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Charge sensitivity of the inductive single-electron transistor

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We calculate the charge sensitivity of a recently demonstrated device where the Josephson inductance of a single Cooper-pair transistor is measured. We find that the intrinsic limit to detector performance is set by oscillator quantum noise. Sensitivity better than 10^{-6} e/$\sqrt{\text{Hz}}$ is possible with a high Q value $\sim 10^3$, or using a superconducting quantum interference device amplifier. The model is compared to experiment, where charge sensitivity 3×10^{-5} e/$\sqrt{\text{Hz}}$ and bandwidth 100 MHz are achieved. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2034096]

Remarkable quantum operations have been demonstrated in the solid state.[1–3] As exotic quantum measurements known in quantum optics are becoming adopted for electronic circuits,[4] sensitive and desirably nondestructive measurement of the electric charge is becoming even more important.

A fast electrometer, the inductive single-electron transistor (L-SET) was demonstrated recently.[5] Its operation is based on gate charge dependence of the Josephson inductance of a single Cooper-pair transistor (SCPT). As compared to the famous rf-SET,[6] where a high-frequency electrometer is built using the control of single-electron dissipation, the L-SET has several orders of magnitude lower dissipation due to the lack of shot noise, and hence also potentially lower back action.

Charge sensitivity of the sequential tunneling SET has been thoroughly analyzed. However, little attention has been paid to the detector performance of the SCPT, probably because no real electrometer based on SCPT had been demonstrated until invention of the L-SET. Some claims have been presented[5,7] that the performance of SCPT in the L-SET setup could exceed the shot-noise limit of the rf-SET,[8] $s_q \simeq 10^{-6}$e/$\sqrt{\text{Hz}}$, but no accurate calculations have appeared.

In this letter we carry out a sensitivity analysis for L-SET in the regime of linear response. We find that (neglecting $1/f$ background charge noise) the intrinsic limit to detector sensitivity is set, unlike by shot noise of electron tunneling in a normal SET, by zero-point fluctuations.[9]

A SCPT has the single-junction Josephson energy E_J, and the total charging energy $E_C=e^2/(2C_\Sigma)$, where C_Σ is the total capacitance of the island. At the lowest energy band the energy is E_0, the effective Josephson energy is $E_J^* = \partial^2 E_0(q,\varphi)/\partial\varphi^2$, and the effective Josephson inductance is $L_J=(\Phi_0/2\pi)^2(E_J^*)^{-1}$. These have a substantial dependence on the (reduced) gate charge $q=C_gV_g/e$ if $E_J/E_C \lesssim 1$. Here, φ is the phase across the SCPT. With a shunting capacitance C, SCPT forms a parallel oscillator. We further shunt the oscillator, mainly for practical convenience, by an inductor $L \simeq L_J$. Hence we have the resonator as shown in Fig. 1, with the plasma frequency $f_p=\omega_p/(2\pi)=1/(2\pi)(L_{\text{tot}}C)^{-1/2} \approx 1$ GHz, where $L_{\text{tot}}=L\|L_J$.

The coupling capacitor, typically $C_c \ll C$, allows, in principle, for an arbitrarily high loaded quality factor Q_L. If directly coupled to the feedline, $Q_L=Z_0\sqrt{C/L_{\text{tot}}}\approx 1$, which is clearly intolerable. With a coupling capacitor, however, $Q_L=1/2Q_i$ in the optimal case (as shown later) of critical coupling $Z=Z_0$. Here, Q_i is the internal Q value, which indicates the dissipation residing within the resonator. The internal losses can be modeled as being due to a shunting resistor R as in Fig. 1: $Q_i=R/(\omega_p L_{\text{tot}})$.

We consider only the regime of harmonic oscillations of the phase φ around the Josephson potential minimum at $\varphi=0$, where the detector works by converting charge to resonant frequency. A second mode, the "anharmonic" operation mode[5] uses nonlinear oscillations of φ with an amplitude of 2–10 periods of 2π, and the gate charge now affects Q_i through a control of nonlinear dynamics. The anharmonic mode, which, in fact, yields better sensitivities in experiment, will be discussed in detail in a forthcoming publication.

