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Quantum Nucleation of Vortices in the Flow of Superfluid $^4$He through an Orifice

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Flow measurements in ultrapure $^4$He through a micron-size orifice at millikelvin temperatures show, for the first time, the transition from thermal to quantum nucleation of nanometer-size vortices below a crossover temperature of 0.147 K. These observations establish the close relationship between this type of critical flow and negative-ion motion in superfluid $^4$He and strongly suggest that the underlying mechanism, the nucleation of vortex half rings, is identical.

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There is continuing interest in the problem of critical velocities in superfluid $^4$He and the nucleation of quantized vortices [1–5]. From a number of independent studies, it has become apparent that the critical velocity $v_c$ through apertures less than about one micron in size increases in a near-linear fashion as the temperature is reduced below 1 K, following a relation of the form $v_c = v_c(1 - T/T_0)$ with $T_0 = 2.45$ K. Slightly different temperature dependences have been reported recently [6–8], indicating possible variations with the nature of the substrate and the measuring technique. This temperature variation of $v_c$ in a range where the internal variables of the superfluid such as the superfluid density $\rho_s$ or the vortex core radius $a_0$ are independent of temperature is a distinctive feature which we have attributed [1,9] to the nucleation of vortices by thermal activation. The rate of such processes is expressed by the Kramers formula:

$$\Gamma_\alpha = \frac{\omega_0}{2\pi} \exp \left( - \frac{E_a(u_s)}{k_B T} \right).$$

(1a)

The problem here, tackled first in the vicinity of the lambda point by Fordanski and by Langer and Fisher (1LF) [10], is to determine the activation energy $E_a$ in terms of the superfluid velocity at the nucleation site $u_s$, and the attempt frequency $\omega_0/2\pi$.

As the energies involved are small, the nucleated vortices are of very small size, of the order of a few nm. It can then be expected that, below some crossover temperature $T_q$, thermal nucleation will give way to quantum tunneling, in which case the rate becomes temperature independent and is given by [11]

$$\Gamma_q = \frac{\omega_0}{2\pi} \left( \frac{864 \pi E_a}{\hbar \omega_0} \right)^{1/2} \exp \left( \frac{-36}{5} \frac{E_a}{\hbar \omega_0} \left[ 1 + \frac{45 \zeta(3)}{2\pi^3 \omega_0 T} \right] \right).$$

(1b)

$\omega_0$ being related, within small corrections in the case of low-damping systems ($\omega_0 T \ll 1$), to the thermal attempt frequency [11]. The crossover temperature $T_q$ between the thermal and the quantum regimes is quite generally related to the attempt frequency $\omega_0$ [11]:

$$k_B T_q = \hbar \omega_0/2\pi.$$

(2)

The existence of such thermal and quantum nucleation phenomena has been established both theoretically [12] and experimentally [13] for negative ions moving at high speed (~55 m/s) through superfluid $^4$He. We report here the first observation of the saturation of the critical velocity $v_c$ as $T$ is lowered below 150 mK in ultrapure $^4$He at zero pressure. We interpret this effect as evidence for vortex nucleation by quantum tunneling, bringing significant and novel information on this problem, in particular on $\omega_0$ which is otherwise an elusive quantity to determine.

These new experimental results were obtained using the same double-hole hydrodynamic resonator, operated at a frequency $\omega/2\pi$ of 12.5 Hz, with the same microrifice and procedures as those described in Ref. [9], but with two significant improvements which may explain why the saturation of $v_c$ went unnoticed in our earlier work [14]. First, the entire experimental setup has been moved to new, more massive and rigid surroundings with improved vibration damping and sound proofing to achieve a very high degree of decoupling from external mechanical disturbances [15]. Second, refined signal filtering and processing techniques have been implemented which permit a full analysis of the cell operation, including a drastic rejection of spurious signals and a determination of both the trapped bias current and of the initial state of circulation of the loop between the two holes of the resonator [15,16].

The $^4$He samples used in these experiments were purified in the laboratory using both conventional distillation and the heat flush effect [17] to a concentration $x_1$ of $^3$He impurities of 0.9 parts per 10$^3$ (ppb) measured [18] on a gaseous sample collected after completion of the experiments. Typical results for $v_c(T)$ from ~15 mK up to the highest temperature at which our superconducting displacement sensor operates are shown in Fig. 1. The plateau in $v_c$, at low temperature is the characteristic and reproducible behavior of ultrapure $^4$He in this cell, as is the constant slope decrease at high temperature, $v_c$, extrapolating to zero at $T_0 = 2.45$ K as in all our previous
measurements. The inset in Fig. 1 displays the effect of
~ 7 ppb of $^3$He impurities [19]. The mean critical veloc-
ity and the width of its statistical distribution (see below)
for the highest purity sample ($x_1 = 0.9$ ppb) are shown in
Fig. 2. It is experimentally established that the plateau
is due neither to the lack of thermal equilibrium nor to res-
idual impurities.

