One can see that the spectrum of fermions and bosons in anisotropic superfluid ^{3}He-A also satisfies the Nambu conjecture written in the form

$$E_1^2 + E_2^2 = 4\bar{\Delta}^2 \tag{28}$$

(for each of the two pairs listed above) with the "average fermion gap" given by Eq. (25).

Alternatively these values may be obtained if, in Eq. (2.16) of Ref. [52], the values of $\Delta^2(\theta)$ are substituted by their averages $\bar{\Delta}^2 \equiv \langle\Delta^2(\theta)\rangle = \frac{2}{3}\Delta_0^2$. Then, the integrals are omitted, and we obtain the above-listed values of the gaps.

As it was mentioned above, in the anisotropic systems in which the fermionic energy gap has zeroes, the spectrum of massive collective modes has an imaginary part due to radiation of the gapless fermions. That is why the Nambu rule is not obeyed for the pole masses but is obeyed for the mass parameters that are real, since they are determined at $\omega = 0$. In the systems, in which radiation is absent, such as isotropic fully gapped superfluid ^{3}He-B, the pole masses of the collective modes coincide with their mass parameters.

D. Superfluid phases in 2 + 1 films

The same relations (26) and (27) take place for the bosonic collective modes in the quasi-two-dimensional superfluid ^{3}He films. There are two possible phases in thin films, the a phase and the so-called planar phase (b phase in the terminology of Ref. [55]). Both phases have isotropic gap Δ in the 2D case, as distinct from the 3D case in which such phases are anisotropic with zeroes in the gap.

We have the effective action for the bosonic degrees of freedom, Eqs. (5) and (6) with the 2×3 matrix $c_{i\alpha}$. The following two forms of these matrices correspond to the a and b phases [55]:

$$\begin{aligned} c_{i\alpha}^{(0)}(p) &= (\beta V)^{1/2} C\delta_{p0}\begin{pmatrix} 1 & 0 & 0 \\ i & 0 & 0\end{pmatrix}(\text{a-phase}) \\ c_{i\alpha}^{(0)}(p) &= (\beta V)^{1/2} C\delta_{p0}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0\end{pmatrix}(\text{b-phase}). \end{aligned} \tag{29}$$

Let us consider the second possibility (the planar b phase). We have the symmetry-breaking pattern $SO(2)\otimes SO(3)\otimes U(1) \to SO(2)$. Correspondingly, there are four gapless Goldstone modes. Among them, there are u_{13} and u_{23} modes. Modes v_{13} and v_{23} are their partners with the energy gaps 2Δ. The analysis is similar to that of the s-wave superconductor.

As for the modes u_{ij}, v_{ij} with $i, j = 1, 2$, the analysis is similar to that of the ^{3}He-B phase. The spectral densities $\rho_{u,v}$ differ from those of Eq. (12) by the kinematic factor $\sqrt{1/t}$ instead of $\sqrt{1-4\Delta^2/t}$. Next, we substitute $D = 2$ into Eq. (13) and get

$$E_{u,v}^{(J)} = \sqrt{2\Delta^2(1\pm\eta^{(J)})}, \quad J = 0, 1, 2. \tag{30}$$

Direct calculation of Eq. (14) gives $\eta^{J=0} = -\eta^{J=1} = 1$, and $\eta^{J=2} = 0$. (In this case, J is not the total momentum of the Cooper pair.)

Lemma II.3.—The resulting spectrum in the b phase is

$$\begin{aligned} E_{u,v}^{(0)} &= 2\Delta, 0; \qquad E_{u,v}^{(1)} = 0, 2\Delta; \\ E_{1,u,v}^{(2)} &= \sqrt{2}\Delta; \qquad E_{2,u,v}^{(2)} = \sqrt{2}\Delta. \end{aligned} \tag{31}$$

In the a phase, the symmetry breaking is $SO(2)\otimes SO(3)\otimes U(1) \to U(1)_Q \otimes SO(2)$ with three Goldstone modes. Acting as above, for the b phase (in this case, the u and v modes are mixed unlike the b phase), or applying the results of Ref. [55], one obtains

Lemma II.4.—These modes of the a phase form two pairs of Nambu partners (triply degenerated), with $Q = 0$ and $|Q| = 2$:

$$E_1^{(Q=0)} = 0, \qquad E_2^{(Q=0)} = 2\Delta \quad \text{and}$$
$$E^{(Q=+2)} = \sqrt{2}\Delta, \qquad E^{(Q=-2)} = \sqrt{2}\Delta. \tag{32}$$

Note that, since masses of $Q = +2$ and $Q = -2$ modes are equal, the Nambu sum rule necessarily leads to the definite value of the masses of the "charged" Higgs bosons.

It is worth mentioning that, in principle, the derivation of the energy gaps for the a phase with minor modifications may be applied also for the evaluation of the real parts of the energy gaps of the 3D A phase. In such calculations dealing with the equations that are the analogs of Eq. (14), we need to substitute the angle averaged fermionic gap (25).

Because of the common symmetry-breaking scheme in the electroweak theory and in ^{3}He-A, we consider the energy gaps listed above as an indication of the existence of the Higgs boson with mass

$$M_H = \sqrt{2}M_T. \tag{33}$$

This mass is about 245 GeV, which is roughly twice the mass of the lowest-energy Higgs boson.

