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Periodic elastic medium in which periodicity is relevant

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We analyze, in both $(1+1)$ and $(2+1)$ dimensions, a periodic elastic medium in which the periodicity is such that at long distances the behavior is always in the random-substrate universality class. This contrasts with the models with an additive periodic potential in which, according to the field-theoretic analysis of Bouchaud and Georges and more recently of Emig and Nattermann, the random manifold class dominates at long distances in $(1+1)$ and $(2+1)$ dimensions. The models we use are random-bond Ising interfaces in hypercubic lattices. The exchange constants are random in a slab of size $L^{d-1}\times\lambda$ and these coupling constants are periodically repeated, with a period λ, along either {10} or {11} [in $(1+1)$ dimensions] and {100} or {111} [in $(2+1)$ dimensions]. Exact ground-state calculations confirm scaling arguments which predict that the surface roughness w behaves as $w\sim L^{2/3}, L\ll L_c$ and $w\sim L^{1/2}, L\gg L_c$ with $L_c\sim\lambda^{3/2}$ in $(1+1)$ dimensions, and $w\sim L^{0.42}, L\ll L_c$ and $w\sim\ln(L), L\gg L_c$ with $L_c\sim\lambda^{2.38}$ in $(2+1)$ dimensions.

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I. INTRODUCTION

Periodic elastic media arise in a surprising array of problems, including spin or charge density waves, flux line lattices, and random magnets. A model frequently used [1–3] to describe a manifold, defined by the single-valued height variable $h(\vec{r})$ in a periodic elastic medium (PEM), is

$$\mathcal{H}_{\mathrm{PEM}}=\int d\vec{r}\left\{\frac{\gamma}{2}[\nabla h(\vec{r})]^2+\eta[h(\vec{r})]+V_p[h(\vec{r})]\right\}, \quad (1)$$

where V_p is a periodic potential in the height direction and the random potential η is not periodic. This is directly analogous to the model used to study lattice effects in thermal roughening and in field-theoretic studies of commensurate phases in Ising magnets with competing interactions. In the model (1), the periodic potential is nonrandom and tends to pin the interface while the quenched random pinning $\eta[h(\vec{r})]$ tends to make the interface wander. The surface tension term $\gamma/2[\nabla h(\vec{r})]^2$ seeks a flat interface and also competes with the quenched random pinning. Field-theoretic calculations [1–3] suggest that at long distances, for (1+1)- and (2+1)-dimensional interfaces, the periodic pinning potential is irrelevant, and hence the interface scaling behavior is in the random-bond Ising universality class where width $w^2=\langle h^2\rangle-\langle h\rangle^2\sim L^{2\zeta}$ with the roughness exponent $\zeta=2/3$ in $(1+1)$ and $\zeta\approx 0.21(4-D)$ in $(D+1), D\geq 2$ [4–8]. Note that lattice calculations are strongly affected by a lattice pinning potential and have a flat phase even for large lattice sizes [8].

Another problem which has been heavily studied is the random substrate problem [9–11]. This was introduced to model the effect of a random substrate on layers of absorbed atoms, and also serves as a model for the effect of a p-fold random field on the XY model [9]. There is now a consensus that there is a disorder-dominated glassy phase in this model (in two substrate dimensions) at low temperatures that is reflected in long-distance correlations which behave as $C(r)\sim\ln^2|r|$ [in contrast to thermally rough correlations in dimension $(2+1)$, which grow as $C(r)\sim\ln|r|$]. There has been some uncertainty about whether the leading-order correlations found by Cardy and Ostlund (CO) [9] are correct, with functional renormalization-group calculations agreeing with CO [10,11], and variational calculations disagreeing. The substrate roughness is randomly drawn from the interval (0,1) (in lattice units). This corresponds to a different sort of periodic elastic medium from that described in Eq. (1). Here, the random substrate leads to a periodically repeated disorder seen by an interface lying above the random substrate. This arises due to the fact that the first, third, fifth, etc. atoms deposited at the same position on the random substrate see exactly the same disorder when they land. This corresponds to a random-bond Ising magnet in which the disorder is repeated with period $\lambda=2$ along the growth direction. In general, the disorder may range over a scale $(0,\lambda-1)$, and this leads to a periodic variation in the disorder on length scale λ. The continuum model for this system is simply

$$\mathcal{H}_p=\int d\vec{r}\left\{\frac{\gamma}{2}[\nabla h(\vec{r})]^2+\eta[h(\vec{r})]\right\}, \quad (2)$$