The impedance of the L-SET circuit as illustrated in Fig. 1 is

$$Z=\frac{1}{i\omega C_c}+\left(i\omega C+\frac{1}{i\omega L}+\frac{1}{i\omega L_J}+\frac{1}{R}\right)^{-1}. \quad (1)$$

The circuit is probed by measuring the voltage reflection coefficient $\Gamma=|\Gamma|\exp[i\arg(\Gamma)]=(Z-Z_0)/(Z+Z_0)$ to an incoming voltage wave of amplitude V_0. The reflected wave amplitude is $V_1=|\Gamma|V_0$. Here, $Z_0=50\ \Omega$ is the wave impedance of coaxial lines.

The spectral density of noise power at the output of the first stage amplifier, referred to the amplifier input, is $k_BT_N^*$, where the effective temperature T_N^* is due to amplifier noise and sample noise: $T_N^*=T_N+T_S$. The sample is supposed to be critically coupled, and hence its noise is like that of a 50-Ω resistor at the temperature T_S (note that the Josephson effect

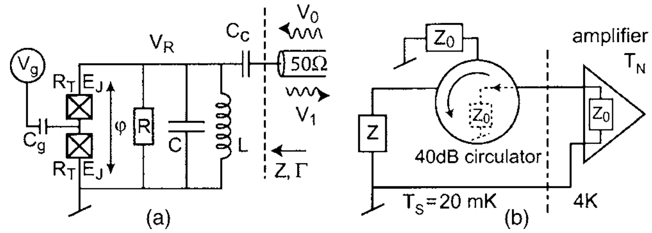

FIG. 1. The L-SET resonator (a), and its equivalent circuit (impedance Z) coupled to cabling (b).

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is a system ground-state property and hence it contributes no noise). Typically, $k_BT_S \lesssim \hbar\omega_p$, and thus sample noise is already in the quantum limit.

The noise of contemporary rf amplifiers, however, remains far from the quantum limit, i.e., $T_N \gg T_S$. The best demonstrated superconducting quantum interference device (SQUID)-based rf amplifiers have reached $T_N \sim 100-200$ mK.[10] Therefore, added noise from the sample can be safely ignored when analyzing detector performance.

The charge sensitivity for *amplitude modulation* (AM) of the rf-SET was calculated in detail in Ref. 11 assuming detection of one sideband. It was assumed that the sensitivity is limited by the general equivalent noise temperature similarly as here, and hence the formula applies as such:

$$s_q = \sqrt{2k_BT_NZ_0} \Bigg/ \left(V_0\frac{\partial|\Gamma|}{\partial q}\right). \tag{2}$$

In the linear regime, the best sensitivity of the L-SET is clearly at the largest acceptable value of V_0, where linearity still holds reasonably well. This is the case when an ac current of critical current peak value flows through the SCPT, and the phase swing is π *p-p*. Then, voltage across the SCPT, and the resonator (later we discuss important quantum corrections to this expression), $V_R = |2Z_R/(Z+Z_0)|V_0$ equals a universal critical voltage of a Josephson junction,[12] $V_C = \pi\hbar\omega/(4e) \simeq 3\ \mu$V at $f_p = 1$ GHz. Here, Z_R is impedance of the parallel resonator.

We decompose the derivative in Eq. (2) into terms due to the circuit and SCPT: $\partial|\Gamma|/\partial q = (\partial|\Gamma|/\partial\omega_p)(\partial\omega_p/\partial L_J) \times(\partial L_J/\partial q)$. We define a dimensionless transfer function $g' = (\partial L_J/\partial q)(1/L_{J0})$ scaled according to a minimum (with respect to the gate) of L_J. The gate value that yields the maximum of g', denoted g, is the optimum gate dc operation point of the charge detector. In what follows, L_J should be understood as its value at this point. With a given E_J/E_C ratio, we compute the values of g and L_J numerically from the SCPT band structure (g is plotted in Fig. 4 in Ref. 5). If $E_J/E_C \ll 1$, one can use the analytical result $L_{J0} = (\Phi_0/\pi)^2(1/E_J)$.

