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Anisotropic fracture behavior of the 3rd generation advanced high-strength – Quenching and Partitioning steels: Experiments and simulation

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ABSTRACT

Advanced high-strength steels (AHSS) have revolutionized the automotive industry by reducing weight without compromising crashworthiness. The new third-generation steels, such as quenching and partitioning (QP) steels, offer exceptional strength and ductility. However, despite the extensive strength-ductility studies, there is a wide knowledge gap in the literature on the fracture behavior of QP steels under a large range of stress states and loading conditions for material forming operations. This study aims to systematically investigate the fracture behavior of QP1000 sheet metal through a combination of experimental and numerical approaches. In addition to the classic fracture dependency on stress states, we particularly focus on the anisotropic behavior in terms of both plasticity and fracture. Mechanical tests with digital image correlation are performed along three loading directions covering stress states from simple shear to plane-strain tension. The evolving non-associated Hill48 (enHill48) model is employed to describe anisotropic plasticity, while the fracture behavior is represented by a partially coupled anisotropic fracture model and a fully anisotropic fracture model. It is concluded that the investigated QP steel shows moderate anisotropic plasticity behavior yet strong fracture anisotropy, which intensifies with the increase of stress triaxiality. The partially coupled anisotropic fracture model, which has shown success for materials with minor anisotropic plasticity, fails to describe the anisotropic fracture and a fully anisotropic model provides significantly improved predictive capability.

1. Introduction

Advanced high-strength steels (AHSS) have been developed in the automotive industries to effectively improve fuel economy by reducing the weight without compromising the crashworthiness of the structure [1]. The strength level of the first-generation (1st G) AHSS has been elevated beyond 800 MPa from traditional high-strength steels. However, excellent strength usually comes with the sacrifice of ductility. Such a dilemma has been solved with the significantly promoted third-generation (3rd G) steels [2]. One typical example is the quenching and partitioning (QP) steels, offering a superior combination of strength and ductility due to their high amount of tempered martensite and the phase transformation-induced plasticity [3–10].

Among the widely adopted grades, QP1000 has attracted the most

attention due to its high relevance in automotive applications [11]. With similar strength to dual-phase or complex-phase steels currently used in the market but more than doubled elongation (25%–30%), it shows an obvious advantage in cold forming processes and crashworthiness properties. However, tensile properties cannot represent the complete required performance in terms of manufacturing particularly when it involves local deformation, e.g., shearing, bending, edge forming, etc. Therefore, numerous experimental tests have been developed and conducted in the past decades to characterize the local fracture behavior of AHSS, e.g., bendability [12], edge formability [13], and general ductile fracture [14] under various stress states. However, for the 3rd G steels, especially for QP steels, while a large amount of research effort has been evident in the design of the heat treatment processes [15] and understanding of the correlation between microstructure and tensile

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Table 1
Chemical composition of QP1000 (mass content in %).

C	Si	Mn	Al	P	S	Ti + Nb
0.15	1.67	2.42	0.074	<0.002	<0.002	0.011

properties [16], the studies focusing on the local fracture behavior are extremely limited, which hinders accelerating the use of these high-performance steels in industry. Therefore, a thorough characterization of the fracture behavior of this steel grade under a wide range of stress states that represent all possible forming scenarios for sheet metals is urgently needed.

Based on extensive studies focusing on this topic of the 1st G AHSS and many other engineering alloys, the fracture behavior of these high-strength steels shows distinct and diverse dependency on the stress states, characterized by stress triaxiality and Lode angle [12,17–23]. Correspondingly, the coupled (the damage-induced softening is considered in the flow behavior), e.g., the Gurson-type models [24–33], uncoupled (damage is not coupled with flow behavior), e.g., the Modified Mohr-Coulomb (MMC) model [21], and the Hosford-Coulomb model [19] and hybrid (switchable between coupled and uncoupled) models, e.g., the hybrid damage mechanics (HDM) model [34], have been developed to account for the dependency of ductile fracture on stress states [35]. For a more comprehensive review of the model development, readers are referred to Pineau et al. [36] oriented on more materials side, and Tekkaya et al. [37] for material forming related applications. In the context of the 3rd Gen steels, only recently, Wang et al. [38] made one of the first attempts to investigate the ductile fracture behavior of a QP980 steel sheet in a wide range of stress states. In the study, several phenomenological uncoupled models, including the Brozzo criterion [39], the Oh criterion [40], the Rice-Tracey criterion [41], and the DF2012 [42] were calibrated and used to describe the fracture behavior of QP980. However, material forming involves many more factors than stress states, such as temperature, strain rates, loading directions and paths. Among them, the anisotropic fracture has been pointed out as a dominant factor in deciding both global formability in for example Nakajima tests [43–46] and edge formability under uniform out-of-plane loading, such as hole expansion and flanging [35].

While the anisotropic fracture behavior of QP steels has received little attention in the current literature, several recent studies showed that this grade of steel possesses significant plasticity anisotropy. Chen et al. [47] showed that QP980 possessed pronounced plastic anisotropy and anisotropic hardening and proposed an analytical yield function under the non-associated flow rule to capture the anisotropic hardening behavior. Hou et al. [48] further discovered that QP980 and later QP1180 [49] also show noticeable tension-compression asymmetry due to the stress-state-dependent austenite-to-martensite phase transformation [50]. The same observation was confirmed by Zhang and Lou [51]. Under non-proportional loading conditions, Hou et al. [52] further showed that QP steels also demonstrate pronounced strain-dependent Bauschinger effect and permanent softening. With these very recent findings of QP steels demonstrating strong deformation-relevant anisotropy, it is urgently needed to investigate the anisotropic fracture behavior of the QP steels in a wide range of stress states that various sheet metal forming processes encounter.

This study, therefore, aims to investigate the anisotropic fracture behavior of a QP1000 sheet metal covering the stress states from simple shear, uniaxial tension, to plane strain tension via experiments conducted along the rolling, transverse, and diagonal directions. A hybrid experimental and numerical procedure will be employed to quantify the anisotropic fracture behavior. The anisotropic hardening and evolution of the plastic potential will be described by the evolving non-associated Hill 48 (enHill48) model [53]. The anisotropic fracture behavior will be modeled by the partially coupled anisotropic fracture model as well as a full anisotropic fracture model based on linear tensor transformation. The performance of both formulations will be comprehensively analyzed

and compared. Guidance will be provided based on the quantitative data to be employed for the prediction of the anisotropic fracture behavior of QP steels in terms of both accuracy and effort in calibration procedures.

2. Material and experiments

2.1. Material and experimental program

An as-received high-strength QP1000 steel with a measured thickness of about 1.4 mm was studied in this work. The chemical composition of the material is summarized in Table 1.

The mechanical testing program is designed to comprehensively investigate the anisotropic effects on plasticity and ductile fracture, using different specimen geometries to cover the stress states relevant to sheet metal forming, as shown in Fig. 1 (a). The anisotropic plastic flow properties were experimentally characterized by performing uniaxial tensile tests at room temperature (RT, 25 °C) under a quasi-static (QS, 0.0002 s⁻¹) strain rate, using smooth dog-bone (SDB) specimens cut along the rolling direction (RD), diagonal direction (DD), and transverse direction (TD). The equal-biaxial flow behavior was characterized by cruciform tests [48] at a small strain level (true strain <0.1), and hydraulic bulge tests at large strains. The 230 mm × 230 mm bulge specimens were clamped by a blank holder with a diameter of 105 mm together with a 123 mm-diameter die and pressed under the velocity of 3 mm/min with a clamping force of 600 kN.

The anisotropic fracture properties, from shear to plane-strain tension, were investigated by applying remote uniaxial tension along RD, DD and TD to five types of specimens with featured shapes, including one shear (SH) specimen in optimized shape, one central-hole (CH) with a diameter of 6 mm (CHD6), and three notched dog-bone (NDB) specimens with radii of 25 mm (NDBR25), 10 mm (NDBR10) and 6 mm (NDBR6). The schematic drawing of the stress states covered by these fracture specimens is illustrated in Fig. 1 (b). The geometrical features and dimensions are adopted from our previous geometry design on a dual-phase (DP) steel at the same strength level (DP1000). The detailed optimization approach can be referred to study [54]. The sketches of all the fracture specimens are illustrated in Fig. 1 (c). The specimens were cut from electrical discharge machining (EDM) with a relatively high surface quality. The overall QS condition in the local deformation region was achieved by applying a crosshead velocity of 0.18 mm/min in all fracture tests. Three parallel tests were conducted for each geometry.

A stereo digital image correlation (DIC) system was applied to measure the strain fields for fracture tests using two cameras, with a detecting resolution of 50 pixels/mm and a frame rate of 10 Hz. The image size is 45 mm in width and 82 mm in height. The surface of the specimen was lacquered with an even layer of white paint and then sprayed to form speckle patterns with a speckle diameter of 5–10 pixels. A parallel filter was placed in front of the spotlight to minimize the reflections from the material surface and achieve optimal lighting conditions. The DIC setup of the hydraulic bulge test can be referred to Ref. [55].

2.2. Experimental results

The engineering and true stress–strain curves along RD, DD, and TD from uniaxial tensile tests are shown in Fig. 2 (a) and (b), respectively. A very good repeatability is obtained from the repeat tests. The anisotropic uniaxial tensile properties, including yield strength (YS), ultimate tensile strength (UTS), uniform elongation (UE), and total elongation (TE), are shown in Fig. 2 (c). The results reveal that QP1000 exhibits existing anisotropy in the strength values, while the ductility properties show moderate dependence on the loading direction. The characterized biaxial flow curve of the material obtained from the bulge test is shown in Fig. 2 (b), which is used for calibrating the parameters in constitutive models. Lankford coefficients (*r*-values) in all three loading directions are smaller than one and keep evolving during deformation, as shown in

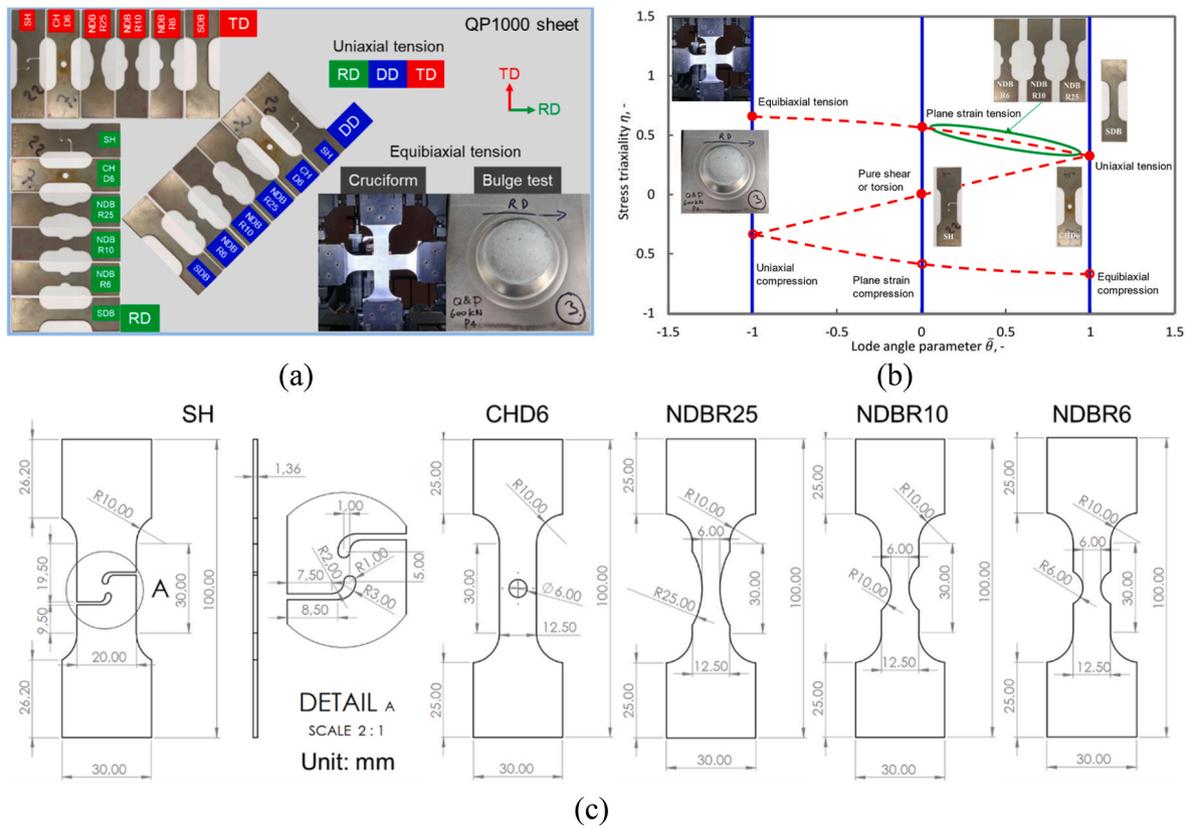


Fig. 1. Overview of the experimental program on (a) material characterization tests, (b) the schematic drawing illustrating the range of stress triaxiality and Lode angle parameters covered from the tests, and (c) the sketches of fracture specimen geometries with dimensions.

Fig. 2 (d). The detailed uniaxial tensile properties of three directions after averaging over repeating tests are summarized in [Table 2](#).

