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Prediction of burst pressure of corroded thin-walled pipeline elbows subjected to internal pressure

Changqing Gong, Shihua Guo, Rui Zhang, Dan M. Frangopol

1. Introduction

Pipeline integrity is a critical concern in the domain of infrastructure and industrial safety. The vulnerability of energy pipelines poses a significant challenge in ensuring societal safety and community well-being. Considering the investigation into the mapping of failure causes, corrosion emerges as a primary factor contributing to failures [1]. Given that pipes are consistently subjected to elevated internal pressure and affected by the surrounding physical and chemical conditions, as well as the nature of the substances being transported, the pressurized oil and gas pipelines can deteriorate due to both external and internal corrosion on the pipeline surface. Accurately evaluating the burst pressure of corroded pipelines is imperative, as it helps identify critical defects and prioritize maintenance resources to mitigate the risk of pipeline failure. In the past, extensive research efforts have focused on assessing the burst capacity of pipelines containing single semi-elliptical corrosion, based on extensive three-dimensional elastic-plastic finite element (FE) analysis validated by full-scale burst tests of straight and elbow pipelines. The Pipe CORRosion Criterion (PCORRC) method, originally developed for straight pipelines, was re-calibrated to predict the remaining strength of corroded pipeline elbows. A revised factor was introduced to account for hoop stress variations caused by elbow curvatures. Results indicate that the revised model offers observable accuracy improvement over the existing models reported in the literature. The proposed model will improve the integrity management of corroded elbows.

The thinning of pipeline walls, resulting from corrosion, reduces their capacity to withstand internal pressures. Accurate prediction of the remaining strength of corroded pipelines is important to prioritize mitigation efforts for critical defects. While burst pressure models for straight pipelines have been well established, the research on the burst capacity of pipeline elbows is still limited, and most studies were established on rectangular-shaped pipe elbows. Pipeline elbows are more vulnerable to corrosion due to flow direction changes, which are subject to shear and impact forces causing erosion and cavitation corrosion. Unlike straight pipelines with uniformly distributed stress under pressure, pipeline elbows experience higher stress on the intrados and lower stress on the extrados. As a result, the existing burst models of straight pipelines cannot directly apply to pipeline elbows without accounting for this distinct stress distribution feature. While there have been recent advancements in evaluating the strength of corroded elbows through various approaches [6,7], a thorough examination of current models and the integration of an adjusted factor to accommodate variations caused by elbow curvatures remains limited. Particularly, there is a need to expand and strengthen research for the impacts of semi-elliptical corrosion on pipeline elbows.

As the research on factors affecting the burst capacity of corroded elbows deepened, an increasing number of scholars started to develop burst models specifically for corroded pipe elbows. In 1978, Goodall [8] suggested a closed-form solution to estimate the burst capacity of defect-free elbows under internal pressure, accounting for curvature...
effects. Many subsequent studies have adapted existing burst models for corroded straight pipelines to predict the burst pressure of a corroded elbow by incorporating the Goodall model. For example, Zhang et al. [9] combined the Goodall model with the B31G. Due to the inherent conservatism of B31G in predicting the burst pressure of the corroded straight pipeline, the model by Zhang et al. [9] is inherently conservative when applied to corroded elbows. To improve prediction accuracy, Khalajastani et al. [10] integrated the DNV model, which is more accurate than B31G, with the Goodall model. Parametric analysis revealed that the prediction results agree well with those by FE. Lee et al. [11] evaluated the performance of several burst models, including B31G, DNV, and PCORRC, among others, when used in conjunction with the Goodall model to evaluate the burst pressure of corroded elbows. They found that in some cases incorporating PCORRC leads to similar accuracy with the use of DNV in comparison to FE results. It is noted that the Goodall model for intact elbows is derived from the Tresca criterion and is therefore a lower-bound solution [12]. When coupled with standard models developed for straight pipelines, additional conservatism is introduced into the prediction of the burst capacity of corroded pipeline elbows. On this basis, Wang and Zhou [13] proposed the von Mises stress-based Goodall model validated by full-scale burst tests and FE analysis of intact pipelines, then integrated it with the DNV model to enhance corroded pipeline elbow assessment. Extensive FE results demonstrated that the developed model has markedly superior accuracy compared to those by simply integrating B31G and DNV models. Utilizing the von Mises stress-based Goodall model, Shuai et al. [14] proposed an empirical revision expression that accounts for the elbow curvature effects on the burst capacity of corroded elbows. The parameters of this model were obtained by fitting FE analysis results. However, despite the promising results achieved by existing models in the reported cases, there is still a need for further improvement in burst models of corroded pipeline elbows. Existing models for elbows were developed on rectangular corrosion profiles, and their applicability to elliptical corrosion defects has not been well explored. Elliptical idealization has been shown to be a more realistic representation of naturally occurring corrosion defects than rectangular approximation [15]. Through a comprehensive exploration of existing models and the incorporation of a revised factor to account for variations induced by elbow curvatures, the research reported herein seeks to enhance the accuracy of burst pressure predictions. The outcomes presented are expected to not only advance the current understanding of semi-elliptical corrosion effects on pipeline elbows but also provide practical implications for pipeline integrity management.

