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Comment on “$T$ Dependence of the Magnetic Penetration Depth in Unconventional Superconductors at Low Temperatures: Can It Be Linear?”

In a recent Letter Schopohl and Dolgov (SD) suggested that a pure $d_{x^2-y^2}$-pairing state becomes invalid in the zero temperature limit, $T \to 0$ [1]. Their arguments are based on thermodynamics: if the magnetic penetration length depends linearly on $T$ at low $T$, the Nernst theorem—the third law of thermodynamics—is violated. We show here that this conclusion is the result of the incorrect procedure of imposing the limit $T \to 0$ in the electromagnetic response. To illustrate their reasoning let us consider a simplified case of the uncharged Fermi superfluid with lines of zeros in the quasiparticle spectrum, the $d_{x^2-y^2}$ pairing being an example. In superfluids the density of the superfluid component $\rho_s(T)$ corresponds to the magnetic penetration length in superconductors, $1/\lambda^2(T) \propto \rho_s(T)$. In the case of the nodal lines it has linear dependence on $T$ at low $T \ll T_c$: $\rho_s(T) = \rho - \rho_s(T)$, where the normal component density in such liquid is $\rho_n(T) \propto \rho T/T_c$. The kinetic energy contribution to the free energy of the liquid flowing with the superfluid velocity $\mathbf{v}_s$ along the channel is

$$F = \frac{1}{2} \rho_s(T) \mathbf{v}_s^2.$$  

(1)

We consider the superflow circulating in an annular channel. This circulation is fixed, if one discards the negligibly small decay of the supercurrent via vortex formation, so one can consider $\mathbf{v}_s$ as temperature independent. This results in the finite entropy in $T = 0$ limit:

$$S(T = 0) = -\frac{\partial F}{\partial T} \bigg|_{T=0} = \frac{1}{2} \frac{\partial \rho_n}{\partial T} \bigg|_{T=0} v_s^2 \propto \frac{\rho}{T_c}.$$  

(2)

If one follows the argumentation in Ref. [1], such a violation of the Nernst theorem suggests that the superfluid density $\rho_s$ (or the related penetration length $\lambda$ in superconductors) cannot be a linear function of $T$, which would mean that the pairing states with nodal lines are prohibited at $T = 0$ by the Nernst theorem.

There is, however, a loophole in this argumentation. The superfluid density $\rho_s(T)$ is the linear response function of the current $j$ to the superfluid velocity $\mathbf{v}_s$, and thus is obtained in the limit $\mathbf{v}_s \to 0$. On the other hand the Nernst theorem requires the limit $T \to 0$ at finite $\mathbf{v}_s$. These two limits are not commuting for the kinetic energy $F$. The crossover parameter, $x = T/\rho T_c$, regulates the scaling behavior of $F$ in different limiting cases: $F(T, x) = f(x) \rho v_s^2 T/T_c$, where $f(x)$ is the dimensionless function of $x$ [2]. The regime $x \gg 1$ corresponds to the linear response to the superfluid velocity, i.e., to the order of limits when $v_s \to 0$ first. In this “high temperature” case, $T \gg \rho T_c$, the scaling function $f(x) \to \text{const}$ and one obtains the finite entropy,

$$S(T) = \lim_{v_s \to 0} \lim_{T \to 0} -dF/dT \propto \rho v_s^2 T/T_c,$$  

(2)

in complete agreement with the Nernst theorem. Thus the linear $T$ dependence of the linear response function $\rho_s(T)$ does not violate the third law of thermodynamics: the Nernst principle does not prohibit a pure $d_{x^2-y^2}$-pairing state to exist at $T = 0$ in an uncharged Fermi liquid.

The same can be immediately applied to the charged case, where the superfluid velocity $\mathbf{v}_s$ is to be substituted by the external electric current $j$ discussed by SD [1]. Considering the true $T = 0$ limit of the energy,

$$\lim_{v_s \to 0} \lim_{T \to 0} -dF/dT \propto jT,$$  

one satisfies the Nernst principle. This does not contradict to the linear $T$ dependence of the electromagnetic response, which for the wave vector $k \neq 0$ gives

$$\lim_{T \to 0} \lim_{j \to 0} \frac{d\lambda(k = 0, T)}{dT} = \text{const}.$$  

(4)

For $k \neq 0$ there is another scaling parameter, $y = T/\rho_F k$, which regulates the dependence of the electromagnetic response on the wave vector $k$ and produces the $T^2$ dependence of the penetration length at finite $k$, i.e., at $y \ll 1$ [3]. In the opposite case, $y \gg 1$, Eq. (4) is restored.

In conclusion, lines of nodes in clean superconductors are not in conflict with the Nernst theorem. The answer to the question in the title of their paper [1] is yes.

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