III. NAMBU SUM RULES IN THE RELATIVISTIC MODELS OF TOP-QUARK CONDENSATION

A. Effective NJL model for the dynamical electroweak symmetry breaking

In this section, we consider the extended NJL model of top-quark condensation. This model was suggested in Ref. [9] by Miransky and coauthors and generalizes the more simple models (see, for example, Refs. [3,56,57]). It includes all six quarks. At the present moment, we do not wish to define the realistic theory aimed to explain dynamical electroweak symmetry breaking and the formation of fermion masses. Our objective is to demonstrate how the Nambu sum rule (probably, in the modified form) may appear in the relativistic models of the general kind.

The most general form of the four-fermion action (for the model with six quarks) has the form

$$S = \int d^4x (\bar{\chi}[i\nabla\gamma]\chi + g(\bar{\chi}_{\alpha A,L}\chi_R^{\beta B})(\bar{\chi}_{\bar{\beta}\bar{B},R}\chi_L^{\bar{\alpha}\bar{A}}) \times [YY^+]^{\alpha A\bar{\beta}\bar{B}}_{\bar{\alpha}\bar{A}\beta B} + g(\bar{\chi}_{\alpha A,L}\chi_R^{\beta B})(\bar{\chi}_{\bar{\beta}\bar{B},L}\chi_R^{\bar{\alpha}\bar{A}}) \times [WW^+]^{\alpha A\bar{\beta}\bar{B}}_{\bar{\alpha}\bar{A}\beta B}). \tag{34}$$

Here, $\chi^T_{\alpha A} = (u, d); (c, s); (t, b)$ is the set of the doublets with the generation index α. Tensors W, Y contain coupling constants. We consider the particular case of this model, when $W = 0$, while tensor Y is factorized:

$$Y^{\alpha A\bar{\beta}\bar{B}}_{\bar{\alpha}\bar{A}\beta B} = L^{\alpha}_{\bar{\alpha}}R^{\bar{\beta}}_{\beta}I^{\bar{B}}_{B}\delta^{A}_{\bar{A}}, \tag{35}$$

where L, R, I are Hermitian. Here, it is taken into account that the electroweak symmetry has to be preserved. The given four-fermion action approximates the microscopic theory. We suppose that this unknown microscopic theory has the approximate $U(2\times 3)_L \otimes U(2\times 3)_R$ symmetry that is broken softly down to $U(2)_L \otimes U(2)_L \otimes U(2)_L \otimes U(1)_R \otimes \ldots \otimes U(1)_R$. In the zeroth-order approximation, all eigenvalues of matrices L, R, I are equal to each other. In the next approximation, this symmetry is violated softly, and the eigenvalues of L, R, I receive small corrections.

Remark III.1.—The field theory with action Eq. (34) is not renormalizable. The ultraviolet divergences become stronger and stronger when the number of loops is increased. Therefore, Eq. (34) describes the phenomenological low-energy theory. It has sense only when a finite ultraviolet cutoff Λ is specified. The predictions of this model become independent of the regularization scheme only for the characteristic energies $\mathcal{E}$ of the processes much smaller than Λ. The physical quantities may, in principle, be evaluated using the $1/N_C$ perturbation theory. The leading terms in the expansion in the powers of $1/N_C$ correspond to the mean field approximation and are limited to the one-loop diagrams. Most of the practical calculations in the NJL-like models are performed in this approximation. (For the review of the calculations in NJL approximation applied to the models of top-quark condensation, see, for example, Ref. [4] and references therein.) The NTL order approximation corresponds to the number of fermion loops larger than one, or, equivalently, to the appearance of meson loops [5]. It has been shown that the NTL contributions to various dimensional quantities are small compared to the leading-order $1/N_C$ results only for $M_t \sim \Lambda$. For $5M_t \le \Lambda$, due to the NTL contribution, the Higgs condensate vanishes [5]. Rough consideration of the NTL (and higher) contributions to the scalar meson (Higgs boson) masses gives the values of the order of Λ unless these mesons are protected from being massive by symmetry. (For example, the Goldstone theorem protects the Goldstone bosons from masses if the chiral symmetry is broken spontaneously.)

The model considered in this paper corresponds to the condition $\Lambda \gg M_T$. Correspondingly, the one-loop results in the complete field model with action Eq. (34) are not valid either for the fermion masses or for the masses of the bosonic excitations if one considers the model nonperturbatively or sums higher-loop contributions. The one-loop prediction of the appearance of the dynamical chiral symmetry breaking may be incorrect as well. However, *we suggest considering action* Eq. (34) *as the action of the effective theory, in which only the leading* $1/N_C$ *(i.e., one-loop) contribution is taken into account, while the higher-loop corrections are to be disregarded*. Strictly speaking, this means that the quantum field theory considered here is not the one with the action of Eq. (34). Namely, the complete action of this theory is to contain additional terms that cancel the quadratically divergent contributions to the fermion and meson masses. For example, the dominant contributions to the meson masses of the diagrams with K four-fermion vertices are $\delta M_H^2 \sim g^K \Lambda^{2K-2}$. Assuming that the one-loop gap equation works, we get $g \sim \frac{1}{\Lambda^2}$. As a result, the higher-loop contributions to the Higgs boson masses are $\delta M_H^2 \sim \Lambda^2$ like in the Weinberg-Salam model, in which the loop corrections give quadratically divergent contributions to the Higgs boson masses. This results in the so-called hierarchy problem. One can see, therefore, that the hierarchy problem of the Standard Model is reflected by the effective theory with the action of Eq. (34). In the same way, the linear divergent contributions to the fermion masses appear due to the higher loops. The above-mentioned additional terms to be added to this action are to cancel these divergent contributions to meson and fermion masses. If so, the higher-loop contributions both to the fermion and boson masses are suppressed by the powers of $\frac{\mathcal{E}}{\Lambda}$, where $\mathcal{E}$ is the characteristic energy of the considered processes.