For the fracture anisotropy, the logarithmic axial strain contours of all specimens were obtained with stereo-DIC. As demonstrated with CHD6 (the last frame right before fracture) in [Fig. 3 \(a\)](#), the fracture strain values show a clear dependence on the loading direction, which is the highest along DD and the lowest along TD. The characteristic force and displacement (corresponding to a gauge length of 30 mm) curves are plotted in [Fig. 4 \(a\) to \(d\)](#), where the last point is identified as the fracture moment. The anisotropic fracture behavior is significant in the current material, as fracture force and displacement vary with the loading directions. The force and displacement curves of RD and DD almost overlap until the fracture point, while there is a noticeable difference in the values of fracture displacement and force. A general trend noticed for all geometries is that the specimens loaded along RD have the lowest fracture force and the longest fracture displacement. In contrast, the specimens loaded along TD have the highest fracture force and the shortest fracture displacement. It is observed that the variation in fracture displacement and force with loading direction is also affected by the stress states, i.e., specimen geometries. As shown in [Fig. 4 \(e\)–\(f\)](#), the normalized fracture displacement and force (where results of RD are taken as reference) show dependency on the stress states. In particular, the anisotropic effects on the fracture force and displacement values are intensified with increasing stress triaxiality from SH to NDBR6.

Furthermore, the anisotropic fracture behavior can also be observed in local strains. The first measure is the logarithmic axial strain ϵ_{yy} at a local maximum pixel on the surface of the specimens from DIC measurement at the last frame right before fracture. This gives information about local surface strains. The second strain measure relies on thickness reduction. The initial thickness t_0 of all the specimens before testing was first measured with a caliper. After the tests, the fracture thickness t_f was measured at the thinnest position of the fractured specimens, as shown

in [Fig. 3 \(b\)](#). The true thickness strain at fracture ϵ_{3f} is then calculated with $\epsilon_{3f} = \ln(t_0/t_f)$ as discussed in Ref. [56]. Both measures, the logarithmic axial strain ϵ_{yy} from DIC and the fracture strain measured based on the true thickness reduction at fracture ϵ_{3f} are plotted in [Fig. 4 \(g\)](#) and [\(h\)](#), respectively. Both strains show clear anisotropic behavior with the same tendency as the normalized fracture displacement. In particular, the anisotropic extent of the axial strain at fracture also increases with higher stress triaxiality.

3. Constitutive models

The precise description of plastic flow behavior is an important prerequisite in the successful prediction of stress-state-dependent fracture properties of anisotropic materials. Depending on how the anisotropy effects are coupled in the failure criterion, anisotropic fracture models can be classified as partially [57] and fully anisotropic [58] models. For partially coupled anisotropic models, only the influence from anisotropic plasticity is taken into account, meaning the fracture criterion is isotropic, which showed success for both aluminum alloys [59] and pipeline steels [60]. For fully anisotropic fracture models, the influence of loading direction on the fracture strain [61] or stress [62] is captured by formulating a unified fracture criterion together with an anisotropic yield criterion.

In partially coupled anisotropic fracture models, the orientation-dependent fracture properties are mainly attributed to the anisotropic plastic flow during deformation. Based on this assumption, the failure criterion parameters are typically calibrated independently for one loading direction, and their applicability to other loading directions needs to be verified. The fully anisotropic fracture models are devoted to formulating a unified criterion that can be applied to arbitrary loading directions, which typically requires the experimental results of fracture tests along three loading directions. Both partially and fully anisotropic

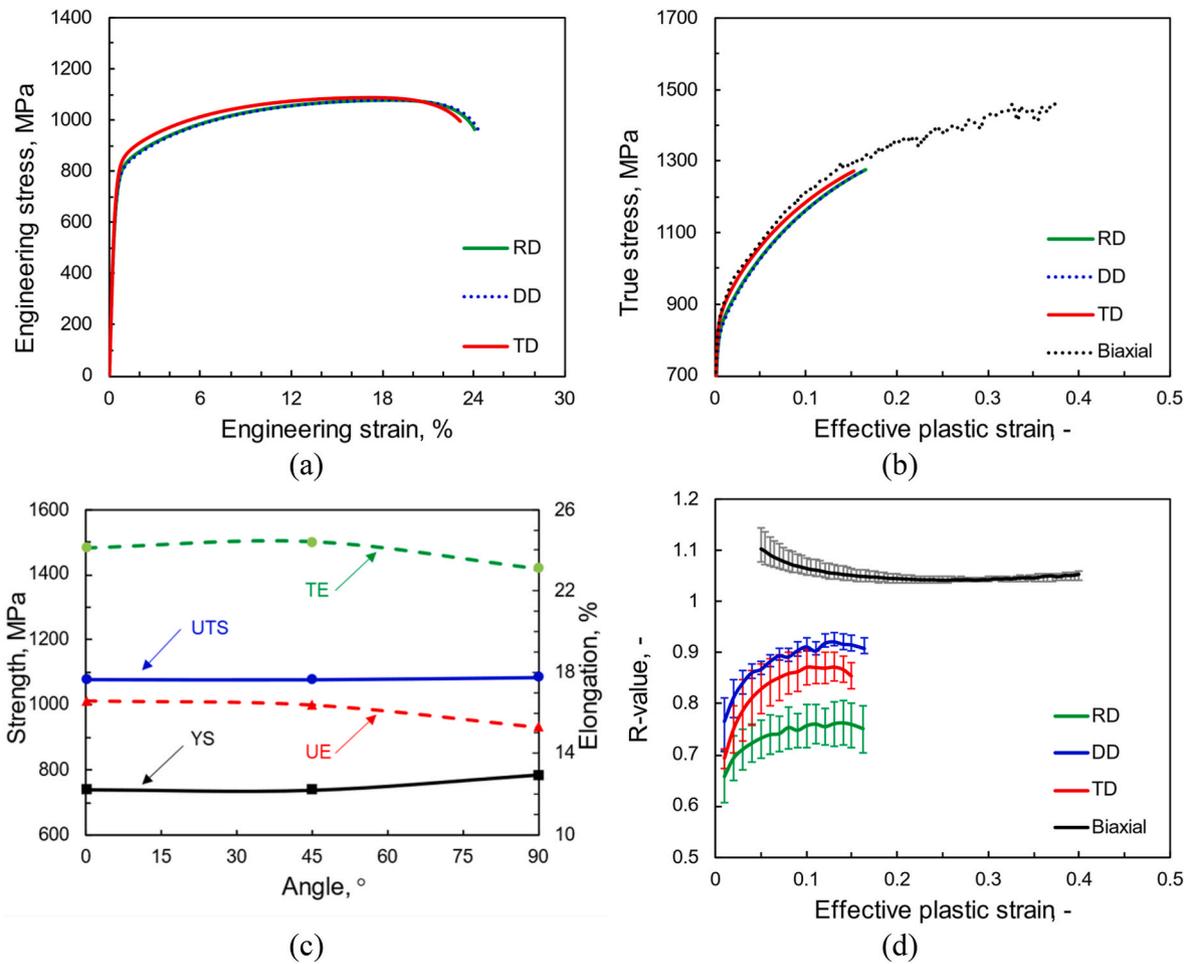


Fig. 2. The characteristic tensile properties of QP1000 along three loading directions, (a) engineering stress–strain curves of uniaxial tension, (b) true stress–strain curves of uniaxial and biaxial tension (c) distribution of tensile properties, and (d) averaged r-value curves of uniaxial tension and biaxial tension with scatters.

Table 2
Summary of uniaxial tensile properties at room temperature for QP1000.

Directions	YS, MPa	UTS, MPa	UE, %	TE, %	r-value at UE, -
RD	739.6	1077.4	16.6	24.1	0.75
DD	738.0	1076.7	16.4	24.4	0.91
TD	783.1	1085.3	15.3	23.1	0.86

approaches have been used to investigate the fracture behavior of QP1000 steel in this study.

3.1. Partially coupled anisotropic fracture model

A partially coupled anisotropic fracture model has been proposed by Shen et al. [63] to describe the fracture properties of high-strength pipeline steels. The partially coupled anisotropic fracture model is formulated by integrating the enHill48 plasticity model [53] into the

hybrid damage mechanics framework proposed by Lian et al. [34], which can be flexibly applied for either damage-dominant fracture in modern advanced high-strength steels [64] or damage-negligible fracture [65,66]. By adopting the non-associated flow rule and considering the evolution of anisotropic stress and r-value, it has been shown that the enHill48 model provides improved accuracy in the final prediction of localization [67] and ductile fracture [63] of different steels.

The yield criterion f and flow potential g are separately defined in the partially coupled anisotropic fracture model, where a scalar parameter D is adopted to quantify the damage-induced softening after reaching the damage initiation criterion.

$$f = \bar{\sigma}_\sigma(\sigma) - (1 - D) \cdot \sigma_Y(\bar{\epsilon}^p) \leq 0 \tag{1}$$

$$g = \bar{\sigma}_r(\sigma) - (1 - D) \cdot \sigma_Y(\bar{\epsilon}^p) \leq 0 \tag{2}$$

where the quadratic Hill48 equivalent stresses $\bar{\sigma}_\sigma(\sigma)$, $\bar{\sigma}_r(\sigma)$ are calculated as below:

$$\bar{\sigma}_\sigma(\sigma) = \left\{ \frac{1}{2} \left[F_\sigma(\sigma_{22} - \sigma_{33})^2 + G_\sigma(\sigma_{33} - \sigma_{11})^2 + H_\sigma(\sigma_{11} - \sigma_{22})^2 \right] + L_\sigma\sigma_{23}^2 + M_\sigma\sigma_{13}^2 + N_\sigma\sigma_{12}^2 \right\}^{\frac{1}{2}} \tag{3}$$

$$\bar{\sigma}_r(\boldsymbol{\sigma}) = \left\{ \frac{1}{2} \left[F_r(\sigma_{22} - \sigma_{33})^2 + G_r(\sigma_{33} - \sigma_{11})^2 + H_r(\sigma_{11} - \sigma_{22})^2 \right] + L_r\sigma_{23}^2 + M_r\sigma_{13}^2 + N_r\sigma_{12}^2 \right\}^{\frac{1}{2}} \quad (4)$$

The onset of plastic deformation is described by the condition when $D = 0$, $f = 0$, and $g = 0$. The evolution of anisotropy is captured by considering the change of anisotropic parameters with the equivalent plastic strain $\bar{\epsilon}^p$. In the yield function, the anisotropic parameters ($F_\sigma, G_\sigma, H_\sigma, N_\sigma$) are calculated according to stresses determined from uniaxial tension (σ_0, σ_{45} and σ_{90}) and equibiaxial tension σ_b . The anisotropic parameters (F_r, G_r, H_r, N_r) in the flow potential are determined by the r-values. The parameters L and M in both yield function and flow potential are set as 3, assuming only in-plane anisotropy according to Aretz [68]. The equations for determining the anisotropic parameters in yield criterion and flow potential can be found in Appendix A. For a detailed derivation of the enHill48 model, the readers can refer to our previous study [53].

For fracture, stress triaxiality η and Lode angle $\bar{\theta}$ are widely used for the general characterization of the stress states, which are related to the invariants of the Cauchy stress tensor $\boldsymbol{\sigma}$ and the deviatoric stress tensor \boldsymbol{s} . These two variables are related to each other under plane stress conditions [17]. For proportional loading conditions, a damage initiation criterion is defined by a critical value of the equivalent plastic strain, and a ductile fracture criterion, which are both dependent on stress triaxiality and Lode angle. The symmetric damage initiation locus (DIL) $\bar{\epsilon}_{\text{ddi}}(\eta, \bar{\theta})$ and ductile fracture locus (DFL) $\bar{\epsilon}_{\text{df}}(\eta, \bar{\theta})$ are described by four

parameters ($D_{1\sim4}$ and $F_{1\sim4}$), respectively. A cut-off value of stress triaxiality ($\eta_c = -1/3$) [69] is assumed that the damage and fracture will not be triggered when the stress triaxiality is below η_c ($\bar{\epsilon}_{\text{ddi}}(\eta, \bar{\theta}) = +\infty, \eta \leq \eta_c$).

$$\bar{\epsilon}_{\text{ddi}}(\eta, \bar{\theta}) = (D_1 \cdot e^{-D_2 \cdot \eta} - D_3 \cdot e^{-D_4 \cdot \eta}) \cdot \bar{\theta}^2 + D_3 \cdot e^{-D_4 \cdot \eta} \quad (5)$$

$$\bar{\epsilon}_{\text{df}}(\eta, \bar{\theta}) = (F_1 \cdot e^{-F_2 \cdot \eta} - F_3 \cdot e^{-F_4 \cdot \eta}) \cdot \bar{\theta}^2 + F_3 \cdot e^{-F_4 \cdot \eta} \quad (6)$$

As a proportional loading with a constant stress state is not easy to achieve in many cases, the material usually undergoes a nonlinear evolution during the plastic deformation. When non-proportional loading conditions are concerned, the average values of both stress state variables ($\eta_{\text{avg}}, \bar{\theta}_{\text{avg}}$) until the specific point instead of the instantaneous ones are used to represent the general stress states considering the evolution history [14,70,71].

$$\eta_{\text{avg}} = \frac{1}{\bar{\epsilon}} \int_0^{\bar{\epsilon}} \eta d\bar{\epsilon}^p \quad (7)$$

$$\bar{\theta}_{\text{avg}} = \frac{1}{\bar{\epsilon}} \int_0^{\bar{\epsilon}} \bar{\theta} d\bar{\epsilon}^p \quad (8)$$