The primary objective of this study is to develop new burst models for pipeline elbows containing semi-elliptical corrosion. These proposed models follow similar forms to previous studies based on the Goodall models while integrating the PCORRC models. A revision factor expression for the PCORRC model is introduced to improve its predictability for corroded elbows. Extensive parametric 3D FE analyses are performed on elbows containing isolated semi-elliptical defects to determine revision factor expression. Full-scale burst test results of two corroded pipeline elbows and two straight pipelines are employed to validate the FE modeling. The rest of this paper is organized as follows. Section 2 describes the modeling details of the FE of corroded elbows and the validation by full-scale burst tests; Section 3 presents the new burst model of corroded elbows and compares the new model with existing models; to further validate the developed new burst model, the FE analysis results from two recent studies using ANSYS are adopted to provide benchmark results in Section 4 and conclusions are provided in Section 5.

2. Finite element (FE) model

2.1. General description

The geometry characteristics of a 90 degree pipeline elbow is illustrated in Fig. 1, where \( D \) is the outer diameter of the pipe (mm), \( R \) is the bending radius of the pipe (mm), and \( L \) is the length of the straight pipe section. To mitigate the adverse impact of boundary conditions, the length of the straight pipe is assumed four times the outer diameter of the elbow.

Fig. 2 describes the geometry parameters of a pipeline elbow containing semi-elliptical-shaped corrosion: the maximum corrosion length in the elbow longitudinal direction is associated with the plane angle, \( \gamma \); while the corrosion width along the circumferential direction is determined by the angle, \( \beta \); \( t \) is the wall thickness of the pipe (mm); \( d \) is the maximum corrosion depth (mm). Wherein, the outer arc side of the elbow corresponds to the position of 9:00, the inner arc side corresponds to the position of 3:00, and the positions of the central axis correspond to the positions of 0:00 and 6:00 respectively. It is noted that the corroded defect of the elbow does not adhere to a strict mathematical definition of a semi-ellipse. Rather, it takes the form of an approximate semi-ellipse, obtained by the coordinate transformation of an ideal semi-ellipse in a straight pipeline.

2.2. Constitutive properties and failure criteria

The FE models of the corroded elbows are generated using the commercial software ANSYS to investigate the failure behavior of the elbow under internal pressure. The model uses a fully integrated 8-node solid element for elastic-plastic analysis considering both geometric and material nonlinearity. The numerical simulation employs the Von Mises yield criterion, associated flow rule, and isotropic hardening criterion.

The widely used power-law model is employed to express the true stress–strain relationship of pipeline elbows:

\[
\begin{align*}
\sigma &= E\varepsilon & \sigma < \sigma_y \\
\sigma &= K\sigma^n & \sigma \geq \sigma_y
\end{align*}
\]

where \( \sigma \) represents the true stress of the material, \( \varepsilon \) represents the true strain of the material, \( E \) represents the elastic modulus, \( \sigma_y \) represents the yield strength, and \( K \) and \( n \) are the strength coefficient and the strain hardening exponent of the material, respectively.