With a general choice of parameters of the tank resonator, Eq. (2) needs to be evaluated numerically. However, when the system is critically coupled, $Z = Z_0$, a simple analytical formula can be derived. Numerical calculations of Eq. (2) over a large range of parameters show that the best sensitivity occurs when $Z = Z_0$. This is reasonable because it corresponds to the best power transfer. All the following results are for critical coupling. Later, we examine the effects of detuning from the optimum. Initially, we also suppose the oscillator is classical, i.e., its energy $E \gg \hbar\omega_p$.

The optimal value of the coupling capacitor is calculated using $Q_L = 1/2Q_i$, and we get $C_c = \sqrt{C/(\omega_pQ_iZ_0)}$.

Since it was assumed $Z = Z_0$, it holds that $Z_R = Z_0 + i/(\omega_pC_c)$. Voltage amplification by the resonator then becomes $V_R = V_0\sqrt{Q_i/(\omega_pZ_0C)}$, which holds for a reasonably large Q_i. We thus have $V_0 = \pi\hbar\omega_p^{3/2}\sqrt{Z_0C}/(4e\sqrt{Q_i})$.

With $\omega_p = (L_{\text{tot}}C)^{-1/2}$, we get immediately $(\partial\omega_p/\partial L_J)^{-1} = 2\sqrt{C}L_J^2\sqrt{1/L+1/L_J}$. Using the fact[13] that the full width at half maximum (FWHM) of the loaded resonance absorption dip at critical coupling is $\omega_p/(2Q_L)$, we get $\partial|\Gamma|/\partial\omega_p = 2Q_L/\omega_p = Q_i/\omega_p$.

Inserting these results into Eq. (2), we get an expression for the AM charge sensitivity in the limit the oscillator is classical

$$s_q = \frac{8eL_J^2\sqrt{\dfrac{1}{L}+\dfrac{1}{L_J}}\sqrt{2k_BT_N}}{g\pi\hbar L_{J0}\sqrt{\omega_pQ_i}} \tag{3}$$

in units of $[e/\sqrt{\text{Hz}}]$. Clearly, the shunting inductor is best omitted, i.e., $L \to \infty$. The classical result, Eq. (3), improves without limit at low E_J/E_C.

We will now discuss quantum corrections to Eq. (3). Although the *spectral density* of noise in the resonator is negligible in output, the *integrated* phase fluctuations even due to quantum noise can be large. Integrated phase noise in a high-Q oscillator is $\langle\Delta\varphi^2\rangle = 2\pi^2\hbar L_{\text{tot}}\omega_p/\Phi_0^2$.[14] When $\langle\Delta\varphi\rangle$ exceeds the linear regime $\sim\pi$, which happens at high inductance (low E_J/E_C), plasma resonance "switches" into the nonlinear regime, and the gain due to the frequency modulation vanishes. If $L \gg L_J$, and $f_p \sim 1$ GHz, we have ultimate limits of roughly $E_J/E_C \sim 0.06$, or ~ 0.02, for a SCPT made out of Al or Nb, respectively.

Even before this switching happens, the quantum noise in the oscillator $E_Q = \frac{1}{2}\hbar\omega_p$ has an adverse effect because less energy can be supplied in the form of drive; that is, V_0 is smaller. This can be calculated in a semiclassical way as follows. Energy of the oscillator is due to drive (E_D) and noise (we stay in the linear regime): $E = (\Phi_0\varphi)^2/(8\pi^2L_{\text{tot}}) = E_D + E_Q = (\Phi_0\varphi_D)^2/(8\pi^2L_{\text{tot}}) + \frac{1}{2}\hbar\omega_p$, where the phases are in RMS, φ is the total phase swing, and φ_D is that due to drive. Solving for the latter, we get $\varphi_D = \sqrt{\varphi^2 - 4\pi^2\hbar\omega_pL_{\text{tot}}/\Phi_0^2}$. The optimal drive strength $V_R = V_C$ corresponds to $\sqrt{2}\varphi = \pi/2$, and hence the maximum probing voltage V_0 is reduced by a factor $\beta = \sqrt{1 - 32\hbar\omega_pL_{\text{tot}}/\Phi_0^2}$ due to quantum noise in the oscillator.

