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Bayesian Network Approach for Geotechnical Risk Assessment in Underground Mines

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Abstract

Mining at great depths gives rise to geotechnical hazards. Formal geotechnical risk assessment can help to forecast and to mitigate these hazards. While conventional probability methods provide a good background to carry out risk assessment work with variable and uncertain data, the probability of failure calculation becomes difficult as the number of variables increase or the available data is scarce. The aim of this paper is to demonstrate the decision making capabilities of Bayesian networks for the purpose of risk assessment by combining expert judgement and available data. The general structure of BN and ways to elicit probability of uncertain variables for risk assessment are presented. Roof fall frequency forecasting using parameter learning is demonstrated using 1,141 roof fall data across 12 coal mines in the USA. A hybrid approach of combining multiple probability distribution curves from historical data with expert opinion from empirical methods is proposed along with financial quantification of risk values. The BN method demonstrates that a proposed normal distribution curve is twice as likely to fit the observed data compared to the initial Poisson distribution. It is concluded that Bayesian network forms a good real time risk assessment tool by combining expert knowledge with available data.

Keywords	geotechnical risk; Bayesian networks; parameter learning; roof fall risk; incident forecasting; expert opinion models
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Dr. Georgios Boustras
Editor-in-Chief
Safety Science

October 6, 2017

Dear Dr. Boustras:

I am pleased to submit an original research article entitled Bayesian network approach for geotechnical risk assessment in underground mines for consideration for publication in Safety Science.

In this manuscript, we show the potential of using Bayesian network based graphical models in geotechnical risk assessment. Parameter learning capability of Bayesian networks in forecasting geotechnical incidents is shown by using roof fall data from 12 coal mines in USA. The manuscript further demonstrates the potential of combining expert opinion in lack of data to carry out risk assessment and incident forecasting. The results from the Bayesian network prediction are compared with conventional statistical methods to showcase the flexibility of graphical models in risk assessment.

We suggest reviewers who are familiar with the field of geotechnical risk assessment in the underground mining industry. Knowledge of empirical and probability based risk assessment methods may be useful in reviewing the paper as we compare the utility of different methods for carrying out geotechnical risk assessment in mines

This manuscript has not been published and is not under consideration for publication elsewhere. We have no conflicts of interest to disclose.

Thank you for your consideration!

Sincerely,

Ritesh Kumar Mishra
Doctoral Student, Department of Civil Engineering
Aalto University

Abstract

Mining at great depths gives rise to geotechnical hazards. Formal geotechnical risk assessment can help to forecast and to mitigate these hazards. While conventional probability methods provide a good background to carry out risk assessment work with variable and uncertain data, the probability of failure calculation becomes difficult as the number of variables increase or the available data is scarce. The aim of this paper is to demonstrate the decision making capabilities of Bayesian networks for the purpose of risk assessment by combining expert judgement and available data. The general structure of BN and ways to elicit probability of uncertain variables for risk assessment are presented. Roof fall frequency forecasting using parameter learning is demonstrated using 1,141 roof fall data across 12 coal mines in the USA. A hybrid approach of combining multiple probability distribution curves from historical data with expert opinion from empirical methods is proposed along with financial quantification of risk values. The BN method demonstrates that a proposed normal distribution curve is twice as likely to fit the observed data compared to the initial Poisson distribution. It is concluded that Bayesian network forms a good real time risk assessment tool by combining expert knowledge with available data.

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Bayesian network approach for geotechnical risk assessment in underground mines

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1. Introduction

Geotechnical incidents and accidents are common events resulting in property loss and bodily harm in the mining industry. As the demand of raw material increases to keep pace with the supply, mining is taking place at increasing depths reaching down to 4km and beyond. Mining at such depth increases the probability of geotechnical incidents because of the increase in *in situ* stresses. Variability of rock mass condition compounds the problem as operations can run into prior unknown geological structures. Mining at great depth also means that collecting geotechnical information through conventional methods such as core drilling is very expensive. Non-intrusive geophysical methods pose the limitation of range and accuracy. All the above reasons create the need for a formal geotechnical risk assessment methodology for underground mining. Risk assessment can help to identify vulnerable areas of the mine so mitigation measures can be planned in advance and employees can be made aware of the hazard. Proper risk assessment also helps justify additional expenses when compared with the financial cost of the risk. Mishra and Rinne (2015) have proposed that mining projects should be evaluated for their geotechnical risk levels through Geotechnical Risk Classification (GRC) from as early as pre-feasibility study and these values should be updated as the project progresses and more data becomes available.

The biggest challenge with risk assessment is the lack of suitable data to be used for the probability of failure assessment and lack of historical evidence in case of a new project. In such cases, the risk assessment often needs to be subjective and qualitative based on expert judgement. While qualitative assessment helps to carry out a quick risk assessment in absence of data, it can be very broad and vague and can be highly influenced by personal opinion and bias. The aim of this paper is to present the use of Bayesian networks (BN) as an alternative to existing risk assessment methods in underground mines by combining expert knowledge and available data. This paper discusses the use of BN as a hybrid approach to include a qualitative assessment from domain experts combined with data when available to build a risk management framework. This can be used not only to calculate the probability of failure but also to assist in incident investigation and to carry out scenario analysis for decision making.

2. Background

Risk and its definition have been discussed extensively in the past (Paté-Cornell and Dillon, 2006). Generally, risk is defined as a product of the likelihood of a hazard and the severity of the consequence if the hazard were to be realized. By extension of this definition, geotechnical risk (GR) can be defined as per Equation 1 below:

$$GR = GL \; X \; GS \qquad [1]$$

Where GL represents the likelihood of a geotechnical hazard while GS represents the severity of the geotechnical hazard. Formal risk assessment at work places for accident prevention is becoming increasingly common in mining industry owing to increased awareness of accident related costs and stringent guidelines set by local legislative bodies regarding injury to people and environment damage. The risk assessment tools used in mining, therefore, has been borrowed from similar industries such as construction, oil and gas, civil infrastructure. Mishra and Rinne (2014) have discussed that these popular risk assessment tools such as Work Place Risk Assessment and Control, Failure Mode and Effect analysis, Bow-Tie analysis should be called hazard identification tool as they cover how an accident can be broken down to its causes but gives little to no details on how the likelihood of these causes should be established. It was therefore proposed that risk assessment should be segregated into the following steps: identification of hazards, selection of risk assessment parameters, selection of risk assessment approach, consequence assessment, and risk representation.

Hazard identification involves breaking down a process being evaluated into smaller fragments and evaluating ways in which these processes can be affected adversely. Once the hazards have been identified, appropriate likelihood values need to be assigned to them. These values can either be qualitative ranging from high to low or quantitative with numeric values or probability distribution curves. While qualitative values can be assigned based on an expert's opinion, quantitative values are derived from historical occurrences, monitoring of site, lab tests etc. This is a challenge in mining and especially in underground excavations because collecting data through drilling prior to project commencement is an expensive exercise and extrapolation of sample data always leaves room for error. Even if the presence of a challenging geological structure is known, uncertainties regarding its design and extent may have an impact on the risk. This becomes more evident when dealing with a non-homogenous rock type. These uncertainties can be grouped into spatial variability, measurement errors, model uncertainty, and uncertainty due to omissions (Einstein and Baecher, 1983).