At a first glance, it is difficult to imagine the reasonable mechanism for the appearance of such terms. However, there exists the theory in which, in a similar situation, such terms do exist. Namely, in quantum hydrodynamics [10], there formally exist the divergent contributions to various quantities (say, to vacuum energy) due to the quantized sound waves. The quantum hydrodynamics is to be considered as a theory with finite cutoff Λ. The loop divergences in the vacuum energy are to be subtracted just like for the case of the NJL model of this section. In hydrodynamics, the explanation of such a subtraction is that the microscopic theory to which the hydrodynamics is an approximation works both at the energies smaller than and larger than Λ, and this microscopic theory contains the contributions from the energies larger than Λ. These contributions exactly cancel the divergences that appeared in the low-energy effective theory. This exact cancellation occurs due to the thermodynamical stability of vacuum. In Ref. [11], it was suggested that a similar pattern may provide the mechanism for the cancellation of the divergent contributions to vacuum energy in quantum gravity and divergent contributions to the Higgs boson mass in the Standard Model. Namely, the contributions of the trans-Λ degrees of freedom to the given quantities exactly cancel the divergent contributions of the effective low-energy theories (correspondingly, of gravity and of the Weinberg-Salam model). *We suppose that, in our case of the NJL model, the contributions of the trans-Λ degrees of freedom cancel the dominant divergences in the bosonic and fermionic masses leaving us with the one-loop approximation as an effective tool for the evaluation of physical quantities.*

B. One-loop effective action for the bosonic modes

Via the suitable redefinition of the fermions, we make matrices L, R, I diagonal. We denote

$$\begin{aligned}
&L = \mathrm{diag}(L_{ud}, L_{cs}, L_{tb}); \qquad R = \mathrm{diag}(R_{ud}, R_{cs}, R_{tb});\\
&I = \mathrm{diag}(I_{\rm up}, I_{\rm down}); \qquad y_u = L_{ud}R_{ud}I_{\rm up},\\
&y_d = L_{ud}R_{ud}I_{\rm down}, \qquad y_c = L_{cs}R_{cs}I_{\rm up},\\
&y_s = L_{cs}R_{cs}I_{\rm down}, \qquad y_t = L_{tb}R_{tb}I_{\rm up},\\
&y_b = L_{tb}R_{tb}I_{\rm down} \qquad y_{ud} = L_{ud}R_{ud}I_{\rm down},\\
&y_{du} = L_{ud}R_{ud}I_{\rm up}, \qquad y_{uc} = L_{ud}R_{cs}I_{\rm up},\\
&y_{cu} = L_{cs}R_{ud}I_{\rm up}, \qquad y_{us} = L_{ud}R_{cs}I_{\rm down},\\
&y_{su} = L_{cs}R_{ud}I_{\rm up}, \ldots\ldots
\end{aligned} \tag{36}$$

We can rescale the coupling constants in such a way that

$$y_q = 1 + \delta y_q, \qquad y_{q_1q_2} = 1 + \delta y_{q_1q_2}, \tag{37}$$

where $|\delta y_q|$, $|\delta y_{q_1q_2}| \ll 1$. The values of δy_q, $\delta y_{q_1q_2}$ satisfy

$$\delta y_{q_1q_2} + \delta y_{q_1q_2} = \delta y_{q_1} + \delta y_{q_2}. \tag{38}$$

The whole symmetry of Eq. (34) is $U_{L,1}(2) \otimes U(2)_{L,2} \otimes U(2)_{L,3} \otimes U(1)_u \otimes \ldots \otimes U(1)_b$. As in the previous sections, we introduce the bosonic variable $c^{\beta B}_{\alpha A}$ and insert into the functional integral the expression $1 \sim \int D\bar{c}Dc \exp\left(-\frac{i}{g}\sum_p \bar{c}^{\beta B}_{\alpha A}(p)c^{\alpha A}_{\beta B}(p)\right)$. We arrive at the action

$$S_{\rm eff} = -\frac{1}{g}\sum_p \bar{c}^{\beta B}_{\alpha A}(p)c^{\alpha A}_{\beta B}(p) + \log \mathrm{Det} M(\bar{c}, c), \tag{39}$$

where

$$\begin{aligned}
\bar{\chi}_{p_1}M(\bar{c}, c)\chi_{p_2} = {}& \bar{\chi}_{p_1}\hat{p}\gamma\chi_{p_2}\delta_{p_1p_2} - \frac{1}{(\beta V)^{1/2}}\\
&\times (Y^{\alpha A\bar{\beta}\bar{B}}_{\bar{\alpha}\bar{A}\beta B}c^{\bar{\alpha}\bar{A}}_{\bar{\beta}\bar{B}}(p_1 - p_2)\\
&\times \bar{\chi}_{p_1,\alpha,A,L}\chi^{\beta B}_{p_2,R} + \mathrm{H.c.}).
\end{aligned} \tag{40}$$

The equation that defines the vacua of the model is

$$\frac{\delta}{\delta c^{\alpha A}_{\beta B}} S_{\rm eff} = 0. \tag{41}$$

The solution of this equation corresponds to the stable vacuum if, at the vacuum value of c, we have $\mathrm{Det}\frac{\delta^2}{\delta c^{\alpha A}_{\beta}\delta c^{\alpha' A'}_{\beta'}} S_{\rm eff} \geq 0$. This occurs if the masses of all Higgs bosons are real. Suppose that the vacuum is CP invariant and the vacuum value of c is equal to

$$c^{(0)\alpha A}_{\beta B}(p) = (\beta V)^{1/2}C^{\alpha A}_{\beta B}\delta_{p0} \in R. \tag{42}$$

