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Internal Magnus Effects in Superfluid $^3$He-A

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The orbital angular momentum of the coherently aligned Cooper pairs in superfluid $^3$He-A is encountered by an object immersed in the condensate. We evaluate the associated quasiparticle-scattering asymmetry experienced by a negative ion; this leads to a measurable, purely quantum-mechanical reactive force deflecting the ion's trajectory. Possible hydrodynamic Magnus effects are also discussed.

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The discovery of superfluidity in $^3$He was followed by evidence for other novel types of superconductivity for which the Cooper pairs may also exist in states with nontrivial internal structure. Therefore, it is important to find probes sensitive to the internal degrees of freedom of the Cooper pairs in condensates with complicated broken symmetries, such as heavy fermions and high-$T_c$ superconductors. The superfluid $^3$He order parameter $A_{ai}$ is a complex $3 \times 3$ matrix where $a$ and $i$ denote the spin and orbital parts. The spin angular momentum in $^3$He-A1 has been probed in experiments on magnetically driven superflow by Ruel and Kojima. We suggest that the orbital angular momentum of the Cooper pairs in $^3$He-A (and $^3$He-A1) could also be detected.

The Magnus force is well known within hydrodynamics. This effect arises when a classical viscous fluid flows past a rotating body, and a Bernoulli pressure difference is created across the object for points where the fluid flow reinforces or counteracts its circulation; the deflecting force is perpendicular to both the direction of motion and the axis of rotation. In $^3$He-A, the picture is reversed: the condensate itself possesses spontaneous angular momentum, which is experienced by a moving nonrotating object. Here the circulation is an inherent property of the superfluid—thus we suggest to call the ensuing phenomena internal Magnus effects.

In $^3$He-A, there exist Cooper pairs with the spin projections $\uparrow \uparrow$ and $\downarrow \downarrow$; both correspond to the same eigenvalue of the orbital angular momentum, $L_z = 1$. A common direction, $\hat{l}$, can thus be ascribed to the internal circulation; the special nature of this direction is manifested in the energy gap $\Delta(\hat{p}) = \Delta_s(T)\sin\theta \exp(\phi)$, which displays nodes (pointlike vortices on the Fermi sphere) along $\hat{l}$. Here $\theta$ is measured from $\hat{l}$, while $\phi$ is the azimuthal angle; $\Delta_s(T)$ is the maximum gap and $\hat{p}$ is the direction of a quasiparticle momentum on the Fermi surface. The nature of the interaction between the superfluid and the foreign body is determined by the size of the impurity with respect to the coherence length $\xi$ over which the order parameter may vary; $\xi \approx 20 \text{ nm}$ at $T_c$. The various scenarios are illustrated in Fig. 1.

(i) The motion of a tiny particle with radius $R \ll \xi$, such as a negative ion (pressure-dependent radius $R \approx 1\text{-}2 \text{ nm}$), is determined by the dissipative quasipar-

\begin{figure}
(a) intrinsic Magnus effect  \hspace{1cm} (b) orbital currents  \hspace{1cm} (c) $\hat{l}$ texture around a sphere

FIG. 1. Internal Magnus effects on different scales in $^3$He-A: (a) For tiny objects ($R \ll \xi$), there is a quantum-mechanical asymmetry in the $^3$He quasiparticle scattering, i.e., an intrinsic Magnus effect. (b) Small particles ($R \ll \xi$), only slightly perturbing the order parameter, generate orbital currents (Ref. 3) which lead to a hydrodynamical current-induced Magnus effect. (c) For a large object ($R > \xi$) (Ref. 4), the order-parameter texture is essentially redistributed by making $\hat{l}$ (arrows) normal to its surface. A virtual vortex is formed (heavy curve with circular arrows), which experiences a hydrodynamical topological Magnus force.

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cle collisions. When an ion moves in the quantum liquid, an intrinsic Magnus effect arises: The asymmetric differential scattering cross section $d\sigma/d\Omega$ in Fig. 1(a) illustrates the broken reflection symmetry in the plane perpendicular to $\hat{l}$, due to the uneven quasiparticle “pressure” created by the perpetual circulation of the Cooper pairs.