Once a decision is made between qualitative and quantitative parameters, the next step is to calculate the risk values. This calculation can be grouped into four broad approaches. For comparative explanation, a scenario of reinforcement design to prevent a wedge fall is considered here. The first approach is a prescriptive or empirical method. Prescriptive methods rely on past case studies to establish a correlation between observable mine parameters and hazard being prevented. Barton's Q system (Barton, 1988) and Mathews stope stability graph method (Mathews et al., 1981) are examples of prescriptive methods. While these methods prescribe a suitable reinforcement design to prevent roof/wall/stope collapse, they do not assign probability of failure values to the mine area where the prescription is used. Another method is Roof Fall Risk Index (RFRI) (Iannacchione et al., 2007a) which assigns an index value similar to Q based on observable parameters and converts them into qualitative risk ranking from highly likely to very unlikely. Empirical methods are not inherently designed to take geotechnical variability into account and therefore are difficult to use when the observed parameters have a range of possible values.

The second approach to assigning risk values is called the deterministic or analytical method. In order to design a reinforcement design against a wedge collapse, the deterministic method will require that the wedge size and weight and the support capacity of the roof bolt are known. The factor of safety (FOS) for the roof reinforcement can then be calculated using Equation 2 below:

$$FOS = \frac{C}{W \times \square_s \times \square_g} \qquad [2]$$

Where C is the bolt capacity and W is the block weight while $\square_s$ and $\square_g$ are partial safety factors for bolt capacity and block weight respectively. For estimated block weight is 280 KN, measured bolt capacity is 220 KN, $\square_s$ and $\square_g$ of 1.1 and 1.35 respectively will result in a FOS of 1.06 if 2 bolts are used to support the wedge. For a true deterministic analysis, the FOS will either need to be individually calculated for all known wedges or the reinforcement design will have to be carried out for the largest possible block size. While the first method requires extensive data collection to evaluate every possible block weight, the second method leads to an ultra-conservative reinforcement design which is expensive. Additionally, the FOS values do not give the probability of failure (POF) but prescribed tables can be used to compare FOS and POF (Frank, 2004).

The third approach to likelihood estimation is the use of probabilistic methods. Probabilistic methods give the advantage of using probability distribution curves instead of single values to arrive at a probability of failure. This works well, when limited sample data is available and it needs to be extended to make decisions for a larger population. Use of classical probabilistic analysis in geotechnical engineering has been covered extensively in the past 40 years (Baecher and Christian, 2005). This method has been used in the past to predict mine subsidence (Stewart and O'Rourke, 2008), pillar collapse in coal mines (Galvin, Hebblewhite and Salamon, 1999), roof fall in underground mines (Duzgun and Einstein, 2004) and several other engineering applications (Paté-Cornell, 2007). In case of the above mentioned wedge collapse example, instead of using an individual value for block weight and bolt capacity, the probability distribution curve for both the parameters can be defined based on collected data. Table 1 shows the calculated probability of failure in three scenarios where the block weight and bolt capacity are assumed to be truncated normal distribution with same lower bound, upper bound and mean but different variance. The block is assumed to fail when block weight exceeds bolt capacity.

Scenario	Parameter	Lower Bound	Upper Bound	Mean	Variance	Probability of Failure with 2 Bolts
1	Block Weight	260 KN	320 KN	280 KN	10	0.0%
	Bolt Capacity	210 KN	230 KN	220 KN	1	
2	Block Weight	260 KN	320 KN	280 KN	500	25.4%
	Bolt Capacity	210 KN	230 KN	220 KN	1	
3	Block Weight	260 KN	320 KN	280 KN	1000	31.7%
	Bolt Capacity	210 KN	230 KN	220 KN	1	

Table 1: Impact of variance on probability of failure (POF)

As can be seen from the Table 1, the variability in data as accounted by variance has a large impact on the probability of failure for the same mean values and probabilistic methods offer the advantage of accommodating this in the risk assessment. Additionally, probabilistic methods directly give the probability of failure values. The disadvantage of the conventional probabilistic method is that the complexity of the model grows exponentially as the number of uncertain variables grows beyond 2 (Matarawi and Harrison, 2017).

The fourth approach to likelihood estimation is the use of graphical models to carry out a probabilistic risk assessment. Graphical models offer the advantage of conventional risk assessment along with the freedom to have a large number of uncertain variables in the risk assessment model. Graphical models can be further subdivided into Artificial Neural Networks (ANN) and Bayesian Networks (BN). The key difference between these two models is, that Bayesian networks use pre-defined nodes to represent uncertain variables while ANN assigns a variable type to its nodes depending on the data set (Yegnanarayana, 2009). This paper focuses on the use of BN to carry out risk assessment as ANN requires a large amount of existing data to train the model which is often unavailable in case of geotechnical incidents. Bayesian networks (BN) are decision models which combine expert judgement and available data with the use of conditional probability tables to arrive at failure probabilities (Fenton and Neil, 2012). Bayesian networks have been used in the past in dam risk assessment (Smith, 2006), tunnel risk assessment (Sousa and Einstein, 2007, Spackova and Straub, 2011), and modeling uncertainties in rock-fall hazards (Straub and Schubert, 2008) This paper focuses on the parameter learning capability of Bayesian Networks to learn from historical data and forecast future occurrences by using roof fall in coal mines as an example. It further combines parameter learning with expert judgement in the form of Roof Fall Risk Index (RFRI) to form a hybrid risk assessment model.

For the wedge collapse example, the factor of safety (FOS) can be expanded to incorporate additional variables as shown in Equation 3 below:

$$FOS = \frac{f_{yk} \times \frac{A_b}{Y_s}}{Y_g \times \left(\frac{1}{3}\right) \times s^2 \times h \times \rho \times g} \quad [3]$$

Where f_{yk} is steel strength, A_b is the area of bolt, Ys is the partial safety factor for steel strength, s and h are base length and height of the wedge respectively, ρ is the density of the rock, g is the acceleration due to gravity and Y_g is the partial safety factor for block weight. Figure 1 shows the probability of failure calculation using the Bayesian network with s, h, and f_{yk} as uncertain variables and FOS using Equation 3. Table 2 shows the distribution properties of the variables used. The details of how to construct a BN and elicit probability values are discussed further in the paper.

Parameter	Distribution	Lower Bound	Upper Bound	Interval	Mean	Variance
Steel Strength (fyk)	Normal	490 MPa	550 MPa	10 MPa	530 MPa	100
Wedge base length (s)	Uniform	3.1 m	4 m	0.1 m		
Wedge Height (h)	Uniform	2.6 m	3 m	0.1 m		

Table 2: Probability distribution parameters used for factors affecting wedge collapse