We also require that the mass matrix for the fermions $\mathbf{M}^{\alpha A}_{\beta B} = Y^{\alpha A\bar{\beta}\bar{B}}_{\bar{\alpha}\bar{A}\beta B}C^{\bar{\alpha}\bar{A}}_{\bar{\beta}\bar{B}}$ is Hermitian; then, Eq. (41) in one-loop approximation has the form

$$C^{\alpha A}_{\beta B} = \frac{ig}{2(2\pi)^4} Y^{\alpha A \bar{\beta} \bar{B}}_{\bar{\alpha} \bar{A} \beta B} \int \mathrm{Tr}\left(\frac{1}{l\gamma - \mathbf{M}}\right)^{\bar{\alpha}\bar{A}}_{\bar{\beta}\bar{B}} d^4 l = \frac{2igN_C}{(2\pi)^4} Y^{\alpha A \bar{\beta} \bar{B}}_{\bar{\alpha} \bar{A} \beta B} \int \left(\frac{\mathbf{M}}{l^2 - \mathbf{M}^2}\right)^{\bar{\alpha}\bar{A}}_{\bar{\beta}\bar{B}} d^4 l. \tag{43}$$

Here, $N_C = 3$ is the number of colors.

This equation has many different solutions that correspond to different vacua. We consider here only the case in which the matrices C and $\mathbf{M}$ are diagonal, so that there exist the condensates $\langle q\bar{q}\rangle$ and nonzero masses for all quarks. We also imply that the t-quark mass and the t-quark condensate dominate. The quark masses $M_q = y_q C_q$ satisfy the equations

$$0 = \frac{1}{gN_C} - \frac{2i}{(2\pi)^4} y_q^2 \int \left(\frac{1}{l^2 - M_q^2}\right) d^4 l = \frac{1}{gN_C} - \frac{y_q^2}{8\pi^2}\left(\Lambda^2 - M_q^2 \log \frac{\Lambda^2}{M_q^2}\right), \tag{44}$$

where Λ is the ultraviolet cutoff. If we set

$$g = \frac{8\pi^2}{\Lambda^2 N_C}, \tag{45}$$

then at $\Lambda \gg M_q$ from the gap equations, it follows that

$$\frac{M_q^2}{\Lambda^2} \log \frac{\Lambda^2}{M_q^2} = 2\delta y_q. \tag{46}$$

The given vacuum does not exhaust all possible vacua of the model. Further, we shall imply that the external conditions (and the values of couplings) are such that the given vacuum wins the competition between all possible vacua.

The symmetry-breaking pattern is $U_{L,1}(2) \otimes U(2)_{L,2} \otimes U(2)_{L,3} \otimes U(1)_u \otimes \ldots \otimes U(1)_b \to U(1)_u \otimes \ldots \otimes U(1)_t \otimes U(1)_b$. Therefore, we expect the appearance of 12 Goldstone bosons. Only three of them are to be eaten by Z and W bosons. In order to make the other modes massive, the gauge fields may be added, which become massive due to the symmetry breaking and absorb the mentioned extra Goldstone bosons.

C. Higgs bosons masses

The bosonic masses can be calculated as follows. As in Sec. II, at $k = 0$, the polarization operator can be represented as

$$\Pi(\omega) = \int_0^\infty dz \frac{\rho(z)}{z + \omega^2}, \tag{47}$$

with the spectral function ρ.

In the scalar/pseudoscalar $q\bar{q}$ channel, we have $((\sqrt{t}, 0) = p_+ + p_-;\ p_\pm^2 = M_q^2)$:

$$\begin{aligned} \rho^S_{q\bar{q}}(t) &= \frac{1}{32\pi^2}\theta(t - 4M_q^2)\sqrt{1 - \frac{4M_q^2}{t}}\mathrm{Sp}(\gamma p_- + M_q)(\gamma p_+ - M_q) = \frac{1}{16\pi^2}\sqrt{1 - \frac{4M_q^2}{t}}(t - 4M_q^2)\theta(t - 4M_q^2) \\ \rho^P_{q\bar{q}}(t) &= \frac{1}{32\pi^2}\theta(t - 4M_q^2)\sqrt{1 - \frac{16M_q^2}{t}}\mathrm{Sp}i\gamma^5(\gamma p_- + M_q)i\gamma^5(\gamma p_+ - M_q) = \frac{1}{16\pi^2}\sqrt{1 - \frac{4M_q^2}{t}}t\theta(t - 4M_q^2). \end{aligned} \tag{48}$$

Integrals in Eq. (47) are ultraviolet divergent. The regularization may be introduced in such a way that the upper limit in each integral is substituted by the finite cutoff (which may depend on the channel). Next, the $(q\bar{q})$ condensate provides the symmetry breaking. There should be Goldstone bosons corresponding to the broken symmetry. This provides that the P excitation in the $(q\bar{q})$ channel is massless [the corresponding bilinear appears via the application of the generator of the broken symmetry to $(q\bar{q})$]. Then, we have $\Pi^P_{q\bar{q}}(0) = \Pi^S_{q\bar{q}}(2iM_q)$, which means that the massive scalar excitation appears with mass $2M_T$. The same result can be obtained in the neutral channels $q\bar{q}$ via the direct calculation of the polarization operator:

