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Network traffic management via exclusive roads for altruistic vehicles under mixed traffic equilibrium

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ABSTRACT

We present a computational study of network ensembles with two types of coexisting vehicle classes: an altruistically routing vehicle (ARV) class – potentially automated vehicles that are routed to reduce total system travel time – and a selfishly routing vehicle (SRV) class, corresponding to human-driven vehicles. We investigate the performance of these networks when some links are reserved for exclusive use by the ARVs. The goal of these interventions is to avoid or mitigate the detrimental effects of the SRVs on the costs of the ARVs. We formulate the problem as a bi-level network design problem, where the upper level deals with optimising the choice of ARV-exclusive links minimising the statistical dispersion of used-route costs, while the lower level finds the corresponding traffic equilibrium under static traffic assignment conditions. We tackle the ARV-exclusive link selection with a genetic algorithm, where the fitness of solutions is based on the dispersion of the costs of routes used by ARVs. The mixed equilibrium is found by solving a multi-class static traffic assignment problem, with constraints on the SRV flows on the ARV-exclusive links. SRVs attempt to minimise their personal travel time, whilst ARVs attempt to drive the flows to system optimal. Our approach is effective in reducing the per-vehicle travel cost of the ARVs to below that of the SRVs, making altruistic routing a more attractive option on average. Our results are consistent across networks with different structures and demand levels.

1. Introduction

Connected and automated vehicles (CAVs) are being presented as one of the main components of future mobility systems; however, market projections of their introduction to the vehicle fleet suggest decades of coexistence with human-driven vehicles (Department for Transport, 2021; Lavasani et al., 2016). Depending on new vehicle ownership models that may develop as well as implemented policies, the impact of automation on urban traffic remains highly uncertain. Furthermore, technologies such as vehicle-to-infrastructure communication, may enable novel and much more flexible management strategies of the existing infrastructure; for example, a traffic management authority may dynamically allocate exclusive right-of-way to different types of vehicles (whether automated or not) along specific links in the network.

It has been recognised that CAVs can also be used as actuators in control strategies to regulate network traffic throughput (Chen et al., 2020) by enabling adherence to different routing behaviours and strategies. In particular, this could be in the context of managing a fleet of altruistically routing vehicles (ARVs) that aim to reduce system costs, or alternately improve emissions or livability (see Vol et al. (2023), for other types of altruistic objectives). Thus, in considering the coexistence of human-driven vehicles with CAVs it is important to also consider mixed traffic conditions where vehicle fleets with different routing behaviours interact.

In the 80s and 90s, when route guidance systems were developing and reaching maturity, the possibility of nudging the system closer to optimal by providing drivers with routing options that lower system costs became apparent (Watling and van Vuren, 1993). Even at this stage, the challenge of adherence to route guidance was clearly identified (van Vuren et al., 1990) and ways of incentivising adherence to routes with larger travel times are still not resolved (Vol et al., 2023).

Navigation apps are now ubiquitous and thanks to crowd-sourced information, can provide real-time travel time estimates as well as suggest alternative routes. Currently, however, they are mainly used in conjunction with selfish routing. Also when considering the future deployment of CAVs, the underlying assumption is frequently that CAVs will be routed selfishly (Alfaseeh et al., 2018; Huang et al., 2020; Mehr and Horowitz, 2020; Wang et al., 2019) or at least for the benefit of a fleet operator.

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In this paper, we investigate a mixed equilibrium (ME) traffic scenario where connected and automated vehicles are part of a single fleet that is centrally managed, journeys are routed with the goal of optimising the average travel time of all vehicles on the network. A proportion of the vehicles are ARVs, and the remaining human-driven vehicles are selfishly routing vehicles (SRVs). In order to ameliorate the travel times of the ARVs and make them an attractive mode choice we allocate a subset of links in a network for exclusive use by the ARVs.

1.1. Background on mixed traffic equilibrium

Wardrop's two criteria (Wardrop, 1952) specify two (generally) different traffic equilibria, sometimes seen as representing opposite behaviour: *user equilibrium* (UE) and *system optimum* (SO). The traffic pattern reached at UE arises by drivers (selfish) choosing their paths to minimise their individual travel times, which results in all used paths between any given origin–destination pair having the same cost. Conversely, at SO, vehicles are altruistic and seek to minimise the average travel time of all vehicles in the network. A consequence of the SO assignment is that used routes can present a wide spread of costs, making the higher-cost routes particularly unattractive.

A ME can arise when different user classes follow different routing principles. Considering a multi-class traffic assignment problem, van Vuren et al. (1990) model a mixed flow system with two classes of vehicles. One class follows the UE principle, while the other composed of ARVs attempts to drive the system to SO. This results in a ME; however, the ARVs experience higher per-vehicle costs. Their main result is that while system cost improvements happen already at moderate penetration rates of ARVs (of around 30%), very high penetration rates (70% and above) are required for the ARVs to experience lower travel times than at pure UE.

Vehicle classes with different routing behaviours emerge naturally in route guidance contexts. The types of equilibria used to model them attempt to capture different aspects of possible scenarios. For example, van Vuren and Watling (1991) use stochastic assignment to include perception errors of SRVs, as well as possible estimation errors by guidance systems. They obtain similar results to van Vuren et al. (1990).

Yang et al. (2007) developed a more complex model with an additional behavioural class, that follows the same core concept. In addition to the UE and SO classes, a third type of vehicle class is introduced that seeks to minimise the average cost to the vehicles in its own fleet. The results of Yang et al. (2007) reiterate findings by van Vuren et al. (1990); ME can indeed improve costs significantly with respect to UE. As a caveat, improvements depend on the network and demand. It is possible to introduce system-optimising vehicles without any cost improvements to the system until a threshold penetration rate is reached, since ARVs absorb the cost gains made by the selfish vehicles. Additionally, ARVs experience an uneven distribution of route costs.

Whilst UE and SO traffic equilibria are extremely idealised they serve as boundary cases and illustrate some challenges faced by altruistic routing. In order to minimise the total system cost, some vehicles incur much larger travel times than those on the shortest path.

1.2. Background on management strategies enabled by CAVs

With the advent of CAVs (e.g., at SAE levels 4 or 5) and vehicle-to-infrastructure communication, more flexible use of the road networks appears to be possible (Pompigna and Mauro, 2022; Touko Tcheumadjeu et al., 2022).

In terms of more dynamic infrastructure control, Ampountolas et al. (2020) propose a real-time lane control method for managing motorways with reversible lanes. Schemes like this could further be improved by making use of CAV capabilities, for example by incorporating ramp metering and lane control (Roncoli et al., 2015). Roncoli et al. (2017) develop a feedback control strategy that assigns lane changes to CAVs to maximise throughput at bottlenecks in motorways and redistributes vehicle density.

Special dedicated links for CAVs in conjunction with platooning and headway reduction have been proposed (see, for example, Papamichail et al. (2019)), which avoid the mixture of CAVs and SRVs that might disrupt the efficiency gains, or simply be unsafe. Conversely, Bahrami and Roorda (2020) observe that CAV-exclusive links can lead to significant improvements.

From the network perspective, dedicated CAV links open up routes for the CAVs that are unavailable for SRVs. If the CAVs are centrally routed to follow SO, these paths through the network that are unimpeded by SRVs have the potential of ameliorating the extra cost absorbed by vehicles attempting to follow a SO assignment. Thus, if automated ARVs are considered, restricting some links exclusively for them can provide them with better paths from which they are not displaced.

However, exclusive right of way for a vehicle class in a network can result in complex effects. Removing links from a network can improve system costs, such as the well-known Braess paradox (Braess, 1968), even in the case where they are still available to a subset of the vehicle classes (e.g., Acemoglu et al. (2018)).

Choosing the roads in a network to be designated as ARV-exclusive is a network design problem (NDP) which has received ample study in transportation research (Farahani et al., 2013). The NDP can be seen as a bilevel programming problem (Suh and Kim, 1992) where at the upper level, the network is altered to optimise an objective whose value is the result of a network equilibrium problem; the lower level problem.

Due to the computational complexity of the problem, usually, a candidate set of links that can be modified is pre-selected, and how they are to be modified is also specified. This can range from modifications to their capacity, in the continuous case, to the complete removal or addition of the links to the network in the discrete case. Since we are selecting existing links in the networks to be of exclusive use for one class of vehicles, we can consider the problem at hand a discrete NDP. This can be framed as a mixed integer program, and can be solved with, for example, the algorithms proposed by Gao et al. (2005) and Zhang et al. (2009). However, in practice, the number of decision variables used with these methods is small.

Both the NDP and the ME static traffic assignment problem have received ample study. In recent years, due to the seeming inevitability of an automated vehicle future, there has been a resurgence in the study of ME flow in various settings. These range from developing new capacity models taking into account short headways of CAVs (Lazar et al., 2020), mixed flows of system-optimising vehicles (Barzegari et al., 2023), to network control scenarios. For example, Zhang et al. (2009) investigate the optimal ratio of ARVs to SRVs required per origin–destination (OD) pair for reducing total network travel time. Even real-time control schemes are being devised that make use of machine learning for controlling individual paths of ARVs in order to optimise network flow (Lazar et al., 2021).

Table 1 shows existing works thematically related to this article, arranged to show their differences and their scope. In summary, there are many studies dealing with mixed traffic flow; however, their settings and methods vary widely and in general simulations tend to cover a reduced number of networks. CAVs have previously been identified as possible enablers of altruistic routing and effort has gone into understanding how to improve system costs using them. For example, Zhang and Nie (2018), focus on optimising the ARV ratio; however, they do so per OD pair, and Lazar et al. (2021) as well as Chen et al. (2020) attempt to control the paths directly.

In methodological terms, GAs have been shown to be effective in tackling the NDP, especially when using large numbers of decision variables. As for the networks that have been used in these studies, they tend to be small example networks, slightly larger networks with regular structures, or networks that are used as canonical examples in the literature such as Sioux Falls. To the best of the authors' knowledge, there are no studies similar to this one that cover a large ensemble of networks.

Table 1
Existing literature related to the current study. References are loosely organised around the three key themes: ME (with a main focus on altruistic routing), CAVs, and the importance of the studied Networks.

Theme	Focus	Key results	Networks	Refs.
Mixed equilibrium (altruistic routing)	Co-existing routing classes. UE, SO, *Cournot-Nash*, *stochastic UE*.	High proportion of ARVs (and Cournot-Nash veh.) needed for SO costs. Stackelberg equilibrium is better.	Small/medium test networks. Sioux falls (Wang et al., 2019).	van Vuren et al. (1990), van Vuren and Watling (1991), Yang et al. (2007), Wang et al. (2019)
	Optimal ARV to SRV ratio to reduce system costs.	Small ARV proportion almost achieves optimum. Network topology can dominate demand.	Sioux Falls. Anaheim	Zhang and Nie (2018)
	Control of individual ARV paths.	Reinforcement learning routing outperforms other schemes (Lazar et al., 2021)	Small example networks (Lazar et al., 2021)	Lazar et al. (2021), Chen et al. (2020)
CAVs	Road capacity increase	Road cost functions for mixed flows. Bounds on UE costs.	General networks	Lazar et al. (2020)
	Traffic flow stability in mixed flows	CAVs can help stabilise traffic. But in general specific CAV densities are needed.	No network (linear flow)	Huang et al. (2020)
	Capacity dependent on (Selfish) CAV proportions.	Capacity function conditions to decrease total costs	General networks	Mehr and Horowitz (2020)
	Capacity with CAVs (mixed traffic). Exclusive links.	AV exclusive links can improve system.	Small test network. Sioux Falls	Bahrami and Roorda (2020)
	Mixed SO flows	Trade-offs between cost and capacity	Simple networks. Sioux Falls.	Barzegari et al. (2023)
Network level	Mixed integer programming for NDP	Development of algorithms. (few decision variables).	Lattice test network (Gao et al., 2005). Small network. Sioux Falls. Hull.	Gao et al. (2005), Zhang et al. (2009)
	GAs for NDP	GAs are efficient, but require careful tuning. Useful ranges for GA parameters.	Melito Porto Salvo. Simplified Los Angeles	Cantarella et al. (2006) Pinninghoff et al. (2008)

1.3. Objective and contributions

The two contributions of this paper are related to mixed equilibria and altruistic routing with a focus on the role of the network. The first contribution is a proposed method to design interventions allocating existing links in road networks for use by ARVs (which could be CAVs) that make altruistic routing an attractive choice. The second contribution consists of empirically showing that having exclusive links for ARVs can consistently yield lower per-vehicle costs for ARVs relative to selfishly-routing vehicles in ME scenarios across networks with different topologies.