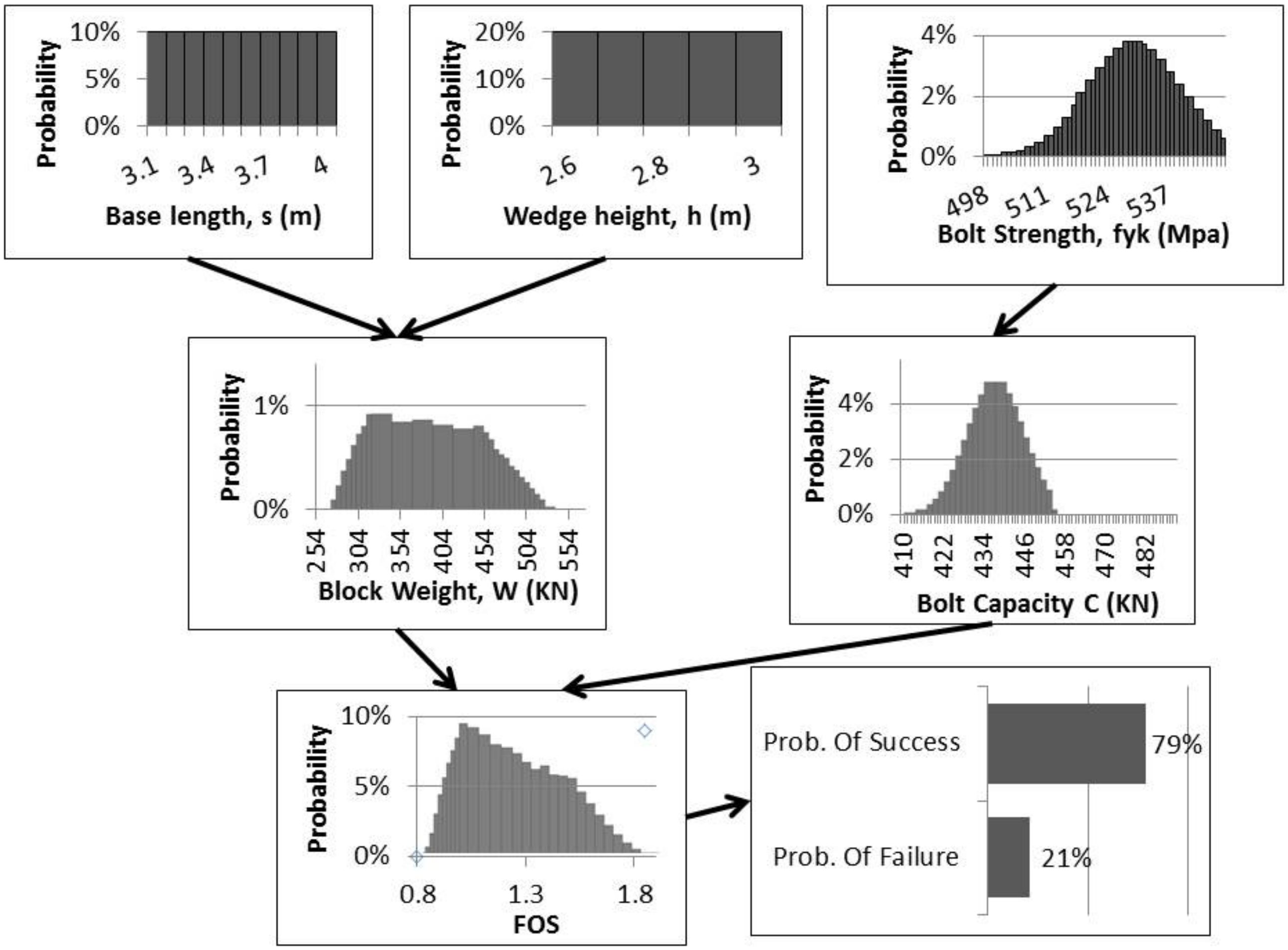

Figure 1: Bayesian Network model to forecast wedge failure using Equation 3

This BN can be expanded to include other variables such as rock density and bolt diameter to carry out a 'what if' scenario analysis. BN can also learn probability distributions using parameter learning from available sample data to carry out fully automated risk assessments (Fenton and Neil, 2012). Where numerical data is unavailable, subjective opinion of experts can be incorporated as nodes in the network. The advantage of a BN based risk assessment is that it can evolve from qualitative assessment to quantitative assessment as more data becomes available and can, therefore, provide a framework for a mine wide risk assessment from pre–feasibility stage to operations stage. Unlike probabilistic methods, the probability distribution parameters don't need to be defined but the network can learn them from available data and improve over time (Fenton and Neil, 2012). Due to the flexibility of BNs to work with varying extent of data, modeling uncertainties in the data and mapping failure processes, it provides a platform for geotechnical risk assessment in deep mines where data acquisition through invasive methods such as core drilling is difficult and expensive.

Based on the above comparison of likelihood estimation, the suitability of each method can be grouped based on the extent of current data and historical data available. While current data represents knowledge available at present such as values for roof convergence, information about geological structures, rock strength etc. historical data represents the information regarding correlation between geotechnical parameters and geotechnical risk obtained through case studies and incident investigations in the past. This comparison has been shown in Figure 2. The capability of BN to incorporate expert makes it suitable for use when limited current and historical data is available. However, its capability to learn from data and update prior assumptions make it useful as the extent of data available increases. BN can, therefore, be used for risk assessments for the entire spectrum of current and historical data availability.

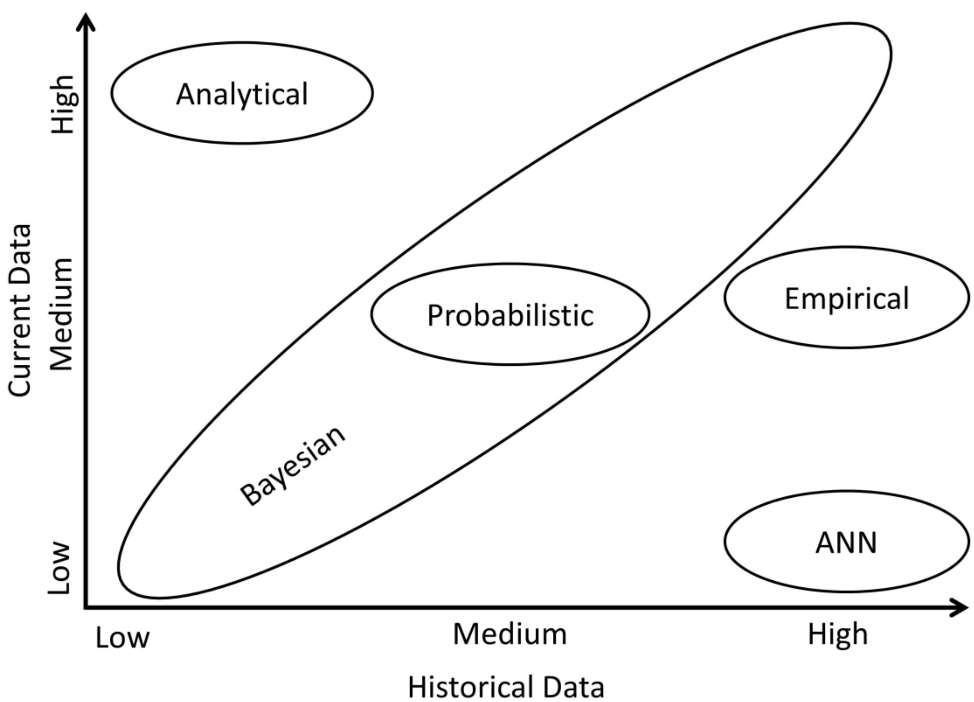

Figure 2: Comparison of likelihood estimation methods with respect to data availability

3. Risk modeling using Bayesian networks

This section describes the process of drawing a Bayesian network for geotechnical risk assessment, eliciting node probabilities and using the risk model to carry out incident investigation and arrive at the most likely cause. Bayesian Networks (BN) or Bayesian belief networks are based on Bayes theorem as developed by Thomas Bayes (Bayes, Price and Canton, 1763). It describes the conditional relationship between 2 or more variables using probability as defined in Equation 4

$$P(A|B) = [P(B|A) \times P(A)]/P(B) \tag{4}$$

Where P(A) and P(B) are the probability of event A and B occurring while P(A|B) and P(B|A) are conditional probabilities of A occurring given B has already happened and B occurring when A has already happened respectively. A different version of the above theorem can be expressed in terms of hypothesis and evidence as shown in Equation 5.