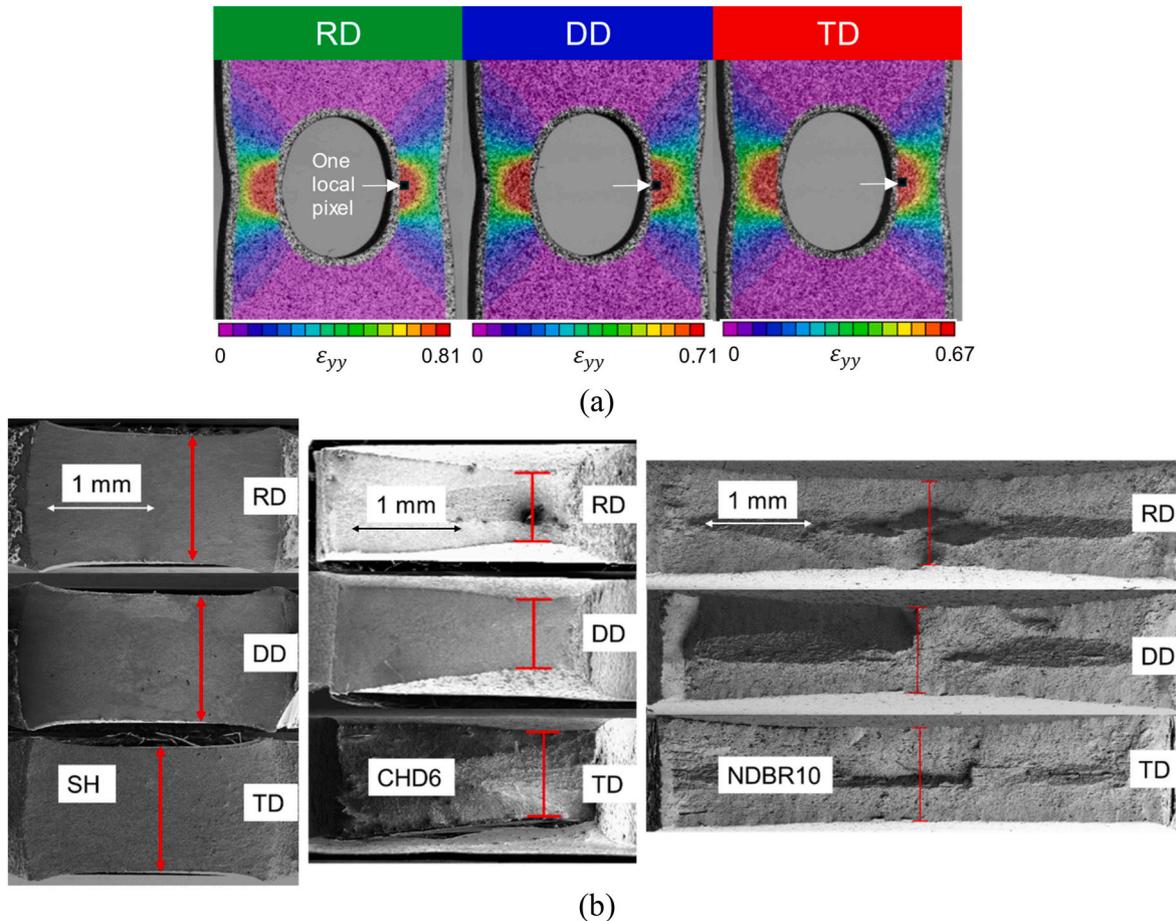


Fig. 3. (a) Contour plots of logarithmic axial strain at the last frame before fracture displacements along three tensile directions on CHD6 specimens, and (b) measurement of fracture thickness on SH, CHD6, and NDBR10 specimens.

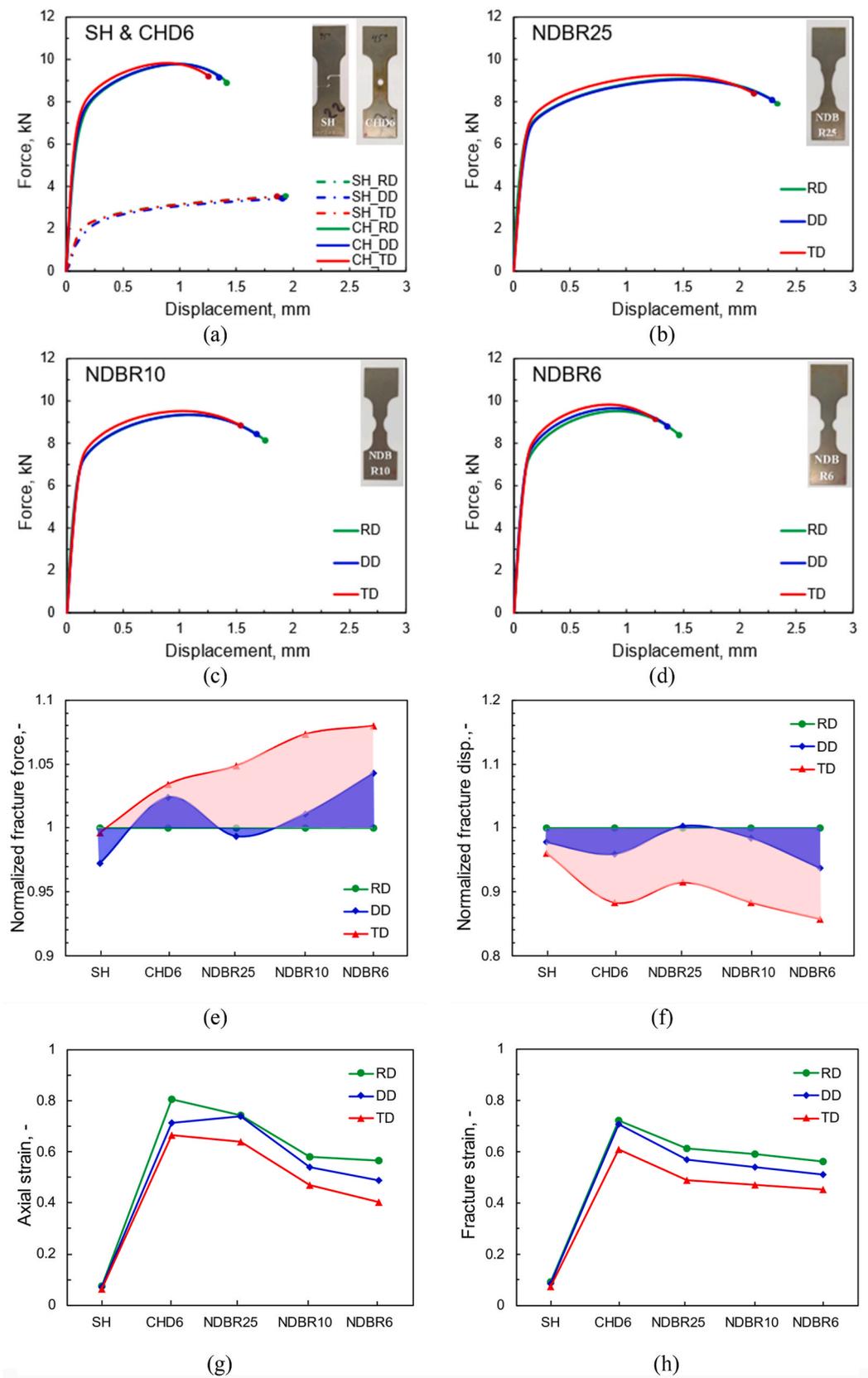


Fig. 4. The force–displacement curves of QP1000 on RD, DD, and TD for (a) SH together with CHD6, (b) NDBR25, (c) NDBR10, (d) NDBR6. (e) Normalized fracture force, and (f) normalized fracture displacement of DD and TD with respect to RD. (g) Logarithmic axial strain from one local pixel on the surface from DIC measurement, and (h) the true thickness strain at fracture.

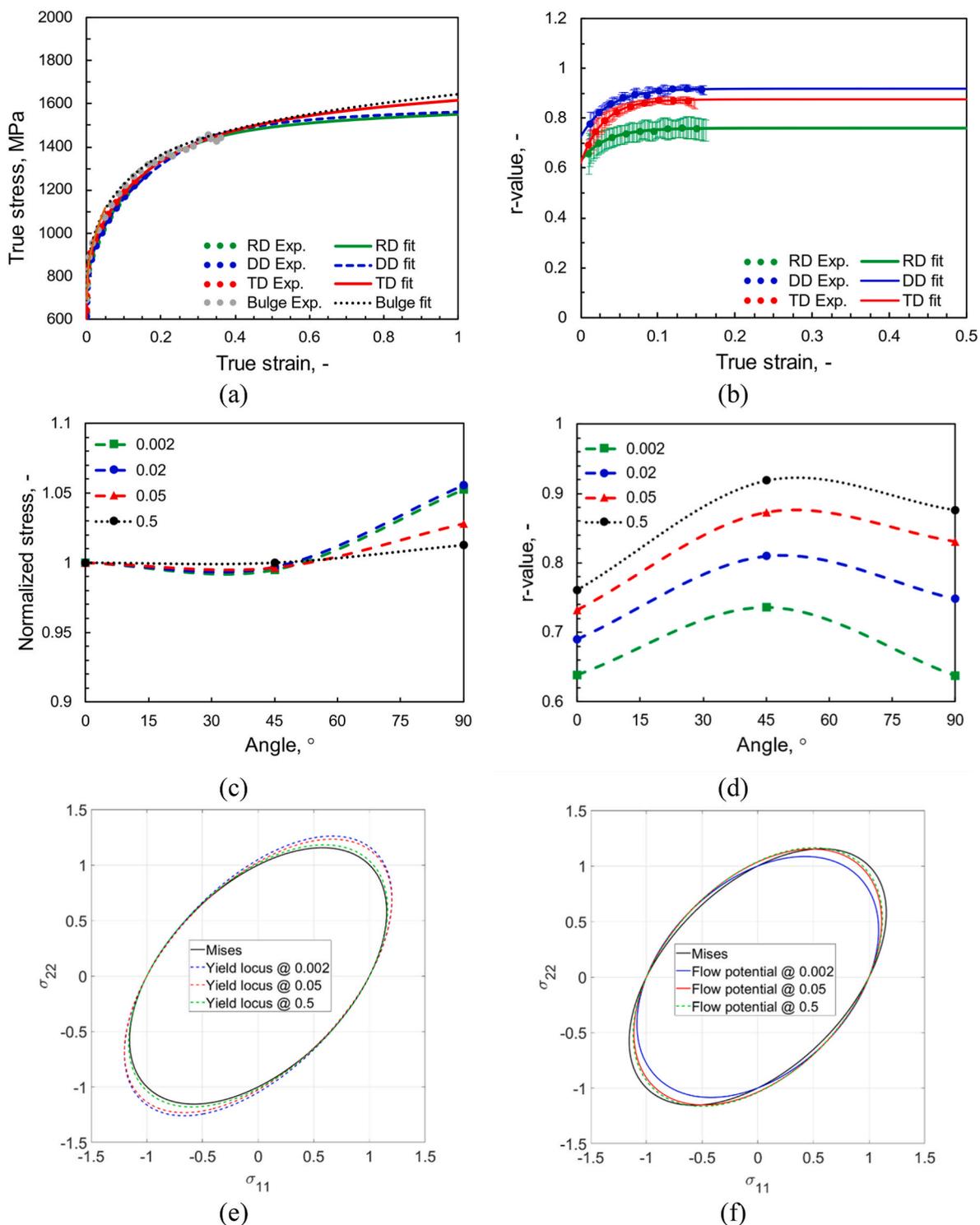


Fig. 5. The flow stress and r-value evolution from tests and model calibration: (a) true stress–strain curves of tensile tests along three loading directions and Bulge test with extrapolation using piecewise Swift-Voce law, (b) r-value evolution curves of tensile tests along three loading directions fitted by Voce law, (c) directionality of the normalized stress, (d) r-value, (e) yield locus and (f) flow potential at different strain levels from the enHill48 model.

Table 3
Calibrated parameters in hardening functions for the flow curve extrapolation.

Plastic strain range	Directions	$\sigma = w \cdot \sigma_{\text{Swift}} + (1 - w) \cdot \sigma_{\text{Voce}}$						
		$\sigma_{\text{Swift}} = A \cdot (\epsilon_0 + \bar{\epsilon}^P)^n$			$\sigma_{\text{Voce}} = k_0 + Q \cdot (1 - \exp(-\beta \cdot \bar{\epsilon}^P))$			
		A	ϵ_0	n	k_0	Q	β	w
$\bar{\epsilon}^P \leq 0.02$	RD	1303	2e-5	0.10	0.5	1125	1197	0.85
	DD	1295	4e-5	0.11	0.2	1346	1546	0.86
	TD	1320	1e-5	0.10	0.3	1286	1410	0.87
	Biaxial	2616	0.04	0.38	0.1	1700	423	0.91
$\bar{\epsilon}^P > 0.02$	RD	1766	6e-5	0.06	0.7	1225	7	0.60
	DD	1606	3e-5	0.07	0.7	1442	7	0.66
	TD	1598	0.01	0.10	0.8	1686	6	0.81
	Biaxial	1722	0.5	0.12	0.5	1195	10	0.52

Table 4
Calibrated parameters in Voce law for r-value curve extrapolation.

