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Reply to the comment of Makse and Amaral

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Maunuksela *et al.* Reply: The Comment of Amaral and Makse (AM) [1] addresses an interesting question regarding the role of quenched noise in the paper burning experiments reported in our recent Letter [2]. The early experimental work by Zhang *et al.* [3] indicated that scaling of slow combustion fronts in paper is dominated by directed percolation depinning (DPD) effects. The scaling exponents of the DPD model are $\chi = \beta \approx 0.633$ in the pinned and $\chi \approx \beta \approx 0.75$ in the moving phase near the percolation threshold, respectively [4]. However, by carefully controlling the experimental conditions we were able to show [2] that asymptotically, i.e., for not very short length and time scales, exponents belonging to the Kardar-Parisi-Zhang (KPZ) universality class [5] ($\chi = 0.5$, $\beta = 0.33$) could be obtained. The Comment does not criticize this main conclusion.

Instead, AM claim that our data can be explained by the simple DPD model near the percolation threshold. This is based on the observation that before KPZ asymptotics, the *short-range* (SR) behavior of the interface roughness (as measured through the saturated correlation function $G(r) \sim r^{2\chi}$) has a higher effective exponent that may not be incompatible with DPD. This may indeed not be an unreasonable suggestion but one should also notice that, using the other saturated correlation function $C_s(t) \sim t^{2\beta}$, we estimated [2] the short-time exponent to be in the range $\beta_{SR} \approx 0.40$ – 0.46 , which *did not* agree with DPD close to the threshold. In our Letter we specifically refrained from drawing firm conclusions about the SR behavior at the interface. The resolution of the data was not sufficiently good (as is evident also from Fig. 1 of the Comment), nor could we exclude many other factors that may strongly affect SR roughness (see [2] for discussion).

More recently, we have modified the experimental setup so that the SR behavior can be studied with enhanced spatial resolution. Detailed data analysis will be presented elsewhere [6]. We now observe better SR scaling for $G(r)$ with r smaller than about 3 mm, and estimate that $\chi_{SR} = 0.86(2)$ by averaging over 60 independent burns. Choosing twenty of the burns that show the best scaling, we find $\chi_{SR} = 0.87(2)$. From the data it is clear that χ_{SR} is definitely bigger than 0.75; the whole range of variations for the 20 best burns is 0.82–0.89. Furthermore, subtracting the intrinsic width (convolution ansatz [7]) gives $\chi_{SR} = 0.88(2)$ demonstrating the robustness of the data.

The new data for $C_s(t)$ with 1 s sampling time (the propagation speed of the fronts was approximately 0.5 mm/s) also gives improved SR scaling. Fitting up to about 10 s gives $\beta_{SR} = 0.71(2)$ for six burns. However, after subtracting the intrinsic width the SR data do not scale very well any more, and $\beta_{SR} \approx 0.65$ – 0.7 . It remains to be seen if further increase in the sampling frequency will improve the SR scaling.

In our simulations [8] of a simple DPD lattice model [9] this combination of effective exponents could not be re-

produced. As in earlier similar model calculations [4], in the moving phase close to the directed percolation threshold, both χ_{SR} and β_{SR} were ≈ 0.73 – 0.75 when measured from their respective saturated correlation functions.

To summarize, for the SR scaling of the interfaces we find a value of χ_{SR} that seems to be clearly larger than that of the DPD models, while β_{SR} is not inconsistent with the (effective) DPD value. Therefore, simple DPD models alone do not seem to explain the SR behavior of our experimental data. We are currently trying to probe a different regime where effects arising from an underlying percolation transition would be dominant.

J. Maunuksela, M. Myllys, and J. Timonen
Department of Physics, P.O. Box 35
University of Jyväskylä
FIN-40351 Jyväskylä, Finland

M. Kuittu¹ and T. Ala-Nissila^{1,2,*}
¹Helsinki Institute of Physics, P.O. Box 9
FIN-00014 University of Helsinki
Helsinki, Finland
²Department of Physics, Brown University
Providence, Rhode Island 02912

M. J. Alava
NORDITA, Blegdamsvej 17
DK-2100 Copenhagen, Denmark
and Helsinki University of Technology
Laboratory of Physics, P.O. Box 1100
FIN-02015 HUT, Espoo, Finland

N. Provatas
Department of Physics, Loomis Laboratory of Physics
University of Illinois at Urbana-Champaign
1110 West Green Street
Urbana, Illinois 61801-3080

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*Permanent address: Helsinki University of Technology,
Laboratory of Physics, P.O. Box 1100, FIN-02015 HUT,
Espoo, Finland.

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