$$P(H_i|E) = [P(E|H_i) \times P(H_i)]/[\sum_i P(E|H_i) \times P(H_i)] \tag{5}$$

Where $P(H_i|E)$ is called the posterior probability which is the probability of a hypothesis (H) being true and i representing the different states for the Hypothesis possible given a particular evidence (E). $P(E|H_i)$ is called the likelihood which is the probability of observing an evidence (E) if the hypothesis were true. $P(H_i)$ is called the priori which is the prior belief in the hypothesis. A prior belief can be an expert opinion such as the probability of strain burst given the rock type and local stress. Prior belief can also be a statistical summary such as average roof fall per year given the historical roof fall frequencies. P(E) is called the marginal likelihood which represents the prevalence of the evidence in the base population expressed through product rule as $P(E|H_i) \times P(H_i)$. This process of updating our prior belief in a hypothesis in light of new evidence is known as Bayesian inference. Bayesian inference, therefore, relies on both data and subjective assessment to make decisions and this makes it a better fit for mining related assessments given the limited amount of real time data that is available and the complex failure mechanisms behind geotechnical incidents.

3.1. Structure of a Bayesian Network

A Bayesian network (BN) is an explicit description of the direct dependencies between a set of variables. This description is in the form of a directed acyclic graph (DAG) and a set of node probability tables (Fenton and Neil, 2012). Bayesian networks consist of a set of nodes which represent variables incorporated in the model. These variables are connected with arcs which generally indicate the direction of cause to effect. This direction of the arc depends on what the risk analyst is trying to model. Figure 3 shows a decision making model using a Bayesian network which has been modified from the work done by Smith (2006). For the rock burst example, it is assumed that the two primary causes are rock type and local stress. The directions of the arrows indicate that rock type and local stress cause rock burst making them the parent node and rock burst the child node. The relationship between the parent and child nodes is defined by conditional probability tables which have been discussed later in the paper. Before the BN is solved, prior belief or knowledge is used to update the parent nodes. In case of the parent nodes rock type and local stress, this can be current best understanding of the rock type and local stress distribution. This network can now be used to forecast rock burst probability with current prior knowledge. The parent and child nodes along with their relationship arcs form the causal model. The use of prior knowledge and conditional probability tables to carry out risk assessment forms the risk assessment Bayesian network. The completed BN can now be updated with actual evidence from the mine. For example, if a rock burst happens and the rock type is known, the model can back calculate the likely local stress which may have caused the incident in light of the new evidence available. Use of evidence to update existing knowledge forms the decision making model.

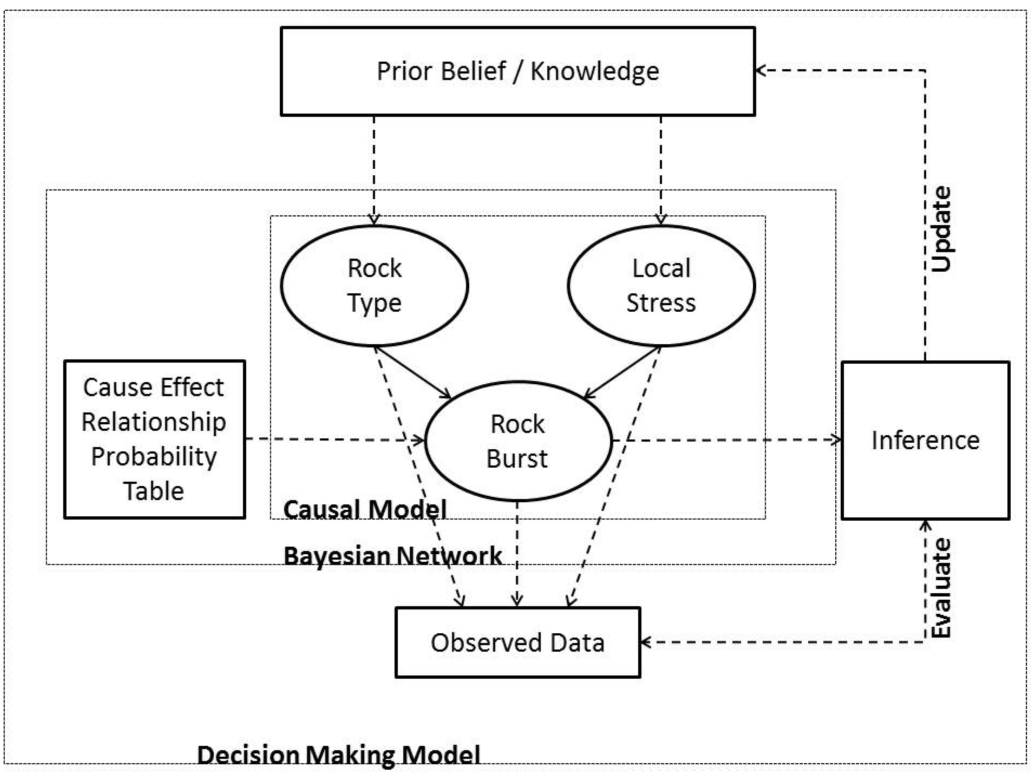

Figure 3: Decision making model using the Bayesian network. Modified from Smith (2006)

3.2. Eliciting node probability

Once the Bayesian network is constructed by mapping the connected variable, the next step is to define the probabilistic relationship between the parent and the child nodes. This is done by the means of node probability tables (NPT). Node probability tables depend on the various states in which a node can exist. The decision to choose the appropriate number of states for the nodes depends on the granularity of the model, the extent of data available and whether the node in question is the final decision node or not. The node states for parent nodes can be qualitative such as ‘brittle’ and ‘not brittle’ in case of the ‘Rock Type’ node in Figure 3. BN can also use numeric nodes in the form of probability density and probability mass functions when sufficient data is available to construct the appropriate statistical distribution. For the parent nodes ‘Rock Type’ and ‘Local Stress’ in Figure 3, the assumed qualitative node states and node state prior probabilities are shown in Table 3:

Rock Type (R)	**Prior Probability**
Brittle	10%
Not Brittle	90%
Local Stress (S)	**Prior Probability**
< 90 MPa	20%
90 - 120 MPa	60%
> 120 MPa	20%

Table 3: Example of a node probability table and a labelled node showing rock type and local stress state probabilities

For child nodes with one or more parents, the relationship between child and parent node states is defined using a conditional probability table. Conditional probability table needs to consider all possible state combinations of parent and child nodes. In order to define the relationship between nodes ‘Rock Burst’, ‘Rock Type’ and ‘Local Stress’, some prior understanding of the relationship is required. Rock burst is a failure mode, where deformation energy is stored in the rock and then released rapidly resulting in local damage. The ability to store energy is reflected by how much induced stresses the rock is able to sustain. After the capacity is exceeded, the post-critical behavior can be categorized to class I ductile rock or class II brittle rock (Wawersik and Fairhurst, 1970). Rock bursting does not occur in ductile rock mass or fractured rock mass. Field observations of rock bursting should be used if available. If no suitable data is available, then an expert opinion can be used (Figure 4).