We aim for a breadth of scope in covering many networks with diverse morphologies. Thus, our approach consists of using simple models to uncover stylised facts, for which we present results for ensembles of randomly generated networks. The ensembles differ in the morphological parameters of the networks they are composed of. We use a genetic algorithm (GA) to solve the NDP of allocating ARV-exclusive links and evaluate ensemble performance for different demand values whilst covering a range of ARV penetration rates for each demand level. To keep the approach as parsimonious as possible, we consider fixed demands in order to avoid introducing additional parameters in the form of demand functions, which necessitate additional specifications and implicit assumptions.

The remainder of the paper is structured as follows: Section 2 presents the synthetic network model and the methods used in framing the ME traffic assignment as well as the NDP of selecting exclusive links for the ARVs; details of the implementation of the GA used in solving the NDP and the set-up for our computational experiments are given in Section 3. In Section 4 our results are presented and discussed. Finally, our conclusions are summarised in Section 5.

2. Methods

As expressed above, the goal of this paper is to understand how reserving some links for exclusive use by altruistically routing vehicles can be used as a general traffic management scheme across different networks. To make our results generalisable to some degree, we perform numerical experiments on ensembles of networks with different morphological features, rather than rely on popular example networks or exemplar networks of specific morphologies.

Our methodology consists of three components. The first consists of the *generation of the synthetic networks* that are used, as well as selecting parameters for their cost functions and a suitable OD pair. We aim at generating networks that are comparable amongst themselves even though they can contain different numbers of links and have different structures and topologies. The second component consists of the *selection of network links* that will be reserved for ARV use. Crucial in this step, is the need to ensure that all nodes in a network remain connected and reachable by both classes, regardless of their origin. The third component covers the *calculation of the ME traffic assignment*. We express the Wardrop equilibrium (Wardrop, 1952) of both vehicle classes (ARVs and SRVs) as the solution to an optimisation problem with a single objective. Since each vehicle class – when considered independently – has different equilibrium conditions, we refer to it as a mixed equilibrium static traffic assignment problem (ME-STAP).

The experimental workflow, from network generation to obtaining ensemble results for the link-segregated ME is represented in Fig. 1. In the remainder of this Section, we cover the methodological details and explain the rationale of the model that we then use for the numerical experiments presented in Section 4.

2.1. Network model

We use the $\alpha\beta$-network model developed in Espinosa Mireles de Villafranca et al. (2017) and Espinosa Mireles de Villafranca (2020)

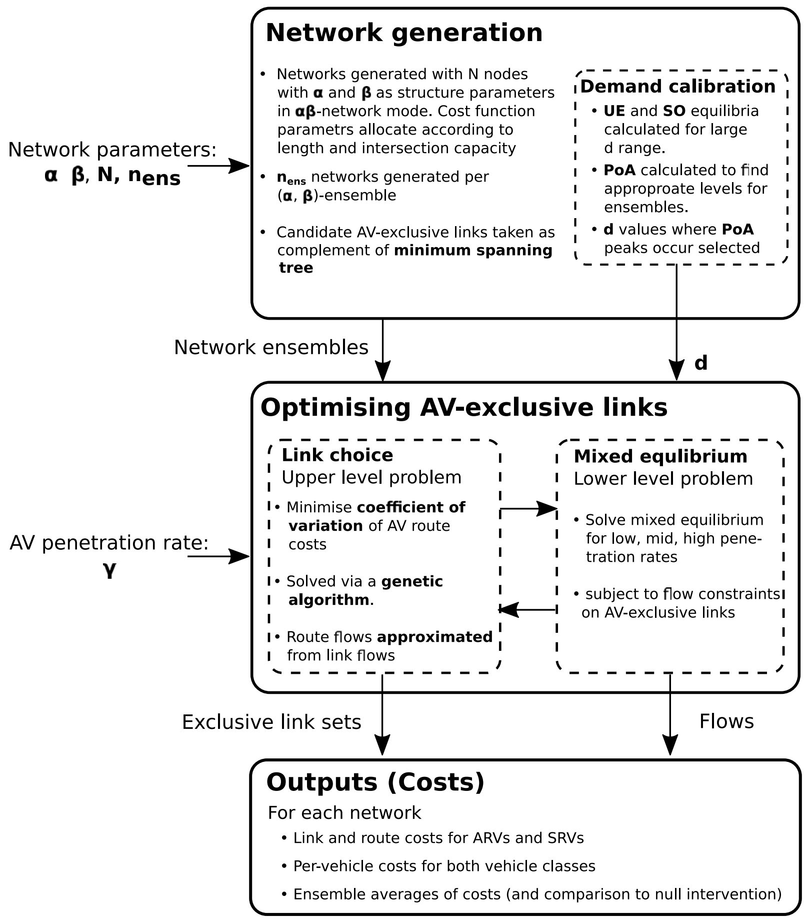


Fig. 1. Components of the proposed methodology, with input parameters as well as relevant outputs from each step.

to generate synthetic road networks that are directed random planar graphs, which resemble realistic urban networks to some extent. The model generates networks according to two structural parameters α and β that control for the randomness of node distribution and the density of links, respectively. Fig. 2 shows some network instances for different values of α.

The node-placement parameter α changes the distribution of the nodes between a perfect lattice ($\alpha = 0$) and a uniformly random distribution ($\alpha = 1$) inside the unit square. Therefore the model can capture features of networks that have different morphologies, from very grid-like, to networks that have much more random intersection distributions. The links of the network are determined by constructing their β-skeleton (Jaromczyk and Toussaint, 1992). Broadly speaking, β parametrises whether a given pair of nodes are considered 'closest' neighbours, and thus connected. More precisely, for every pair of nodes, β parametrises a region which must remain empty of any additional nodes in order for the original pair to be connected with a link.

The networks generated this way are initially obtained as undirected networks; to obtain the directed networks that we use, we simply replace each undirected link with two directed links with opposite orientations. Thus, each link from the $\alpha\beta$-networks models a two-way street.

2.1.1. Affine cost function parameters

For each link i in the network we use flow-dependent affine cost functions, c_i of the form

$$c_i(f_i) = a_i + b_i f_i, \tag{1}$$

where f_i is the vehicular flow on the link, while a_i and b_i are link-specific parameters that represent the free flow travel time and the sensitivity to congestion, respectively. The free-flow travel time, a_i, is set proportional to the (euclidean) length of the link, while the b_i are allocated with a heuristic that ensures that networks with different amounts of links are comparable. The idea behind the heuristic is that all networks have the same supply of infrastructure, which can be set to unity (in the dimensionless units that we are working with), so that

$$\sum_{i\in\mathcal{A}} \frac{a_i}{b_i} = 1, \tag{2}$$

where $\mathcal{A}$ is the set of links in the network. Additionally, we require that all nodes in a network have the same 'node capacity' and that this capacity is shared equally amongst its incoming links, yielding (after some algebraic manipulation)

$$b_i = k_i \sum_{v\in\mathcal{V}} \frac{1}{k_v} \sum_{j\in\mathcal{I}_v} a_j, \tag{3}$$

where $\mathcal{I}_v$ is the set of links that are incoming at node v and k_v is the in-degree of v. Note that, with a slight abuse of notation, k_i is the in-degree of the node to which link i is incident. In simpler words, for a given road network, all intersections are allocated the same node capacity, which is then evenly split amongst the incoming links at each node (for more details of this heuristic see: Espinosa Mireles de Villafranca et al. (2017), Espinosa Mireles de Villafranca (2020)).

2.2. Mixed assignment

We express the ME-STAP as a single-objective convex optimisation problem. Since our cost functions are affine, thus polynomial, we can

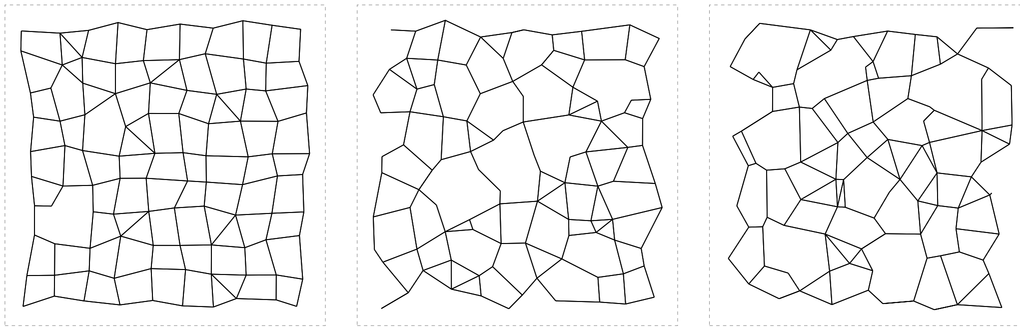


Fig. 2. Some examples of $\alpha\beta$-networks composed of 100 nodes each. They are all generated with $\beta = 1.5$ with (a) $\hat{\alpha} = 0.5$, (b) $\hat{\alpha} = 0.75$, and (c) $\hat{\alpha} = 1$.

follow van Vuren et al. (1990) and combine the different classes' experienced costs in a single Beckmann-like functional. Details of this formulation are found in Appendix C.

Since the ARVs are attempting to minimise the total system cost, their perceived link costs are higher, incorporating the imposed costs on others due to their presence on the link. Thus, their cost functions are the *marginal* cost functions derived from Eq. (1).

The resulting objective function for the mixed equilibrium assignment is

$$T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}) = \sum_{i\in\mathcal{A}} \int_0^{f_i^{\mathrm{ARV}}+f_i^{\mathrm{SRV}}} b_i s\, ds + \sum_{i\in\mathcal{A}} a_i f_i^{\mathrm{SRV}} + \sum_{i\in\mathcal{A}} \frac{a_i}{2} f_i^{\mathrm{ARV}}, \tag{4}$$

where the first term captures the flow-dependent term of the costs, while the second and third terms capture the (transformed) free flow costs for the SRVs and ARVs, respectively.

We solve the problem in the link-node formulation (Patriksson, 2015), where the flow conservation constraints are imposed at each node for each vehicle's class (ARVs and SRVs). That is

$$\begin{aligned} A\,\mathbf{f}_k^{\mathrm{SRV}} &= \mathbf{d}_k^{\mathrm{SRV}}, \qquad \forall k \in \mathcal{C}^{\mathrm{SRV}} \\ A\,\mathbf{f}_k^{\mathrm{ARV}} &= \mathbf{d}_k^{\mathrm{ARV}}, \qquad \forall k \in \mathcal{C}^{\mathrm{ARV}}, \end{aligned} \tag{5}$$

where A is the link-node incidence matrix of the road network and the vector $\mathbf{d}_k^{\mathrm{class}}$ is a vector that indicates the sink and source demands for each OD pair, defined as

$$(\mathbf{d}_k^{\mathrm{class}})_i = \begin{cases} -d_k, & \text{if } i \text{ is origin of } k \\ d_k, & \text{if } i \text{ is destination of } k \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

In the case of dealing with multiple ODs, we have that the link flows are the aggregate link flows for each class,

$$\begin{aligned} f_i^{\mathrm{SRV}} &= \sum_{k\in\mathcal{C}^{\mathrm{SRV}}} f_{ki}^{\mathrm{SRV}} \\ f_i^{\mathrm{ARV}} &= \sum_{k\in\mathcal{C}^{\mathrm{ARV}}} f_{ki}^{\mathrm{ARV}}, \qquad \forall i \in \mathcal{A}. \end{aligned} \tag{7}$$

The remaining constraints needing to be enforced pertain to restricting SRVs from using the ARV-exclusive links. We, therefore, impose zero SRV flow on the ARV-exclusive links, $\mathcal{A}^{ARV} \subset \mathcal{A}$,

$$f_{ki}^{\mathrm{SRV}} = 0, \quad k \in \mathcal{C}^{SRV},\ \forall i \in \mathcal{A}^{\mathrm{ARV}}. \tag{8}$$

In summary, the assignment is found by solving the link-node formulation of the ME-STAP, where the costs of both classes are encompassed by the objective function $T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{SRV}})$ (Eq. (4)), with constraints (5), (7), and (8):

$$\begin{aligned} \min_{\mathbf{f}^{\mathrm{SRV}},\, \mathbf{f}^{\mathrm{ARV}}} \quad & T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}) \\ \text{s.t.} \quad & \mathbf{f}^{\mathrm{SRV}} \geq \mathbf{0} \\ & \mathbf{f}^{\mathrm{ARV}} \geq \mathbf{0} \\ & A\,\mathbf{f}_k^{\mathrm{SRV}} = \mathbf{d}_k^{\mathrm{SRV}}, \quad \forall k \in \mathcal{C}^{\mathrm{SRV}} \\ & A\,\mathbf{f}_k^{\mathrm{ARV}} = \mathbf{d}_k^{\mathrm{ARV}}, \quad \forall k \in \mathcal{C}^{\mathrm{ARV}} \\ & f_i^{\mathrm{SRV}} = \sum_{k\in\mathcal{C}^{\mathrm{SRV}}} f_{ki}^{\mathrm{SRV}}, \quad \forall i \in \mathcal{A} \\ & f_i^{\mathrm{ARV}} = \sum_{k\in\mathcal{C}^{\mathrm{ARV}}} f_{ki}^{\mathrm{ARV}}, \quad \forall i \in \mathcal{A}, \\ & f_{ki}^{\mathrm{SRV}} = 0, \quad k \in \mathcal{C}^{SRV},\ \forall i \in \mathcal{A}^{\mathrm{ARV}}. \end{aligned} \tag{9}$$

There are two significant differences with the classic formulation of traffic assignment problems. The first is that the different vehicle classes have access to different subsets of the network. The second is that the different classes also experience different costs. A proof that problem (9) satisfies equilibrium conditions is given in Appendix D.