The values of \( n \) and \( K \) in Eq. (1) are determined using the following empirical expressions [16]

\[
\begin{align*}
n &= 0.224 \left( \frac{\sigma_y}{\sigma_t} \right)^{-0.604} \\
K &= \frac{\sigma_t}{n^n}
\end{align*}
\]
elbows, the failure behavior used in this study can be defined as follows: when the maximum Von Mises stress within the corroded defect region reaches the true stress corresponding to the ultimate tensile strength of the material, the burst failure is assumed to occur. At this point, the internal pressure applied to the pipeline is considered as the limit burst pressure of the pipeline. The validation of this failure criterion for predicting burst pressure has been confirmed by comparison with a series of full-scale burst tests of corroded straight pipelines [17].

2.3. FE mesh

The meshing density of the pipeline elbows significantly impacts the accuracy of the FE results. As the number of elements increases, the computational time increases drastically and the improvement in accuracy diminishes gradually. To achieve a balance between computational cost and accuracy, optimal meshing density should be explored through controlled analysis.

Fig. 3 depicts the schematic diagram of the mesh configuration. A high-density mesh is utilized to discretize the corrosion region in order to accurately characterize the localized stress and strain distribution. The mesh sizes of the corrosion region in the longitudinal and angular directions are set equal to \( \min(L, W)/(\pi D)\cdot(\delta/360) \), where \( L \) and \( W \) are the maximum length and width of a single corrosion defect along the elbow longitudinal and circumferential direction, respectively; and \( \delta \) (\(^{\circ}\)) is the angular angle corresponding to the mesh size. To achieve a good balance between computational efficiency and accuracy, a transition mesh is built to smoothly increase the mesh size of the defect-free region and ensure the continuity of the mesh configuration. Convergence studies show that \( \delta = 1^{\circ} \) is sufficient to achieve numerical convergence. The mesh size of the defect-free region is assumed to be dictated by the times the mesh size increases from the corrosion region outwards. Empirical results indicate that augmenting the mesh size by 1 or 2 times demonstrates no significant influence on the FE burst pressure results. The wall thickness is divided into four layers within the corrosion region, while for the non-corroded area, the number of mesh layers is reduced to 2 layers. By comparison of burst pressures for different number of mesh transitions for different conditions, it can be concluded that the number of grid transitions has almost no influence on the accuracy of FE results. To improve the computational efficiency of the FE model, the selection of the number of transition layers is determined based on the number of grids in the non-corrosion area in the circumferential direction. If the number of grids in the non-corrosion area is higher than 60, the number of transitions is increased by one; if the number of circular grids in the non-corrosion area is less than 30, the number of transitions will be reduced by 1. This wall thickness mesh is adapted from Zhang and Zhou [15], and the adequacy of this strategy has been confirmed by their convergence studies.

2.4. FE validation

Reports of full-scale burst tests of corroded pipeline elbows are very limited in the literature. The burst tests on elbows performed by Kim et al. [18] represent one of few reports that can provide data of sufficient efficiency of burst assessment. As such, they are employed to validate the FE modeling. To further validate the proposed FE model, computational results are also compared with experimental results of two straight pipeline tests containing isolated circular corrosion defects as conducted by Al-Owaisi et al. [19]. The FE models of the corroded elbows and straight pipelines are constructed in a similar manner, including element meshing strategy and failure criterion, aside from the difference in boundary conditions, and geometric, and material properties. Comparison with straight pipeline tests can also contribute to improving confidence in FE validation.

2.4.1. Pipeline elbow burst tests

The geometric and material properties of two full-scale burst tests of corroded pipelines conducted by Kim et al. [18] are summarized in Table 1. The size and position of the corrosion are provided in Table 2. The elbow test configuration is discussed in [18] and the FE modeling is shown in Fig. 4. Both ends of the test elbow are closed with end caps. To prevent the rigid body displacement of the pipe, displacement constraints are applied at both ends during the test. To maintain consistency with the experimental conditions during FE analysis, this section establishes the end caps using solid elements, and the displacements of all
Table 2
The size and position of the elbow corrosion in [18].

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>ρ/π</th>
<th>σ/π</th>
<th>d/t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Extrados</td>
<td>0.44</td>
<td>0.57</td>
<td>0.786</td>
</tr>
<tr>
<td>2</td>
<td>Intrados</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

nodes on the pipe ends are linked to the two reference pins. The internal pressure load is applied on the surface of the pipeline elbow.