Constants	$r_{\text{Voce}}(\bar{\epsilon}^P) = C_1 + C_2(1 - e^{-C_3 \bar{\epsilon}^P})$		
	C_1	C_2	C_3
r_{0°	0.63	0.13	30.26
r_{45°	0.73	0.19	28.74
r_{90°	0.62	0.26	34.59

$$D = \begin{cases} 0 & I_{dd} < 1 \\ \frac{\bar{\sigma}_{ddi}^c}{G_f} I_{df} & I_{dd} \geq 1 \wedge I_{df} < 1 \\ 1 & I_{dd} \geq 1 \wedge I_{df} \geq 1 \end{cases} \quad (11)$$

3.2. Fully anisotropic fracture model

The indicator of ductile damage initiation I_{dd} and indicator of ductile fracture I_{df} are applied to quantify the effects of loading paths on damage and fracture. The initiation of damage and fracture occurs when the corresponding indicator reaches unity. A linear damage accumulation law is assumed, which is controlled by the equivalent stress at the damage initiation point $\bar{\sigma}_{ddi}$ and the energy dissipation parameter G_f .

$$I_{dd} = \int_0^{\bar{\epsilon}^P} \frac{1}{\bar{\epsilon}_{ddi}(\eta, \theta)} d\bar{\epsilon}^P \quad (9)$$

$$I_{df} = \int_{\bar{\epsilon}_{ddi}}^{\bar{\epsilon}^P} \frac{1}{\bar{\epsilon}_{df}(\eta, \theta) - \bar{\epsilon}_{ddi}(\eta, \theta)} d\bar{\epsilon}^P \quad (10)$$

To enrich the flexibility in presenting the anisotropy in fracture, the fully anisotropic fracture criteria have been formulated, in which the linear transformation concept from the anisotropic plasticity models, such as Yld2000-2D [72], Yld2004-18p [73], Yld2011-27p [74] has been adopted to unify either the strain components [75] or the stress components [76]. Recently, a scaling method by formulating the fracture parameters as a function of loading angles has also been proposed and applied to steels [77] and aluminum alloys [78].

In this study, the fully anisotropic fracture model manipulating the strain components based on linear transformation is employed. For any arbitrary stress state, the plastic strain rate tensor $\dot{\epsilon}^P$ contains five independent components considering the incompressibility of plastic deformation. The anisotropy effects are described by the transformation tensor C , which is identical to the one in the Yld2004 plasticity model. The plastic incompressibility is considered by using the modified transformation tensor T , following the recommendation by Lou and

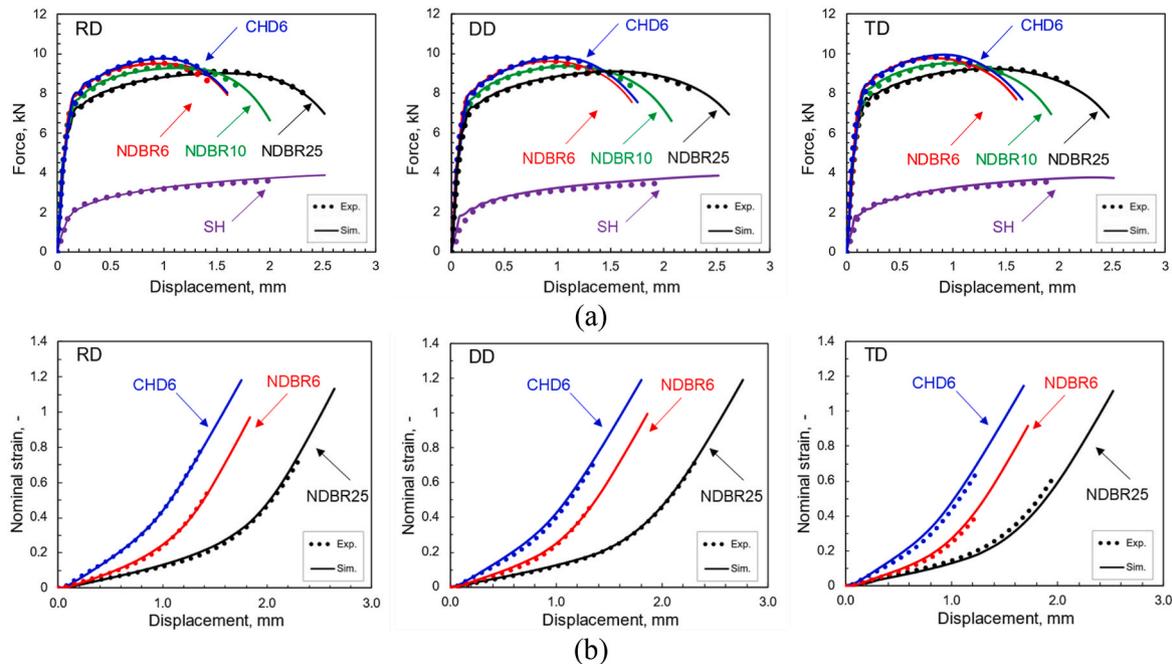


Fig. 6. Validation of the anisotropic plasticity model compared with experimental results of tensile tests in RD, DD, and TD on (a) the force–displacement curves, and (b) the evolution of nominal strain.

Table 5
The local stress state variables and equivalent plastic strain at fracture moment.

Loading direction	Specimen	PEEQ	η	η_{avg}	$\bar{\theta}$	$\bar{\theta}_{avg}$
RD	SH	0.76	0.17	0.04	0.18	0.08
	CHD6	0.96	0.33	0.32	0.91	0.93
	NDBR25	0.85	0.75	0.51	0.25	0.61
	NDBR10	0.79	0.76	0.54	0.19	0.50
	NDBR6	0.76	0.77	0.57	0.14	0.41
DD	SH	0.78	0.16	0.04	0.17	0.08
	CHD6	0.82	0.32	0.33	0.91	0.95
	NDBR25	0.68	0.65	0.53	0.38	0.66
	NDBR10	0.63	0.67	0.53	0.29	0.57
	NDBR6	0.58	0.67	0.55	0.24	0.47
TD	SH	0.80	0.18	0.05	0.15	0.06
	CHD6	0.82	0.32	0.33	0.92	0.95
	NDBR25	0.61	0.59	0.45	0.46	0.71
	NDBR10	0.55	0.61	0.48	0.38	0.60
	NDBR6	0.50	0.62	0.51	0.32	0.50

Yoon [75]. The anisotropic strain rate tensor $\dot{\epsilon}$ is formulated by applying the linear transformation L to the plastic strain rate tensor $\dot{\epsilon}^p$.

$$\dot{\epsilon} = C \cdot T \cdot \dot{\epsilon}^p = L \cdot \dot{\epsilon}^p \tag{12}$$

$$C = \begin{bmatrix} 0 & -c_{12} & -c_{13} & 0 & 0 & 0 \\ -c_{21} & 0 & -c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}; T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Hence the components of $\dot{\epsilon}$ are calculated as:

$$\dot{\epsilon} = \begin{bmatrix} \dot{\epsilon}_{11}^a \\ \dot{\epsilon}_{22}^a \\ \dot{\epsilon}_{33}^a \\ \dot{\epsilon}_{23}^a \\ \dot{\epsilon}_{13}^a \\ \dot{\epsilon}_{12}^a \end{bmatrix} = L \cdot \dot{\epsilon}^p = \begin{bmatrix} c_{13} & -c_{12} + c_{13} & 0 & 0 & 0 \\ -c_{21} + c_{23} & c_{23} & 0 & 0 & 0 \\ -c_{31} & -c_{32} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \cdot \begin{bmatrix} \dot{\epsilon}_{11}^p \\ \dot{\epsilon}_{22}^p \\ \dot{\epsilon}_{23}^p \\ \dot{\epsilon}_{13}^p \\ \dot{\epsilon}_{12}^p \end{bmatrix} \tag{14}$$

When all the anisotropic parameters c_{ij} take the value of one, then the model is reduced to isotropic. The corresponding anisotropic equivalent strain rate $\dot{\epsilon}^a$ of the anisotropic strain rate tensor $\dot{\epsilon}$ is determined according to expression in the classical Mises plasticity theory:

Table 6
Calibrated damage initiation parameters of QP1000 in the partially coupled anisotropic fracture model for RD and TD.

Loading direction	D_1	D_2	D_3	D_4
RD	1.03	0.10	0.81	0.10
TD	1.04	0.40	0.86	1.50

$$\dot{\epsilon}^a = \sqrt{\frac{2}{3}} \dot{\epsilon} : \dot{\epsilon} \tag{15}$$

The anisotropic ductile failure criteria are formulated as a critical value of the anisotropic equivalent strain at the ductile damage initiation $\bar{\epsilon}_{ddi}^a$, and ductile fracture $\bar{\epsilon}_{df}^a$, which depend on the stress states whereas independent of loading orientations.

$$\bar{\epsilon}_{ddi}^a(\eta, \bar{\theta}) = (D_1^a \cdot e^{-D_2^a \cdot \eta} - D_3^a \cdot e^{-D_3^a \cdot \eta}) \cdot \bar{\theta}^2 + D_3^a \cdot e^{-D_4^a \cdot \eta} \tag{16}$$

$$\bar{\epsilon}_{df}^a(\eta, \bar{\theta}) = (F_1^a \cdot e^{-F_2^a \cdot \eta} - F_3^a \cdot e^{-F_4^a \cdot \eta}) \cdot \bar{\theta}^2 + F_3^a \cdot e^{-F_4^a \cdot \eta} \tag{17}$$

$D_{1\sim4}^a$ and $F_{1\sim4}^a$ are material parameters to be calibrated for the anisotropic damage initiation locus and ductile fracture locus, respectively. Two different sets of anisotropic parameters c_{ij} can be determined for damage initiation and ductile fracture criteria using experimental results under different stress states and loading orientations. Similar to the isotropic formulation, the indicator of anisotropic damage initiation I_{dd}^a and anisotropic ductile fracture I_{df}^a are used for non-proportional loading conditions, as shown in Eq. (18) and (19). Based on the formulated anisotropic loci, it is assumed that damage evolution is independent of loading directions, i.e., the energy dissipation parameter G_f stays as a constant.

$$I_{dd}^a = \int_0^{\bar{\epsilon}^p} \frac{1}{\bar{\epsilon}_{ddi}^a(\eta, \bar{\theta})} d\bar{\epsilon}^a \tag{18}$$

$$I_{df}^a = \int_{\bar{\epsilon}_{ddi}^a}^{\bar{\epsilon}^a} \frac{1}{\bar{\epsilon}_{df}^a(\eta, \bar{\theta}) - \bar{\epsilon}_{ddi}^a(\eta, \bar{\theta})} d\bar{\epsilon}^a \tag{19}$$

$$D = \begin{cases} 0 & I_{dd} < 1 \\ \frac{\bar{\sigma}_{ddi}^c \cdot I_{df}^a}{G_f} & I_{dd} \geq 1 \wedge I_{df} < 1 \\ 1 & I_{dd} \geq 1 \wedge I_{df} \geq 1 \end{cases} \tag{20}$$

4. Plasticity parameters calibration and validation

4.1. Anisotropic plasticity parameters calibration

The anisotropy of the material plasticity evolves in terms of both stress and r-value during plastic deformation. The strain hardening behavior of QP1000 is described using the combined Swift-Voce hardening law, where the individual parameters for each loading condition are fitted based on experimental stress–strain curves, as shown in Fig. 5 (a). The r-value curves from three uniaxial tension directions are fitted by Voce law to capture the anisotropic evolving feature as depicted in Fig. 5 (b). To illustrate the pattern of the anisotropic evolution, normalized stress and r-values at yielding points (approximately strain level of 0.002), 0.02, 0.05, the onset of necking (approximately strain level of 0.15), and at larger strain (0.5) are plotted in Fig. 5 (c) and (d). From the results, it is observed that the anisotropic pattern of r-value distribution over loading direction is amplified with increased plastic strain, which necessitates the application of evolving plasticity models. Both the yield locus and flow potential are evolving during plastic deformation, as shown in Fig. 5 (e) and (f) at strain levels of 0.002, 0.05, and 0.5, together with Mises as a reference. The evolving characteristics of anisotropy can be well captured by the enHill48 anisotropic plasticity

model. The calibrated parameters for hardening and r-values are listed in Table 3 and Table 4.

4.2. Anisotropic plasticity parameters validation

To validate the plasticity model, finite element (FE) simulations of uniaxial tension with five geometries along three directions are performed. Since the deformation is mainly concentrated around the notch area, the FE models are built only to the extensometer measurement region of the specimens. Since the in-plane anisotropy is dominating, a half symmetry through the thickness direction is applied to the model. First-order reduced-integration solid elements (C3D8R) are used with the explicit method in Abaqus. Elements with a size of 0.05 mm in three directions are used to capture the deformation around the critical area, while the rest of the model is featured with a coarser mesh for computational efficiency. The presented meshing strategy has been optimized via a parametric study with a finer mesh size. The comparison of the cost-effectiveness is shown in Appendix B, which verifies that 0.05 mm mesh size is enough to reach the local variable (in terms of PEEQ) convergence. The constitutive model is implemented via a VUMAT user material subroutine in Abaqus/Explicit.

The performance of the anisotropic plasticity model is firstly validated based on the force–displacement responses predicted by the numerical simulations and the experimental results in each direction, as shown in Fig. 6 (a). The anisotropic plasticity model provides accurate prediction of force–displacement curves until the fracture points in all

three directions during plastic deformation. The prediction quality is especially high for CH and NDB specimens with large radii in both the uniform deformation and post-necking deformation towards the end. In addition, a high agreement with experimental results is achieved by the plasticity model in the description of local strain fields. Virtual extensometers with a length of 1 mm are placed across the fracture region for both the DIC reference image and the FE model. Local strain is measured along the virtual extensometer and compared between FE simulation and experimental results, as shown in Fig. 6 (b). The prediction on the history of nominal strain until fracture matches well with DIC results for different geometries and loading directions. The enHill48 model shows a good capability in accurately describing both the anisotropic plasticity and the anisotropic evolving features of the investigated QP1000 steel.