Since routes that contain ARV-exclusive links remain unused by SRVs, they can have lower costs than other used routes, which is inconsistent with the conditions for traditional UE (Beckmann et al., 1956) where used routes have lower or equal cost than unused ones. However, ARV-exclusive links do not actually form part of available routes for the SRVs. This is standard for capacitated traffic assignment (Patriksson, 2015) and does not prevent the equilibrium from existing or being unique in terms of route flows. Furthermore Acemoglu et al. (2018) explicitly consider multiple classes whose difference is the available subnetwork they can use, and they discuss the equilibrium in detail. Furthermore, restricting a class from links is equivalent to both the *side constrained* traffic assignment problem, and the traffic assignment problem with generalised costs (an extensive discussion of this can be found in Patriksson (2015)).

2.3. Selection of ARV-exclusive links

Selection of ARV-exclusive links is a discrete NDP. It can be seen as a bi-level programme, where the upper-level problem deals with the selection of ARV-exclusive links – subject to the traffic flow on the network – and the lower level problem consists of solving the ME-STAP (9) to find the equilibrium traffic flow (Farahani et al., 2013).

From previous studies (van Vuren et al., 1990; van Vuren and Watling, 1991; Yang et al., 2007), we know that optimising the total travel time does not guarantee a positive outcome for the ARVs. Additionally, in a mixed equilibrium scenario, as the penetration rate increases ARVs initially start filling routes that are close in cost to UE cost routes (Espinosa Mireles de Villafranca, 2020). For low demands, UE and SO costs and flows coincide, however, as UE and SO costs diverge the spread of costs along the routes used by ARVs increases. Even when some ARVs achieve equal per-vehicle costs to SRVs, the distribution of costs that the ARV fleet experiences can include very large route costs (Jahn et al., 2005, see for example).

A large cost spread is both indicative of a large deviation from UE (low per-vehicle costs) as well as travel time unreliability (Pu, 2011; Taylor, 2013). Thus, reducing the dispersion of the distribution of path costs reduces the unreliability of ARVs as a transport mode by ensuring larger proportions of the vehicles have path costs closer to the mean (and the lowest cost path).

Thus, our proposed objective is to minimise the *coefficient of variation* (CV), a measure of statistical dispersion, of the route costs for the ARVs, $\mathrm{CV}_{\boldsymbol{\kappa}}$, over the choice set of ARV-exclusive links $\mathcal{A}^{\mathrm{ARV}}$.

Concisely, the problem can be expressed as

$$\begin{aligned} \min_{\mathcal{A}^{\mathrm{ARV}}} \quad & \mathrm{CV}_{\boldsymbol{\kappa}}(\mathbf{h}^{\mathrm{ARV}} ; \mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}) \\ \text{s.t.} \quad & \mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}} \text{ solutions to (9)} \\ & \mathbf{h}^{\mathrm{ARV}} = \Lambda^{-1}\mathbf{f}^{\mathrm{ARV}}, \end{aligned} \tag{10}$$

where $\mathbf{h}^{\mathrm{ARV}}$ is the vector of route flows of ARVs; as above, $\mathbf{f}^{\mathrm{ARV}}$ and $\mathbf{f}^{\mathrm{SRV}}$ are the ARV and SRV link flows; Λ is the route-link incidence matrix, which encodes the links used by each path through the network. The coefficient $\mathrm{CV}_{\boldsymbol{\kappa}}$ depends indirectly on the link flows of both ARVs and HVs, which are the flow vectors obtained from solving problem (9), since it involves the link travel costs. Thus we have subscripted the CV with $\boldsymbol{\kappa}$ to indicate explicitly that it measures the dispersion of route costs.

In principle, $\mathbf{h}^{\mathrm{ARV}}$ is obtained from the link flows via the inverse of the route-link incidence matrix Λ^{-1}; however, this is a slight abuse of notation; in general, Λ is not invertible and route flows obtained from the link-node based formulation of the STAP, which we use, are not unique (Patriksson, 2015). Nevertheless, the route flows, $\mathbf{h}^{\mathrm{ARV}}$, can be approximately determined replacing Λ^{-1} with the pseudoinverse of Λ (details in Section 3.1.1).

Our approach to the solving problem (10) is employing a GA (full details are included in Section 3.1) mainly because we consider large amounts of candidate ARV-exclusive links. Since we are using the link-node-based formulation of the STAP, we also approximate (or determine) the ARV route flows from the link flows using a pseudoinverse of matrix Λ.

3. Implementation and experimental set-up

This Section focuses on the computational aspects of this study. We motivate the use of a GA for solving problem (10) and give an overview of the details of its implementation (Section 3.1) with special attention to the fitness calculations in Section 3.1.1. We also describe the experimental set-up for the network ensembles used (Section 3.2).

3.1. GA implementation

As mentioned above, the NDP has a high degree of complexity which causes difficulties such as large computational times, especially when considering large numbers of decision variables. Possel et al. (2018) have shown that GAs can be more efficient than other meta-heuristic methods such as simulated annealing methods and Deb et al. (2002) show that GAs can be very effective at exploring the solution space.

Since we look to keep a large number of decision variables, as opposed to pre-defining a small selection of possible links, in addition to keeping the procedure flexible enough to allow significant changes to the objective functions (of either Eq. (4) or (10)), a GA fits our experimental requirements.

In what follows in this section (Section 3.1), we give an overview of the specific GA we use. Further, more in-depth, details on parameter choices and values are given in Appendix E.

The main building blocks of a GA are (i) the mapping of the decision variables to the chromosomes of the individuals in the solution population, (ii) the selection procedure for mating pairs, (iii) the reproduction mechanism, and (iv) the propagation mechanism of the population of solutions from generation to generation. Underpinning the GA is the choice of the fitness function, which is in line with the objective function of the optimisation problem.

The decision variables for a given network are the candidate links for exclusive use by the ARVs. We seek to keep as many candidate links as possible as well as to maintain generality in the way they are chosen in order to allow future modifications and adaptations of the algorithm. We allow all links to be candidates except those belonging to a minimum spanning tree (MST) of the network (where the weights of the links used, a_i, are equivalent to their free-flow travel time). This ensures that all nodes remain reachable by SRVs. Since $\alpha\beta$-networks are based on rectangular grids, an MST contains about half of the links, which leaves the remainder (for our networks, more than 100 links, see Section 3) as decision variables for the chromosomes.

We use tournament selection to choose reproducing individuals. Where we choose two individuals uniformly at random and select the fittest of them for reproduction. A mating pair is obtained by repeating the procedure twice. Thus each individual in a reproducing pair has been deemed fitter than an alternative choice. Tournament selection is not as sensitive to the actual fitness value of the individuals as other methods, such as fitness-proportional selection, but still ensures that fitter individuals reproduce more than their lower-fitness counterparts (Sastry et al., 2014).

In terms of genetic operations, we use a one-point crossover which consists of randomly choosing a point on the mating chromosomes, splitting both chromosomes in two at the crossover point and swapping the respective ends. The two possible offspring chromosomes then receive two blocks of the genome of each of the parents; we keep both resulting new chromosomes in the offspring population, after allowing each gene to mutate with a non-zero probability. The one-point crossover is considered an adequate general-purpose genetic operation (Sastry et al., 2014).

We use an elite-preserving mechanism to improve the performance of the GA (Rudolph, 1998). The lowest-fitness offspring individuals are replaced with the fittest individuals of the parent population as long as they are fitter than the best-performing offspring solutions. This serves as a ratchet mechanism keeping the maximum fitness of progressive populations monotonically increasing. To counteract decreases in diversity of the population and to increase the search space of the GA a proportion of the lowest-fitness individuals is replaced with individuals with random chromosomes. This ensures that in every generation, there are new individuals with previously nonexistent combinations of ARV-exclusive links.

3.1.1. Fitness (objective function)

As discussed above (Section 2.3), in order to reduce the average per-vehicle costs for the ARVs, we minimise the CV of their route costs, $\mathrm{CV}_{\boldsymbol{\kappa}}$. In the GA, the objective function of problem (10) has to correspond to the fitness of the individuals. Correspondingly, we use the inverse of the CV as the fitness to be maximised,

$$\omega = \frac{\mu}{\sigma}, \tag{11}$$

where ω is the fitness of an individual, μ is the mean, and σ is the variance of the ARV path costs under the ME assignment.

We are solving the ME problem in the link-node formulation (Patriksson, 2015), as this avoids having to explicitly specify usable paths. This means that to calculate ω for an individual solution we have to recover the path flows. In general, due to the large number of possible paths in comparison to network links, route flows are underdetermined and there are multiple path flows that satisfy the ME link flows. Therefore, to obtain an estimate of the mean path cost we must first reconstruct a used path set.

For a given network $\mathcal{N} = (\mathcal{V}, \mathcal{A})$ with set of ARV-exclusive paths $\mathcal{A}^{\mathrm{ARV}}$ and ARV flows $\mathbf{f}^{\mathrm{ARV}}$, we find the induced subgraph $\tilde{\mathcal{N}} = (\mathcal{V}, \tilde{\mathcal{A}})$, where $\tilde{\mathcal{A}}$ is the subset of links that carry non-zero ARV flow, $\tilde{\mathcal{A}} = \{a \in \mathcal{A} \mid f_a^{\mathrm{ARV}} > 0\}$. In practice, we use a tolerance value of 10^{-8}.

ARVs are attempting to solve SO, which means that all the used paths have equal marginal costs and those of unused paths are equal or greater (Patriksson, 2015; Roughgarden, 2006). Thus, to reconstruct the used path set for network $\tilde{\mathcal{N}}$, we use Yen's algorithm to find the k-shortest simple paths (Yen, 1971), where the link weights are the marginal costs of the links, $c_i'(f_i)$. In preliminary experiments, it was determined that $k = 20$ yielded good path sets.