(ii) For a small particle with radius $R < \xi_0$, in Fig. 1(b), introducing only a minor distortion of the order parameter, there also exists a nondissipative hydrodynamic current-induced Magnus effect. In this case, first discussed close to $T_c$ by Rainer and Vuorio, there results a superflow and also a countersuperflow at a distance $r \approx \xi$, such that the total net angular momentum of the liquid surrounding the sphere amounts to zero. However, a net hydrodynamic contribution to the Magnus force from the superflow, $F_m$, remains: $F_m \approx (m^* / h) \sigma N \times \xi N(0) \Delta l^2 v$, where $m^*$ is the effective mass of a quasiparticle, $\sigma N$ is the transport cross section in the normal phase, $N(0)$ is the density of states on the Fermi surface, and $v$ is the perpendicular (to $\hat{l}$) component of the drift velocity for the particle. In the Knudsen limit, the drag force experienced by an impurity, e.g., an ion, is $F_{\text{drag}} = (e / \mu_\perp) v = \rho F \sigma N (\mu N / \mu_\perp) v$. Here $\rho F$ is the $^3$He Fermi momentum, $\rho$ is the mass density, and $e$ is an electron’s charge; $\mu_\perp$ is the isotropic normal-phase mobility. For clarity, we only discuss motion in the plane perpendicular to $\hat{l}$; the corresponding mobility $\mu_\perp$ and the mobility component $\mu_\parallel$ along $\hat{l}$ were calculated in Ref. 5. For a small particle, the ratio between the hydrodynamic Magnus force and the drag force is, for $T \rightarrow T_c$:

$$\frac{F_m}{F_{\text{drag}}} = \left( \frac{\Delta l}{k_B T_c} \right)^2 \frac{k_B T_c}{E_F} \mu_\perp \mu_\parallel (R < \xi_0).$$

(iii) A macroscopic body with $R > \xi_0$, shown in Fig. 1(c), redistributes both the phase and the amplitude of the order parameter. The object aligns the angular momentum vector $\hat{l}$ perpendicular to its surface, which is a topological dilemma: A virtual vortex line emerges. The topological Magnus force exerted on this vortex may be estimated by noting that its circulation is of order one quantum of circulation, $h/m$; hence $F_m \approx \rho \times (h/m) (l \times v) R$, where $\rho$ denotes the superfluid density. We use Stokes’s formula $F_{\text{drag}} = \pi \eta R v$, with $\eta$ the $^3$He shear viscosity, to compare $F_m$ with the drag force in the hydrodynamic regime; we find

$$\frac{F_m}{F_{\text{drag}}} \approx \frac{k_B T_c}{E_F} \left( \frac{\Delta l}{k_B T_c} \right)^2 \mu_\perp \mu_\parallel (R > \xi_0).$$

Therefore, this hydrodynamic Magnus force is negligible close to $T_c$. In the Knudsen regime where $\xi_0 < R$ and, moreover, $R \ll L$ with $L$ the quasiparticle mean free path, of order 1 $\mu$m, we estimate

$$\frac{F_m}{F_{\text{drag}}} \approx \frac{a \rho_s \mu_\parallel}{R \mu_\parallel} \left( \frac{\Delta l}{k_B T_c} \right)^2 \frac{k_B T_c}{E_F} \left( \frac{\Delta l}{k_B T_c} \right) \frac{k_B T_c}{E_F} \left( \xi_0 < R \ll L \right),$$

where $a$ denotes an interatomic distance. Because of the divergence of the mobility, the Magnus and drag forces may become comparable for $T \approx 0.3 T_c$ in the $A$ phase, this would require applied magnetic fields of order 1 T.

The most profound Magnus-type phenomenon is, however, the purely quantum-mechanical effect depicted in Fig. 1(a). To observe the maximum deflection, one must move ions perpendicular to $\hat{l}$ in an external electric field $\hat{E}$. This situation allows for a precise calculation, with no adjustable parameters; it directly measures the microscopic quasiparticle-scattering anomaly from tiny objects.

The nodes in the energy gap lead to the principal anisotropy $-\mu_\parallel vs \mu_\perp$—first measured by Roach, Ketterson, and Roach, for the mobility of negative ions in $^3$He-$A$. Here we discuss an additional anisotropy in the plane perpendicular to $\hat{l}$, due to the nonzero angular momentum of the condensate fraction, manifested in the exp($\phi$) factor of the energy gap. The accurate computation of the matrix $T(p, \hat{p}, \hat{E})$ for a quasiparticle scattering off the ion requires a careful, separate treatment of the initial and final directions of the quasiparticle momentum, $\hat{p}(\theta, \phi)$ and $\hat{p}(\theta', \phi')$, respectively. In the plane perpendicular to $\hat{l} \parallel \hat{z}$, only the relative projection $(\phi' - \phi)$ is relevant. The $\phi$ term in the $A$-phase energy gap, however, breaks the reflection symmetry, making the directions $(\phi' - \phi)$ and $-(\phi' - \phi)$ disparate.