5. Partially coupled anisotropic approaches

5.1. Calibration and validation of partially coupled anisotropic fracture model

Based on the failure mechanisms analysis of the fracture surfaces in Ref. [16], the damage initiation in QP1000 takes place at a rather late stage of plastic deformation with a rapid damage evolution. Therefore, the uncoupled damage mechanics approach flexibly offered by the HDM model [34] is employed for this QP1000 steel by taking the critical strain values at fracture points (corresponding to the sudden force drop) to determine the damage initiation parameters (D_{1-4}) and assuming the

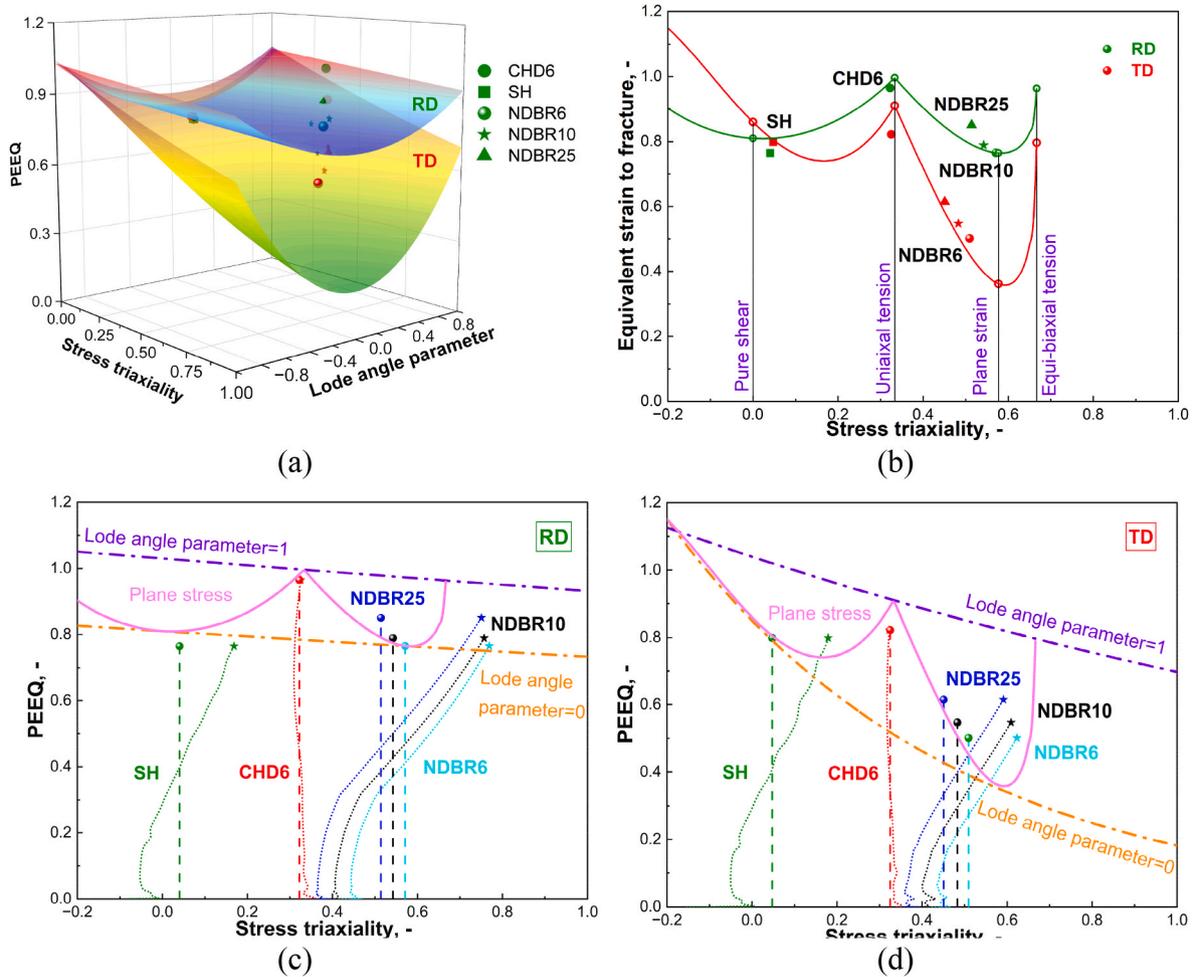


Fig. 7. The 3D damage initiation loci (a) in the partially coupled anisotropic fracture model, (b) the fracture strain at stress states of shear, uniaxial tension, plane strain, biaxial tension, the 2D projection of the damage initiation loci with loading paths for (c) RD and (d) TD, in which the cross indicates the fracture points and the dots are corresponding to the averaged values.

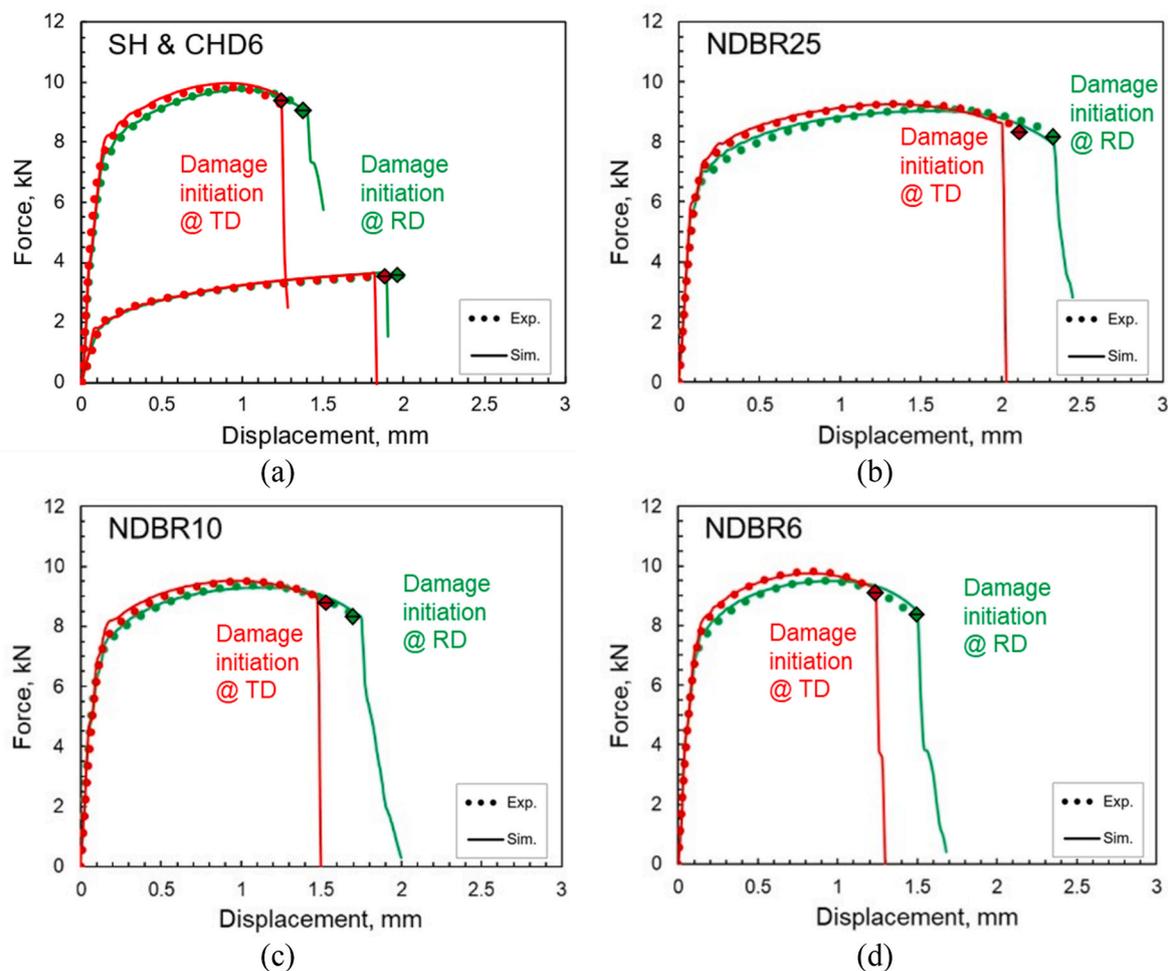


Fig. 8. Failure prediction in tensile tests of QP1000 using partially coupled anisotropic fracture model with independently calibrated damage initiation criterion for RD and TD in (a) SH and CHD6, (b) NDBR25, (c) NDBR10, and (d) NDBR6.

DIL converges with the DFL. Therefore, the two sets of damage/fracture parameters are reduced to only the four damage initiation parameters. In addition, the damage evolution parameter G_f is also omitted due to the uncoupled use of the HDM model for rapid damage evolution.

With one set of plasticity parameters determined for the QP1000 steel for all loading directions, the damage initiation parameters (D_{1-4}) in the partially coupled anisotropic fracture model have been calibrated separately for RD and TD based on the local stress and strain variables in critical elements extracted from simulations, as listed in Table 5 and the calibrated parameters are listed in Table 6. The locations of the critical elements were shown in detail in the previous investigation [35]. It is evident that in Fig. 7 the local stress state in the critical elements of all specimens is rather not constant during deformation. Therefore, the values of local stress state variables (η_{avg} , $\bar{\theta}_{avg}$) are averaged along deformation until the fracture moment, shown as solid dots in Fig. 7 (b)–(d), to represent the overall stress state and characterize non-proportional loading effects. The calibrated fracture parameters for RD and TD are shown in Fig. 7. The two ductile fracture loci for RD and TD are constructed in the 3D space of equivalent plastic strain (PEEQ), stress triaxiality, and Lode angle parameter in Fig. 7 (a), with the plane stress conditions relevant for sheet metal forming highlighted by a solid curve.

Noticeable anisotropic effects on the fracture condition have been observed for the investigated QP1000 steel. It is noticed that the impacts of the stress state on fracture are significantly affected by the loading direction. For each Lode angle value, the stress triaxiality has a more pronounced influence on the failure strain in TD than RD. Similarly, the

failure strain in TD is more sensitive to the Lode angle than RD for a given stress triaxiality. In particular, the anisotropic effect on damage/fracture strain is enhanced with increasing stress triaxiality. The performance of the partially coupled anisotropic fracture model in predicting the global force–displacement response for all five fracture specimens is presented in Fig. 8. The experimental (dotted curves) and numerical (solid curves) results of force–displacement response in tensile tests are represented by green and red curves for RD and TD, respectively. The fracture points from experimental data are marked with the corresponding-colored quadrilaterals for each direction. When the independently determined fracture criterion for RD and TD is applied to predict the fracture behavior along the corresponding direction, very accurate predictions of fracture force and displacements can be achieved using the partially coupled anisotropic fracture model.

5.2. Limitations of partially coupled anisotropic approaches

The limitations of partially coupled anisotropic fracture models are mainly located in the blind prediction of fracture behavior in arbitrary loading directions, which are not involved in parameter calibration. These observed strong variations of failure strain with loading direction indicate that significant deviations will be obtained when using the failure criterion determined for one specific direction to predict the fracture behavior along other loading directions, even for materials showing minor anisotropic effects in the plastic flow behavior. Cross-prediction using the partially coupled anisotropic fracture model with different fracture criteria has been performed to elaborate on the limitations of partially coupled anisotropic approaches. In the first cross-

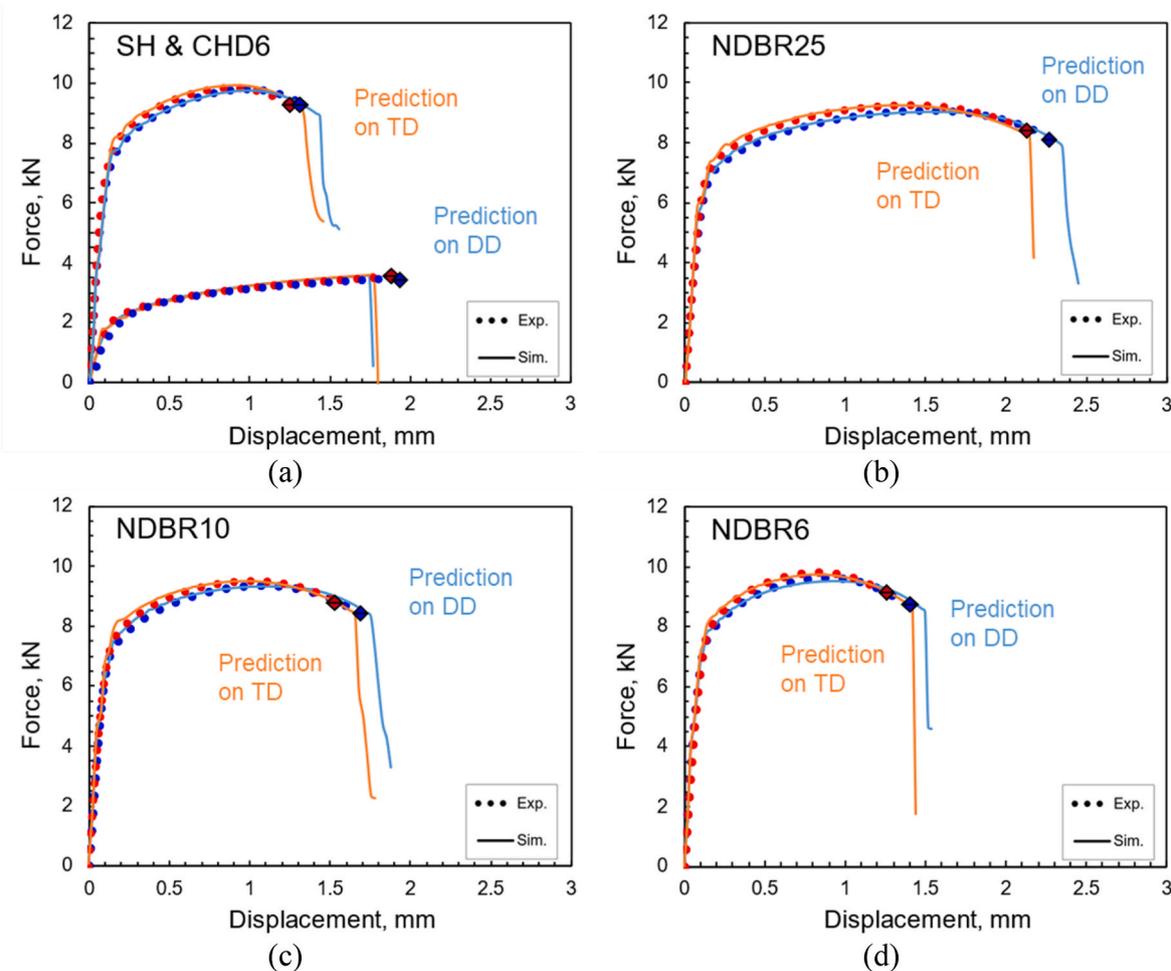


Fig. 9. Prediction of fracture in specimens loaded along DD and TD using the partially coupled anisotropic fracture model with failure criterion determined based on RD in (a) SH and CHD6, (b) NDBR25, (c) NDBR10, and (d) NDBR6.

evaluation, the fracture criterion of the RD is applied to predict the fracture behavior along TD and DD in tensile tests, which is adopted in many investigations on fracture properties of anisotropic materials. As shown in Fig. 9, a significant overestimation of the fracture displacements is observed in all geometries (except shear) along other directions when only the failure criterion of the RD is applied. Noticeable underestimation of fracture displacements is obtained in the second cross-evaluation, as shown in Fig. 10, where the failure criterion of TD is used to predict the fracture in other loading directions (RD and DD).