Once the shortest paths are found, the path flows need to be determined. The link flows, $\mathbf{f}$, can be defined by the following equation in terms of path flows, $\mathbf{h}$, via the link-route incidence matrix Λ (Patriksson, 2015),

$$\mathbf{f} = \Lambda \mathbf{h}, \tag{12}$$

where the elements of Λ are

$$\Lambda_{ij} = \begin{cases} 1, & \text{if link } i \text{ is in path } j, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Since the system is under-determined, we obtain an approximate solution for the path flows for the ARVs $\hat{\mathbf{h}}^{\mathrm{ARV}}$ by solving (12) using the Moore–Penrose pseudoinverse (Ben-Israel and Greville, 2013; Penrose, 1955) of Λ,

$$\hat{\mathbf{h}}^{\mathrm{ARV}} = \Lambda^{\dagger} \mathbf{f}^{\mathrm{ARV}}. \tag{14}$$

The solution obtained with the pseudoinverse is a least-square solution (Penrose, 1956), since it does not necessarily satisfy the non-negativity condition of flows, thus we project it onto the positive orthant by rounding negative values to zero. Furthermore, we require that the path flows add up to the ARV demand, which we enforce by taking the new path estimate $\tilde{\mathbf{h}}^{\mathrm{ARV}}$,

$$\tilde{h}_j^{\mathrm{ARV}} = \frac{d\gamma}{\sum_r \left[\hat{h}_r^{\mathrm{ARV}}\right]_+} \left[\hat{h}_j^{\mathrm{ARV}}\right]_+, \tag{15}$$

where the operation $[\,\cdot\,]_+ = \max(0, \,\cdot\,)$ rounds negative values to zero, d is the total demand, and γ is the proportion of ARVs. Paths with zero flow are removed from the path set and the matrix Λ is updated accordingly.

Finally, the ARV path costs are calculated by adding their respective link costs, where, for simplicity, the SRV flows from the original assignment, f_i^{SRV}, are kept. Thus, κ_p, the cost of path p is

$$\kappa_p(\mathbf{f}^{\mathrm{SRV}}, \tilde{\mathbf{f}}^{\mathrm{ARV}}) = \kappa_p(\mathbf{f}^{\mathrm{SRV}}, \Lambda\tilde{\mathbf{h}}^{\mathrm{ARV}}) = \sum_{i \in p} c_i(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}}), \tag{16}$$

where index i runs over the links that belong to the path (the non-zero elements in column p of Λ). The ARV link flows can be once again obtained using Λ.

In preliminary tests, the discrepancy between total travel time for flows from the original ME assignment and the one with $\tilde{\mathbf{h}}^{\mathrm{ARV}}$ was less than 1% for a variety of networks and demand conditions. The links with the highest flow discrepancy (both relative and absolute) were links connected to the origin and the destination.

Finally, estimates for the mean and variance of the path costs can be calculated,

$$\hat{\mu} = \frac{1}{d\gamma} \sum_{p \in \mathcal{P}} \tilde{h}_p \kappa_p(\mathbf{f}^{\mathrm{SRV}}, \tilde{\mathbf{f}}^{\mathrm{ARV}}), \tag{17}$$

$$\hat{\sigma}^2 = \frac{1}{d\gamma} \sum_{p \in \mathcal{P}} \tilde{h}_p \left[\kappa_p(\mathbf{f}^{\mathrm{SRV}}, \tilde{\mathbf{f}}^{\mathrm{ARV}}) - \hat{\mu}\right]^2, \tag{18}$$

from which the fitness $\omega = \hat{\mu}/\hat{\sigma}$, follows.

Table 2
Parameters used in numerical experiments. All networks have 100 nodes. $\hat{\alpha}$ and β are the morphological parameters of the network ensembles. Both d and γ cover low, mid and high values. $|\mathcal{E}|$ is the number of networks per ensemble.

Nodes	$\hat{\alpha}$	β	d	γ	$\lvert\mathcal{E}\rvert$
100	0.5, 0.75, 1	1.5	0.0001, 0.0151, 0.03	0.25, 0.5, 0.75	20

3.2. Experimental set-up

We now specify the details of the numerical experiments performed, whose results are presented in the following Section. The parameters chosen are summarised in Table 2 and described below.

In particular, we solve the link segregated ME-STAP, specified in (9), for three network ensembles with different values of $\hat{\alpha}$: 0.5, 0.75, and 1. The network structure in the different ensembles ranges from fairly gridded ($\hat{\alpha} = 0.5$) to much more random, where the networks with $\hat{\alpha} = 1.0$ have a wider distribution of "block" sizes (see exemplar networks in Fig. 2). For all the ensembles, we use $\beta = 1.5$, since this value matches real-world networks to some extent (Osaragi and Hiraga, 2014).

Each ensemble, $\mathcal{E}$, consists of 20 networks with 100 nodes each, while the number of links is variable, and depends on the positions of the nodes, whose randomness is parametrised by $\hat{\alpha}$, and more directly on β which is being kept constant at $\beta = 1.5$.

In our experiments, we use a single OD pair for each network. In order to have the equilibrium flow pattern utilise as much of each network as possible, we choose the nodes that are closest to opposing corners of the unit square in which the networks are defined. The origin is the node closest to the bottom left corner, while the destination node is the closest node to the top right corner.

We evaluate nine different demand scenarios which cover all combinations of low, medium and high levels of total demand d, as well as ARV penetration rate γ. The appropriate values of d for the networks are determined by the procedure outlined in Section 3.2.1, are d = 0.0001, 0.0151 0.03. The penetration rate values are $\gamma = 0.25, 0.5, 0.75$.

For each (d, γ)-pair and each ensemble (i.e., 27 experimental ensembles), we run the GA detailed in Section 2.3 for each of the 20 networks, for 900 generations with a population size of 150 individuals. The GA parameters are summarised in Table E.4. This means that 540 instances of the GA are run, which amounts to 72,900,000 individual solutions being evaluated.

3.2.1. Ensemble demand range

Disregarding the flow constraints on the ARV-exclusive links, the ME assignment (in terms of both costs and flows) lies between UE and SO (Espinosa Mireles de Villafranca, 2020; van Vuren et al., 1990). Thus, the largest benefits from ME are expected when the discrepancy between the costs of UE and SO is the largest. This can be measured by the *price of anarchy* (PoA) (Roughgarden, 2006), which is defined as the ratio of the costs between the UE and SO assignments,

$$\mathrm{PoA} = \frac{\sum_{i \in \mathcal{A}} f_i^{\mathrm{UE}} c_i(f_i^{\mathrm{UE}})}{\sum_{i \in \mathcal{A}} f_i^{\mathrm{SO}} c_i(f_i^{\mathrm{SO}})}, \tag{19}$$

where f_i^{UE} and f_i^{SO} are the link flows at UE and SO respectively.

For general cost functions and hard capacities on links, the PoA can be arbitrarily large; however, for well-behaved cost functions, there is a theoretical upper bound (Roughgarden, 2006) that depends on the type of cost functions, which, for affine functions, is known to be 4/3.

In practice, for polynomial cost functions and feasible flows, the UE does not differ too much from SO and the PoA remains much lower than the upper bound (Youn et al., 2008, see for example), which is what we observe in our networks as well. Furthermore, for low ($\mathbf{f} \to \mathbf{0}$) and high flows ($\mathbf{f} \gg \mathbf{0}$) the PoA tends to unity (Colini-Baldeschi et al., 2020). That is, for low flows or highly congested networks, the UE cost approaches or even attains SO values, with a region in between where

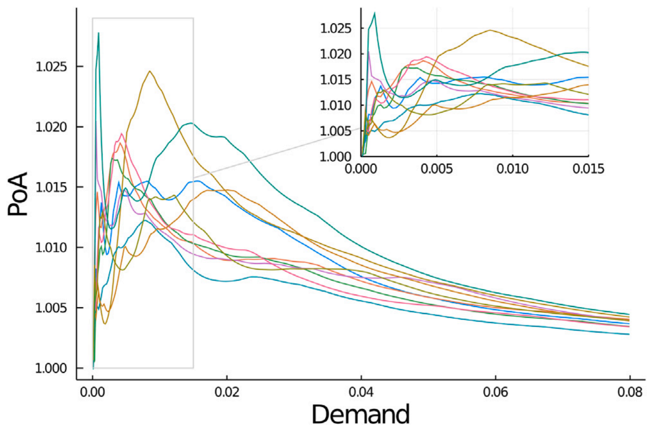


Fig. 3. Demand calibration with PoA profiles for different networks in the same ensemble ($N = 100$, $\hat{\alpha} = 0.75$, $\beta = 1.5$).

the PoA achieves a maximum at 'medium' flow levels. What 'medium' is, depends on the demand, the network, and the cost functions.

In our case, since we are studying network ensembles and are interested in the performance of the networks across a range of demands, the appropriate demand range for use in our numerical experiments must be determined. The $\alpha\beta$-network model and choice of parameters for the cost functions are designed to be able to compare assignment results between networks. However, costs and traffic patterns are still very sensitive to the network structure, and the maximum peaks of the PoA – while in the same general demand region – do not exactly line up.

Therefore, we calibrate the demand range for our ensembles graphically; Fig. 3 shows the PoA as a function of demand for 10 networks in an ensemble. Both the general overlap due to the choice of a_i and b_i, as well as the variation due to network-specific structure, can be seen. We aim to determine a demand region that captures the PoA peaks of the networks.

The qualitative behaviour of the PoA is the same for all convex polynomial cost functions, while the main difference is that the upper bound increases with the degree p of the polynomial (Correa et al., 2008; Roughgarden, 2006), as well as the empirical PoA values experimentally observed. Thus, affine functions serve the purpose since the possible gains in efficiency only increase with the polynomial order.

Since the PoA captures the relative difference between SO and UE in terms of the system costs, it can serve as a measure to see whether there are possible gains achievable by shifting the traffic equilibrium closer to SO. By selecting the experimental demands for this study as described above, we ensure that the demand region where the difference is bigger between UE and SO is explored by the ME.

3.2.2. Evolution of GA solutions

As with other heuristic methods, GAs require some experimentation and parameter tuning to be effective at arriving at good solutions. In particular, it is essential to have a balance between increasing the fitness of individuals as the generations progress, with maintaining a diverse set of individuals to effectively explore the search space.

Due to the large number of decision variables we consider, exhaustively exploring combinations of parameters is impracticable, and the parameters were selected by progressive improvement and iteration. In this Section, we give some detail on the performance and behaviour of our GA.

In Fig. 4 we show an example network with its MST and a solution obtained by the GA within 300 generations. We can compare the fitness of this solution $\omega = 4{,}746$ with the ensemble average of the fitness of $\langle\omega\rangle$ of 1,361.9 after 300 generations. This suggests that to further increase the fitness, an increase in the number of generations is probably needed; however, as is shown in Fig. 5 the fitness rarely increases significantly past 600 generations with the exception of a few outliers whose fitness keeps increasing. Thus we select 900 generations as the stopping point, to balance optimisation results with computational time.

4. Results

For each network in an ensemble, we take the best GA solution after the 900th generation as the intervention. We calculate the ME-STAP, as well as the UE and SO assignments. For each assignment, we calculate the total costs (in terms of their travel times), as well as per-vehicle costs.

As there can be significant variation in the networks for an ensemble, to have a more rigorous check whether the sample size is appropriate, for the most important metric, the ratio of per-vehicle costs between the classes, we calculate the bootstrap estimate (Davison and Hinkley, 1997) of the standard error as well as the 95% confidence intervals (see Appendix F), to check if there is any reason to suspect that the sample mean is different than the theoretical population mean, or that the sample size of $|\mathcal{E}| = 20$ networks per ensemble is too small.

The effectiveness of the interventions can be evaluated by the relationship between the per-vehicle cost of the ARVs to the per-vehicle costs of the SRVs, as well as the comparison between the ME costs before and after adding the ARV-exclusive lanes. For each ensemble, we calculate the mean ratio between the per-vehicle costs of the ARVs ($C_{\mathrm{pv}}^{\mathrm{ARV}}$) and SRVs($C_{\mathrm{pv}}^{\mathrm{SRV}}$)

$$\left\langle \frac{C_{\mathrm{pv}}^{\mathrm{ARV}}}{C_{\mathrm{pv}}^{\mathrm{SRV}}} \right\rangle = \left(\frac{1}{\gamma} - 1\right) \left\langle \frac{\sum_{i\in\mathcal{A}} c_i(f_i) f_i^{\mathrm{ARV}}}{\sum_{i\in\mathcal{A}} c_i(f_i) f_i^{\mathrm{SRV}}} \right\rangle_{\mathcal{N}\in\mathcal{E}}, \tag{20}$$

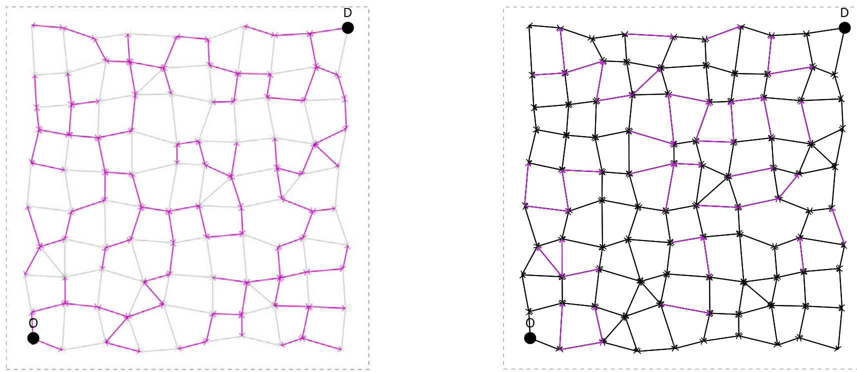
where the first term is the simplified quotient of the respective ARV and SRV demands. We also calculate the mean ratio of ME costs with and without the ARV-exclusive road intervention, $\left\langle C_{\mathrm{ME}}^{\mathrm{sol}} / C_{\mathrm{ME}}^{\mathrm{null}} \right\rangle$, where $C_{\mathrm{ME}}^{\mathrm{sol}}$ is the total ME cost for the best-performing GA solution, and $C_{\mathrm{ME}}^{\mathrm{null}}$ the total ME cost for the null intervention. In order to compare our intervention to the null intervention (no ARVs and no changes to the networks) the ratio of mean costs between these scenarios is also calculated, $\left\langle C_{\mathrm{ME}}^{\mathrm{sol}} / C_{\mathrm{UE}}^{\mathrm{null}} \right\rangle$.