Table 3 lists the comparison results between the FE model and the full-scale pipeline elbow burst tests. The test burst pressure is denoted by \( P_{\text{TEST}} \) while the burst pressure predicted by FE is expressed by \( P_{\text{FE}} \). It is shown that the predictions by FE and burst tests are in good agreement, with \( P_{\text{TEST}} \) being slightly higher than \( P_{\text{FE}} \). This minor difference can potentially be attributed to the imperfect semi-elliptical shape of the corrosion manufactured in the test compared to the idealized FE model. Since only one axial and circumferential wall thickness measurement is given in [18], the actual corrosion profile is difficult to ascertain. Nevertheless, the closely matched results between the test and FE provide confidence in the FE analysis as an accurate prediction tool for pipeline elbow failure.

2.4.2. Straight pipeline burst tests

Table 4 provides the geometric properties of the two straight pipeline specimens with semi-sphere corrosion defects reported by Al-Owaisi et al. [19]. The specimens are end-capped during the burst test to contain the internal pressure. Since the endcap causes axial deformation of the pipeline, axial forces are applied to one end of the pipeline while fixing the other end in FE analysis. The FE model of one corroded pipeline specimen is illustrated in Fig. 5. Table 5 compares the burst pressures obtained from the pipeline test and those predicted by the FE. It is shown that the FE predictions closely match the experimental results with \( P_{\text{TEST}} / P_{\text{FE}} \) being close to unity. This demonstrates the validity of the FE for modeling the corroded pipelines.

3. Burst pressure model for corroded pipeline elbows

3.1. Review of existing models

(a) Zhang et al. [9] proposed a burst pressure model for corroded pipeline elbows by combining the B31G model with the Goodall model, expressed by

\[
P_{Zhang} = k \cdot P_{\text{B31-M}}
\]

\[
P_{\text{B31-M}} = \frac{r \sigma_f}{r + \frac{0.85 \sigma_f}{\pi}}
\]

where \( P_{\text{B31-M}} \) represents the burst pressure of the corroded straight pipeline by the B31G model; \( D \) is the pipeline outer diameter; \( t \) represents the wall thickness; \( r = (D - t) / 2 \) is the mean elbow radius; \( L \) is the length of the corrosion defect; \( M_1 \) is the Folias factor; \( d_{\text{max}} \) is the maximum depth of the corrosion defect; \( \sigma_f \) represents the flow stress which is calculated as \( \sigma_f = (\sigma_g + \sigma_u) / 2 \); and \( k \) is the curvature correction factor, defined by

\[
k = \frac{R + \frac{1}{2} \rho \sin \theta}{R + \frac{1}{2} \rho \sin \theta}
\]

in which \( R \) is the elbow bend radius. \( k \) essentially reflects the variation of the hoop stress in the circumferential direction. As a reminder, the lower

![Fig. 4. FE modeling of the test pipe elbow.](image-url)

bound to the burst pressure of an intact pipeline elbow by the Goodall model is \( P_{\text{Goodall}} = g \frac{D t}{R} \), where \( g \) is the value of \( k \) when \( \theta = -90^\circ \).

(b) Khalajestani et al. [10] adapted the DNV burst model for the corroded straight pipeline to predict the burst pressure of corroded pipeline elbows, as:

\[
P_{\text{Khal}} = k \cdot P_{\text{DNV}}
\]

\[
P_{\text{DNV}} = \frac{r \sigma_f}{r + \frac{0.85 \sigma_f}{\pi}}
\]

\[
M_2 = \sqrt{1 + \frac{0.31L^2}{Dt}}
\]

where \( P_{\text{DNV}} \) is the burst pressure associated with the original DNV model, and \( M_2 \) is the Folias factor of the DNV model.
Table 5
Comparison of burst pressures of the straight pipelines by the FE and test.