$$\begin{aligned} \frac{1}{gN_C} - \Pi^S_{q\bar{q}}(iE) &= \frac{1}{gN_C} + \frac{iy_q^2}{2(2\pi)^4}\int d^4 l \mathrm{Sp}\frac{1}{l\gamma - M_q}\frac{1}{(p - l)\gamma - M_q} \\ &= (p^2 - 4M_q^2) y_q^2 I(M_q, M_q, p) \\ \frac{1}{gN_C} - \Pi^P_{q\bar{q}}(iE) &= \frac{1}{gN_C} + \frac{iy_q^2}{2(2\pi)^4}\int d^4 l \mathrm{Sp} i\gamma^5\frac{1}{l\gamma - M_q} i\gamma^5 \frac{1}{(p - l)\gamma - M_q} \\ &= (p^2) y_q^2 I(M_q, M_q, p), \\ I(m_1, m_2, p) &= \frac{i}{(2\pi)^4}\int d^4 l \frac{1}{(l^2 - m_1^2)[(p - l)^2 - m_2^2]}. \end{aligned} \tag{49}$$

Here, the gap equation is used. We get

$$M^P_{q\bar{q}} = 0; \qquad M^S_{q\bar{q}} = 2M_q \tag{50}$$

for $q = u, d, c, s, t, b$.

Remark III.2.—The calculations of the bosonic spectrum in the NJL models suffer from the ambiguity that appears when the shift of the variable is performed in the integral $\int \frac{d^4 l}{(l-p)^2 - m^2} \to \int \frac{d^4 l}{l^2 - m^2}$. In fact, this change of variables is not rigorous and results in the appearance of the new surface terms. This is a very well-known problem of the NJL models. (See, for example, Ref. [58].) The resulting contributions due to the surface terms were evaluated in Eq. (37) of Ref. [58]. From Ref. [58], it follows that, in the limit of large cutoff Λ, the extra contributions to Eq. (50) vanish. It should be stressed that this problem does not appear in the dimensional regularization. In the lattice regularization, momentum space is the four-dimensional torus, and the shift of the integration variable in the integrals like $\int \frac{d^4 l}{(l-p)^2 - m^2}$ is performed easily. Instead, in both cases, their own problems appear like the fermion doubling problem. However, in the approach through the dispersion relation, Eq. (48), this problem does not appear, while the final result is again $2M_q$. This justifies indirectly the shift of the variable in Eq. (49), $l - p \to l$. Anyway, the phenomenologically justified value of the NJL calculations in QCD and in the other models allows us to disregard the mentioned problem and to perform the shift of the integration variables in the calculation of the bosonic spectrum.

In each channel that includes two different quarks, q_1, q_2, the polarization operator is a 2×2 complex matrix $\mathcal{P}$. For its components, we have

$$\begin{aligned}
\frac{1}{gN_C} - \Pi^{q_{1,L}\bar{q}_{2,R}}_{q_{1,L}\bar{q}_{2,R}}(iE) &= \frac{1}{gN_C} + \frac{iy^2_{q_1q_2}}{4(2\pi)^4}\int d^4 l \mathrm{Sp}\frac{1}{l\gamma - M_{q_1}}(1 \pm \gamma^5)\frac{1}{(p-l)\gamma - M_{q_2}}(1 \mp \gamma^5)\\
&= (p^2 - M^2_{q_1} - M^2_{q_2})y^2_{q_1q_2} I(M_{q_1}, M_{q_2}, p) - \frac{iy^2_{q_1q_2}}{(2\pi)^4}\int d^4 l\frac{1}{l^2 - M^2_{q_1}} - \frac{iy^2_{q_1q_2}}{(2\pi)^4}\int d^4 l\frac{1}{l^2 - M^2_{q_2}}\\
&\quad + (1 + \zeta_{q_1q_2})\frac{iy^2_{q_1q_2}}{(2\pi)^4}\int d^4 l\frac{1}{l^2 - M^2_{q_1}} + (1 - \zeta_{q_1q_2})\frac{iy^2_{q_1q_2}}{(2\pi)^4}\int d^4 l\frac{1}{l^2 - M^2_{q_2}}\\
&= (p^2 - M^2_{q_1} - M^2_{q_2})y^2_{q_1q_2} I(M_{q_1}, M_{q_2}, p) + \zeta_{q_1q_2} y^2_{q_1q_2} J(M_{q_1}, M_{q_2}),\\
&\quad \text{where } \frac{1}{y^2_{q_1q_2}} = \frac{1 + \zeta_{q_1q_2}}{2y^2_{q_1}} + \frac{1 - \zeta_{q_1q_2}}{2y^2_{q_2}}.
\end{aligned} \tag{51}$$

For the cross-terms,

$$\begin{aligned}
-\Pi^{q_{2,L}\bar{q}_{1,R}}_{q_{1,L}\bar{q}_{2,R}}(iE) &= \frac{iy_{q_1q_2}y_{q_2q_1}}{4(2\pi)^4}\int d^4 l \mathrm{Sp}\frac{1}{l\gamma - M_{q_1}}(1 \pm \gamma^5)\frac{1}{(p-l)\gamma - M_{q_2}}(1 \pm \gamma^5)\\
&= 2M_{q_1}M_{q_2}y_{q_1q_2}y_{q_2q_1} I(M_{q_1}, M_{q_2}, p).
\end{aligned} \tag{52}$$