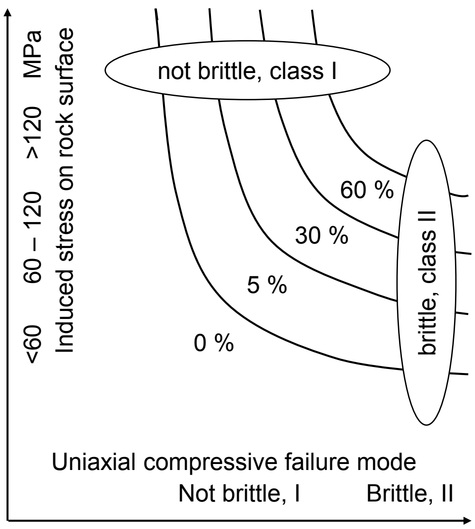

Figure 4: The spalling probability as a function of failure mode and induced rock stress. Modified from Diederichs (2007)

If a qualitative Boolean state for node 'Rock Burst' is assumed, the conditional probability of 'Rock Burst' is shown in Table 4.

Rock Burst Prior probability	Local Stress	< 60 MPa		60 - 120 MPa		> 120 MPa	
	Rock Type	Brittle	Not Brittle	Brittle	Not Brittle	Brittle	Not Brittle
	TRUE	5%	0%	30%	5%	60%	30%
	FALSE	95%	100%	70%	95%	40%	70%

Table 4: Prior belief of rock burst probability conditional on rock type and local stress

Each cell in the table defines a proposed conditional probability. It assumes that when rock type is brittle and the local stresses exceed 120 MPa then there is a 60% chance that there will be a rock burst event for a unit time interval. This time interval can range from weeks to years depending on the extent of risk assessment. Defining a node probability table manually as shown in Table 4 can become challenging as the number of parents for a child node grows and the number of possible states for all the nodes grow. Complex causal processes can often have more than three parent nodes causing the event, in which case the number of cells in the NPT grows. When using a BN processing software, these problems can be dealt with by using comparative expressions as shown in Equation 6. The actual syntax used for comparative equations will differ for each software (Murphy, 2017).

If "Rock type" = "Brittle" and "Local Stress" > 100 MPa $then$ "Rock Burst" = "True" $else$
"Rock Burst = False"

[6]

3.3. Solving Bayesian network and inferencing

A completed Bayesian network is solved using Bayes law as per Equation 3. For BN in Figure 3 with variable Rock Burst (RB), Local Stress (S) and Rock Type (R), the prior probability of a rock burst occurring is solved through the process or marginalization using Equation 7.

$$P(RB = True) = \sum_{R,S} P(RB = True|R,S) \times P(R) \times P(S) \quad [7]$$

The expanded version of Equation 5 for solving the marginal probability is shown in Equation 8

$$\begin{aligned}P(RB = True) = {} & P(RB = True \mid R = \text{"}Brittle\text{"},\ S = \text{"}<90\ MPa\text{"}) \times P(R = \text{"}Brittle\text{"}) \times P(S = \text{"}<90\ MPa\text{"}) \\ & + P(RB = True \mid R = \text{"}Brittle\text{"},\ S = \text{"}90-120\ MPa\text{"}) \times P(R = \text{"}Brittle\text{"}) \times P(S = \text{"}90-120\ MPa\text{"}) + P \\ & (RB = True \mid R = \text{"}Brittle\text{"},\ S = \text{"}>120\ MPa\text{"}) \times P(R = \text{"}Brittle\text{"}) \times P(S = \text{"}>120\ MPa\text{"}) + \\ & (RB = True \mid R = \text{"}Not\ Brittle\text{"},\ S = \text{"}<90\ MPa\text{"}) \times P(R = \text{"}Not\ Brittle\text{"}) \times P(S = \text{"}<90\ MPa\text{"}) + P \\ & (RB = True \mid R = \text{"}Not\ Brittle\text{"},\ S = \text{"}90-120\ MPa\text{"}) \times P(R = \text{"}Not\ Brittle\text{"}) \times P(S = \text{"}90-120\ MPa\text{"}) \\ & + P(RB = True \mid R = \text{"}Not\ Brittle\text{"},\ S = \text{"}>120\ MPa\text{"}) \times P(R = \text{"}Not\ Brittle\text{"}) \times P(S = \text{"}>120\ MPa\text{"})\end{aligned}$$

[8]

Solving Equation 6 with values from Table 4 gives the probability of rock burst at 10%. The interval of this rock burst is the same as the unit time interval assumed when defining the conditional probability. This BN can now be used for decision making by updating it with observed data. If for a given region of the mine it is confirmed that the rock type is brittle then the equation is solved using Equation 9 to give revised probability of failure at 31 %.

$$P(RB = True|R = \text{"}Brittle\text{"}) = \sum_{R,S} P(RB = True|R = \text{"}Brittle\text{"},S) \times P(R = \text{"}Brittle\text{"}) \times P(S) \quad [9]$$

As is evident from Equation 8, manual calculation of probabilities of BN nodes becomes complex and time consuming with more nodes. To counter this problem and to take advantage of better computing power, complex Bayesian networks can be solved using software available both under commercial and open source license. This paper uses AgenaRisk - Version 7 software to construct, calculate and carry out inference using Bayesian networks (Agena, 2017). The above mentioned BN is a simplified example of how a complete BN can be used to do quick risk assessments and draw inference for decision making as more data is presented to the model. The next section discusses how complex geotechnical risk in the mining industry can be modeled using Bayesian networks.

4. Roof fall risk forecasting using Bayesian Networks

This section will consider an example of roof fall risk estimation using real data as collected by MSHA in United Stated for roof fall risk in coal mines. The data collected is between a period of 1979 to 1997 across 12 anonymized underground mines in the Appalachian region (Anon, 2000). This data has been analyzed in the past using curve fitting the accident frequency to an appropriate probabilistic distribution to forecast failure (Duzgun and Einstein, 2004, Einstein, 1997). Table 5 shows the number of annual incidents in Mine ID 4601816.

Roof fall in Mine ID 4601816			
Year	Number of Roof Fall events	Year	Number of Roof Fall events
1	7	11	3
2	3	12	10
3	7	13	6
4	1	14	5
5	8	15	8
6	10	16	3
7	6	17	5
8	7	18	3
9	1	19	3
10	12		

Table 5: History of annual roof fall in Mine ID 4601816 (Anon, 2000)

Duzgun and Einstein (2004) used exponential distribution for the time between failures and Poisson distribution for the annual roof fall frequency (NOF) to fit the roof fall data for roof fall prediction. Poisson distribution is a one parameter probability distribution as shown in Equation 10 for annual roof fall frequency.

$$P(NOF) = e^{-\lambda} \times (\lambda^{NOF} / NOF!) \quad [10]$$

Where λ is the average roof fall per year and NOF is the annual roof fall frequency whose probability of occurrence is being evaluated. The Poisson parameter λ for the roof fall frequency for the above mentioned mine was 5.68 using data from Table 5. The classical approach to risk assessment is to use one value of λ to plot the probability distribution. Any probability distribution curve drawn using a single value for a statistical parameter is referred as the classical approach in this paper. The probability distribution of roof fall frequency using Equation 10 is shown in Figure 5 with annual roof fall frequency (NOF) in the x-axis and probability density (PD) in the y-axis. The probability of n number of roof falls can then be read from this distribution.