<table>
<thead>
<tr>
<th>No.</th>
<th>( P_{\text{test}} )</th>
<th>( P_t )</th>
<th>( P_{\text{test}}/P_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.55</td>
<td>19.58</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>24.67</td>
<td>24.68</td>
<td>0.99</td>
</tr>
</tbody>
</table>

(c) Lee et al. [11] also investigated the applicability of the revised PCORRC based on the Goodall model to evaluate the burst pressure of corroded elbows, expressed as

\[
P_{\text{Lee}} = k \cdot P_{\text{PCORRC}}
\]

(11)

\[
P_{\text{PCORRC}} = 0.95 \frac{2 \sigma_{ut}}{D} \left( 1 - \frac{d_{\text{max}}}{t} \left( 1 - \exp \left( \frac{-0.224 \lambda}{\sqrt{D(t - d)/2}} \right) \right) \right)
\]

(12)

where \( P_{\text{PCORRC}} \) is the burst pressure of the corroded straight pipeline associated with the PCORRC model. Note that the DNV and PCORRC models are fundamentally different from the B31G in that the former were developed based on curve fitting FE analysis of moderate to high toughness pipes, while the latter was empirically determined on experimental burst data of old low-grade steels. The DNV and PCORRC models are more applicable to modern pipelines.

(d) Considering that under the Tresca yield criterion, the Goodall model may underestimate the failure pressure of the intact pipeline elbow. Wang and Zhou [13] improved the burst predictability for intact elbows by incorporating the von Mises criterion. A new DNV-based burst model for corroded pipeline elbows was proposed as:

\[
P_{\text{Wang}} = \frac{2k}{\sqrt{4 - 2k + k^2}} P_{\text{DNV}}
\]

(13)

(e) Shuai et al. [14] introduced a correction factor, dependent on the geometric and material properties of the corroded pipeline elbow, to evaluate the burst pressure of the corroded elbow. Regression analyses were performed on FE results to determine the parameters of the correction factor equation. The resulting model allows the burst pressure of a corroded pipeline elbow to be calculated by

\[
P_{\text{Shuai}} = \frac{2k}{\sqrt{4 - 2k + k^2}} \cdot P_c
\]

(14)

\[
P_c = \frac{\tau}{\pi} \left( 1 - \frac{d}{t} \left( 1 - \frac{0.7151 \exp \left( -1.0887 L \sqrt{D(t - d)/2} \right)}{1 - \frac{d}{t}^{0.3621}} \right) \right)
\]

(15)

where \( P_c \) is the correction factor that accounts for the geometry effect of the corroded elbow.

3.2. Proposed burst capacity model

In this study the burst capacity model of a corroded elbow is computed by the product of a geometric revision factor, \( f_{ge} \), in terms of \( k \) in Eq. (4), and the predicted burst pressure given by the corroded straight pipeline model, i.e., \( P_{b,\text{elbow}} = f_{ge} P_b \), where \( f_{ge} \) accounts for geometric characteristics of the pipeline elbow, and \( P_b \) is the reference burst pressure computed by the straight pipeline model. The PCORRC model is selected to evaluate \( P_b \). Instead of using the PCORRC model described by Eq. (12), this study adopts the original PCORRC model by Lee et al. [11] as follows:

\[
P_{b, \text{PCORRC}} = \frac{2 \sigma_{ut}}{D} \left( 1 - \frac{d_{\text{max}}}{t} \left( 1 - \frac{0.157 L}{\sqrt{D(t - d)/2}} \right) \right)
\]

(16)

Notably, the PCORRC model was originally developed by Stephens and Leis [5] for pipe steels (i.e., X52–X70), and was subsequently recalibrated by Yeom et al. [20] specifically for X70. Eq. (12) adopted by Lee et al. [11] defines the recalibrated model. Statistical analysis of model errors associated with burst models for corroded straight pipelines using full-scale burst test results of pipelines with real corrosion indicated that the original PCORRC has higher prediction accuracy than the B31G and DNV [21].

A total of 288 parametric FE cases of representative pipelines with single isolated corrosion defects are analyzed to determine \( f_{ge} \). The outside diameter (\( D \)), wall thickness (\( t \)), yield strength (\( \sigma_y \)), engineering ultimate tensile strength (\( \sigma_u \)), and pipeline steel grade of the parametric analyses are summarized in Table 6. For each type of pipeline elbows, an analysis matrix is created with \( d/t = 30 \% \) or 60 \%, \( R/D \) equal to 1.5, 2, 3, or 5, \( \gamma/\pi \) equal to 0.3 or 0.6, and \( \beta/\pi \) = 0.09 or 0.18. To simulate the actual pipeline operating conditions accurately, both ends of the
Corrosion defects are assumed to be on the external pipeline surface, which is justified by empirical evidence that the corrosion type (i.e., external, or internal) has negligible impact on the pipeline burst capacity. Both pipeline elbow ends are fixed to represent the actual boundary restraints of in-service pressurized pipeline elbows. It is further assumed that the corrosion defect is located at either extrodos or introdos. Note that the presence of corrosion defects at the extrodos does not necessarily imply failure at the extrodos. For the intact pipeline elbow under internal pressure, the maximum Von Mises stress is higher at the introdos compared to the extrodos. Thus, elbows may fail at the introdos despite extrodos corrosion. Of 288 cases, 18 cases experienced this type of failure mode with failure locations different from the defect locations and are excluded from the discussion of the results.