At $\Lambda \gg M_{q_1} > M_{q_2}$, we get $J(M_{q_1}, M_{q_2}) \approx [M^2_{q_1} - M^2_{q_2}] I(M_{q_1}, M_{q_2}, p) \approx \frac{M^2_{q_1} - M^2_{q_2}}{16\pi^2}\log \Lambda^2/M^2_{q_1}$ (here, Λ is the ultraviolet cutoff of the NJL model). Therefore,

$$\begin{aligned}
\frac{1}{gN_C} - \mathcal{P}_{q_1q_2}(iE) &= \begin{pmatrix} [E^2 - M^2_{q_1}(1 - \zeta_{q_1q_2}) - M^2_{q_2}(1 + \zeta_{q_1q_2})]y^2_{q_1q_2} & 2M_{q_1}M_{q_2}y_{q_1q_2}y_{q_2q_1}\\ 2M_{q_1}M_{q_2}y_{q_1q_2}y_{q_2q_1} & [E^2 - M^2_{q_1}(1 + \zeta_{q_2q_1}) - M^2_{q_2}(1 - \zeta_{q_2q_1})]y^2_{q_2q_1} \end{pmatrix}\\
&\quad \times \frac{1}{16\pi^2}\log \Lambda^2/M^2_{q_1}
\end{aligned} \tag{53}$$

Each state is doubly degenerate (we mark the corresponding states by + or −). We come to

Lemma III.3.—In the considered toy model, we have two excitations in each $q\bar{q}$ channel for $q = u, d, c, s, t, b$,

$$M^P_{q\bar{q}} = 0; \qquad M^S_{q\bar{q}} = 2M_q, \tag{54}$$

and four excitations (i.e., two doubly degenerated excitations) in each $q_1\bar{q}_2$ channel (for $q_1 \neq q_2 = u, d, c, s, t, b$) with masses:

$$M^2_{q_1q_2} = M^2_{q_1} + M^2_{q_2} \pm \sqrt{(M^2_{q_2} - M^2_{q_1})^2\zeta^2_{q_1q_2} + 4M^2_{q_1}M^2_{q_2}}, \tag{55}$$

where $\zeta_{q_1q_2}$ are given by

$$\zeta_{q_1q_2} = \frac{2\delta y_{q_1q_2} - \delta y_{q_2} - \delta y_{q_1}}{\delta y_{q_2} - \delta y_{q_1}} = \zeta_{q_2q_1}. \tag{56}$$

The Nambu sum rule has the form

$$[M^+_{q_1\bar{q}_2}]^2 + [M^-_{q_1\bar{q}_2}]^2 + [M^+_{q_2\bar{q}_1}]^2 + [M^-_{q_2\bar{q}_1}]^2 \approx 4[M^2_{q_1} + M^2_{q_2}], \qquad q_1 \neq q_2$$
$$[M^P_{q\bar{q}}]^2 + [M^S_{q\bar{q}}]^2 \approx 4M^2_q. \tag{57}$$

Remark III.4.—At $|\zeta_{q_1q_2}| < 1$, all considered bosonic masses are real, and there are no tachyons, which means that the vacuum is stable.

Since the top quark mass is much larger than the other fermion masses, the Nambu sum rule in the form of Eq. (1) with the top quark mass in the rhs holds in all channels that include the top quark. The other boson masses are much smaller.

Remark III.5.—Among the mentioned Higgs bosons, there are 12 Goldstone bosons that are exactly massless [in the channels $t(1 \pm \gamma^5)\bar{b}$, $t\gamma^5\bar{t}$, $c(1 \pm \gamma^5)\bar{s}$, $c\gamma^5\bar{c}$, $u(1 \pm \gamma^5)\bar{d}$, $u\gamma^5\bar{u}$, $b\gamma^5\bar{b}$, $s\gamma^5\bar{s}$, $d\gamma^5\bar{d}$]. There are Higgs bosons with the masses of the order of the t-quark mass [$t(1 \pm \gamma^5)\bar{b}, t\bar{t}, t(1 \pm \gamma^5)\bar{s}, t\gamma^5\bar{c}, t(1 \pm \gamma^5)\bar{d}, t\gamma^5\bar{u}$]. The other Higgs bosons have masses much smaller than the t-quark mass.

As it was mentioned above, the simplest relativistic models of the kind discussed in this section were considered in Refs. [1,3,56]. In these models, the neutral Higgs bosons have masses 0 or $2M_T$. However, the model considered in Ref. [56] has the charged Higgs bosons with masses $\approx\sqrt{2}M_T$. Actually, our derivation of the masses in $t\bar{q}$ channels is similar to that of Ref. [56] for the charged Higgs bosons.

A further generalization of the model of Ref. [56] was considered in Ref. [57], in which three scalar Higgs doublets are to be introduced; the fourth generation of quarks with large masses is involved. In this model, there are two charged scalar Higgs modes with masses $M^{H^\pm}_1$, $M^{H^\pm}_2$ and two pseudoscalar modes with masses M^A_1, M^A_2 that satisfy the relation $2\sum_i([M^{H^\pm}_i]^2 - [M^A_i]^2) \approx 4M^2_T$.

D. Nambu sum rules in dense QCD

Among the other relativistic systems, where the analogues of the Nambu sum rules were observed, we would like to mention QCD in the presence of finite chemical potential. First, let us notice the normal phase with the broken chiral symmetry (both T and μ are small compared to the QCD scale $\Lambda_{\rm QCD}$). We already mentioned in the introduction that, in this phase, the NJL approximation leads to the Nambu sum rule in the trivial form $M_\sigma = 2M_{\rm quark}$.