The optimal sensitivity is finally

$$s_q^{QL} = \frac{64\sqrt{2}eL_J^2\sqrt{2k_BT_N}}{g\pi\sqrt{\hbar}\Phi_0L_{J0}\sqrt{Q_i}}, \tag{4}$$

which depends only weakly on operation frequency. We optimized Eq. (2) (replacing V_0 by βV_0), assuming similar tunnel junction properties as in the experiment, $E_JE_C = 1.8\ \text{K}^2$ (Al) and $E_JE_C = 10\ \text{K}^2$ (Nb). The results are plotted in Fig. 2 together with corresponding power dissipation $(V_C/\sqrt{2})^2/R = \pi^2\hbar^2\omega_p/(32e^2Q_iL_J)$.

The optimal sensitivity is reached around $E_J/E_C \simeq 0.1, \ldots, 0.3$, where the curves in Fig. 2 almost coincide with Eq. (4). C_c should be chosen so that critical coupling results. Typically it should also hold $L \gg L_J$ (see the analytical curve in Fig. 2). However, sensitivity decreases only weakly if these values are detuned from their optimum (Fig. 3).

By numerical investigation we found that readout of $\arg(\Gamma)$, with mixer detection, offers within accuracy of numerics the same numbers than the discussed AM (readout of $|\Gamma|$).

In experiment, we measured the charge sensitivity for the following sample and resonator: $R_T \simeq 11\ \text{k}\Omega$, $E_J \simeq 0.7$ K, $E_C \simeq 2.6$ K, $E_J/E_C \simeq 0.3$, $Q_i \simeq 16$, $L \simeq 28$ nH, $C \simeq 1.2$ pF, $C_c \simeq 0.5$ pF. In all samples so far, $Q_i \lesssim 20$, which is currently not understood. The measurements were done as described in Ref. 5, with $T_N \sim 5$ K.[15] We measured $s_q = 7$

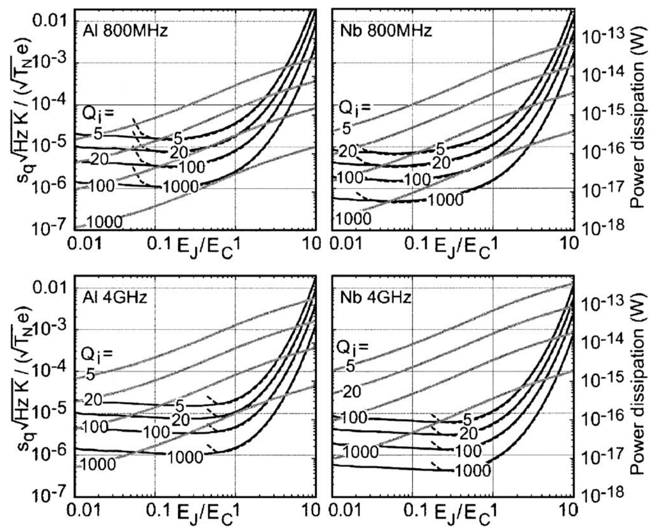

FIG. 2. Charge sensitivity of the L-SET optimized from Eq. (2) (black lines). The analytical result [Eq. (3) multiplied by β^{-1}], with $L=\infty$, is shown with dashed lines. The gray lines are the corresponding power dissipation. All the graphs have the same scales, which are indicated for s_q (left) and dissipation (right). The curves are for different Q_i as marked. All graphs have $Z \simeq Z_0$.

$\times 10^{-5}$ e$/\sqrt{\mathrm{Hz}}$ by AM at 1 MHz, while a prediction with the present parameters is $s_q=3\times 10^{-5}$ e$/\sqrt{\mathrm{Hz}}$ (see also Fig. 3).