We also calculate the average cost ratio of the intervention with that of the *null intervention*, with the same proportion γ of ARVs but where no exclusive links are provided for altruists, $\langle \mathrm{sol}\ C_{\mathrm{ME}} / \mathrm{null}\ C_{\mathrm{ME}} \rangle$. For completeness, we also report the average cost ratio of the intervention to the cost of the UE assignment, also for the case where no flow constraints are imposed on any links in the networks, $\langle \mathrm{sol}\ C_{\mathrm{ME}} / \mathrm{null}\ C_{\mathrm{UE}} \rangle$.

To compare the total costs of the interventions with respect to UE and SO costs in a way that facilitates comparison across the ensembles, we calculate the normalised costs with respect to the UE-SO cost gap. From the normalised costs for each network, we obtain the average ensemble normalised cost by calculating the arithmetic mean of C_{norm} of the best-performing solutions in each ensemble. For a single network, the normalised cost is

$$C_{\mathrm{norm}} = \frac{C_{\mathrm{ME}} - C_{\mathrm{SO}}}{C_{\mathrm{UE}} - C_{\mathrm{SO}}}, \tag{21}$$

where C_{UE} is the cost of UE, C_{SO} is the cost at SO, and $C_{\mathrm{ME}} = C(\mathbf{f}^{\mathrm{ME}})$ is the cost of the flows resulting from the ME assignment with the ARV-exclusive links from the best GA solution. It should be emphasised that C_{norm} is a relative measure of the ME cost in terms of the size of the UE-SO cost gap, which in practice, both in this paper and more widely in the literature (Youn et al., 2008, see for example), is generally small. Thus, even large values do not necessarily imply particularly large absolute costs compared to the SO cost. Note that for very low demands, where the PoA is unity and no cost gap exists C_{norm} is not well defined.



(a) MST (b) ARV-exclusive links

Fig. 4. A 100-node $\alpha\beta$-network ($\hat{\alpha} = 0.5$, $\beta = 1.5$) with (a) its MST and (b) its corresponding GA solution after 300 generations (with a fitness $\omega = 4{,}746$) $d = 0.0151$ and $\gamma = 0.5$. The origin and destination nodes are marked O and D, respectively.

Table 3
Results from ensemble runs, averages are of the best performing GA solution for each network in the ensemble. $\langle\omega\rangle$ is the average ensemble fitness, $\left\langle C_{\mathrm{pv}}^{\mathrm{ARV}}/C_{\mathrm{pv}}^{\mathrm{SRV}}\right\rangle$ is the average of the ratio of ARV per-vehicles cost to SRV per-vehicle cost, $\langle \mathrm{sol}\ C_{\mathrm{ME}}/\mathrm{null}\ C_{\mathrm{ME}}\rangle$ is the ratio of total ME cost for the solution to the ME cost under no intervention, $\langle \mathrm{sol}\ C_{\mathrm{ME}}/\mathrm{null}\ C_{\mathrm{UE}}\rangle$ is the ratio of total ME cost for the best solution to the UE cost with no intervention. The values of C_{norm} marked with a '-' are undefined due to the denominator of (21) being zero.

$\hat{\alpha}$	d	PoA	γ	$\langle\omega\rangle$	$\langle AV\,links\rangle$	$\left\langle \frac{C_{\mathrm{pv}}^{\mathrm{AV}}}{C_{\mathrm{pv}}^{\mathrm{SV}}} \right\rangle$	$\left\langle \frac{\mathrm{sol}C_{\mathrm{ME}}}{\mathrm{null}C_{\mathrm{ME}}} \right\rangle$	$\left\langle \frac{\mathrm{sol}\ C_{\mathrm{ME}}}{\mathrm{null}\ C_{\mathrm{UE}}} \right\rangle$	$\langle C_{\mathrm{norm}}\rangle$
0.5	0.0001	1.0000	0.25	85.720	43.9	0.932	1.072	1.077	–
			0.5	104.67	42.7	0.933	1.039	1.045	–
			0.75	96.749	51.5	0.913	1.022	1.028	–
	0.0151	1.0037	0.25	6399.3	40.6	0.751	1.183	1.180	63.19
			0.5	1361.9	45.5	0.801	1.089	1.086	25.88
			0.75	646.37	43.65	0.943	1.015	1.012	5.01
	0.03	1.0016	0.25	4330.5	41.45	0.640	1.255	1.253	187.7
			0.5	1525.0	45.5	0.768	1.113	1.111	84.5
			0.75	729.58	48.7	0.942	1.013	1.012	10.29
0.75	0.0001	1.0000	0.25	84.871	31.75	0.956	1.058	1.063	–
			0.5	97.013	30.6	0.942	1.043	1.049	–
			0.75	95.854	38.2	0.935	1.017	1.023	–
	0.0151	1.0052	0.25	1297.4	26.15	0.814	1.110	1.106	20.75
			0.5	868.89	31.85	0.867	1.065	1.061	13.08
			0.75	341.63	32.95	0.965	1.012	1.009	2.68
	0.03	1.0027	0.25	1489.3	29.35	0.739	1.155	1.153	60.73
			0.5	771.08	34.65	0.813	1.090	1.088	33.56
			0.75	440.55	36.1	0.950	1.013	1.011	5.61
1	0.0001	1.0000	0.25	72.022	24.35	0.960	1.104	1.109	–
			0.5	77.818	28.95	0.914	1.051	1.057	–
			0.75	80.576	27.1	0.909	1.023	1.029	–
	0.0151	1.0052	0.25	651.68	18.75	0.827	1.100	1.095	22.22
			0.5	436.24	25.15	0.904	1.040	1.036	8.58
			0.75	259.83	24.9	0.968	1.011	1.008	2.75
	0.03	1.0027	0.25	776.02	22.5	0.743	1.186	1.183	66.86
			0.5	496.43	25.8	0.867	1.053	1.050	19.42
			0.75	284.27	27	0.975	1.006	1.005	2.74

The above values along with PoA, the mean fitness $\langle\omega\rangle$ of the best GA solution, and the mean number of ARV-exclusive links are shown in Table 3.

Fig. 6 shows flows on an exemplar network from the best-performing ensemble in terms of ARV per-vehicle costs ($\hat{\alpha} = 0.5$, $d = 0.03$, and $\gamma = 0.25$). This particular network has $C_{\mathrm{pv}}^{\mathrm{ARV}}/C_{\mathrm{pv}}^{\mathrm{SRV}} = 0.615$ which is below the ensemble mean, in this case, there is a system cost increase of 20.4% compared to the ME scenario with no intervention.

The emerging traffic patterns show the formation of main corridor routes (see Fig. 6(c)) used by the ARVs which take roundabout routes and cut across the network taking advantage of key ARV-exclusive links near the centre of the network. These main routes carry most of the flow. It is notable that even though the distribution of ARV-exclusive links is fragmented due to the protected MST structure, the result is the formation of a few key routes which take the majority of the ARV flow. On the other hand, the SRVs show a much more homogeneous distribution across the road network.

The first point to note is that for all ensembles and demand levels, the intervention results in lower per-vehicle costs for ARVs than for SRVs. These values can be seen in the 7th column of Table 3 ($\left\langle C_{\mathrm{pv}}^{\mathrm{ARV}}/C_{\mathrm{pv}}^{\mathrm{SRV}}\right\rangle$). Interestingly, this also occurs for low total demands ($d = 0.0001$) where PoA = 1, that is, where the cost gap between UE (completely selfish routing) and SO (completely altruistic routing) is negligible. This means that even for uncongested networks, exclusive lanes for altruists can result in lower per-vehicle costs for SRVs.

It also has to be noted that, in all cases, the total cost after the interventions is higher than for the null intervention. However, in most cases, the increase in total cost is less than 10%. The highest cost

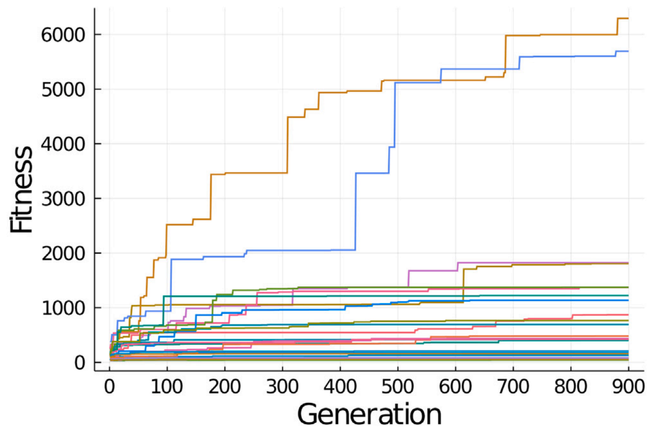


Fig. 5. Fitness evolution. The fitness of the best solution is shown for one graph in each ensemble as a function of the generation number.

increases for each ensemble happen at low penetration rates ($\gamma = 0.25$) with high total demands, that is, when the network is congested and the majority of the vehicles follow selfish routing. However, in these cases, the burden of the increase in cost is mostly carried by the SRVs. These cases, with the highest total cost increase, are also the ones in which the ARVs have the lowest per-vehicle cost relative to the SRVs. For example, for $\hat{\alpha} = 0.5$, when the demand is $d = 0.03$ with a $\gamma = 0.25$, we see the highest increase in total cost post-intervention at 25.5%. Yet the SRV-to-HV per-vehicle cost ratio is the lowest at 0.640. Fig. 7 compares the per-vehicle cost ratio against the total cost ratio between the solutions and the null intervention for all experimental ensembles.

As can be seen in Fig. 7, the relationship between the cost ratios is dominated by the penetration rate γ. In terms of the per-vehicle cost ratio, unsurprisingly, the best-performing networks are those with low penetration rates of ARVs. For high penetration rates, we observe that the total costs of the GA solutions are the most similar to the null intervention costs. This is also expected, as ARVs are able to use the whole network, therefore we can expect the actual traffic patterns to be similar to the unrestricted ME when large proportions of the fleet are ARVs.

Focusing only on the medium and low penetration rates ($\gamma = 0.25, 0.5$), we note that for low demands, the per-vehicle cost ratio is always above 90%. For medium and high demand values the ARVs are a considerably more attractive mode choice with most exhibiting improvements of 15% or greater when compared to SRVs.

Interestingly enough, the morphology of the networks does not seem to be important in the performance of the ME under this infrastructure management scheme. The penetration rate and the demand matter much more in the potential attractiveness of altruistic routing choices. It should be noted, that when thinking of applying a scheme like this on actual road networks, the combination of realistic demand patterns together with the network morphology is expected to play a significant role in the potential benefits. However, as a first instance, we can broadly say that networks with high demand and medium to low penetration rates can expect to see the biggest impact.