In dense QCD with $\mu > \Lambda_{\rm QCD}$, there may appear several phases with different diquark condensates. Among them, there is, for example, the CFL phase. In the phenomenological models of this phase, the three quarks u, d, s are supposed to be massless. The condensate is formed [59,60]:

$$\langle[\psi^i_\alpha]^t i\gamma^2\gamma^0\gamma^5\psi^j_\beta\rangle \sim \Phi^I_J\epsilon_{\alpha\beta J}\epsilon^{ijI} \sim (\beta V)^{1/2}C\epsilon_{\alpha\beta I}\epsilon^{ijI}. \tag{58}$$

There are 18 scalar fluctuations of Φ around this condensate (there are also 18 pseudoscalar fluctuations with the same masses [61]). The symmetry-breaking pattern is $SU(3)_L \otimes SU(3)_R \otimes SU(3)_F \otimes U(1)_A \otimes U(1)_B \to SU(3)_{CF}$. That is why there are $9+9$ massless Goldstone modes. Among the remaining $9+9$ Higgs modes, there are two octets of the traceless modes and two singlet trace modes. Correspondingly, the quark excitations also form singlets and octets. The singlet fermionic gap Δ_1 is twice larger than the octet fermionic gap Δ_8 (see Sec. 5.1.2. of Ref. [60]). Applying the technique similar to that which we developed for the consideration of ^{3}He-B, we get the scalar singlet and octet masses $M_1 = 2\Delta_1$, $M_8 = 2\Delta_8$. This may also be derived from the results presented in Refs. [62,63]. Thus, for the CFL phase of the color superconductor, we have the Nambu sum rules in the trivial form.

E. Nambu sum rule and Veltman identity

The condition for the cancellation of the quadratic divergences in the mass of the Higgs boson was discussed in a number of papers (see, for example, Refs. [64–68] and references therein). In the case of the single Higgs boson and in the absence of the gauge fields, this condition reads $3M^2_H = 4\sum_f M^2_f$. Here in the left-hand side, the scalar boson mass stands, while in the rhs, the sum is over the fermions. If the model contains only triply degenerated quarks, this relation is reduced to

$$M^2_H = 4\sum_f M^2_f. \tag{59}$$

There is an obvious analogy between this identity and the Nambu sum rule, Eq. (1). Let us consider this analogy in more details.

In this particular case, the bare action for the model that involves the scalar doublet c and the fermion fields χ^q has the form

$$\begin{aligned} S_{\rm eff} = {} & \frac{1}{2}\sum_p \bar{c}(p)\left[p^2 + \frac{M^2_H}{2}\right]c(p) \\ & - \frac{\lambda^2}{8}\sum_{p_1-p_2=p_3-p_4}\bar{c}(p_1)c(p_2)\bar{c}(p_4)c(p_3) \\ & + \sum_p \bar{\chi}_p p\gamma\chi_p \\ & - \sum_{q,p=p_1-p_2}(y_q c(p)\bar{\chi}_{p_1,q,L}\chi^q_{p_2,R} + \text{H.c.}). \end{aligned} \tag{60}$$

M_H is equal to the bare mass of the scalar. Masses of the fermions are related to this value as $M_q = \frac{y_q M_H}{\lambda}$ The origin of Eq. (59) is in the expression for the one-loop correction to the Higgs mass

$$\Pi^{(2)} \sim \frac{i3\lambda^2}{2(2\pi)^4}\int d^4l \frac{1}{l^2 - M_H^2} + \frac{i3\lambda^2}{2(2\pi)^4}\int d^4l \frac{1}{l^2} + \sum_q \frac{iN_C y_q^2}{(2\pi)^4}\int d^4l \mathrm{Sp}\frac{1}{l\gamma - M_q}\frac{1}{(p-l)\gamma - M_q}\Big|_{p=0}$$
$$= 2N_C\sum_q(-4M_q^2)y_q^2 I(M_q, M_q, 0) - \lambda^2\sum_q \frac{4N_C iM_q^2}{M_H^2(2\pi)^4}\int d^4l \frac{1}{l^2 - M_q^2} + \frac{3i\lambda^2}{2(2\pi)^4}\int d^4l \frac{1}{l^2 - M_H^2} + \frac{3i\lambda^2}{2(2\pi)^4}\int d^4l \frac{1}{l^2}$$
$$\approx \frac{\Lambda^2}{16\pi^2}\frac{\lambda^2}{M_H^2}\Big(3M_H^2 - 4N_C\sum_q M_q^2\Big). \tag{61}$$

This expression looks similar to Eq. (49). However, their origins are different. For example, the condition for the cancellation of quadratic divergences relies on the identity $N_C = 3$, while, in the derivation of the Nambu sum rule, this was never used. The Nambu sum rule, Eq. (1), in the models considered above works for any number of colors. Also, in Eq. (61), the number of the components of the scalar is important. Therefore, we come to the conclusion that the nature of Veltman identity, Eq. (59), differs from the nature of the Nambu sum rule. Their coincidence at $N_C = 3$ in the absence of the gauge fields is, presumably, an accident.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we have calculated the bosonic spectrum in the particular case of the model suggested by Miransky and coauthors in Ref. [9] that involves all six quarks. Our model appears when the constraints on the values of the coefficients are imposed. These constraints come from the supposition that the microscopic theory approximated by the given NJL model has the large symmetry. This $U(2\times 3)_L \otimes U(2\times 3)_R$ symmetry is broken softly down to $U(2)_L \otimes U(2)_L \otimes U(2)_L \otimes U(1)_R \otimes \ldots \otimes U(1)_R$. In the zeroth-order approximation, the parameters L_{ud}, L_{cs}, L_{tb}, R_{ud}, R_{cs}, R_{tb}, and I_{up}, I_{down} of the Lagrangian Eq. (34) are equal to each other, and all quarks acquire equal masses. In this approximation, the symmetry $U(2\times 3)_L \otimes U(2\times 3)_R$ is preserved. In the next approximation, this symmetry is violated, and the elements of matrices L, R, I receive small corrections that provide the validity of the Nambu sum rule and the difference in quark masses.