It is more difficult to rule out the effect of some extrin-
sic apparatus vibration. All our efforts to permanently
alter the plateau level or the crossover temperature by
changing the experimental conditions have failed: We be-
lieve that we are observing a phenomenon intrinsic to the
superfluid and that the fluctuations taking over below $T_a$
must be of a quantum nature. This point of view is fur-
ther promoted by the consideration of the following vor-
tex nucleation model which describes the transition from
the thermal to the quantum nucleation regimes.

Along the lines of our previous attempts [1, 9] and fol-
owing the ideas of Muirhead, Vinen, and Donnelly [12],
we consider the nucleation of a vortex half ring perpen-
dicular to a smooth plane wall, with an axis opposite to
the local superfluid velocity $u_s$, that is, we simply extend
to wall nucleation the ILF theory [10] in which the vortex
“free” energy is written as

$$\mathcal{E}_v = \mathcal{E}_0 - \mathcal{P}_0 v.$$  \hfill (3)

The half-ring energy and impulse are given in terms of its
radius $r$ by $\mathcal{E}_0 = r \eta/4$ and $\mathcal{P}_0 = r^2/4$ when we use dimen-
sionless units such that lengths are scaled by the core pa-
rameter $b$, energies by $E^* = \rho_s \kappa^2 b$, and velocities by
$v^* = \kappa / 2 \pi b$ so that $v = u_s / v^*$. The core parameter $b$
is related to the core radius $a_0$, for a hollow-core classical
ring [20], by $b = e^2 a_0 / 8$ so that the quantity $\eta$ is $\ln r$.
These classical hydrodynamic expressions are valid for
bulk vortices and for $r \gg 1$. The deviations as $r \to 1$
have been studied by Jones and Roberts [21] who find, as
confirmed by Dalfovo [4], that $\mathcal{E}_0$ and $\mathcal{P}_0$ do not decrease
with $r$ as rapidly as for the classical ring and that vortex
rings in the bulk cease to exist altogether for $r < 1$. More
directly relevant to our problem is the work of Sonin [22]
on the behavior of vortex filaments close to walls, who
reached the conclusion that their energy per unit length
goes to zero at the wall as the square of the distance from
the wall with a certain characteristic length. This dy-
amic healing length, also considered in Ref. [23], is not
well defined in the presence of vortices and is comparable
to and probably larger than the static healing length com-
puted by Hills and Roberts [24]. Thus, for computa-
tional convenience as well as to at least in part take into ac-
count the results of Refs. [21, 22] at $r \geq 1$, we have put in
the following: $\eta = \frac{1}{2} \ln (1 + r^2)$. This regularization
artifice has the important physical consequence that the
vortex self-velocity $\partial \mathcal{E}_0 / \partial \mathcal{P}_0$ goes through a maximum
$v_{1,0} = 0.432$ at $r_{m} = 1.26$ and decreases to zero as $r$. A sur-
facer vorticity sheet in a metastable state builds up at the
wall and constitutes the system which either tunnels
through or hops over the energy barrier [25]. None of
the complexities of the interplay between the imposed
superflow, the surface vorticity, and the nucleating vortex
are included in Eq. (3). We shall adjust $b$ phenom-
ena logically to approximate reality, and hence, we convey
to it a physical significance which clearly differs from that
of the core parameter and which is more like the length
of the vortex.

Vortex “free” energies given by Eq. (3) are shown for
several velocities in the inset of Fig. 3. The activation en-
ergy $\Delta_e (\nu)$ is the difference, when $\nu \leq v_{1,0}$, between the
values of the minimum and the maximum of \( \mathcal{E}_c(r) \). It must be evaluated numerically except for \( \epsilon = (1 - v^2 / v_0^2) \approx 1 \). The dotted curve is the outcome of the present model for half-ring nucleation when using the same approximations as ILF [10]. The inset shows the vortex reduced “free” energy given by Eq. (3) in terms of the reduced half-ring radius \( r \) with \( \nu \) equal to 0.308, 0.320, 0.347, 0.385, and 0.432. These values of \( \nu \) are taken at the temperatures of 2.17, 1.05, 0.147, and 0 K, respectively, in the (classical) case of the dotted-dashed curve.

\[ \mathcal{E}_c = \frac{2}{3} \mathcal{E}_f N, \quad (4) \]

with \( \frac{2}{3} \mathcal{E}_f = \frac{1}{2} v_0 r_m N^{1/2} (r_m^2 + 1)^{1/2} (3r_m^2 - 1)^{-1/2} \). Equation (4) holds in a quite general way about the point where the system ceases to be metastable and “runs away” spontaneously. This situation has been studied extensively [11] in the context of superconducting junctions and macroscopic quantum tunneling, in which case \( \mathcal{E}_f \) is the “Josephson-junction” energy. We can thus use the results obtained in this closely related field to solve the vortex nucleation problem. Then, the nucleation rate \( \Gamma \) at all temperatures between the two limits of Eqs. (1a) and (1b) can be obtained using the work of Grabert, Olchowski, and Weiss [26].