but where η is *periodic in* $h(\vec{r})$, so that we require $\eta[h(\vec{r})+\lambda]=\eta[h(\vec{r})]$. There has been considerable study of the random substrate $(\lambda=2)$ problem, with the early controversy now being resolved in favor of a "super-rough" "Bragg-glass" phase in $(2+1)$ dimensions in which $w\sim\ln(L)$. Exact ground-state calculations have been very useful in resolving this controversy [12–15]. It is quite easy to see (see Sec. III) that in $(1+1)$ dimensions, the random substrate problem behaves as a random walk (RW), so that $w\sim L^{1/2}$. Note, however, that it has been recently argued that although typical dislocations do not destroy the "Bragg-

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glass'' ground state, optimal dislocations have negative energy, and hence are expected to destroy the Bragg glass in $(2+1)$ dimensions [16,17].

In this paper we study the Hamiltonian (2) as a function of the periodicity λ of the disorder. We show that at long length scales in $(1+1)$ and $(2+1)$ dimensions, the periodicity is relevant and the random substrate universality class holds. The paper is arranged as follows. Section II sets up the model and describes the way in which we calculate the exact positions of interfaces in random Ising magnets. The scaling theory describing the behavior of these interfaces is developed and tested in Sec. III. We give a brief conclusion in Sec. IV.

II. DISCRETE MODEL AND EXACT ALGORITHM

The model which we use to analyze the effect of periodic disorder on interface properties is a spin-half Ising system with random bonds (RB) on square and cubic lattices. The Hamiltonian is given by

$$\mathcal{H}_{\rm RB} = -\sum_{\langle ij\rangle} J_{ij} S_i S_j , \tag{3}$$

where $J_{ij} > 0$ are coupling constants and the spin variables S_i take the values ± 1. The spins on two opposite boundaries of the lattices, $h=1$ and $h=L$, are fixed and have opposite signs so that an interface must exist in the lattice. Our calculations are at zero temperature and we find the ground-state interface properties for interfaces whose average normal vectors lie in the $\{10\}$ or $\{11\}$ directions of square lattices and in the $\{100\}$ or $\{111\}$ directions of cubic lattices. The coupling constants are random in a slab of size $L^{d-1}\times\lambda$ and then periodically repeated L/λ times along a chosen direction. The distributions used for the J_{ij}'s vary here from case to case but are always chosen so that the interfaces are rough even for small lattices sizes, and even in the $\{100\}$ orientation cubic systems. In Fig. 1, we illustrate the way in which the periodic disorder is implemented for the $\{10\}$ and $\{11\}$ directions of a square lattice. As is now well known [7,8,18], the ground-state interface of the system (3) can be found *exactly* using the maximum flow algorithm. We have a custom implementation of the push-relabel algorithm for this problem and using it we are able to find the exact ground-state interface in Ising systems of one million sites in about 1min of CPU time on a high-end workstation.