At the present moment, we do not intend to consider this model as realistic. Our aim was to demonstrate how the sum rule Eq. (1) emerges in relativistic models. Nevertheless, in principle, one may try to update this model in order to move it toward a realistic theory. In order to do this, one needs to provide large masses for the light scalar bosons of this model. It is worth mentioning that the energy scale of the microscopic theory that has the considered NJL model as an approximation should be essentially larger than 1 TeV. To pass the existing constraints on the flavor-changing neutral current, we need $[1/g]^{1/2} \geq 10^3$ TeV [7]. This implies that $\Lambda \geq 10^3$ TeV. The large value of Λ is also necessary in order to provide the realistic value of $F_T \approx 245$ GeV. In addition, one must provide that the production cross sections of the composite Higgs bosons with 130 GeV $< M_H <$ 550 GeV are much smaller than that of the Standard Model Higgs.

The unknown microscopic theory should provide that the trans-Λ degrees of freedom give contributions that exactly cancel the dominant divergent higher-loop contributions to the fermion and the boson masses of the effective theory given by Eq. (34) (in order to produce the masses of the excitations much smaller than the cutoff). Such a cancellation may occur due to the mechanism similar to that of quantum hydrodynamics [10]. Namely, in quantum hydrodynamics, there exists the ultraviolet cutoff Λ, and the divergent contributions to vacuum energy are present. These contributions, however, are exactly cancelled by the contributions of the trans-Λ degrees of freedom of the microscopic theory. The cancellation occurs due to the thermodynamical stability of the vacuum. We imply that such a mechanism works in the unknown microscopic theory having the NJL model with action Eq. (34) as a low-energy approximation. This cancellation allows us to use a one-loop approximation to the NJL model for the calculation of various quantities just like the classical hydrodynamics can be used disregarding divergent loop contributions.

Our analysis prompts that, in the realistic model, which inherits the structure of the considered toy model, the Nambu sum rule may appear in the form of Eq. (1). If so, it gives an important constraint on the bosonic spectrum. The Nambu sum rule generalizes the relation noticed by Nambu in Ref. [2]. According to this sum rule, the sum of the composite scalar boson masses squared (within each channel) is equal to $2M_f$ squared, where M_f is the mass of the heaviest fermion that contributes to the formation of the given composite scalar boson. (It is implied that the single fermion dominates in the formation of this state, i.e., its mass is essentially larger than the masses of the other fermions that contribute to the given composite boson.) Originally, this sum rule was considered by Nambu in ^{3}He-B and in the conventional superconductivity. In the present paper, we also consider how the Nambu sum rule emerges in ^{3}He-A including the thin films. We mention the analog of this sum rule in QCD at finite chemical potential.

We feel it is natural to suppose that the top quark contributes to the formation of the composite Higgs bosons. The other composite scalar bosons would have much smaller masses. The fact that such states are not observed means that the formation of these states is suppressed. For example, the light scalar bosons may be eaten by some extra gauge fields that acquire masses due to the Higgs mechanism. It is worth mentioning that the Nambu sum rule alone cannot predict the masses of all composite Higgs bosons. There exist infinitely many possibilities. Below, we list a few of them that seem to us interesting and instructive. In all these cases, it is implied that, in the rhs of Eq. (1), the top quark mass stands.

(1) If there are two (doubly degenerated) Higgs bosons in the channel that contains the 125 GeV Higgs, then the partner of the 125 GeV boson should have mass around 210 GeV.

(2) If there are only two states in this channel, then the partner of the 125 GeV Higgs should have the mass around 325 GeV. Then, the two Higgs masses $M_{H1} = 125$ GeV and $M_{H1} = 325$ GeV satisfy the relations $M_{H1} = \sqrt{1/8}(2M_T)$, $M_{H2} = \sqrt{7/8}(2M_T)$. These relations are to be compared with Eq. (22).

(3) In the channel with two states of equal masses, the 245 GeV Higgs bosons should appear in analogy with ^{3}He-A considered in Sec. II. Again, a certain excess of events in this region has been observed by ATLAS in 2011 (see, for example, Ref. [69]).

(4) There is an interesting possibility that there exist eight Higgs bosons of equal masses in a certain channel. Then, the Nambu sum rule predicts $M_H =$ 125 GeV, i.e., the value of mass reported recently as the candidate for the mass of the Standard Model Higgs boson.

(5) If, in the given channel, there are only two Higgs bosons, and one of them is Goldstone boson, the other one should have mass around 350 GeV. [This is the case of the $t\bar{t}$ channel in the original model of top-quark condensation by Bardeen and coauthors [3]. Thus, the discovery of the 125 GeV Higgs boson excludes this model. This also excludes the majority of the technicolor models considered so far, including the so-called walking technicolor models (substitute the mass of the technifermion instead of the top-quark mass into the Nambu sum rule).]

We did not consider in this paper the possibility that the order parameter in the relativistic NJL model has the structure of the space-time tensor as in ^{3}He (see, e.g., Refs. [70–74]). The simplest models of this kind appear as a modification of our toy model with the action of Eq. (34), where $\bar{\chi}_\alpha(p_+)O_{ij}\chi^\beta(p_-)$ stands instead of $\bar{\chi}_{\alpha,L}(p_+)\chi^\beta_R(p_-)$. Here, O_{ij} is the space-time tensor composed of gamma matrices and momenta $p_\pm$ [75].

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