The ratios of the values of the burst pressure from the 270 parametric FE cases, denoted by $P_{FE}$, relative to those corresponding to $P_{b}$, $P_{CORRC}$ are calculated. The empirical expressions for $f_{ge}$ associated with the PCORRC model are developed by curve fitting as:

$$f_{ge} = 0.146 + k + 0.14d_{max}/t \quad (17)$$

Taking the product of $f_{ge}$ and $P_{b}$, $P_{CORRC}$, one can readily calculate the proposed burst pressure of the corroded elbow, denoted by $P_{MPCORRC}$.

**Table 6**

Properties of pipeline elbows in the parametric FE analysis.

<table>
<thead>
<tr>
<th>No.</th>
<th>$D$ (mm)</th>
<th>$t$ (mm)</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>Steel grade</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72</td>
<td>7</td>
<td>275.2</td>
<td>449.2</td>
<td>A234</td>
<td>[18]</td>
</tr>
<tr>
<td>2</td>
<td>168</td>
<td>9.5</td>
<td>290</td>
<td>495</td>
<td>X42</td>
<td>[11]</td>
</tr>
<tr>
<td>3</td>
<td>323.9</td>
<td>9.71</td>
<td>452</td>
<td>542</td>
<td>X60</td>
<td>[22,23]</td>
</tr>
<tr>
<td>4</td>
<td>406</td>
<td>7</td>
<td>359</td>
<td>455</td>
<td>X52</td>
<td>[13]</td>
</tr>
<tr>
<td>5</td>
<td>508</td>
<td>8.9</td>
<td>435</td>
<td>560</td>
<td>X60</td>
<td>[22,23]</td>
</tr>
<tr>
<td>6</td>
<td>610</td>
<td>12.2</td>
<td>483</td>
<td>565</td>
<td>X70</td>
<td>[13]</td>
</tr>
<tr>
<td>7</td>
<td>1219</td>
<td>21.9</td>
<td>589</td>
<td>775</td>
<td>X80</td>
<td>[24,25]</td>
</tr>
<tr>
<td>8</td>
<td>1320</td>
<td>22.9</td>
<td>690</td>
<td>886</td>
<td>X100</td>
<td>[24,25]</td>
</tr>
<tr>
<td>9</td>
<td>762</td>
<td>15.9</td>
<td>532.2</td>
<td>626.8</td>
<td>X70</td>
<td>[26]</td>
</tr>
</tbody>
</table>

**Fig. 6.** Comparison of burst pressures by the FE and empirical models.
based model and Wang et al.’s model are in good agreement with those obtained from the FE analysis. However, there are noticeable differences in burst pressures between FE modeling and the models by Zhang et al. [9] (Fig. 6(a)), Lee et al. [11] (Fig. 6(b)), and Shuai et al. [14] (Fig. 6(e)). Zhang et al.’s and Lee et al.’s models significantly overestimate the burst pressure. To quantify the uncertainty of the model error associated with these models, Table 7 lists the means and COVs (coefficient of variations) of the FE-to-predicted ratios. It is shown that the mean (1.02) and COV (3.3 %) of the PCORRC-based model are less than all existing models reported in the literature. Wang et al.’s model performs second best with the mean and COV of the FE-to-predicted ratio equal to 1.03 and 5.4 %, respectively. Shuai et al.’s model results in the burst capacity with the highest variability, i.e., the COV of the FE-to-predicted ratio up to 12.3 %. Lee et al.’s model gives the most conservative prediction and leads to the largest variability: the mean of the FE-to-predicted ratio is equal to 1.38. These results confirm the improved prediction accuracy of the PCORRC model compared to the existing models.