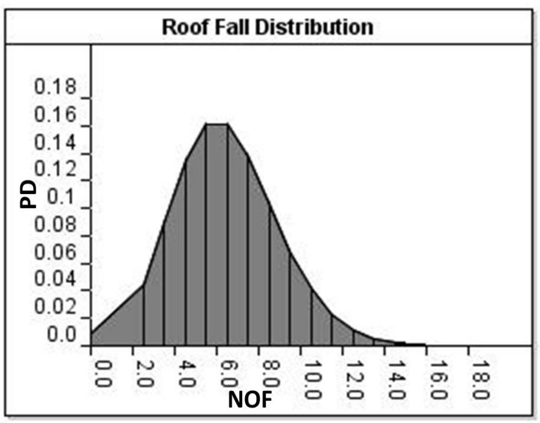

Figure 5: Annual roof fall frequency (NOF) of Mine ID 4601816 shown as a Poisson distribution with λ = 5.68

The Bayesian approach to solving this problem is by considering the λ parameter of the Poisson distribution as a variable itself. Parameter learning capabilities of BN can then be used to learn the lambda parameters from the population. In this BN, the parent node is λ, which was assigned a prior uniform probability of occurring between 0 and 20 as shown by the lambda node in Figure 6. Assuming a uniform probability for lambda implies that any value between 0 and 20 roof falls per year is equally likely for the Mine ID 4601816. The child nodes to this parent are the number of roof failures occurring in different years with annual roof fall frequency in the x-axis and probability distribution (PD) in the y-axis. The conditional probability between lambda and roof fall node is defined by the Poisson equation as shown in Equation 10 Without entering any existing evidence of past roof falls, the roof fall forecast for n number of roof falls will be the same for different years as shown in Figure 6 for roof fall in year 1 (Y1) and year 8 (Y8).

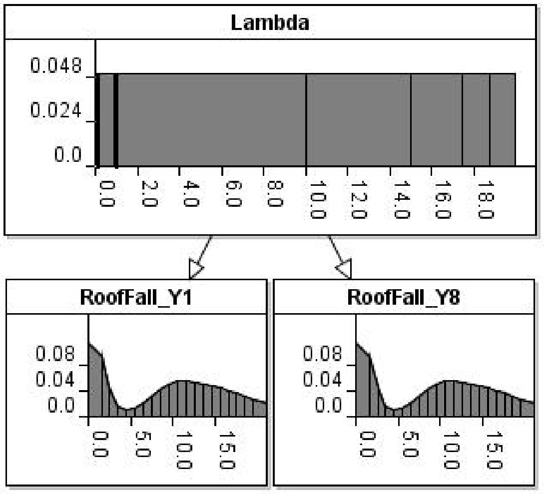

Figure 6: Bayesian network showing relationship between Poisson parameter lambda and roof fall frequency

Figure 7 shows a complete parameter learning BN. The BN was drawn by first creating a parent node lambda which has a uniform prior probability between 0 and 20. 19 child nodes were added to the parent node representing the roof fall in each of the 19 years in mine ID 4601816. The relationship between child node roof fall and parent node lambda was defined based on Equation 10 where λ is obtained from the parent node. An additional roof fall node was created named 'Predicted Roof Fall Frequency'. The roof fall observed in each of the 19 years was entered as evidence in each of the 19 nodes shown as 'Scenario' in Figure 7. The BN was then run to revise the λ distribution from the prior uniform distribution to a new distribution which best fits the entered evidence. The revised λ distribution was then used to obtain the distribution in 'Predicted Roof Fall Frequency Node' as shown in Figure 7. As the evidence for year 20 becomes available, it can then be added as an additional child node which will then revise both the λ and predicted roof fall frequency distribution in light of additional evidence.

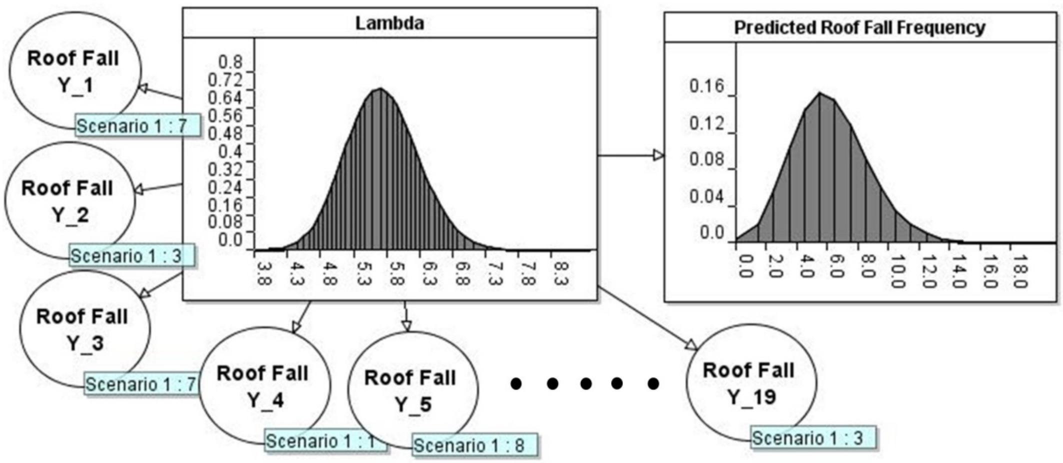

Figure 7: Parameter learning BN to learn lambda from observed roof falls

When comparing the Bayesian distribution with the classical method, the Bayesian prediction is nearly identical. The difference, however, becomes more evident when data from multiple mine sites is used to evaluate the roof fall frequency for a particular region as proposed in the classical method. The BN used for mine ID 4601816 was rerun by using mine data from 8 of the 12 mines which were located in Kentucky. The resulting Bayesian vs. classical comparison is shown in Figure 8 along with the relative frequency of the observed data.

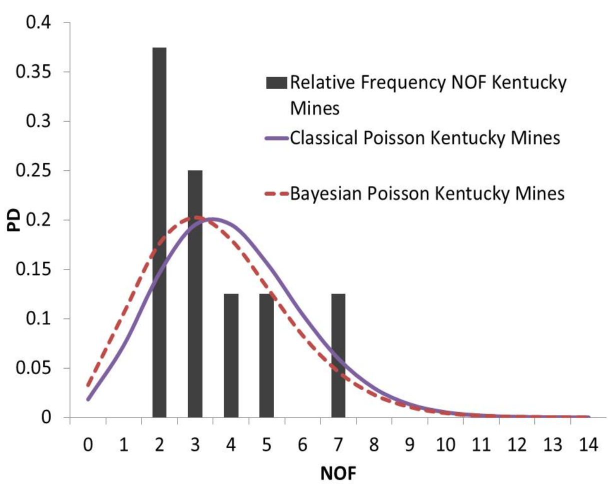

Figure 8: Classical Poisson vs. Bayesian Poisson comparison for annual roof fall (NOF) probability for all Kentucky Mines

Visual comparison of the Bayesian and classical Poisson distribution in Figure 8 indicates that the Bayesian distribution shown in dotted lines is closer to the observed data for NOFs 2, 3, 4 and 5 compared to the classical distribution and classical distribution is better for NOF 7. Overall, the Baysesian distribution is a better fit. A mathematical comparison of goodness of fit of competing distributions is discussed later in the paper. Although the Bayesian curve is a better fit compared to the classical method, when looked individually, both of them are a poor fit to the observed data when assuming a Poisson distribution. The data for Mine ID 460186 was then assumed to follow a normal distribution with mean and variance replacing λ as the parent node. The mean and variance parameters for the normal distribution were learned using parameter learning from the observed data for 19 years similar to the one carried out in Figure 7. The resulting Bayesian network is shown in Figure 9 while the comparison of classical Poisson distribution, Bayesian Normal distribution and the actual observed data is shown in Figure 10.

