Volovik, G.E.

Comment on "T dependence of the magnetic penetration depth in unconventional superconductors at low temperatures: Can it be linear?

Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.81.4023

Published: 01/01/1998

Document Version
Publisher's PDF, also known as Version of record

Comment on "*T* Dependence of the Magnetic Penetration Depth in Unconventional Superconductors at Low Temperatures: Can It Be Linear?"

In a recent Letter Schopohl and Dolgov (SD) suggested that a pure $d_{x^2-y^2}$-pairing state becomes invalid in the zero temperature limit, $T \rightarrow 0$ [1]. Their arguments are based on thermodynamics: if the magnetic penetration length depends linearly on T at low T, the Nernst theorem—the third law of thermodynamics—is violated. We show here that this conclusion is the result of the incorrect procedure of imposing the limit $T \rightarrow 0$ in the electromagnetic response. To illustrate their reasoning let us consider a simplified case of the uncharged Fermi superfluid with lines of zeros in the quasiparticle spectrum, the $d_{x^2-y^2}$ pairing being an example. In superfluids the density of the superfluid component $\rho_s(T)$ corresponds to the magnetic penetration length in superconductors, $1/\lambda^2(T) \propto \rho_s(T)$. In the case of the nodal lines it has linear dependence on T at low $T \ll T_c$: $\rho_s(T) = \rho - \rho_n(T)$, where the normal component density in such liquid is $\rho_n(T) \propto \rho T/T_c$. The kinetic energy contribution to the free energy of the liquid flowing with the superfluid velocity $\mathbf{v}_s$ along the channel is

$$\mathcal{F} = \frac{1}{2}\rho_s(T)v_s^2. \tag{1}$$

We consider the superflow circulating in an annular channel. This circulation is fixed, if one discards the negligibly small decay of the supercurrent via vortex formation, so one can consider $\mathbf{v}_s$ as temperature independent. This results in the finite entropy in $T = 0$ limit:

$$S(T = 0) = -\frac{\partial \mathcal{F}}{\partial T}\bigg|_{T=0} = \frac{1}{2}\frac{\partial \rho_n}{\partial T}\bigg|_{T=0} v_s^2 \propto v_s^2 \frac{\rho}{T_c}. \tag{2}$$

If one follows the argumentation in Ref. [1], such a violation of the Nernst theorem suggests that the superfluid density ρ_s (or the related penetration length λ in superconductors) cannot be a linear function of T, which would mean that the pairing states with nodal lines are prohibited at $T = 0$ by the Nernst theorem.

There is, however, a loophole in this argumentation. The superfluid density $\rho_s(T)$ is the linear response function of the current $\mathbf{j}$ to the superfluid velocity $\mathbf{v}_s$, and thus is obtained in the limit $\mathbf{v}_s \rightarrow 0$. On the other hand the Nernst theorem requires the limit $T \rightarrow 0$ at finite $\mathbf{v}_s$. These two limits are not commuting for the kinetic energy $\mathcal{F}$. The crossover parameter, $x = T/p_F v_s$, regulates the scaling behavior of $\mathcal{F}$ in different limiting cases: $\mathcal{F}(T,x) = f(x)\rho v_s^2 T/T_c$, where $f(x)$ is the dimensionless function of x [2]. The regime $x \gg 1$ corresponds to the linear response to the superfluid velocity, i.e., to the order of limits when $v_s \rightarrow 0$ first. In this "high temperature" case, $T \gg p_F v_s$, the scaling function $f(x) \rightarrow$ const and one obtains the finite entropy, $S(T) = \lim_{T\rightarrow 0}\lim_{v_s\rightarrow 0} -d\mathcal{F}/dT \propto \rho v_s^2/T_c$ in Eq. (2).

In the opposite limit of low T, $x \ll 1$, the scaling function has the asymptote $f(x) \rightarrow \frac{a}{x} + bx$, where a and b are parameters of order unity [2]. In this true Nernst limit the entropy is zero at $T = 0$:

$$\lim_{v_s\rightarrow 0}\lim_{T\rightarrow 0} -\frac{d\mathcal{F}}{dT} \propto v_s T \frac{\rho}{p_F T_c}, \tag{3}$$

in complete agreement with the Nernst theorem. Thus the linear T dependence of the linear response function $\rho_s(T)$ does not violate the third law of thermodynamics: the Nernst principle does not prohibit a pure $d_{x^2-y^2}$-pairing state to exist at $T = 0$ in an uncharged Fermi liquid.

The same can be immediately applied to the charged case, where the superfluid velocity $\mathbf{v}_s$ is to be substituted by the external electric current $\mathbf{j}$ discussed by SD [1]. Considering the true $T = 0$ limit of the energy, $\lim_{j\rightarrow 0}\lim_{T\rightarrow 0} -d\mathcal{F}/dT \propto jT$, one satisfies the Nernst principle. This does not contradict to the linear T dependence of the linear electromagnetic response, which for the wave vector $k = 0$ gives

$$\lim_{T\rightarrow 0}\lim_{j\rightarrow 0}\frac{d\lambda(k = 0,T)}{dT} = \text{const}. \tag{4}$$

For $k \neq 0$ there is another scaling parameter, $y = T/v_F k$, which regulates the dependence of the electromagnetic response on the wave vector k and produces the T^2 dependence of the penetration length at finite k, i.e., at $y \ll 1$ [3]. In the opposite case, $y \gg 1$, Eq. (4) is restored.

In conclusion, lines of nodes in clean superconductors are not in conflict with the Nernst theorem. The answer to the question in the title of their paper [1] is yes.

G. E. Volovik
Helsinki University of Technology
Low Temperature Laboratory
P.O. Box 2200, FIN-02015 HUT, Finland,
and Landau Institute for Theoretical Physics
117334 Moscow, Russia

Received 26 May 1998 [S0031-9007(98)07550-4]
PACS numbers: 74.25.Nf, 74.20.Fg, 74.72.Bk

[1] N. Schopohl and O. V. Dolgov, Phys. Rev. Lett. **80**, 4761 (1998).
[2] G. E. Volovik, JETP Lett. **65**, 491 (1997).
[3] I. Kosztin and A. J. Leggett, Phys. Rev. Lett. **79**, 135 (1997).

0031-9007/98/81(18)/4023(1)$15.00