4. Validation of the proposed burst pressure model

To further validate the proposed PCORRC model, the FE analysis results from two recent studies using ANSYS are adopted to provide benchmark results. The failure criterion and stress–strain relationship in these studies are the same as the assumptions used in this study. By comparing the predictions of the PCORRC model with these FE results as reference, the generalization and robustness of the proposed model can be rigorously assessed.

4.1. Defect-free elbow burst prediction

When corrosion defects are absent, the burst pressure equations by existing elbow burst models and the new proposed model are reduced to

\[ P_{\text{Zhang}} = k \frac{\sigma_u t f}{D} \]  

(18)

\[ P_{\text{Lee}} = k \frac{1.9 \sigma_u t}{D} \]  

(19)

\[ P_{\text{Shuai}} = k \frac{t \sigma_u}{D} \]  

(20)

\[ P_{\text{Wang}} = P_{\text{Shuai}} = \frac{2k}{\sqrt{4 - 2k + k^2}} \frac{t \sigma_u}{D} \]  

(21)

\[ P_{\text{AT-PCORRC}} = (0.146 + k) \frac{2 \sigma_u t}{D} \]  

(22)

These equations predict the burst pressure of intact pipeline elbows when the value of \( k \) is evaluated at \( \theta = -90^\circ \).

Wang and Zhou [12] performed the FE analyses on 40 intact elbows with \( D = 610 \text{ mm}, f = 7.36 \text{ or } 12.2, \text{ and } R/D = 1.5, 2.2, 2.5 \text{ or } 3 \). The proposed model and existing models are utilized to predict the burst pressure of the pipeline elbows. The mean and the coefficient of variation (COV) of FE-to-predicted ratios are provided in Table 8. The results demonstrate that the PCORRC model is the most accurate model to evaluate the burst capacity of corroded pipeline elbows: the mean and COV of the FE-to-predicted burst pressure ratios are 1.00 and 1.7 %, respectively. Zhang et al.’s model is the most conservative one with a mean of the FE-to-predicted burst pressure ratios equal to 1.31.

4.2. Corroded elbow burst capacity prediction

Wang and Zhou [13] performed the FE analyses of 16 elbows involving \( R/D = 3, \beta/\pi = 0.33, \gamma/\pi = 0.056 \text{ or } 0.167, \text{ and } d/t = 30 \%, 40 \%, 50 \% \text{ or } 60 \% \). The pipeline elbow contains single isolate semi-ellipse corrosion defects. Table 9 compares the performance of the proposed PCORRC model and existing models in predicting the burst capacities of the FE cases. Results indicate that the developed model achieves the best accuracy with the mean and COV of the FE-to-predicted ratios equal to 1.09 and 3.5 %, respectively. The accuracy performance of Wang et al.’s model is slightly weaker than the proposed model with the mean and COV of the FE-to-predicted ratios equal to 1.12 and 6.1 %, respectively. In contrast, Lee et al.’s model is highly conservative compared to the FE results: the mean of \( P_{\text{FE}}/P_{\text{Lee}} \) is up to 1.48. While Shuai et al.’s model has a similar mean of the FE-to-predicted ratios to Khalajestani et al.’s model, it has the largest variation among all the models, with the COV of \( P_{\text{FE}}/P_{\text{Lee}} \) equal to 14.0 %. These results provide further evidence of the PCORRC model’s superiority in the accuracy of predicting burst capacities of corroded pipelines compared to the existing models.

5. Conclusions

This study proposes a new burst pressure prediction model for pipeline elbows with semi-elliptical corrosion, addressing the limitation of existing models that were developed for rectangular corrosion profiles and haven’t been well explored for elliptical defects, despite the latter being a more realistic representation of naturally occurring corrosion. The model is developed through extensive three-dimensional nonlinear finite element analysis, verified by full-scale burst tests of corroded pipelines. A revision expression is introduced and coupled with the original PCORRC model for corroded straight pipelines to evaluate the burst pressure of corroded pipeline elbows. A matrix of 288 analysis cases is created to provide the reference burst pressures for determining the revision factor expression. Additional FE results from the literature are collected to further validate the accuracy of the proposed model.