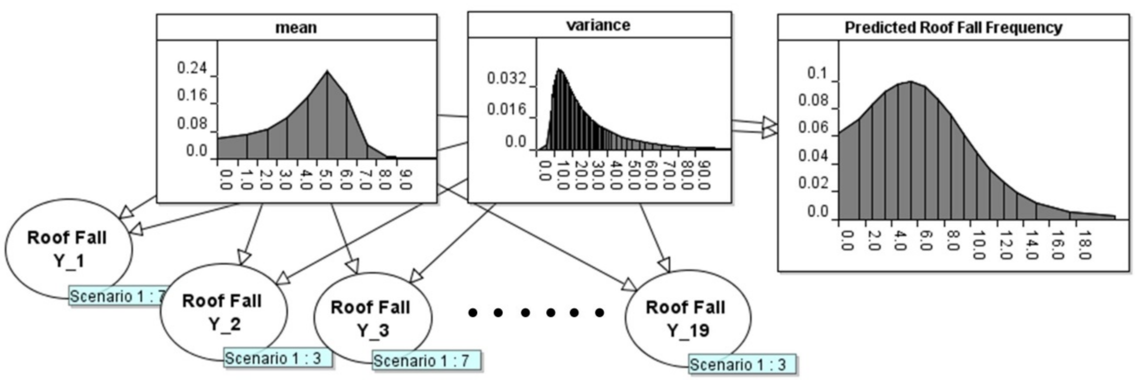

Figure 9: Parameter learning BN to learn normal distribution parameters from observed roof falls

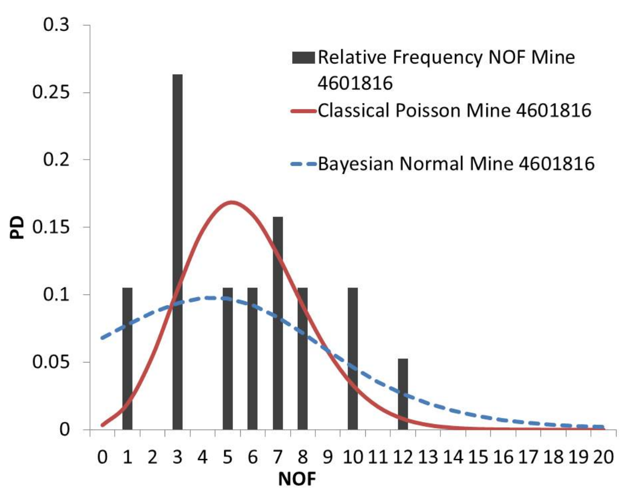

Figure 10: Classical Poisson vs. Bayesian normal comparison for roof fall probability in Mine ID 4601816

The error in the forecasted data compared to the actual observation was carried out using Equation 11 for both Poisson and Normal distribution.

$$E = \frac{\sqrt{(P_n - RF_n)^2}}{RF_n} \qquad [11]$$

Where E is the error in the forecasted data, P_n is the forecasted probability of n roof fall and RF_n is the relative frequency of n roof fall. Using Equation 9 the average error in forecast using classical Poisson distribution was 55%, which was reduced to 37% when using the Bayesian Normal distribution. There are mathematical tests such as the Chi-square goodness of fit test that can be carried out to evaluate how well a proposed distribution fits the data (Ang and Tang, 1984). These tests use statistical parameter such as mean which is not directly observed in the population but is inferred from a sample population. An alternative method using BN is to evaluate the likelihood of historical data (Evidence) given a probability distribution (Hypothesis). This can be modeled using a BN where competing probability distributions form the node states for a parent node. This BN is shown in Figure 11 where the parent node 'Competing Distribution' has two possible labelled nodes namely 'Truncated Normal' and 'Poisson'.

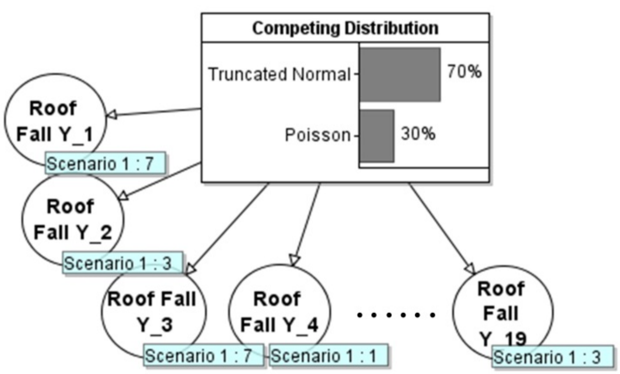

Figure 11: Goodness of fit test for competing distributions using BN

The child nodes are values observed in each of the 19 years for Mine ID 4601816. The prior probability of the parent node is considered 50% for each representing that both the distributions are equally likely to be correct. The NPT for the child node is conditional on the parent nodes as shown in Table 6.

Competing Distribution	Truncated Normal	Poisson
Y1	TNormal (5.75, 16.4, 0, 20)	Poisson(5.68)

Table 6: Node probability table showing relation between competing distributions and roof fall frequency

The values in parenthesis for TNormal represent mean, variance, lower bound and upper bound respectively for the truncated Normal Distribution, which has been obtained earlier from solving the Bayesian network in Figure 8. This BN is solved by entering roof fall values from year 1 to 19 resulting in a posterior probability of 70% for the Bayesian Normal distribution and 30% for the classical Poisson distribution. This implies that we have a 70% probability of observing the given data under the Bayesian Normal distribution compared to 30% under the classical Poisson distribution. In other words, the observed data is more than twice as likely under the Bayesian normal distribution as compared to the Poisson distribution and is, therefore, a better model for forecasting roof fall frequency in Mine ID 4601816. This goodness of fit test using the actual observed parameters was carried out for all the 12 mines and Bayesian normal distribution was found to be a better fit compared to Classical Poisson distribution in 7 of the 12 mines. Results of this analysis are shown in Table 7.

These results indicate that it is unlikely to have one distribution that is the best fit for roof fall data across different mines. One possible cause for this is that, failures across different mines can be triggered due to different geotechnical reasons even if the extent of human and design errors are discounted. Bayesian networks can overcome this problem by creating a hybrid distribution curve weighted on the goodness of fit test results and as discussed in the next section. The percentage probability values indicate which of the 2 distributions are more likely to fit the data better when compared with each other.

Mine ID	Poisson Parameter	Bayesian Normal Parameter		Probability of Observed Data	
	λ	Mean	Variance	Classical Poisson	Bayesian Normal

4601816	5.68	5.75	16.40	30%	70%
1502709	3.26	3.42	5.06	62%	38%
1503178	3.00	6.19	20.83	10%	90%
0100758	5.36	5.39	5.84	58%	42%
1502132	1.89	2.35	4.20	68%	32%
1502502	3.80	3.85	5.31	27%	73%
1504020	1.81	2.51	5.25	81%	19%
1512941	2.43	3.10	7.12	52%	48%
1513920	4.75	5.97	20.71	18%	82%
1514492	7.15	8.21	30.41	1%	99%
3600958	3.53	4.02	10.00	19%	81%
4605978	2.70	2.25	4.21	29%	71%

Table 7: Goodness of fit test results for classical Poisson and Bayesian Normal distribution for roof fall frequency

4.1. Combining models and use of expert judgement

Table 7 reveals that five of the mines are better suited to use the Poisson distribution while the other seven provide a better forecast with Bayesian normal distribution. Bayesian networks provide the flexibility of combining multiple models to create a weighted prediction where the weights to the distribution are provided by the likelihood of observing data for a given distribution. Results from the goodness of fit test as shown in Figure 11 can, therefore, be extended to forecast roof fall frequency in the coming years. Ideally, any probabilistic decision making and underlying distributions should evolve as more observations become available. These observations can be added to the BN to update the prior weights of multiple models, which in this case is a Poisson and Normal distribution, to do an improved forecast in the coming years. When making incident forecast using empirical data alone, one of the basic assumptions is that the prevailing conditions when the failure happened over the years are same. If not, they would become part of different populations and hence cannot be modeled under one distribution curve. This may be true when the failures are primarily induced by geological features which are uniform across the mine and all other factors that can contribute to the failure remain constant. This, however, may not be true across all the incidents in a single mine as roof falls can be triggered by other factors such as over mining of pillars, seismicity, blasting, poor reinforcement etc. In the case of the 12 mines considered here, even though 1,141 roof falls incident may seem like a large data set, it only comes out to 0.4 incident data per mine per month. Therefore if the extent of the forecast for a given mine is limited to a monthly period, the available data is very small to make a statistical prediction. This reduces even further if this needs to be evaluated for different parts of a mine, such as between an access drift and a working face.