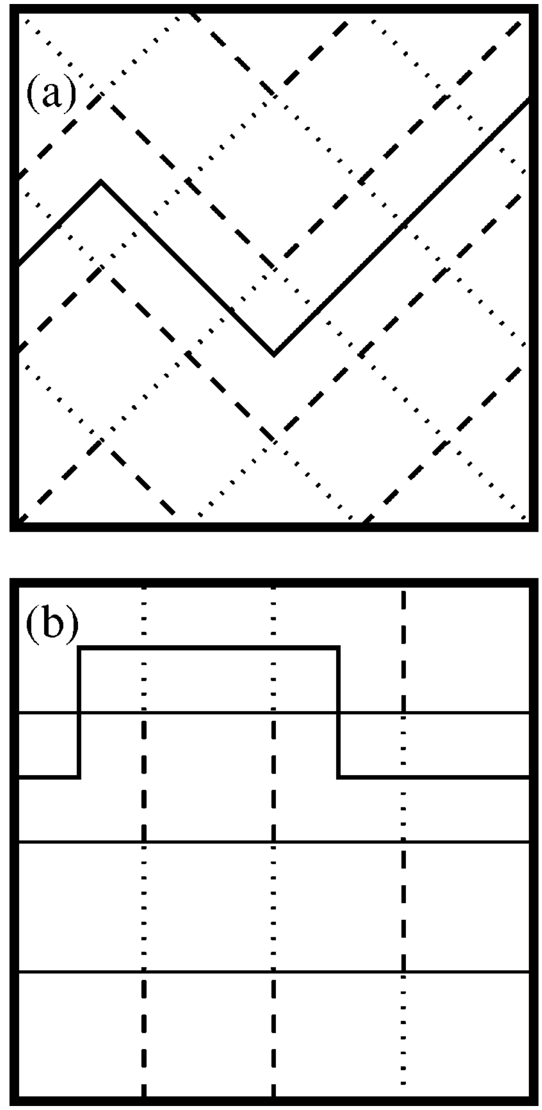

FIG. 1. An example of interface in a random substrate problem, with period $\lambda=2$: (a) in $\{11\}$ orientation; (b) the $\{10\}$ orientation. The dotted line describes the lower energy bond of the two bonds (in the system of period 2), while the dashed line describes the higher energy bond. A minimum energy path through each system is indicated with a thick solid line.

III. SCALING THEORY AND NUMERICAL RESULTS

Consider the ground-state interface of a square lattice in which the bond disorder has period 2 in the $\{11\}$ orientation [e.g., Fig. 1(a)]. It is obvious that the interface is highly degenerate, as the ground-state interface may start in any of $L/2$ equivalent positions. Consider now starting to create a ground-state interface from the left side of Fig. 1(a). To minimize the interface energy one chooses the weakest bond. Having chosen this weakest bond, the interface crosses this weakest bond and chooses the weakest bond in the next column. This process of choosing the weakest bond continues across the sample and, for period 2, the random walk so generated gives the *exact* ground state. The reason this ground state is exact is that at each step, all of the possible random bonds in each column are tested (there are only two). Thus in this limit, $w\sim L^{1/2}$ as for a random walk. In contrast, if the period diverges, the model returns to the random-bond Ising universality class [or equivalently the directed polymer (DP) in a random medium] for which $w\sim L^{2/3}$. For finite λ, we expect that the interface will seek to optimize its global wandering until the roughness reaches the wavelength of the periodicity [19]. After that it has exhausted all possibilities and then returns to a random walk behavior. We thus have

$$w(L,\lambda)\sim\begin{cases} L^{2/3}, & w\ll\lambda, \\ L^{1/2}, & w\gg\lambda. \end{cases} \tag{4}$$

A natural scaling form based on these limiting behaviors is

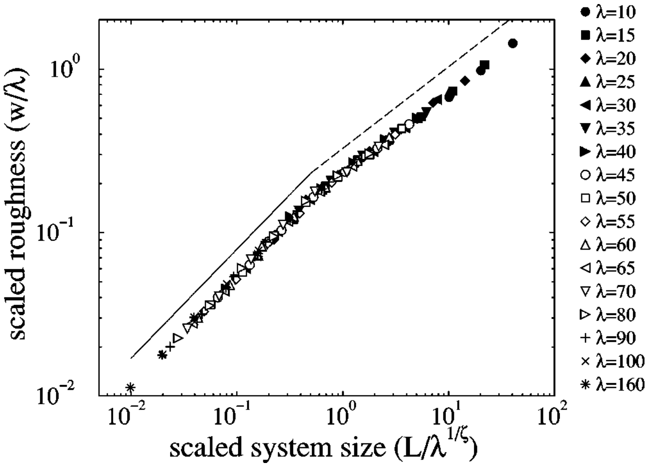

FIG. 2. The roughness (w) of manifolds divided by the wavelength of the periodicity (λ) vs normalized system size ($L/\lambda^{1/\zeta}$), where $\zeta=\zeta_{DP}=\frac{2}{3}$, for $\{10\}$ oriented $(1+1)$-dimensional systems. The random bonds are from a uniform distribution with strength $\Delta J_{ij,\perp}/J_0=1$ in the perpendicular (z) direction, and $\Delta J_{ij,\parallel}/J_{0,\parallel}=0.1$ in the parallel (x) direction in all layers in order to break the degeneracy. $J_{0,\parallel}/J_0=0.2$. The number of realizations $N=200$ for each wavelength $\lambda\in[10,\ldots,160]$ and system size $L^2\in[20^2-1280^2]$. The solid line has a slope $\zeta=\zeta_{DP}=\frac{2}{3}$ and the dashed line has a slope $\zeta=\zeta_{RW}=1/2$.

$$w(L,\lambda)\sim L^{2/3}f\left(\frac{L}{\lambda^{3/2}}\right), \tag{5}$$

where the scaling function $f(z)$ for the roughness has the asymptotic behavior

$$f(z)\sim\begin{cases}\text{const}, & z\ll 1,\\ z^{-1/6}, & z\gg 1.\end{cases} \tag{6}$$

Tests of the asymptotic behaviors (4) and the scaling function (5) and the results are presented in Figs. 2 and 3 for the $\{10\}$ orientation. It is seen that the predictions of the scaling theory are nicely confirmed. Similar results were found for the $\{11\}$ orientation, too.