Theory and experiment thus agree reasonably. The somewhat lower sensitivity in experiment is likely to be due to external noise, which forces a lower V_0 and also smoothes out the steepest modulation. Its origin is not clear. Also the 25% higher values of L_J than expected agree qualitatively with noise.

In the "anharmonic" mode, we measured $s_q=3\times 10^{-5}$ e$/\sqrt{\mathrm{Hz}}$, with a usable bandwidth of about 100 MHz ($s_q\sim 10^{-4}$ e$/\sqrt{\mathrm{Hz}}$ at 100 MHz). Considering both s_q and band, a performance comparable to the best rf-SETs (Refs. 6 and 16) has been reached with the L-SET, though here at more than two orders of magnitude lower power dissipation ($\sim$10 fW).

In the linear regime, the power lost P_Σ from drive frequency $m=1$ to higher harmonics is determined by the sum, for $m\geq 2$, of Josephson junction admittance components $|Y_m|=2J_m(2e/(\hbar\omega)V_1)$. At the critical voltage $V_1=V_C$, this

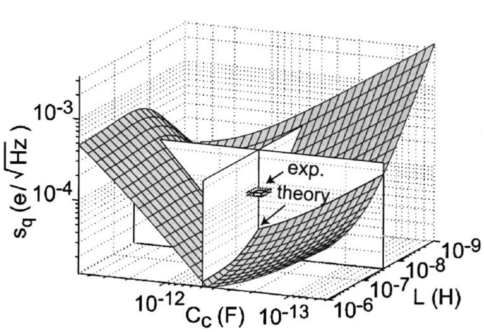

FIG. 3. Measured charge sensitivity (box) compared to calculations. In experiment, $C_c\sim 0.5$ fF, and $L\sim 28$ nH. The graph shows also how the sensitivity would change as a function C_c and L according to the model.

amounts to $Y_\Sigma/Y_1=P_\Sigma/P_1\sim 30\%$. Since charge sensitivity is proportional to square root of power, it thus decreases only $\sim$15% due to nonlinearity. Further corrections due to slightly nonsinusoidal lowest band of the SCPT, as well as asymmetry due to manufacturing spread in junction resistance, we estimate as insignificant.

Next we discuss nonadiabaticity. Interband Zener transitions might make the SCPT jump off from the supposed ground band 0. We make a worst case estimate by assuming that the drive is $2\pi\, p$-p (partially due to noise). The probability to cross the minimum Δ_m of band gap $\Delta=E_1-E_0$ is: $P_Z\simeq\exp[-\pi\Delta_m^2/(2\hbar D\dot{\varphi})]$, where we evaluate the dependence of the band gap on phase $D=\partial\Delta/\partial\varphi$ at $\varphi=\pi/2$. $\dot{\varphi}=2\omega_p$ is determined by the drive.

Zener tunneling is significant if it occurs sufficiently often in comparison to $1\rightarrow 0$ relaxation. The threshold is when $P_Z\sim\Gamma_\downarrow/(2f_0)$, where $\Gamma_\downarrow\gtrsim(1\ \mu\mathrm{s})^{-1}$ is the relaxation rate. Operation of the L-SET can thus be affected above $P_Z\sim 10^{-4}$.

Numerical calculations for P_Z show that Zener tunneling is exponentially suppressed, at the L-SET optimal working point, in the interesting case of low E_J/E_C.[12] This is because Δ_m becomes large and D small. For instance, if $E_J=1$ K and $f_p=1$ GHz, we got that Zener tunneling is insignificant below $E_J/E_C\sim 3$. With $E_J=0.5$ K and $f_p=5$ GHz, the threshold is $E_J/E_C\simeq 1$.

We conclude that with sufficiently high Q_i and using a amplifier close to the quantum limit, even $s_q\sim 10^{-7}$ e$/\sqrt{\mathrm{Hz}}$, order of magnitude better than the shot-noise limit of rf-SET, is intrinsically possible for the L-SET. So far, the sensitivity has been limited by $Q_i\lesssim 20$.

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