In terms of the GA, the solutions are chosen based on their fitness which is calculated based on an estimate of route flows. In our framework, both the route set and the corresponding flows are approximations: the route set because of how paths are recovered from the link flows; and the flows themselves, due to how they are calculated with the pseudoinverse of the link-route incidence matrix. While our results can be improved upon, for example by using a better approximation for the path flows, the simple heuristic of selecting solutions with low dispersion of even approximate route costs (for ARVs) gives consistently lower per-vehicle costs for altruists than for selfishly routing drivers. This happens across a range of network morphologies, for different demand levels, and for different penetration rates of ARVs.

Due to our approach of averaging over ensembles of networks, we see that minimising path-cost dispersion with the choice of ARV-exclusive lanes is an effective heuristic for making the altruistic route choice a better alternative than selfish routing.

In summary, the trade-off is between higher total system costs and unattractive travel times for altruists. By considering the altruistic vehicles to be CAVs one distances passengers further from the operation of the vehicle, and opting to use an automated ARV can be considered a choice transportation mode. Therefore, ensuring a significant penetration rate of altruists could arise naturally out of a selfish mode choice since ARV per-vehicle costs can be made lower than that of SRVs. Our results indicate that if higher system costs are tolerated initially, selfish mode choice leading to an increased penetration rate of altruists could naturally regulate this extra system cost. In this way, CAVs can provide the mechanism to make altruistic routing a viable strategy for implementing these types of management strategies: instead of incentivising users to make altruistic route choices, it can be presented as a convenient modal choice.

5. Conclusions

In this paper, we have studied the impact that allocating exclusive links for use by vehicles that choose routes altruistically has on both the total cost (as measured by travel time) of the system as well as on the average per-vehicle travel costs of the selfish and altruistic vehicle classes.

The equilibrium traffic flows were calculated by solving the static traffic assignment problem in a multiclass setting with two co-existing vehicle classes following altruistic and selfish routing principles, respectively. We solved the network design problem of selecting the links for the ARVs with a GA that seeks to minimise the statistical dispersion of the path costs of the ARVs. The effects of these interventions are studied on ensembles of synthetic networks to evaluate the performance of these measures across many networks that have diverse morphologies, and to explore the generalisability of our strategy to different networks and different levels of demand.

Our main contribution is that the presented framework is effective at reducing the per-vehicle average travel cost of the ARVs to below that of the selfishly-routing vehicles. Although the total system costs increase when compared to not carrying out any intervention, for all our network ensembles the interventions make the altruistic route choice (on average) more attractive than the selfish routes. These results are consistent across networks with significantly different morphologies and for all the demand levels and penetration rates of ARVs that we studied. Furthermore, a main finding is that minimising the dispersion of the path costs of the ARVs as a network design objective is effective in making it a more attractive routing mode than following selfish routing. Additionally, we show that even when considering sub-optimal ARV-exclusive link choices, that are obtained via estimates of the path flows, this is enough to obtain consistent ARV cost improvements across diverse network structures.

In our experiments we find that the overall network costs increase above the original UE costs as improvements are made to the ARVs' costs, this remains an issue to be addressed in further work. However, our traffic equilibrium model is conservative by design as we use a static assignment model due to the speculative nature of this study. We expect that taking into account the dynamical aspects of both demand variation and the possibilities that connectivity and digitalisation of infrastructure will offer in the future, means that greater benefits can be reaped by extending the core ideas of our framework into more sophisticated network management schemes.

In terms of implementation, for ARVs to benefit consistently, an additional mechanism that ensures that ARVs sample the different cost routes evenly over consecutive trips would be necessary. As stated in

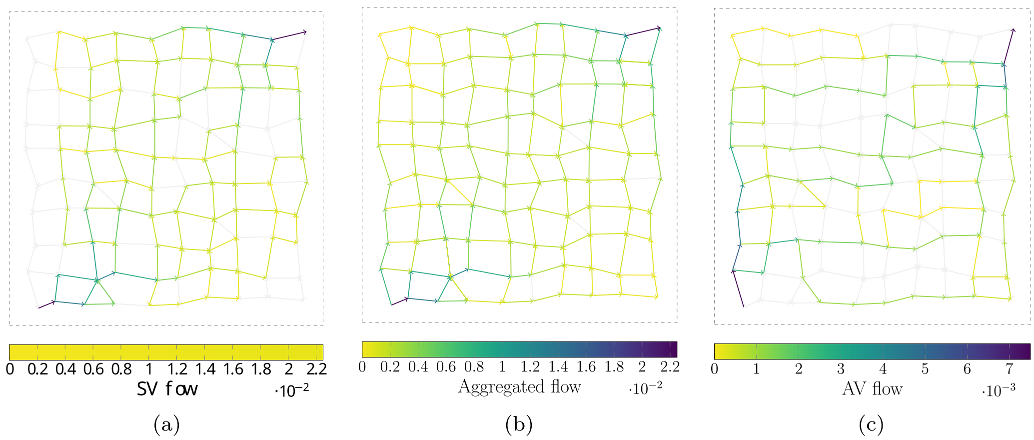


Fig. 6. Equilibrium flows for (a) SRVs, (b) aggregated SRVs and ARVs, and (c) ARVs on a network from the ensemble with parameters $\hat{\alpha} = 0.5$, $d = 0.03$, and $\gamma = 0.25$. Note the different scales for (c).

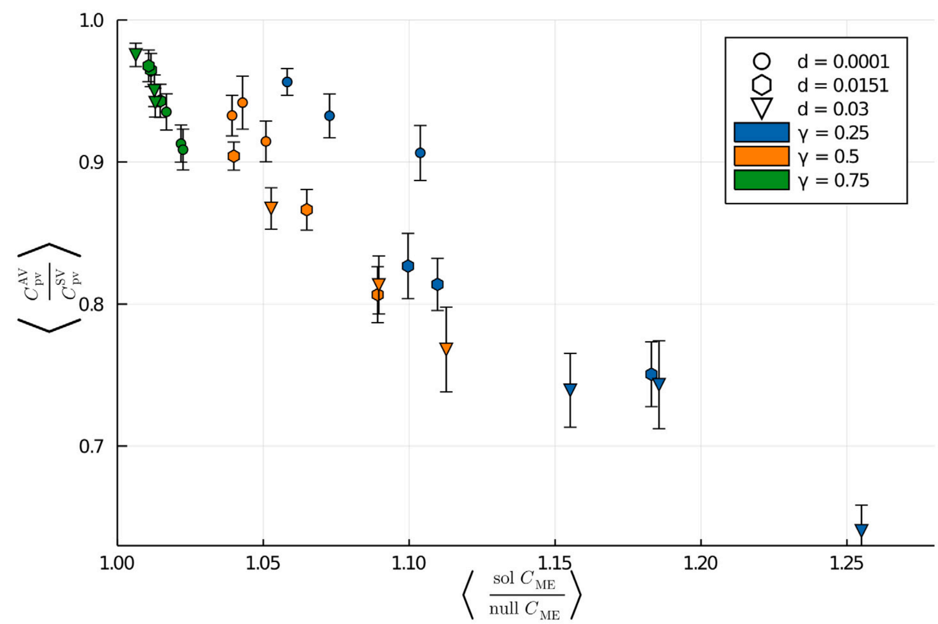


Fig. 7. Relationship between the per-vehicle cost ratio of ARVs to SRVs against the proportional increase in total travel cost after exclusive road intervention. The colours group points with the same γ, while the marker shapes indicate demand levels: circle for $d = 0.0001$, hexagon for $d = 0.151$, and triangle for $d = 0.03$. Error bars show the standard error calculated with bootstrap re-sampling (see Appendix F).

Section 1, CAVs and a system for digital infrastructure management – as emerging technologies – can provide a way for schemes like the one presented in this paper to be applied efficiently. In addition, in possible future implementations, tailoring the GA to a particular network by taking into account realistic demand structure as well as variation across time of day is likely to achieve much better outcomes than the ensemble average presented here.

Future work stemming from this study can lie in at least two directions. In terms of implementability and approaching real-world systems using existing city road networks is an option as well as replacing the static traffic assignment component in the methodology for dynamic assignment. In another vein, as sustainability and environmental concerns keep growing in importance and urgency, genetic algorithms can deal with multiple objectives. In addition to travel time, the sort of objectives that can be considered include CO_2 emissions, fuel consumption, safety, and accessibility metrics. This opens up an avenue for enriching the altruistic routing model with goals that align with current transport policy issues.

CRediT authorship contribution statement

Alonso Espinosa Mireles de Villafranca: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Claudio Roncoli:** Conceptualization, Formal analysis, Methodology, Validation, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Notation

Symbol	Description
$\mathcal{N}$	Network ($\mathcal{N} = (\mathcal{V}, \mathcal{A})$)
$\mathcal{V}$	Set of nodes in a network
$\mathcal{A}$	Set of links in a network
$\mathcal{P}$	Set of simple paths in a network
$\mathcal{A}^{\mathrm{ARV}}$	Set of ARV-exclusive links in a network
$\mathcal{C}^{\mathrm{SRV}}$	Set of OD pairs for SRVs
$\mathcal{C}^{\mathrm{SRV}}$	Set of OD pairs for ARVs
A	Link-node incidence matrix
Λ	Link-route incidence matrix
$\Lambda^{\dagger}$	Moore–Penrose pseudoinverse of Λ
$\hat{\alpha}$	Griddedness parameter in $\alpha\beta$-network
β	Link construction parameter in $\alpha\beta$-network (β-skeleton parameter)
$\mathcal{E}$	Ensemble of networks with same morphological parameters $\hat{\alpha}$, β
$\mathbf{d}_k^{\mathrm{class}}$	Vector with source and sink demands for OD pair k, with class $\in \{\mathrm{SRV}, \mathrm{SRV}\}$ and $k \in \mathcal{C}^{\mathrm{class}}$
f_i	Flow on link i
$\mathbf{f}$	Network flow vector with entries f_i ($\mathbf{f} = \left(f_i\right)_{i\in\mathcal{A}}$)
f_i^{SRV}	SRV flow on link $i \in \mathcal{A}$
$\mathbf{f}^{\mathrm{SRV}}$	Network SRV flow, $\mathbf{f}^{\mathrm{SRV}} = \left(f_i^{\mathrm{SRV}}\right)_{i\in\mathcal{A}}$
$\mathbf{f}_k^{\mathrm{SRV}}$	SRV Network flow due to vehicles of OD pair k, for $k \in \mathcal{C}^{\mathrm{SRV}}$
f_i^{ARV}	ARV flow on link $i \in \mathcal{A}$
$\mathbf{f}^{\mathrm{ARV}}$	Network SRV flow, $\mathbf{f}^{\mathrm{ARV}} = \left(f_i^{\mathrm{ARV}}\right)_{i\in\mathcal{A}}$
$\mathbf{f}_k^{\mathrm{ARV}}$	ARV Network flow due to vehicles of OD pair k, for $k \in \mathcal{C}^{\mathrm{ARV}}$
f_i^{UE}	Flow on link $i \in \mathcal{A}$ arising from UE assignment
f_i^{SO}	Flow on link $i \in \mathcal{A}$ arising from SO assignment
$\mathbf{f}^{\mathrm{ME}}$	Flow of ME solution: $\mathbf{f}^{\mathrm{SRV}} = \mathbf{f}^{\mathrm{SRV}} + \mathbf{f}^{\mathrm{ARV}}$
$\mathbf{h}$	Network path flow with entries h_p ($\mathbf{h} = \left(h_p\right)_{p\in\mathcal{P}}$). Subscripts and superscripts have the same meanings as for $\mathbf{f}$
c_i	Cost (travel time) on link $i \in \mathcal{A}$
c_i'	Marginal cost of link $i \in \mathcal{A}$
κ_p	Cost of path $p \in \mathcal{P}$
$\tilde{c}_i'$	Transformed marginal cost function for ARVs for use in objective function T (re-scaled so that flow-dependent term is the same as for SRVs)
a_i	Free-flow costs parameter (proportional to link length)
b_i	Congestibility parameter for affine travel time function
C	$C = C(\mathbf{f})$: Total cost for flow $\mathbf{f}$, $C(\mathbf{f}) = \sum_{i\in\mathcal{A}} c_i(f_i) f_i$
C_{UE}	Total cost for UE flow: $C_{\mathrm{UE}} = C(\mathbf{f}^{\mathrm{UE}})$
C_{SO}	Total cost for SO flow: $C_{\mathrm{SO}} = C(\mathbf{f}^{\mathrm{SO}})$
C_{ME}	Total cost for ME flow: $C_{\mathrm{ME}} = C(\mathbf{f}^{\mathrm{ME}})$
T	Beckmann-equivalent objective for ME. Has three separable terms: one for the flow-dependent terms of both ARVs and SRVs, and a term each for the free-flow term for each class (see equation (4))
ω	Fitness of individual in GA population
μ	Mean ARV path cost for individual (under ME assignment)
σ	Variance of ARV path cost for individual (under ME assignment)
l_{gen}	Length of genome (number of decision variables)
n_{pop}	Number of individuals in GA population
p_{mut}	Mutation probability for each gene
e_{prop}	Proportion of elite individuals to keep ($e_{\mathrm{prop}} \in [0, 1]$)
r_{prop}	Proportion of individuals to replace with 'diverse' individuals

Appendix B. Abbreviations

Abbreviation	Meaning
ARV	Altruistically routed vehicle
SRV	Selfishly routed vehicle
CAV	Connected and automated vehicle
ME	Mixed equilibrium
UE	User equilibrium
SO	System optimum
SAE	Society of Automotive Engineers
NDP	Network design problem
OD	Origin–destination
GA	Genetic algorithm
STAP	Static traffic assignment problem
CV	Coefficient of variation
MST	Minimum spanning tree
PoA	Price of anarchy

Appendix C. Mixed equilibrium objective function

Here we present, for completeness, the derivation of van Vuren et al. (1990) for the ME cost function (Eq. (4)), where the costs of both classes of vehicles – ARVs as well as SRVs – are incorporated into a single objective function. Key to this is that the flow-dependent terms of the cost functions for the different classes can be normalised to the same units (van Vliet et al., 1986). For the ARV and SRV classes, this means that the flow-dependent term of the cost functions for both user classes should be equal.