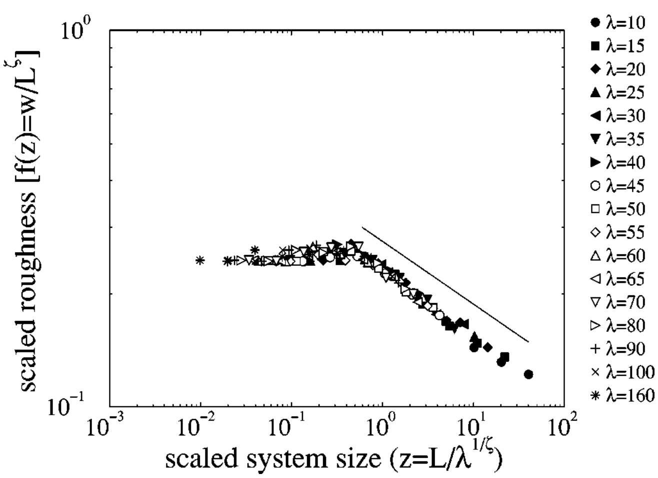

FIG. 3. The scaling function $f(z)=w/L^{\zeta}$ of the roughness $w(L,\lambda)$ vs scaling parameter $z=L/\lambda^{1/\zeta}$, where $\zeta=\zeta_{DP}=\frac{2}{3}$ for the same data as in Fig. 2. The solid line has a slope of $\zeta_{RW}-\zeta_{DP}=-1/6$.