We solve the Lippmann-Schwinger equation $T = V + VGT$ exactly as in Refs. 5 and 10 to find the scattering $T$ matrix. Here $V$ is the scattering potential for a hard sphere and $G$ is the superfluid propagator. $T$ is expressed as a sum of partial waves, whose number is determined by the impact parameter $p_F R$. At $A$-phase pressures of 20, . . . , 29 bars, $p F R = 8.85, . . . , 8.45$, and the inclusion of thirteen partial waves serves as an exact quantum-mechanical solution. Because of the symmetry of the $A$-phase energy gap, it is convenient to express $T$ in terms of states of definite $m$, i.e., as projections of the partial waves on $\hat{l}$. Averaging over the four scattering channels $\pm \hat{p} \leftrightarrow \pm \hat{p}'$, we obtain the effective squared $T$-matrix element $\langle \vec{t}_{\hat{p}} \vec{t}_{\hat{p}'} \rangle^2$. Multiplying this by the momentum transfers $\Delta p_i \Delta p_j$, with $i = x, y, z$, we find that, in addition to the principal anisotropy, there also emerges an anomalous skew-scattering contribution to the momentum-transfer cross section in the plane perpendicular to $\hat{l}$. This term was not considered before.

All the nonvanishing cross sections $\sigma_{lj}$ at quasiparticle
energy \( E \) thus are the following:

\[
\sigma_i(E) = \int d\Omega \int d\Omega' (\cos\theta' - \cos\theta)^2 d\sigma/d\Omega,
\]

\[
\sigma_\perp(E) = \int d\Omega \int d\Omega' \sin^2\theta + \sin^2\theta' - 2 \sin\theta \sin\theta' \cos(\phi' - \phi)] d\sigma/d\Omega,
\]

\[
\sigma_{xy}(E) = \int d\Omega \int d\Omega' [-\sin\theta \sin\theta' \sin(\phi' - \phi) + \sin^2\theta \sin(\phi' - \phi) \cos(\phi' - \phi)] d\sigma/d\Omega,
\]

where

\[
d\sigma/d\Omega = \frac{1}{2} (1/2\pi)^2 (\partial\epsilon/\partial\hat{p})(|t_{\hat{p}\hat{p}'}|^2)(\partial\epsilon/\partial\hat{p}'),
\]

\( \sigma_i = \sigma_{xx} \), and \( \sigma_\perp = \frac{1}{2} (\sigma_{xx} + \sigma_{yy}) \) is an average over the indistinguishable directions \( \hat{x} \) and \( \hat{y} \); \( \sigma_{xx} \) and \( \sigma_{yy} \) equal zero. The dissipative terms \( \sigma_i \) and \( \sigma_\perp \) are discussed in Ref. 4. We shall now consider the consequences of the novel reactive component \( \sigma_{xy} \).

Although the Lippman-Schwinger equation is separable in \( m \), a detailed calculation\(^1\) of \( \sigma_{xy} \) shows important couplings of partial waves \( m-2 \) through \( m+2 \) in the cross section; hence, the intrinsic Magnus effect is a result of intricate interferences between many resonant partial waves; pure \( s \)-wave scattering does not produce the reactive force. The computed \( \sigma_{xy} \) is illustrated in Fig. 2 for \( p = 29 \) bars as a function of the \( ^3\)He quasiparticle energy. The cross section peaks for the resonant states near \( E = \Delta_A \) and disappears for \( E/\Delta_A \to \infty \) where the excitations approach normal quasiparticles, and also for \( E \to 0 \). The scattering resonances for \( E < \Delta_A \) are due to bound states created below the gap edge in the vicinity of the ion. Their width originates from the variation of the energy gap as a function of \( \hat{p} \); it facilitates large momentum transfers for quasiparticles whose energy matches any of these levels; therefore, resonances appear in the cross section.

Each resonant partial wave produces a separate maximum: Eight peaks are formed as one should expect for \( p FR = 8 \). This can be understood to originate from the multiple scattering of the \( ^3\)He quasiparticles. An interesting analogous phenomenon, the Fano effect, appears when there are discrete energy levels in the presence of an equiequilibrium continuum, see Fig. 2(b): A particle in a quasibound energy level may relax into the continuum, thus resulting in resonant interferences for certain energies. We interpret the fine structure and the changes in sign of \( \sigma_{xy} \) at the lowest energies as an indication of Fano-type resonances.