C.1. Conditions for equilibrium

The ARVs are attempting to solve for SO, which makes the natural scaling, the well-known transformation that converts the SO problem to the UE problem by using the so-called marginal cost functions (Roughgarden, 2006; Patriksson, 2015). That is, the SO assignment is obtained if the STAP is solved for UE when considering the transformed cost functions

$$c_i'(f_i) = c_i(f_i) + f_i \frac{d c_i(f_i)}{d f_i}. \tag{C.1}$$

The condition for the solution to be unique is that c_i is strictly increasing. Which means that it is second derivative must exist, in addition the marginal cost function must also be increasing,

$$\frac{d c_i'(f_i)}{d f_i} > 0, \tag{C.2}$$

which requires

$$\frac{d^2 c_i}{d f_i^2} > -\frac{2}{f_i} \frac{d c_i}{d f_i}. \tag{C.3}$$

C.2. Polynomial cost functions

Polynomial cost functions of order p of the form

$$c_i(f_i) = a_i + b_i f_i^p \tag{C.4}$$

yield transformed (marginal) cost functions c_i' whose flow dependent term differs from c_i by a (dimensionless) multiplicative factor,

$$c_i'(f_i) = a_i + b_i(1+p)f_i^p. \tag{C.5}$$

These marginal cost functions can be further transformed into $\tilde{c}_i'$ so that their flow-dependent term is the same as in the actual link cost functions c_i, resulting in

$$\tilde{c}_i'(f_i) = \frac{a_i}{1+p} + b_i f_i^p. \tag{C.6}$$

Furthermore, polynomial functions of the form of Eq. (C.4) satisfy condition (C.3), since

$$\frac{d^2 c_i}{d f_i^2} = \frac{p-1}{f_i}\frac{d c_i}{d f_i} > -\frac{2}{p-1}\frac{d c_i}{d f_i}, \tag{C.7}$$

which holds for $p \geq 0$. Therefore SRVs (the UE class) and ARVs (the SO class), can be modelled to experience the costs from Eqs. (C.4) and (C.6), respectively.

C.3. Objective function

The mixed equilibrium assignment can now be obtained by minimising the following objective function

$$T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}) = \sum_{i\in\mathcal{A}} \int_0^{f_i^{\mathrm{ARV}}+f_i^{\mathrm{SRV}}} b_i s^p\, ds + \sum_{i\in\mathcal{A}} a_i f_i^{\mathrm{SRV}} + \sum_{i\in\mathcal{A}} \frac{a_i}{p+1} f_i^{\mathrm{ARV}}, \tag{C.8}$$

By having transformed the cost functions so that the flow-dependent term is the same for both classes it remains unchanged from the standard Beckmann functional $\sum_{i\in\mathcal{A}} \int_0^f c_i(s)ds$ (Beckmann et al., 1956) (except the upper integral limit in Eq. (C.8) reflects the presence of both classes). The flow-independent terms are then considered separately for each class since they are independent of each other and capture the (transformed) free flow costs for the SRVs and ARVs, respectively.

In our case, $p = 1$, since we consider affine cost functions (Eq. (1)), form which Eq. (C.8) yields Eq. (4).

Appendix D. Equilibrium conditions

The minimisation problem (considering a single OD) is

$$\min_{\mathbf{f}^{\mathrm{SRV}},\, \mathbf{f}^{\mathrm{ARV}}} \quad T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}) \tag{D.1a}$$

$$\text{s.t.} \quad \mathbf{f}^{\mathrm{SRV}} \geq \mathbf{0} \tag{D.1b}$$

$$\mathbf{f}^{\mathrm{ARV}} \geq \mathbf{0} \tag{D.1c}$$

$$A\,\mathbf{f}^{\mathrm{SRV}} = \mathbf{d}^{\mathrm{SRV}} \tag{D.1d}$$

$$A\,\mathbf{f}^{\mathrm{ARV}} = \mathbf{d}^{\mathrm{ARV}} \tag{D.1e}$$

$$f_i^{\mathrm{SRV}} = 0,\ \forall i \in \mathcal{A}^{\mathrm{ARV}}. \tag{D.1f}$$

We form the Lagrangean function:

$$L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}^{\mathrm{SRV}}, \boldsymbol{\pi}^{\mathrm{ARV}}, \boldsymbol{\beta}) = T(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}V}) \tag{D.2a}$$

$$+ \sum_{j\in V} \pi_j^{\mathrm{SRV}} \Big(\sum_{k\in\mathcal{O}_j} f_k^{\mathrm{SRV}} - \sum_{k\in\mathcal{I}_j} f_k^{\mathrm{SRV}} - d_j^{\mathrm{SRV}} \Big) \tag{D.2b}$$

$$+ \sum_{j\in V} \pi_j^{\mathrm{ARV}} \Big(\sum_{k\in\mathcal{O}_j} f_k^{\mathrm{ARV}} - \sum_{k\in\mathcal{I}_j} f_k^{\mathrm{ARV}} - d_j^{\mathrm{ARV}} \Big) \tag{D.2c}$$

$$+ \sum_{l\in\mathcal{A}^{\mathrm{ARV}}} \beta_l f_l^{\mathrm{SRV}} \tag{D.2d}$$

Where $\mathcal{O}_j$ and $\mathcal{I}_j$ are the sets of incoming and outgoing links at node j, respectively. The multipliers π_j^{SRV} and π_j^{ARV} correspond to the constraints for flow conservation at nodes for the different classes (D.1d)–(D.1e), and β_l corresponds to the link-exclusivity constraints (D.3g), which are a form of capacity constraints.

The first order optimality conditions are,

$$f_i^{\mathrm{SRV}} \frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial f_i^{\mathrm{SRV}}} = 0, \quad \forall\, i \in \mathcal{A}, \tag{D.3a}$$

$$f_i^{\mathrm{ARV}} \frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial f_i^{\mathrm{ARV}}} = 0, \quad \forall\, i \in \mathcal{A}, \tag{D.3b}$$

$$\frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial f_i^{\mathrm{SRV}}} \geq 0, \quad \forall\, i \in \mathcal{A}, \tag{D.3c}$$

$$\frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial f_i^{\mathrm{ARV}}} \geq 0, \quad \forall\, i \in \mathcal{A}, \tag{D.3d}$$

$$\frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial \pi_j^{\mathrm{SRV}}} = 0, \tag{D.3e}$$

$$\frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial \pi_j^{\mathrm{ARV}}} = 0, \tag{D.3f}$$

$$\frac{\partial L(\mathbf{f}^{\mathrm{SRV}}, \mathbf{f}^{\mathrm{ARV}}, \boldsymbol{\pi}, \boldsymbol{\beta})}{\partial \beta_l} = 0, \quad \forall\, l \in \mathcal{A}^{\mathrm{ARV}} \tag{D.3g}$$

$$f_i^{\mathrm{SRV}}, f_i^{\mathrm{ARV}} \geq 0, \quad \forall\, i \in \mathcal{A}. \tag{D.3h}$$

Eqs. (D.3e)–(D.3f) and (D.3h) are equivalent to the feasibility constraints (D.1b)–(D.1e), which are the non-negativity constraints of the flows and the flow conservation at nodes, respectively.

Eqs. (D.3a) and (D.3b) yield,

$$f_i^{\mathrm{SRV}} \left[a_i + b_i \left(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}} \right) + \pi_j^{\mathrm{SRV}} - \pi_k^{\mathrm{SRV}} + \gamma_i \right] = 0 \tag{D.4a}$$

and

$$f_i^{\mathrm{ARV}} \left[\frac{a_i}{2} + b_i \left(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}} \right) + \pi_j^{\mathrm{ARV}} - \pi_k^{\mathrm{ARV}} \right] = 0. \tag{D.4b}$$

where link $i = (j,k) \in \mathcal{A}$ (link i goes from node j to node k) and the variable

$$\gamma_i = \begin{cases} \beta_i, & \text{if } i \in \mathcal{A}^{\mathrm{ARV}} \\ 0, & \text{otherwise,} \end{cases} \tag{D.5}$$

accounts for the β_is in the case $i \in \mathcal{A}^{\mathrm{ARV}}$.

The conditions on the sign of the derivatives, (D.3c) and (D.3d) result the following for SRV flows,

$$b_i(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}}) + a_i + \pi_j^{\mathrm{SRV}} - \pi_k^{\mathrm{SRV}} + \gamma_i \geq 0. \tag{D.6a}$$

Similarly for the ARV flows, we have

$$b_i(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}}) + \frac{a_i}{2} + \pi_j - \pi_k \geq 0. \tag{D.6b}$$

Proposition D.1. *The first-order optimality conditions (D.3) guarantee that equilibrium conditions hold.*

Proof. Letting r be any utilised route ($f_i^{\mathrm{SRV}} > 0$, for $i \in p$), with origin p and destination q. We sum (D.4a) over links in r,

$$\sum_{i\in r} \left[a_i + b_i \left(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}} \right) + \pi_j^{\mathrm{SRV}} - \pi_k^{\mathrm{SRV}} \right] = \pi_p^{\mathrm{SRV}} - \pi_q^{\mathrm{SRV}} + \sum_{i\in r} c_i(f_i) = 0 \tag{D.7}$$

note that $\gamma_i = 0$, since for r has to be used (and by (D.3g)), then $f_i^{\mathrm{SRV}} = 0$. Thus used routes cannot have links in $\mathcal{A}^{\mathrm{ARV}}$. The π_j^{SRV} for nodes other than p and q cancel telescopically. The remaining sum is the route cost (from the fact that $c_i(f_i) = a_i + b_i f_i$, with $f_i = f_i^{\mathrm{ARV}} + f_i^{\mathrm{SRV}}$).

Thus, the travel cost for any route used by the SRVs must be equal since $\pi_q^{\mathrm{SRV}} - \pi_p^{\mathrm{SRV}}$ is independent of the links used and (D.7) is valid for any used route.

Similarly, we can sum (D.6a) over the links of any route r,

$$\sum_{i\in r} b_i(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}}) + a_i + \pi_j^{\mathrm{SRV}} - \pi_k^{\mathrm{SRV}} + \gamma_i = \pi_p^{\mathrm{SRV}} - \pi_q^{\mathrm{SRV}} + \sum_{i\in r} c_i(f_i) + \sum_{i\in r\cap\mathcal{A}^{\mathrm{ARV}}} \beta_i \ge 0 \tag{D.8}$$

For routes that do not contain ARV exclusive links, the above expression becomes,

$$\sum_{i\in r} c_i(f_i) \ge \pi_q^{\mathrm{SRV}} - \pi_p^{\mathrm{SRV}}, \tag{D.9}$$

which means that unused routes in the subnetwork defined by the links available to them, $\mathcal{A}\setminus\mathcal{A}^{\mathrm{ARV}}$, must cost at least as much as used routes (by virtue of Eq. (D.7)). This means that the conditions for (UE) Wardrop equilibrium hold for SRVs, restricted to their available network.