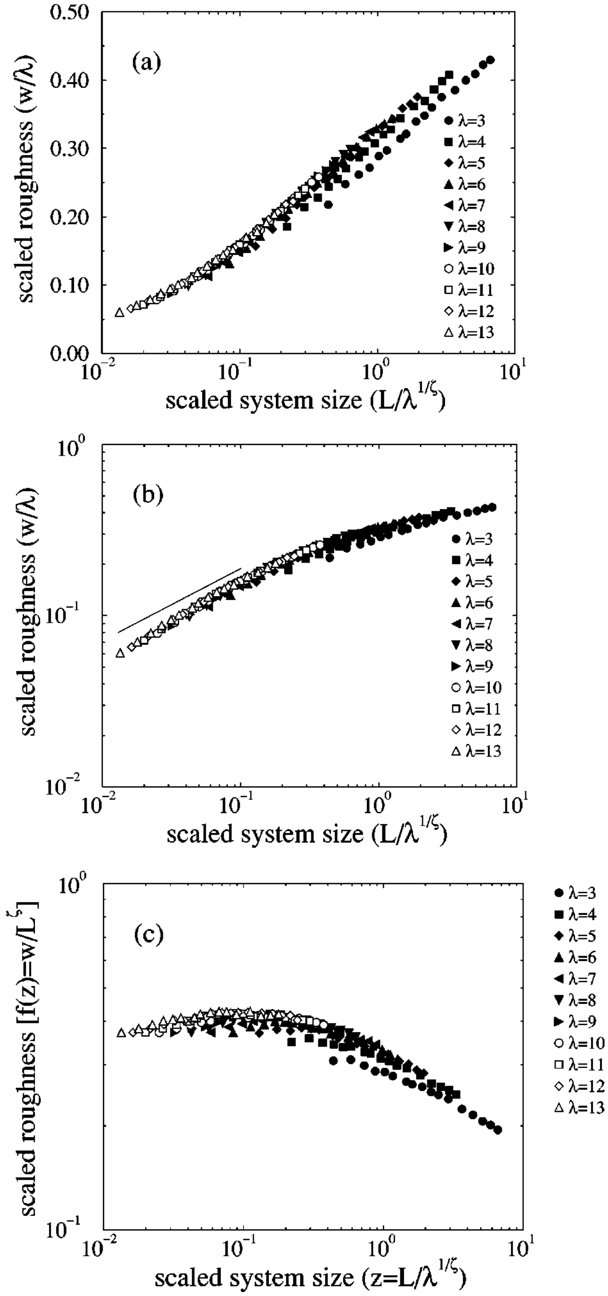

FIG. 4. (a) and (b) The data collapse, w/λ vs $L/\lambda^{1/\zeta}$, $\zeta=\zeta_{RB}=0.42$, for the roughness of $(2+1)$-dimensional $\{100\}$ oriented systems. The random bonds are from uniform distribution with $\Delta J_{ij,\perp}/J_0=1$ in the perpendicular (z) direction and constant $J_{ij,\parallel}/J_0=0.2$ in the parallel (x,y) direction. The number of realizations $N=200$ for each wavelength $\lambda\in[3,\ldots,13]$ and system size $L^3\in[6^3-90^3]$. (c) The scaling function $f(z)=w/L^{\zeta}$ of the roughness $w(L,\lambda)$ vs scaling parameter $z=L/\lambda^{1/\zeta}$. Finite-size effects with logarithmic corrections are visible as a curvature for small L.

We turn now to the behavior of random surfaces in $(2+1)$ dimensions. There, renormalization-group (RG) techniques have been applied to the random-phase sine-Gordon model [9–11], to random-bond interfaces, and to fairly general models of periodic elastic media. Numerically, exact maximum-flow–minimum-cut and minimum-cost-matching algorithms [13] and Monte Carlo methods [20] have been used. In the random substrate problem, there is a low-temperature ''super-rough'' phase where $w^2 \sim \ln^2(L)$, while in the random manifold problem, the surface roughness is found to behave as $w \sim L^{\zeta_{\mathrm{RB}}}$, where $\zeta_{\mathrm{RB}} = 0.42 \pm 0.01$. The qualitative reasoning expressed in the first paragraph of this section also applies to higher dimensions, so that we expect the behavior of $\mathcal{H}_p$ to be in the random substrate universality classes at long length scales $w > \lambda$, while the random manifold universality class is dominant at short length scales $w < \lambda$. The limiting behaviors in dimension $(2+1)$ are then

$$w(L,\lambda) \sim \begin{cases} L^{\zeta_{\mathrm{RB}}}, & w \ll \lambda, \\ \ln L, & w \gg \lambda. \end{cases} \tag{7}$$

We thus expect

$$w(L,\lambda) \sim L^{\zeta_{\mathrm{RB}}} f\left(\frac{L}{\lambda^{1/\zeta_{\mathrm{RB}}}} \right), \tag{8}$$

and that the scaling function in $(2+1)$ dimensions is

$$f(z) \sim \begin{cases} \mathrm{const}, & z \ll 1, \\ \ln z / z^{\zeta_{\mathrm{RB}}}, & z \gg 1, \end{cases} \tag{9}$$

with the scaling parameter $z = L/\lambda^{1/\zeta_{\mathrm{RB}}}$. The asymptotic behaviors of Eq. (7) are illustrated in Figs. 4(a) and 4(b) for interfaces in the $\{100\}$ orientation. The logarithmic asymptotic behavior is clearly confirmed in Fig. 4(a), but the random manifold behavior is still strongly effected by finite-size effects. This is understandable as large system sizes are necessary to see the asymptotic random manifold behavior, even in the $\lambda \to \infty$ limit [8,7]. Though finite-size effects are clearly evident in the scaling plot of Fig. 4(c), the data collapse at large λ is quite satisfying. It is clear that the random substrate (Bragg glass) universality class [12,13] is dominant at large enough length scales. We have tested the behavior in the $\{111\}$ orientations and find that $\{111\}$ interfaces behave in a similar manner.

IV. CONCLUSIONS

We have studied the scaling behavior of an elastic manifold in the presence of a periodically repeated ''strong'' bond disorder. We find that in $(1+1)$ and in $(2+1)$ dimensions, and at long distances, the periodicity is relevant so these interfaces are in the random substrate universality class. This is to be contrasted with an interface in a system with a periodic potential and with random disorder. In the latter problem the periodic potential is claimed to be irrelevant on long length scales in $(1+1)$ and $(2+1)$ dimensions for any disorder [1–3], though at weak disorder numerical work on $\{100\}$ orientation cubic lattices indicates a strong tendency to order due to lattice effects [8,21].

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