Critical velocities are computed as in Refs. [1,10] by assuming that, when \( \Gamma \) exceeds some rate \( \Gamma_{\text{obs}} \) which depends on the method of observation, critical events are being observed and the critical velocity \( v_c \) is reached:

\[ \mathcal{E}_c(v_c) = y k_B T / E^*, \quad (5) \]

with \( y = \ln \Gamma / \Gamma_{\text{obs}} \). For energy barriers of the form (4), \( y \) can be computed explicitly, and to logarithmic accuracy, is equal to \( \ln (0.1 \omega_0 / \omega) \). The reduced critical velocity \( v_c \) is obtained by solving numerically Eq. (5), except when expansion (4) is valid, in which case it can be expressed analytically as

\[ v_c = v_0 \left[ 1 - \frac{3}{2} \left( \frac{k_B T}{E^* \mathcal{E}_f} \right)^{2/3} \right]^{1/2}. \quad (6) \]

The temperature dependence of \( v_c \), shown in Fig. 3, comes in an explicit way from Eq. (5) but also from the quantum corrections [26], and from the implicit dependence of \( r \) and \( \mathcal{E}_f \), taken from Refs. [27,28], respectively. In the numerical calculations, we have adjusted \( E^*(0) \) in such a way that the temperature variation of \( v_c \) between 0.2 and 0.5 K is a near-straight line which intersects the temperature axis at \( T_0 = 2.45 \) K, yielding \( E^*(0) = 98.7 \) K. This value is also used for the computed curve for \( v_c \) in Fig. 2. The lower curve in Fig. 3, whose overall slope between 0.4 and 1.8 K also leads to \( T_0 = 2.45 \) K, is obtained for \( E^*(0) = 493 \) K.

The statistical width of the critical transition is defined as in Ref. [1] and, when expansion (4) holds, is expressed by

\[ \Delta v_c = v_0 \frac{2}{\ln 2} \frac{x(1 - x^2)}{(x^2 + 3) x^2 - 1}, \quad (7) \]

with \( x = v / v_c \). Equation (7) depends only on the critical velocity and on \( \gamma \). As shown in Fig. 2, it yields a fair description of the measured width: The same value of \( \gamma \) accounts for both \( T_q \) and the temperature variation of the width. This constitutes a check on the consistency of our interpretation.

The value of \( b \) at \( T = 0 \) for the top (bottom) curve in Fig. 3 is \( b_0 = 9.44 \) (4.72) A. The corresponding value of \( v^* \) is 16.8 (33.6) m/s and that of the critical velocity on the plateau is 6.5 (12.2) m/s. As discussed in Refs. [1,5], local velocities \( u_s \) are, due to geometrical effects, enhanced with respect to the average velocities \( v_c \) through the orifice and can be expected to reach 10 to possibly 40 m/s. We have at present no direct measurement of \( u_s \) which would pinpoint the value of \( b \) and permit further refinement of the model.

Surface defects of a size comparable to that of the nucleated vortex can be expected to alter the value of \( b \) and thus introduce a measure of variance with experimental situations, as would the change of the van der Waals attraction from substrate to substrate.

In the case of nucleation on negative ions considered in Refs. [12,13], the value of \( b_0 \) was found to be \( -1 \) A at 15 bars [13]: Dynamic healing effects are small on ions. The present analysis yields \( v_0 = 50 \) m/s in agreement with the experimental ion velocity of 55 m/s. The value of \( T_q = 147 \) mK corresponds, by Eq. (2), to \( \omega_0 / 2 \pi = 1.9 \times 10^{10} \) Hz which is of the same magnitude as the vortex “cyclotron” frequency considered by Muirhead, Vinen, and Donnelly [12].

To conclude, we note that, although our model is rudiment-
mentary, the range of values which we find for $b_0$ and $v_{c0}$ certainly falls within expectations and that the qualitative features of the data are well reproduced. This leaves little doubt that we are indeed dealing here with the thermal and quantum nucleation of nanometer-size vortex rings, as is already known to be the case in ion propagation [12,13].

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[4] F. Dalfovo (to be published); (private communication).
[14] Note, however, the top curve in Fig. 1 of Ref. [9] which was, probably incorrectly, attributed to residual $^3$He impurities.
[19] A full discussion of the effect of $^3$He on vortex nucleation is deferred to a forthcoming publication.
[26] H. Grabert, P. Olschowski, and U. Weiss, Phys. Rev. B 36, 1931 (1987). As the plateau of $v_{c}(T)$ is quite flat and the transition region about $T_{x}$ very narrow, the system is weakly damped. Also, the series expansion leading to Eq. (4) is valid only very close to $v_{o}$. In the numerical evaluation of the quantum regime, we have used Eq. (4) with $\bar{\xi}=1.35\xi_{j}$ and $\bar{v}_{o}=0.97v_{c0}$ which yields a more accurate numerical representation for the range of values of $\nu$ spanned when the temperature varies from 0.125 to 0.5 K.