The mobility tensor \( \mu(T) \) is obtained from the cross sections:

\[
e(\mu^{-1})_{ij} = n_3 p_F \int_{-\infty}^{\infty} dE \left( -\frac{\partial f}{\partial E} \right) \sigma_{ij}(E),
\]

where \( n_3 \) is the \( ^3\)He number density and \( f \) is the Fermi function. The drift velocity of an ion, \( v = \mu \mathcal{E} \), now reads

---

FIG. 2. (a) The computed energy dependence of the \( ^3\)He-quasiparticle-negative-ion skew-scattering cross section generating the intrinsic Magnus effect in the plane normal to \( \hat{I} \). Scale is linear for \( E \lessgtr \Delta_A \); for \( E > \Delta_A \) it varies as \( E^{-1} \). This \( \sigma_{xy} \) is a measure of the reactive transfer of Cooper-pair orbital angular momentum to the moving ion; it arises from the asymmetry in Fig. 1(a). (b) Closeup near the node of the gap for the low-energy levels which undergo Fano-type interferences. (c) The magnitude of the intrinsic Magnus effect in the plane \( \Delta \hat{I} \) for negative ions in \( ^3\)He-\( A \). An ion executes transverse motion \( \Delta y \) like in the Hall effect, in addition to that \( \Delta x \) parallel with the applied electric field, \( \mathcal{E} \). The calculated force ratio, \( F_y/F_x \), is shown as a function of \( \Delta x/T \) (lower scale) and \( T/T_c \) (upper scale, weak coupling).
The calculated ratio of the transverse force $F_y (\equiv F_{\text{drag}})$ to the component ($= F_{\text{drag}}$) driving the ion along $\mathcal{E}$ is seen in Fig. 2(a) for a region with uniform $\mathbf{i}$. Note that $F_y/F_x$ is linear in $\Delta_\lambda/k_B T$ for $T \to T_c$. Although $\sigma_{xy}$ in Fig. 2(a) is rather small, of order $10^{-2}\sigma_N$, for motion perpendicular to $\mathbf{i}$, one needs to compare $\sigma_{xy}$ not with $\sigma_N$ but with $\sigma_m$. It is important that Eq. (5) weights the energy-dependent cross sections with the derivative of the Fermi function. Hence, for $T \to 0$, the magnitude of the internal Magnus effect is determined by the ratio of the slopes of $\sigma_{xy}(E)$ and $\sigma_m(E)$. We find that, roughly, $\sigma_{xy} \approx 0.5\sigma_m$ for $E \to 0$ [see Fig. 2(a) and Ref. 5]. This explains why for $T \to 0$ the effect grows as large as 50%.

We may fix the $\mathbf{i}$ texture by applying an external magnetic field $\mathbf{B}$, which serves to orientate the spin anisotropy axis $\mathbf{d} \perp \mathbf{B}$; dipole interaction aligns $\mathbf{i}$ and $\mathbf{d}$. In an experimental situation, the walls also affect $\mathbf{i}$ in their immediate vicinity; the resulting bulk $\mathbf{i}$ texture, however, lies principally in the plane perpendicular to the magnetic field. It follows that the diameter of a cloud of ions moving the distance $\Delta x$, where $\mathbf{i}$ is oriented randomly in the plane perpendicular to the electric field, is widened by $2\Delta \phi$ on reaching the collector. A detectable 5° deflection would be expected already for $T \approx 0.7T_c$, while for $T \approx 0.5T_c$ an expansion of as much as 26° is predicted. It is of interest to extend the ion-mobility techniques to find this manifestation of the internal Cooper-pair angular momentum in superfluid $^3\text{He}-\text{A}$.

While the hydrodynamical Magnus forces measure the asymptotic perturbation, caused by an object in the superfluid, the new quantum-mechanical effect probes the orbital helicity of the condensate at the very site of the impurity. Close to $T_c$, this “core” contribution leads to a Magnus force linear in $\Delta_\lambda/k_B T_c$, seen in Fig. 2(c), thus considerably exceeding the quadratic dependence on $\Delta_\lambda/k_B T_c$ of the hydrodynamical contribution to the Magnus effect, cf. Eqs. (1)–(3).

In conclusion, we note that superfluid $^3\text{He}$ provides a versatile model for exotic pairing states in other condensed-matter systems. The intrinsic Magnus effect seems important for the focusing of ions by continuous vortices in $^3\text{He}-\text{A}$; it is relevant for transport in heavy fermions; in thin films it should also produce a spontaneous internal quantized Hall effect (even in the absence of an external magnetic field). Any unconventional (high-$T_c$) superconductor with Cooper pairing in $L \neq 0$ states and having boojums with noncompensated topological charges on the Fermi sphere should exhibit phenomena akin to the quantum-mechanical asymmetry. Similar effects are generated for $^3\text{He}-\text{B}$ in an applied magnetic field through gap distortion and in the $A$-phase-like core of the $^3\text{He}-\text{B}$ vortices.

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10R. H. Salmelin and M. M. Salomaa (to be published).
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