To quantify the prediction accuracy of the partially coupled anisotropic fracture model, the predicted fracture displacements of different specimens are normalized by experimental results. The normalized fracture displacements using fracture criteria determined based on RD and TD are shown in Fig. 11 (a) and (b), respectively. In the blind predictions on fracture displacements along DD, either significant overestimation or underestimation is obtained using the partially coupled anisotropic fracture model from either RD or TD. It is concluded that for material with minor or moderate plastic anisotropy but pronounced anisotropic fracture behavior, the partially coupled anisotropic fracture models that rely only on the plasticity anisotropy with an isotropic fracture criterion fail to fully describe the anisotropic fracture behavior.

6. Fracture modeling using fully anisotropic criterion

6.1. Parameters calibration of fully anisotropic criterion

Similar to the partially coupled anisotropic fracture model, the fully

anisotropic damage mechanics model is simplified to an uncoupled approach by assuming the damage initiation is identical to the ductile fracture. The main advantage of a fully anisotropic criterion lies in its capacity to predict fracture properties along arbitrary loading directions. To demonstrate and evaluate the performance of fully anisotropic fracture criterion, the anisotropic fracture behavior has been predicted using two different parameter calibration strategies. In the first strategy, the local stress and strain values obtained from RD and TD are used to determine anisotropic fracture parameters, referred to as the 2D case. In the second strategy, the strain and strain data of all three loading directions (RD, DD, and TD) are used to calibrate the anisotropic damage parameters, referred to as the 3D case.

The full plastic strain tensor with six components in the critical elements at the crack initiation instant has been collected from each specimen and in individual directions. A linear transformation is applied to the plastic strain rate tensor $\dot{\epsilon}^p$ to obtain the anisotropic strain rate tensor $\dot{\epsilon}$ according to Eq. (12). The corresponding anisotropic equivalent strain $\bar{\epsilon}^a$ after linear transformation is determined for each specimen and loading direction according to Eq. (17). These anisotropic equivalent strain $\bar{\epsilon}^a$ together with the corresponding stress state variables (η_{avg} , $\bar{\theta}_{avg}$) listed in Table 5 are taken as inputs to determine the parameters of the fully anisotropic fracture criterion, including parameters c_{ij} in the linear transformation tensor and four parameters $D_{1\sim4}^a$ in the anisotropic damage initiation locus. An optimization algorithm has been used to minimize the deviation between the linear transformed values of anisotropic equivalent strain and the predicted ones on the anisotropic damage initiation locus. For such an optimization problem, the objective

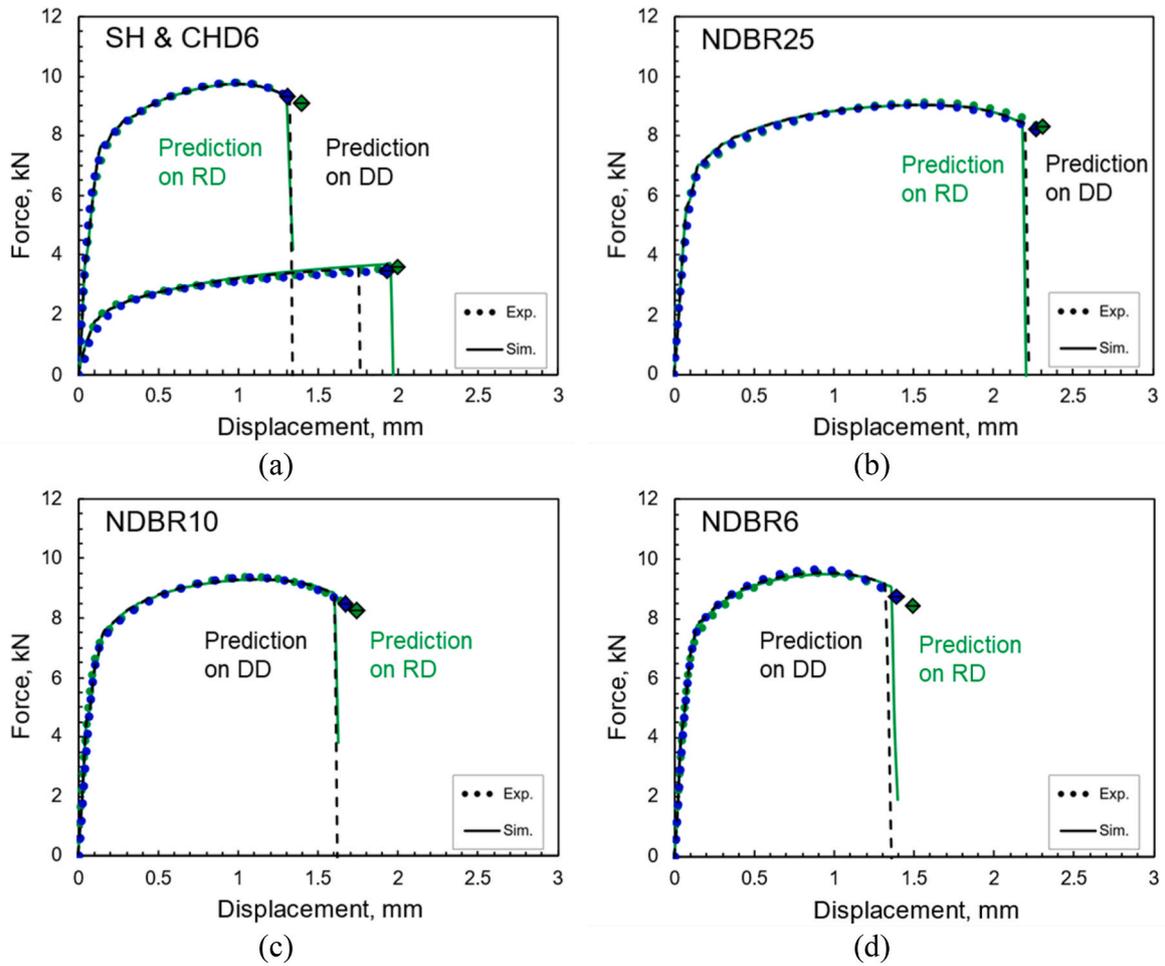


Fig. 10. Prediction of fracture in specimens loaded along RD and DD using the partially coupled anisotropic fracture model with failure criterion determined based on TD in (a) SH and CHD6, (b) NDBR25, (c) NDBR10, and (d) NDBR6.

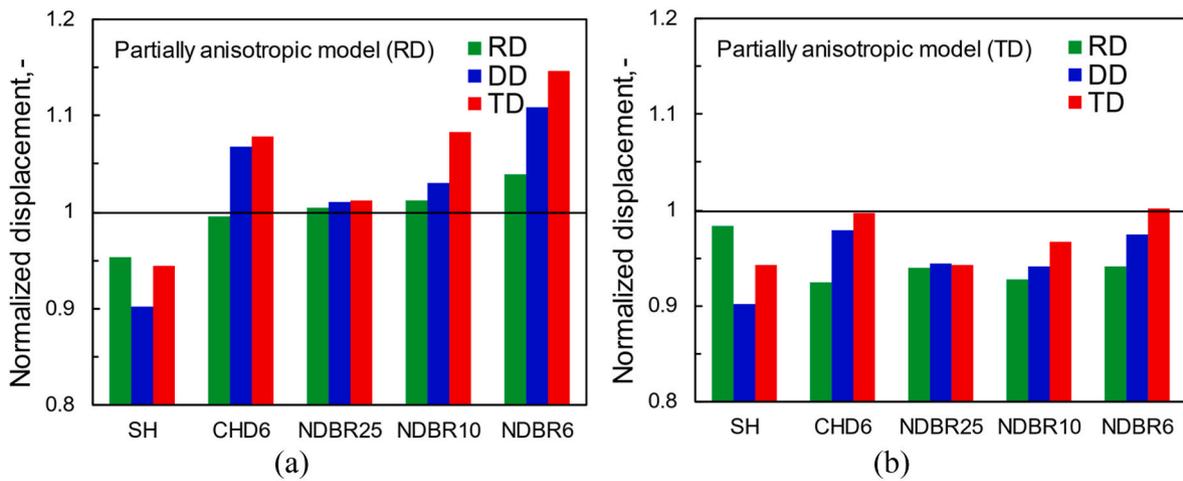


Fig. 11. Evaluation of the prediction capacity of the partially coupled anisotropic fracture model based on predicted normalized fracture displacements using damage criterion of (a) RD and (b) TD.

function is defined to be minimized:

$$err = \frac{1}{N} \sum_{i=1}^N \left(\frac{\bar{\epsilon}_{Pred}^3(\eta_{avg}, \bar{\theta}_{avg})}{\bar{\epsilon}_{Tran}^3(\eta_{avg}, \bar{\theta}_{avg})} - 1 \right)^2 \quad (21)$$

In the objective function, N is the number of data points used for parameter calibration. With five fracture specimens used in this study,

the total number of data points is 10 and 15 in the 2D and 3D calibration cases, respectively. The number of parameters c_{ij} to be determined in the linear transformation tensor according to the first strategy (2D case) is reduced to 6 with the symmetry assumption, $c_{21} = c_{12}$, $c_{31} = c_{13}$ and $c_{32} = c_{23}$, which results in 10 parameters to calibrate. For the 3D case, the 9 parameters in the transformation tensors all take independent values, and together with the failure parameters, there are all 13 pa-

Table 7
Calibrated parameters in the fully anisotropic fracture model based on inputs of RD and TD.

c_{12}	c_{13}	c_{21}	c_{23}	c_{31}	c_{32}	c_{44}	c_{55}	c_{66}
1.03	0.71	1.03	0.85	0.71	0.85	0.72	0.70	0.40
C_1^a	C_2^a	C_3^a	C_4^a					
1.00	0.45	0.78	0.55					

Table 8
Calibrated parameters in the fully anisotropic fracture model based on inputs of three directions.

c_{12}	c_{13}	c_{21}	c_{23}	c_{31}	c_{32}	c_{44}	c_{55}	c_{66}
0.11	0.25	1.52	1.07	1.59	1.10	2.16	1.36	0.50
C_1^a	C_2^a	C_3^a	C_4^a					
1.00	0.45	0.78	0.55					

Table 9
The summary of anisotropic damage initiation strain after linear transformation based on inputs of RD and TD.

Loading direction	$\bar{\epsilon}^a$	SH	CHD6	NDBR25	NDBR10	NDBR6
RD	Tran.	0.75	0.85	0.67	0.59	0.51
	Pred.	0.74	0.85	0.67	0.62	0.62
TD	Tran.	0.74	0.84	0.77	0.58	0.48
	Pred.	0.74	0.85	0.68	0.66	0.64

Table 10
The summary of anisotropic damage initiation strain after linear transformation based on inputs of three directions.

Loading direction	$\bar{\epsilon}^a$	SH	CHD6	NDBR25	NDBR10	NDBR6
RD	Tran.	0.75	0.81	0.71	0.66	0.58
	Pred.	0.74	0.85	0.67	0.62	0.62
DD	Tran.	0.72	0.84	0.85	0.73	0.57
	Pred.	0.74	0.84	0.67	0.64	0.64
TD	Tran.	0.74	0.84	0.84	0.66	0.55
	Pred.	0.74	0.85	0.68	0.66	0.64

rameters to calibrate. The two sets of calibrated parameters are listed in Table 7 and Table 8. The corresponding anisotropic fracture strain values are listed in Table 9 and Table 10, in which $\bar{\epsilon}_{Tran}^a(\eta_{avg}, \bar{\theta}_{avg})$ is the anisotropic equivalent strain after linear transformation and $\bar{\epsilon}_{Pred}^a(\eta_{avg}, \bar{\theta}_{avg})$ is the predicted value on the anisotropic damage initiation locus for the corresponding stress states. A relatively low error

($err = 0.072$) is achieved in both sets of parameters.