Comparison between the model prediction and FE results reveals that the developed PCORRC-based model provides results that closely match the FE results. The proposed model is also compared with existing models (Zhang et al.’s model, Khalajestani et al.’s model, Lee et al.’s model, Wang et al.’s model, and Shuai et al.’s model). It is shown that the developed model outperforms all existing models with minimum bias and model variation. Conclusions are as follows:

1. Comprehensive parametric 3D FE analyses have been conducted on elbows with isolated semi-elliptical defects, aiming to

Table 7
Mean and COV of FE-to-predicted burst pressure ratios in 270 FE cases.

<table>
<thead>
<tr>
<th></th>
<th>( P_{\text{FE}}/P_{\text{Zhang}} )</th>
<th>( P_{\text{FE}}/P_{\text{Lee}} )</th>
<th>( P_{\text{FE}}/P_{\text{Shuai}} )</th>
<th>( P_{\text{FE}}/P_{\text{Wang}} )</th>
<th>( P_{\text{FE}}/P_{\text{Lee}} )</th>
<th>( P_{\text{FE}}/P_{\text{Shuai}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.25</td>
<td>1.38</td>
<td>1.18</td>
<td>1.03</td>
<td>1.19</td>
<td>1.02</td>
</tr>
<tr>
<td>COV</td>
<td>8.8 %</td>
<td>6.2 %</td>
<td>5.3 %</td>
<td>5.4 %</td>
<td>12.3 %</td>
<td>3.3 %</td>
</tr>
</tbody>
</table>

Table 8
Mean and COV of FE-to-predicted burst pressure ratios for 40 intact elbows.

<table>
<thead>
<tr>
<th></th>
<th>( P_{\text{FE}}/P_{\text{Zhang}} )</th>
<th>( P_{\text{FE}}/P_{\text{Lee}} )</th>
<th>( P_{\text{FE}}/P_{\text{Shuai}} )</th>
<th>( P_{\text{FE}}/P_{\text{Wang}} )</th>
<th>( P_{\text{FE}}/P_{\text{Lee}} )</th>
<th>( P_{\text{FE}}/P_{\text{Shuai}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.31</td>
<td>1.23</td>
<td>1.15</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>COV</td>
<td>5.6 %</td>
<td>2.2 %</td>
<td>2.2 %</td>
<td>2.3 %</td>
<td>2.3 %</td>
<td>1.7 %</td>
</tr>
</tbody>
</table>
replicate a pipe elbow with semi-elliptical corrosion and establish the expression for the revision factor. The validity of the established FE model is confirmed through experimental tests conducted on both pipe elbows and straight pipelines. The negligible difference (around 6% for elbows and less than 1% for straight pipes) in burst pressures observed between the pipeline test and the corresponding FE result underscores the accuracy of the FE model in representing corroded pipelines.

(2) For the FE cases used in developing the revision expression, the proposed model achieves a mean FE-to-predict ratio of 1.02 and COV of 3.3%. While Wang et al.’s model is less accurate, it still predicts comparable results to the FE and exceeds other existing models. Lee et al.’s model gives the most conservative prediction and leads to the largest variability: the mean of the FE-to-predicted ratio is equal to 1.38.

(3) To further validate the developed PCORRC model, the FE analysis results from two recent studies are adopted to provide benchmark results. For the case of the defect-free elbow, the developed PCORRC model stands out as the most precise model compared with the existing models, achieving a mean and COV of the FE-to-predicted ratio at 1.00 and 1.7%, respectively; for the corroded elbow case, the developed PCORRC model emerges as the most accurate compared with the existing models, demonstrating a mean and COV of the FE-to-predicted ratio of 1.09 and 3.5%, respectively.

Finally, there exists a limitation within the present research. Geometric characteristics of natural corrosion is typical complex consisting of numerous individual pits due to varying corrosive environmental and pipeline coating conditions. Though the developed method considers a more reasonable elliptical idealization of corrosion pattern, it is critical to account for realistic corrosion profile to achieve more satisfactory accuracy. Further research will focus on the development of semi-empirical models based on FE modeling of realistic shaped corrosion. Moreover, since the burst model is derived by fitting finite element results from representative pipelines within defined parameter ranges, it is important to exercise caution when applying the model to geometries and steel materials that are beyond the scope of the calibration dataset.

Acknowledgments

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[8] Goodall I. Lower bound limit analysis of curved tubes loaded by combined internal pressure and in-plane bending movement. CEGB; (1978).