For routes that contain links in $\mathcal{A}^{\mathrm{ARV}}$, the term containing the β_i does not vanish,

$$\sum_{i\in r} c_i(f_i) + \sum_{i\in r\cap\mathcal{A}^{\mathrm{ARV}}} \beta_i \ge \pi_q^{\mathrm{SRV}} - \pi_p^{\mathrm{SRV}}. \tag{D.10}$$

As noted by Patriksson in Patriksson (2015) one cannot relate the actual travel costs of the unused routes to those of the used ones, however (D.10) relates the generalised costs of these routes (as formulated in the capacitated STAP), to the used route's actual costs (as captured by the SRV OD potentials). In this sense, the sum of β_i for an route unavailable to the SRVs represents the cost gap between these routes and the fastest available routes (that do not contain exclusive links).

Adding (D.4b) over any given route r used by ARVs (considering $f_i^{\mathrm{ARV}} > 0$) gives

$$\sum_{i\in r}\left[\frac{a_i}{2} + b_i\left(f_i^{\mathrm{SRV}} + f_i^{\mathrm{ARV}}\right) + \pi_j^{\mathrm{ARV}} - \pi_k^{\mathrm{ARV}}\right] = 0, \tag{D.11a}$$

which simplifies to

$$\sum_{i\in r} \bar{c}_i'(f_i) = \pi_q^{\mathrm{ARV}} - \pi_p^{\mathrm{ARV}}, \tag{D.11b}$$

where $\bar{c}_i'$ (from Eq. (C.6)) is the transformed marginal cost function of link i. In terms of the marginal cost functions (Eq. (C.5)),

$$\sum_{i\in r} c_i' = 2(\pi_q^{\mathrm{ARV}} - \pi_p^{\mathrm{ARV}}). \tag{D.11c}$$

Eq. (D.11c) shows that the marginal cost of any used route is independent of the route itself and only depends on π_p^{ARV} and π_q^{ARV}. Thus for the ARVs, the conditions for SO hold,

$$\sum_{i\in r} c_i'(f_i) = \sum_{j\in s} c_j'(f_j). \tag{D.11d}$$

For unused routes, we let r be any route and sum (D.6b) over its links, yielding

$$\sum_{i\in r} c_i'(f_i) \ge 2(\pi_q^{\mathrm{ARV}} - \pi_p^{\mathrm{ARV}}). \tag{D.12}$$

Thus, routes that remain unused by ARVs have higher or equal marginal costs than used ones.

Thus we have shown that the first order optimality conditions imply equilibrium in the sense that:

- SRVs are under UE when considering only their available subnetwork which is induced by the edge set $\mathcal{A}\setminus\mathcal{A}^{\mathrm{ARV}}$.
- When considering SRVs and routes on the whole network, the equilibrium conditions of the capacitated problem are met (Patriksson, 2015).
- ARVs are under SO amongst themselves in the sense that the marginal costs of their used routes are equal (which reduces to the standard SO conditions of minimum costs in the case where there are only ARVs). □

Table E.4
Parameter values for the GA described in Section 3.1.

Parameter	Value
Generations	900
n_{pop}	150
p_{mut}	0.05
e_{prop}	0.15
r_{prop}	0.2
k	20

Appendix E. Details on the genetic algorithm

Our GA consists of the following steps:

1. Initialise starting population of individual solutions (n_{pop} individuals).
   (i) Map candidate ARV-exclusive links for a network to genes in the chromosome.
   (ii) Generate a 'diverse' random starting population.
   (iii) Calculate the fitness of individuals.
2. Tournament selection of mating pairs.
   (i) Draw two pairs of individuals from the current population. For each pair draw uniformly random individuals without replacement.
   (ii) For each pair, compare the fitness of the individuals. The fittest individuals of each pair (the winners of their respective tournaments) are matched for reproduction.
   (iii) Repeat steps (i) and (ii) to find each reproducing pair ($n_{\mathrm{pop}}/2$ times).
3. Reproduction (For each reproducing pair).
   (i) Generate two new offspring chromosomes using single-point crossover.
   (ii) Each allele can mutate (flip binary value) with probability p_{mut}
   (iii) Calculate the fitness of each offspring chromosome
   (iv) Modify chromosomes (revert alleles to zero) if corresponding ARV-exclusive links carry no ARV flow.
4. Propagation to next generation.
   (i) Apply elitism. Replace up to e_{prop} fraction of lowest-fitness individuals of the offspring population with the fittest individuals of the parent population as long as the *elite* individuals are fitter than the fittest chromosome of the offspring population.
   (ii) Diversify population. Replace the r_{prop} least fit fraction of the population with randomly generated individuals to preserve 'diversity' in the population.
5. Repeat steps 2–4 until the maximum number of desired generations

A schematic of the population propagation mechanism of our implemented GA, and how the populations propagated to subsequent generations are structured is shown in Fig. E.8. The parameters we use are shown in Table E.4.

The population size n_{pop} is an important parameter, especially in terms of computational time and convergence. Sastry et al. (2014) suggests an initial population of 50 individuals is suitable for many problems. Although for a different formulation of evolutionary algorithms – differential evolution – Storn (1996) proposes that $n_{\mathrm{pop}} = 10 l_{\mathrm{gen}}$ is reasonable. However, Chen et al. (2012) show that large population sizes are not necessarily beneficial. Alander (1992) conclude from empirical results that values between l_{gen} and $2l_{\mathrm{gen}}$ can be suitable

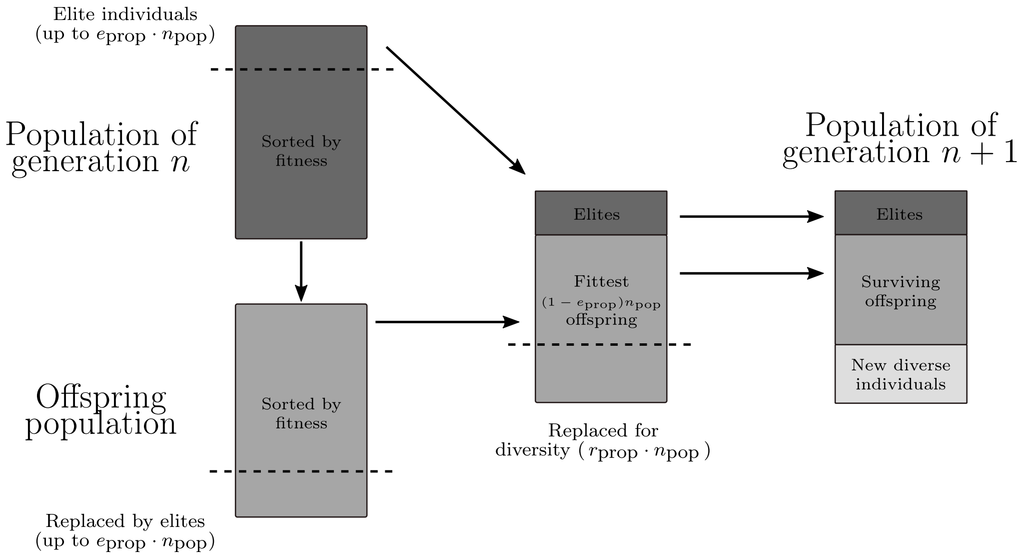

Fig. E.8. Depiction of how the population of individuals of the GA are selected from parent and offspring populations to survive into the next generation.

for binary genomes. In light of the above, we use a population of a similar size to the number of the decision variables, with $n_{\text{pop}} = 150 \approx 1.5 l_{\text{gen}}$.

As well as being in line with general considerations regarding GAs, our chosen parameter values align with applications of GAs to transport network problems. Cantarella et al. (2006), use smaller populations than us (40 individuals), although they also have fewer decision variables (34) by about a factor of four. The mutation rate they use is 0.1. Pinninghoff et al. (2008), consider the possibility of adding new links to networks, for which they require more decision variables (around 500), however they still use smaller populations that us (between 25 and 50 individuals) and while they recognise that larger populations improve results and coverage of the search space, they find their method converges, or at least settles on good solutions in around 500 generations. Their GA also includes elitism and they use a p_{mut} between 0.01 and 0.05.

We implemented the GA and ME solver in the Julia programming language (Bezanson et al., 2017). We solve (9) by using the optimisation package JuMP (Dunning et al., 2017). The open-source numerical solver Ipopt (Wächter and Biegler, 2006) is used as the backend which employs an internal point barrier method to find the solution to the problem.

Appendix F. Sample size and standard error of ensemble mean

In order to check whether the size of the ensembles used in Sections 3 and 4 we estimate the standard error of the ratio of per-vehicle costs, $C_{\text{pv}}^{\text{ARV}}/C_{\text{pv}}^{\text{SRV}}$, by bootstrapping the mean for each ensemble and calculating the standard error of the mean from the bootstrapped samples.

The basic procedure was to sample with replacement 100 sets of samples from the 20 networks in order to calculate an estimate for the standard error of the mean ratio of per-vehicle costs. Table F.5 shows, in addition to $\left\langle C_{\text{pv}}^{\text{ARV}}/C_{\text{pv}}^{\text{SRV}}\right\rangle$ (to more significant figures than in Table 3), the standard error as well as the 95% confidence interval. The confidence interval is given to show the bounds that would have to be exceeded to conclude that a better sample is needed. It should be noted that even for a much narrower bootstrapped confidence interval (e.g. 10%) most of the ensemble means lie within it, strengthening the conclusion that we are not observing outliers.

A permutation test is more appropriate than, for example, a t-test since fewer assumptions are necessary and the values we are testing from are derived from a complex process. We present these statistical details to show that while variance within each experimental ensemble can be high, care has been taken to ensure enough networks were considered per ensemble and that the results presented are significant.

Table F.5
Per-vehicle cost ratio with bootstrapped standard error and 95% confidence intervals.

$\hat{\alpha}$	d	γ	$\left\langle \frac{C_{\text{pv}}^{\text{AV}}}{C_{\text{pv}}^{\text{SV}}} \right\rangle$	std error	95% Conf. interval
0.5	0.0001	0.25	0.9325	0.0175	(0.90237, 0.96202)
		0.5	0.9327	0.0133	(0.90559, 0.95916)
		0.75	0.9129	0.0127	(0.89266, 0.93819)
	0.0151	0.25	0.7506	0.024	(0.70998, 0.79903)
		0.5	0.8066	0.0193	(0.77178, 0.83785)
		0.75	0.943	0.0123	(0.91756, 0.96296)
	0.03	0.25	0.6403	0.019	(0.60599, 0.68037)
		0.5	0.768	0.0243	(0.72058, 0.81117)
		0.75	0.9418	0.0099	(0.92321, 0.95948)
0.75	0.0001	0.25	0.9564	0.0098	(0.93865, 0.97602)
		0.5	0.9418	0.0205	(0.90577, 0.987)
		0.75	0.9353	0.0128	(0.91412, 0.96126)
	0.0151	0.25	0.8139	0.0198	(0.7738, 0.84665)
		0.5	0.8665	0.0169	(0.83691, 0.89822)
		0.75	0.9647	0.0099	(0.94806, 0.98412)
	0.03	0.25	0.7393	0.0267	(0.69604, 0.80733)
		0.5	0.8135	0.0209	(0.76783, 0.85151)
		0.75	0.9502	0.0114	(0.93353, 0.97785)
1.0	0.0001	0.25	0.9064	0.0217	(0.87033, 0.95142)
		0.5	0.9145	0.0173	(0.88538, 0.95226)
		0.75	0.9087	0.0141	(0.88324, 0.93896)
	0.0151	0.25	0.8268	0.0233	(0.78403, 0.8702)
		0.5	0.9041	0.0113	(0.88076, 0.92194)
		0.75	0.9677	0.0144	(0.94217, 0.99527)
	0.03	0.25	0.7432	0.0358	(0.68203, 0.81604)
		0.5	0.8674	0.0143	(0.84242, 0.90097)
		0.75	0.9754	0.0082	(0.96259, 0.99128)

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