The anisotropic damage initiation locus determined based on two different strategies is constructed for the investigated material. The fully anisotropic damage initiation locus determined based on all five geometries of three loading directions in the space of anisotropic equivalent strain, stress triaxiality, and Lode angle parameter, are shown in

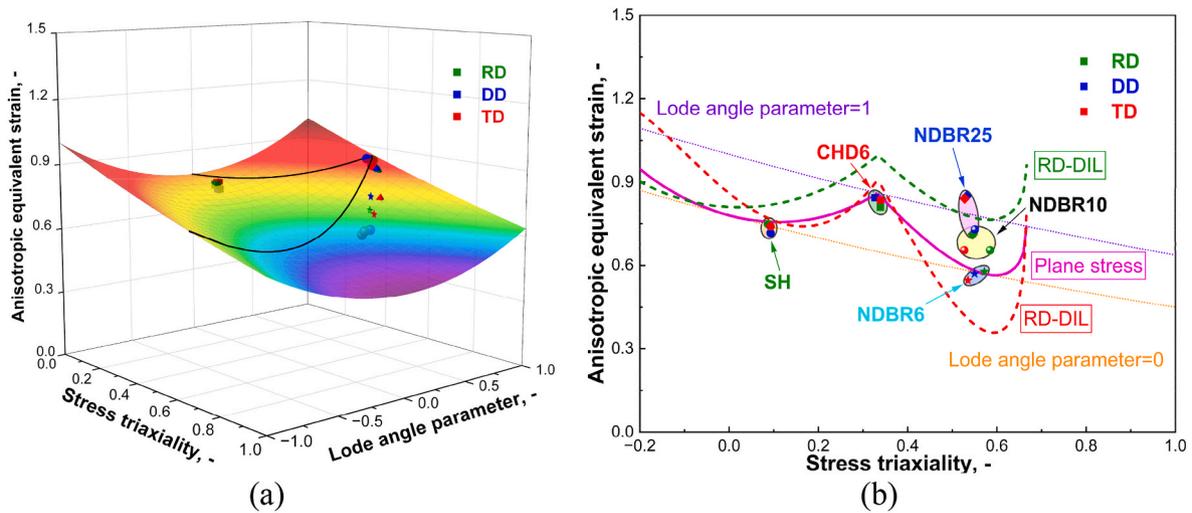


Fig. 12. The fully anisotropic damage initiation locus determined based on inputs of all three loading directions, together with the scatter points of RD and TD after linear transformation in (a) 3D, and (b) the 2D projection, plotted together with the plane stress curves from partially coupled anisotropic fracture model (RD-DIL and TD-DIL).

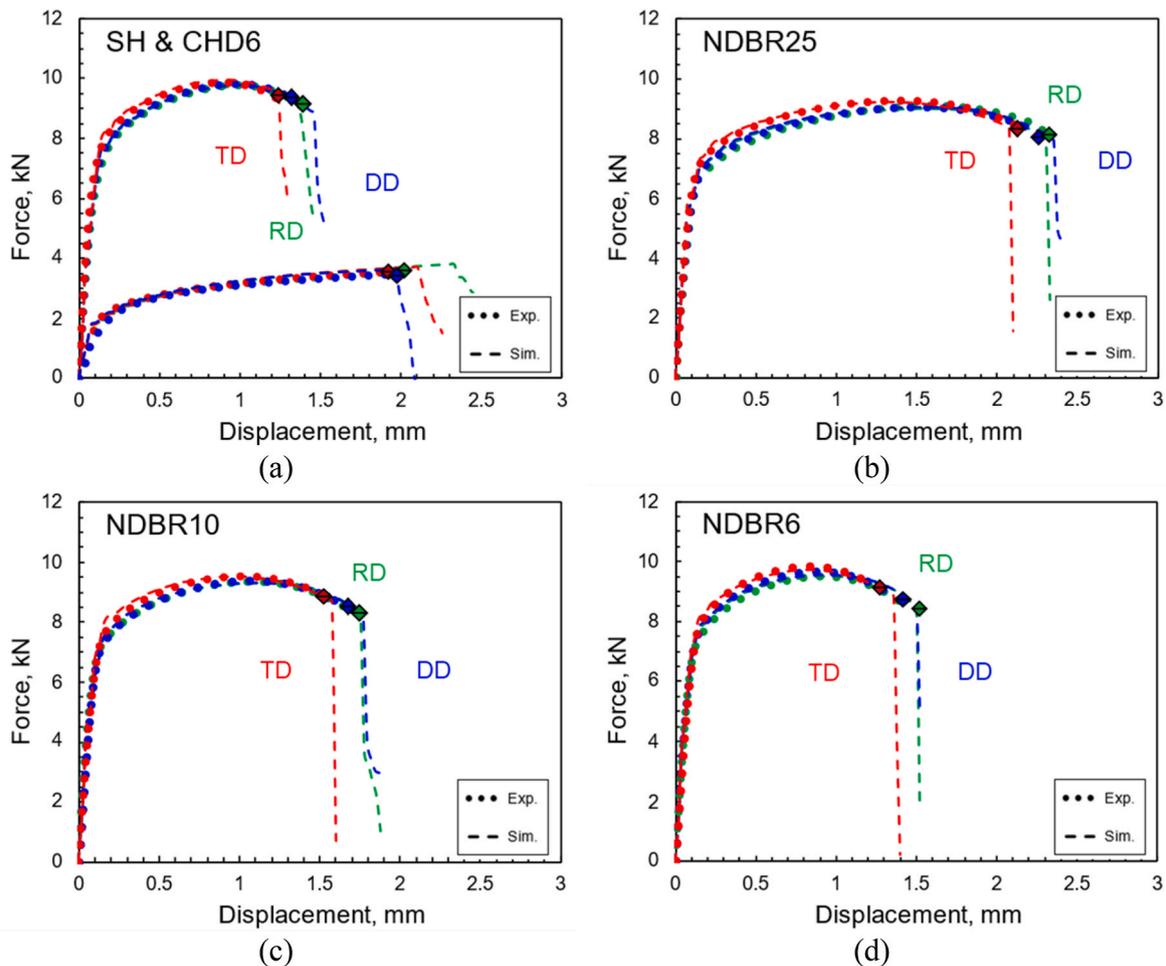


Fig. 13. Prediction of fracture behavior of all tests using the fully anisotropic fracture model calibrated based on inputs of RD and TD (2D): (a) SH and CHD6, (b) NDBR25, (c) NDBR10, and (d) NDBR6.

Fig. 12. The anisotropic equivalent strain after linear transformation is represented by distinct solid symbols with different colors for each loading direction. In general, a good fitting quality of the anisotropic failure criterion is achieved for RD and TD in both calibration strategies. However, there is a noticeable difference between the damage initiation loci.

6.2. Evaluation of fully anisotropic fracture criterion

The performance of the fully anisotropic fracture criterion calibrated using two different strategies, in predicting the global force–displacement response for five fracture specimens along three directions is depicted in Figs. 14 and 13, respectively. The experimental (dotted) and numerical (line) results are represented by three colors for RD (green), DD (blue), and TD (red) respectively. The fracture points indicated with different color quadrilaterals correspond to the fracture points determined from experiments for each direction. For all specimens tested along RD and TD, the fracture properties are accurately predicted using the fully anisotropic fracture criterion with two different parameter calibration strategies. The fracture displacements in specimens along DD are overestimated by the fully anisotropic fracture criterion, where the local strain/stress values of DD have not been taken in the calibration of the parameters. By comparing the performance of fracture prediction along DD, it is noticed that the calibration strategy in the 3D case enhanced the performance, and it can be concluded that the accuracy of the fully anisotropic fracture criterion can be improved by including more data points in the calibration procedures.

The overall performance of the fully anisotropic fracture model with two different parameter calibration strategies is quantified in Fig. 15. The normalized fracture displacement and force from simulation results of all specimens serve as strong evidence of the significance of parameter calibration strategies on the prediction capacity of the failure models. The fully anisotropic fracture model, calibrated based on inputs of three loading directions, is proven to give accurate predictions on the ductile fracture behavior of QP1000 over a broad range of stress states and loading directions, where non-proportional loading effects are involved.

The prediction of the fracture strain for each geometry from the fully anisotropic model based on the inputs from three loading directions is validated compared to the measured logarithmic axial strain ϵ_{yy} extracted from DIC as shown in Fig. 16. The prediction of fracture strain exhibits a comparable level of consistency with DIC measurement as observed in the fracture displacement prediction. The model has the best performance on CH tests, which also agrees with the anisotropic fracture behavior. A relatively large overestimation exists in the axial strain on NDBR6 prediction along TD, owing to the fact that the predicted fracture moment is slightly lagging behind the testing results. Likewise, the underestimation in the axial strain prediction on NDBR25 along RD and DD loading is due to the slight early determination of the fracture point. Overall, the model shows its capability to capture the local strain anisotropy, further proving its applicability in simulating anisotropic fracture behavior of materials.

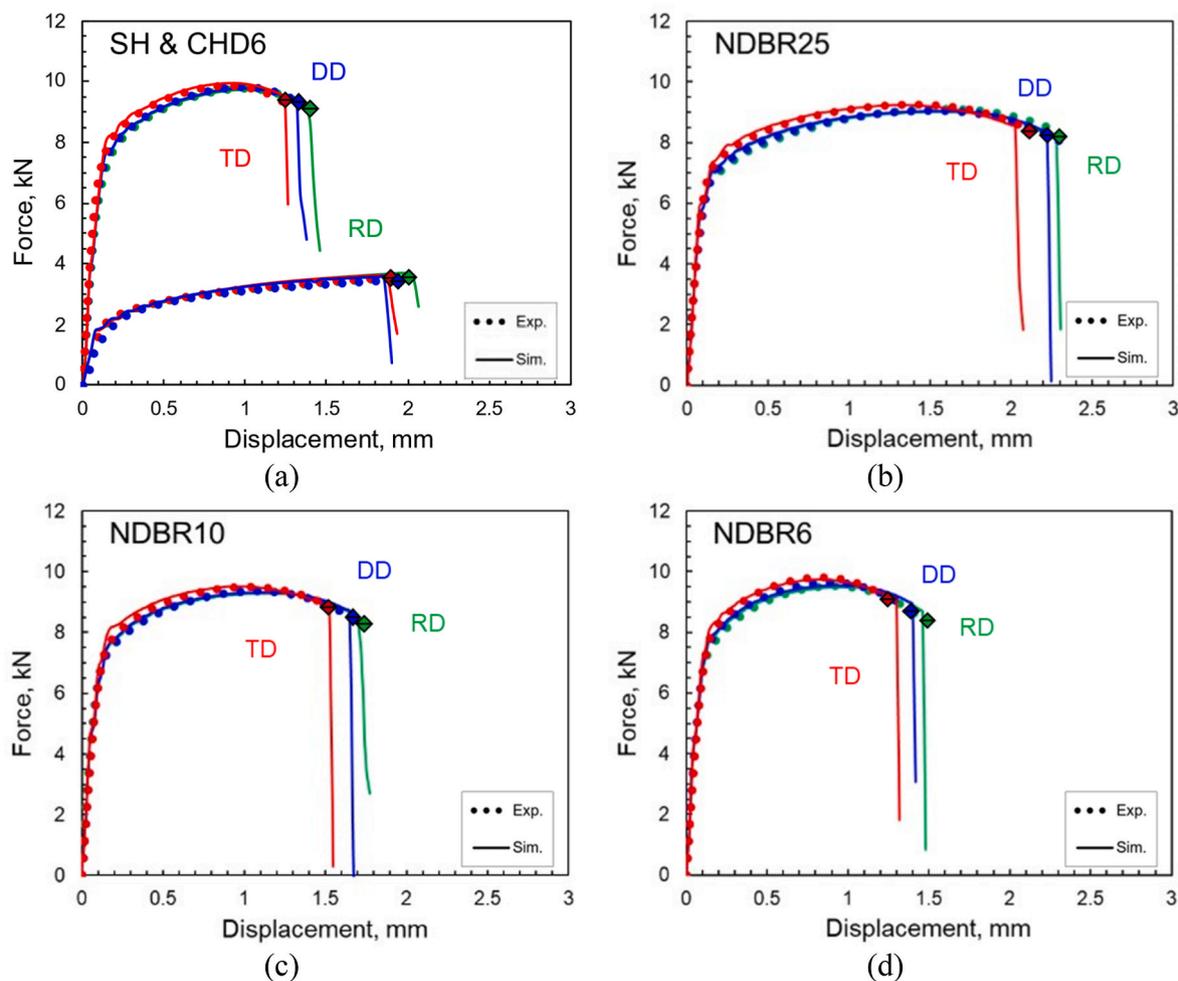


Fig. 14. Prediction of fracture behavior of all tests using the fully anisotropic fracture model calibrated based on inputs of three loading directions (3D): (a) SH and CHD6, (b) NDBR25, (c) NDBR10, and (d) NDBR6.

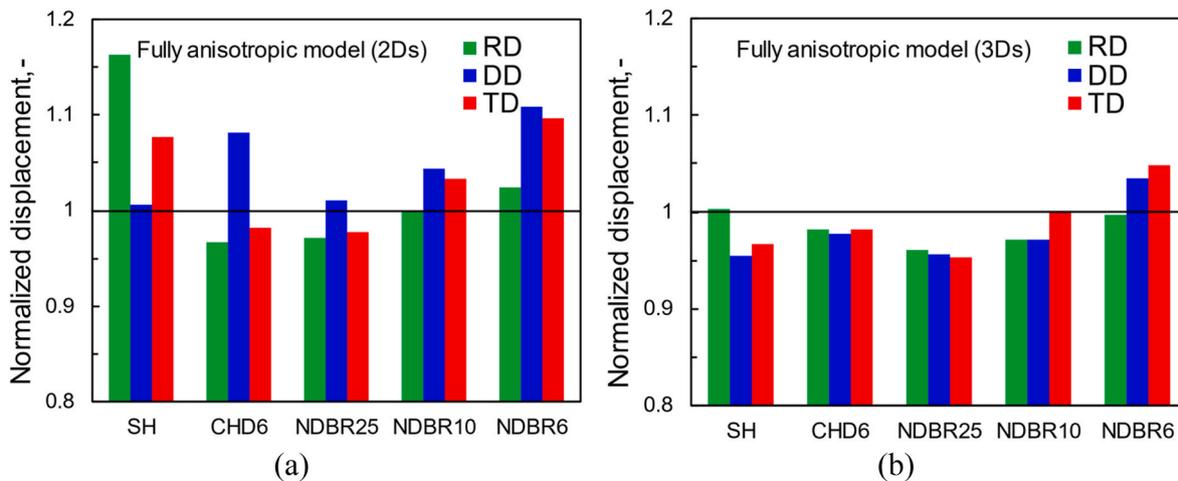


Fig. 15. Evaluation of the performance of the fully anisotropic fracture model on predicted normalized fracture displacements, parameters calibrated based on inputs of (a) RD and TD, and (b) three loading directions.

7. Discussion on the orientation dependency

The loading direction plays an important role in the accurate prediction of the ductile fracture in the investigated material. As shown in Fig. 17, a comparison among all four types of models mentioned above is

plotted. The deviation is calculated between the experimental fracture displacement and predicted value and then averaged among all five geometries for each direction. From the results, for partially coupled anisotropic fracture models, either determined from RD or TD, accurate prediction can be achieved only at the corresponding direction with

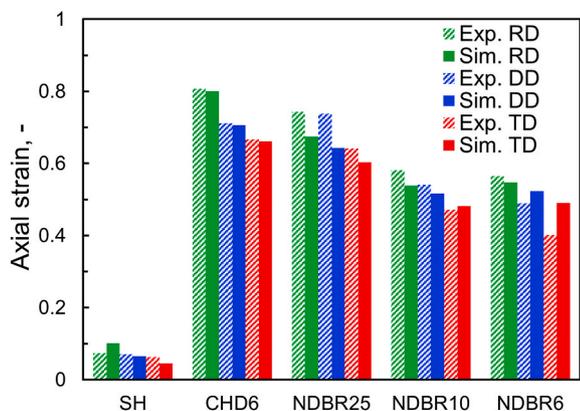


Fig. 16. Comparison of the fracture strain between the prediction from the fully anisotropic model based on the inputs from three loading directions and the measured logarithmic axial strain ϵ_{yy} extracted from DIC.

either 2% or 3%, while for other directions, the deviation could rise beyond 5%. For fully anisotropic fracture models, a large distinction exists between the one determined by only two directions and all three directions. The prediction of the fully anisotropic fracture model with two directional inputs on RD and TD has similar accuracy to the one from partially coupled anisotropic fracture models; however, the prediction of fracture displacement gets more evenly distributed among

three directions with a 5% deviation. On the other hand, the fully anisotropic fracture model with three directional inputs generates the best prediction in all directions. The deviation after averaging among three directions is below 3%.

8. Conclusions

The anisotropic effects on the deformation and fracture properties of QP1000 have been investigated experimentally and numerically under wide loading conditions. Based on the results presented in this study, several conclusions can be drawn.

- The investigated QP1000 steel demonstrates a good combination of strength and formability, with a high yield strength of over 750 MPa averaged among three loading directions and an excellent total elongation of at least 23%.
- The QP1000 steel shows minor to intermediate plastic anisotropy, which evolves clearly over the plastic deformation. However, pronounced anisotropic fracture behavior has been observed. For loading along the rolling direction (RD), diagonal direction (DD), and transverse direction (TD), the worst fracture resistance is along the TD, while the best varies between RD and DD loading.
- The anisotropic fracture behavior of QP1000 is highly dependent on the stress states. It is discovered from the current study that higher stress triaxiality intensifies the level of anisotropic fracture significantly in terms of both the fracture displacement and the failure strain.

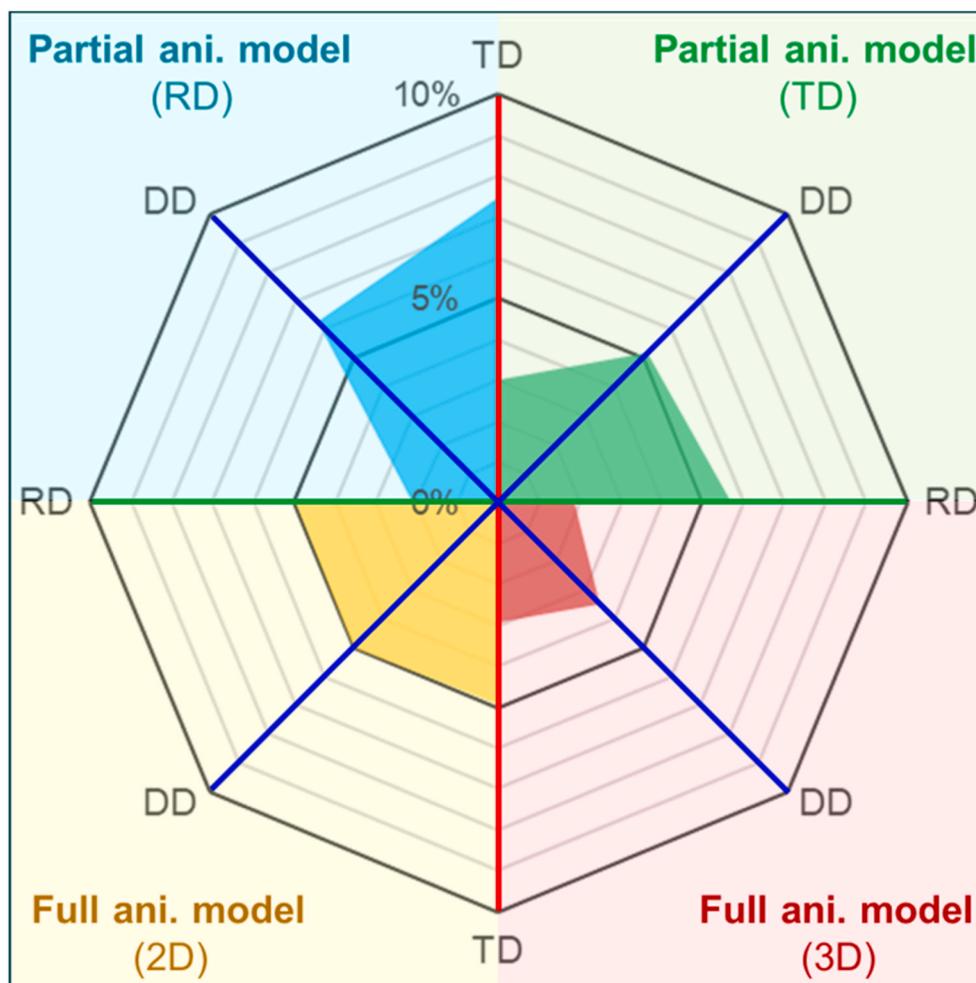


Fig. 17. Comparison of the deviation of the partially and fully anisotropic models on predicted normalized fracture displacement.

- Partially coupled anisotropic fracture models with anisotropic plasticity model but isotropic fracture model can only provide accurate fracture prediction for the loading direction that was used for the calibration of the fracture model but fails in other loading directions due to the pronounced and stress-state-dependent anisotropic fracture behavior of the material.
- A fully anisotropic hybrid damage mechanics model is formulated based on the linear transformation of plastic stains in the damage initiation criterion. A detailed calibration procedure is demonstrated considering testing data along two directions and three directions. It is shown that the latter case provides a satisfactory prediction of the anisotropic fracture across a large range of stress states.

CRediT authorship contribution statement

Zinan Li: Data curation, Formal analysis, Investigation, Software, Validation, Visualization, Writing – original draft. **Fuhui Shen:** Methodology, Investigation, Supervision, Writing – review & editing. **Yi Liu:** Data curation, Formal analysis. **Christoph Hartmann:** Data curation, Formal analysis. **Roman Norz:** Data curation, Formal analysis, Writing – review & editing. **Sebastian Münstermann:** Supervision, Writing –

review & editing. **Wolfram Volk:** Supervision, Writing – review & editing. **Junying Min:** Supervision, Writing – review & editing. **Junhe Lian:** Conceptualization, Methodology, Writing – review & editing, Supervision, Project administration, Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Equations and description of the anisotropic parameters, hardening law and flow rule in enHill48 model

The anisotropic parameters ($F_\sigma, G_\sigma, H_\sigma, N_\sigma$) and (F_r, G_r, H_r, N_r) are calculated as below:

$$F_\sigma = \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_{90}^2(\bar{\epsilon}^p)} - 1 + \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_b^2(\bar{\epsilon}^p)}, \quad G_\sigma = 1 - \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_{90}^2(\bar{\epsilon}^p)} + \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_b^2(\bar{\epsilon}^p)}, \quad H_\sigma = 1 + \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_{90}^2(\bar{\epsilon}^p)} - \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_b^2(\bar{\epsilon}^p)}, \quad N_\sigma = \frac{4 \cdot \sigma_0^2(\bar{\epsilon}^p)}{\sigma_{45}^2(\bar{\epsilon}^p)} - \frac{\sigma_0^2(\bar{\epsilon}^p)}{\sigma_b^2(\bar{\epsilon}^p)}, \quad (A1)$$

$$F_r = \frac{2 \cdot r_0(\bar{\epsilon}^p)}{r_{90}(\bar{\epsilon}^p) \cdot [1 + r_0(\bar{\epsilon}^p)]}, \quad G_r = \frac{2}{1 + r_0(\bar{\epsilon}^p)}, \quad H_r = \frac{2 \cdot r_0(\bar{\epsilon}^p)}{1 + r_0(\bar{\epsilon}^p)}, \quad N_r = \frac{2 \cdot [r_0(\bar{\epsilon}^p) + r_{90}(\bar{\epsilon}^p)] \cdot [r_{45}(\bar{\epsilon}^p) + 0.5]}{r_{90}(\bar{\epsilon}^p) \cdot [1 + r_0(\bar{\epsilon}^p)]} \quad (A2)$$

The reference flow curve $\sigma_V(\bar{\epsilon}^p)$ is obtained from the uniaxial tensile test along RD. The flow curves are described by the combined Swift and Voce hardening law, with hardening parameters ($A, \bar{\epsilon}_0, n, k_0, Q, \beta, \alpha$) calibrated for individual loading directions along RD, DD, TD, and biaxial tension. The evolution of the r-values along three loading directions obtained from uniaxial tensile tests is fitted by the Voce exponential law, with constants ($C_1 \sim C_3$) independently calibrated for each loading direction.

$$\sigma_V(\bar{\epsilon}^p) = \alpha \cdot A (\bar{\epsilon}_0 + \bar{\epsilon}^p)^n + (1 - \alpha) \cdot [k_0 + Q(1 - e^{-\beta \bar{\epsilon}^p})] \quad (A3)$$

$$r(\bar{\epsilon}^p) = C_1 + C_2(1 - e^{-C_3 \bar{\epsilon}^p}) \quad (A4)$$

The flow potential g controls the evolving strain components according to the non-associated flow rule with the equivalent plastic strain rate $\dot{\bar{\epsilon}}^p$, which is not equal to the non-negative plastic multiplier $\dot{\lambda}$ as conventional, and it reads:

$$\dot{\bar{\epsilon}}^p = \dot{\lambda} \frac{\partial g}{\partial \sigma} = \dot{\lambda} \frac{\partial \bar{\sigma}_r}{\partial \sigma} \quad (A5)$$

$$\dot{\bar{\epsilon}}^p = \dot{\lambda} \frac{\partial \bar{\sigma}_r}{\partial \sigma} \quad (A6)$$

Appendix B. Mesh optimization

The mesh size of the local deformed region has been optimized with a parametric study. The cost-effectiveness of the present meshing strategy is illustrated on NDBR6 with the plasticity model in Table B1 and Figure B1.

Table B11

The cost-effectiveness of different meshing strategies on NDBR6 with the plasticity model.

Mesh size in fracture zone (mm)	0.2	0.1	0.05	0.03	0.02
Number of elements	8,561	24,773	181,314	592,733	1,836,986
Size of ODB file (GB)	0.1	0.5	1.76	5.71	13.80
Computational time (CPU hours)	2	10	20	100	460

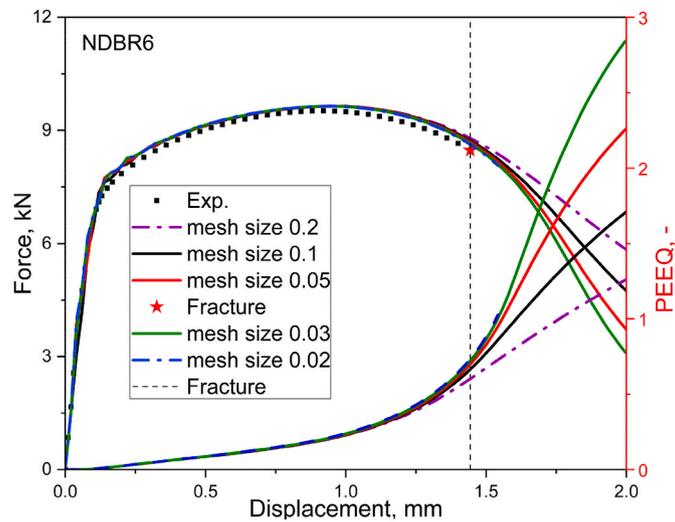


Fig. B1. The evolution of PEEQ value on NDBR6 with plasticity model with different mesh size.

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