Owing to the role of multiple factors which could trigger a collapse, and limited data on actual incidents, a hybrid BN model where expert opinion is combined with empirical data is better suited for incident forecasting. National Institute of Occupational Safety and Health, USA (NIOSH) developed an empirical roof fall risk assessment method using various observable parameters of the mine to do an incident forecast under Roof Fall Risk Index (RFRI) (Iannacchione et al., 2007a). These RFRI parameters can be combined to the existing BN to update an incident forecast. In order to account for the consequences of an incident as per Equation 1, each roof fall incident can be assigned a financial loss value. These values can differ from mine to mine depending on legislation, the extent of work stoppage, tangible and intangible losses etc. (Blumenstein et al., 2011). In case of these 12 mines, an average cost per incident can be obtained from historical data. For representation, it is assumed that the average roof fall incident costs €100,000 per event in Mine ID 4601816.

Expert assessment of roof fall probability under RFRI based on mining parameters has been divided into 6 categories which have been further sub divided in to subcategories (Iannacchione et al., 2007b). To keep the complexity of the BN low, the 6 categories considered are geologic factors, mining induced failure, roof profile, moisture factor, microseismic clustering, and roof deformation. In order to maintain uniformity between data driven and expert model, the RFRI values ranging from 0 to 146 have been instead converted to percentage values from 0 to 100 while incorporating the prescribed weights for each parameter as per RFRI guidelines. The inspection based parameters have ranked nodes on a three level scale from low to high. In monitoring parameters, microseismic clustering is evaluated as a Boolean node of either being present or absent while roof deformation is a labelled node with node states “No Deformation”, “Constant Deformation” and “Accelerated deformation”. RFRI method assumes that lack of deformation and microseismic clustering skews probability of roof collapse slightly to the lower side but their presence heavily skews the probability of failures to the higher side. This was incorporated in the BN using a comparative expression. A combined model of this nature can be designed as a ‘Multiobject Bayesian Network Model’ where elements of the model can be grouped into smaller components for better management (Fenton and Neil, 2012). In this case, the BN can be divided into roof fall frequency forecasted from data and roof fall frequency forecasted from expert judgement. These 2 models can then be combined to do a hybrid forecast. Data driven forecast can be further modeled individually for Bayesian and classical approaches. While data driven model uses actual incident data from Mine ID 4601816, the node values in the expert model have been assumed. Figure 12 shows the hybrid model along with their marginal prior probabilities.

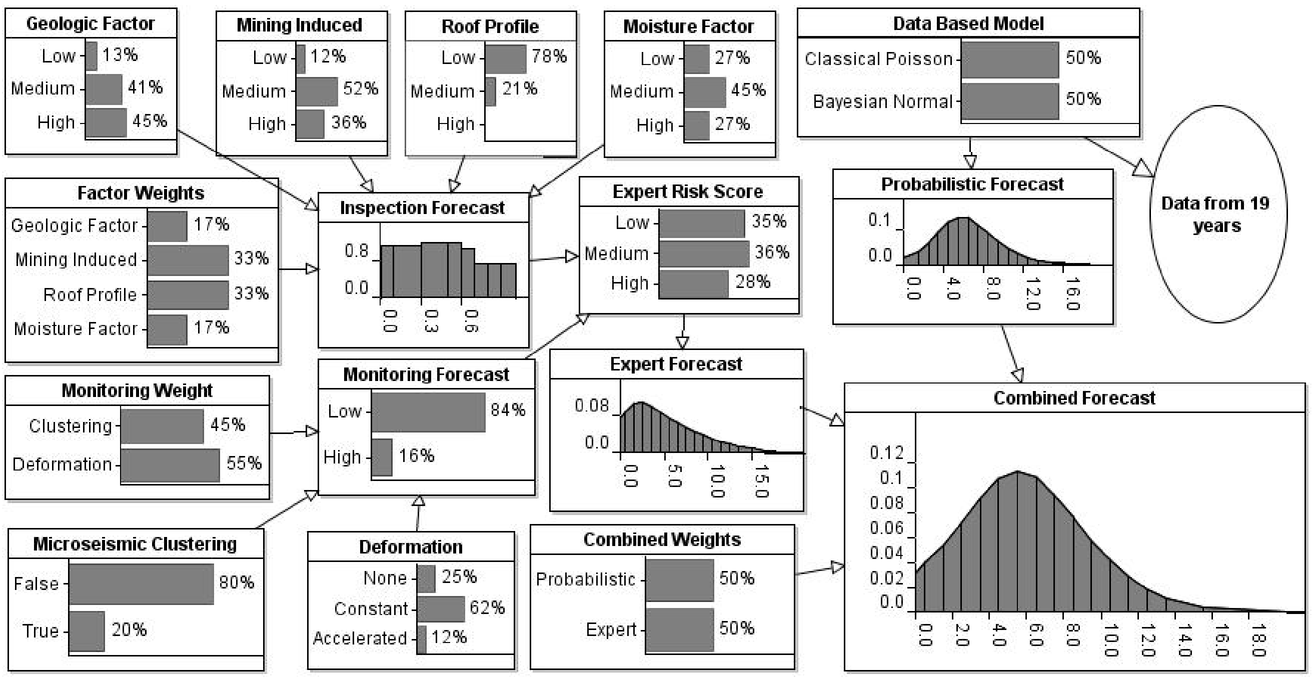

Figure 12: Hybrid BN combining observed data and RFRI method to forecast roof fall probability

This model can now be used by the mine to update as more data becomes available to revise the annual roof fall frequency of the mine. If the marginal probabilities values (Prob.) are multiplied with their corresponding roof fall frequency (NOF) and the average cost per roof fall of €100,000, the resulting value represents current level of financial risk in the mine. These values are shown in Table 8. The expected cost of roof failures is €518,000. It can be used as a reference cost to whether this can be afforded by plotting the quantified risk on an F-N diagram (Mishra et al., 2017). This value also helps justify expenses towards mitigating roof fall risks.

NOF	Prob.	Cost	NOF	Prob.	Cost
0	12%	€ 0	10	4%	€ 41,407
1	7%	€ 7,448	11	3%	€ 33,607
2	9%	€ 17,363	12	2%	€ 26,231
3	9%	€ 28,381	13	2%	€ 19,715
4	10%	€ 39,402	14	2%	€ 23,506

5	10%	€ 49,014	16	0%	€ 5,597
6	9%	€ 55,377	17	1%	€ 10,467
7	8%	€ 57,210	18	0%	€ 0
8	7%	€ 54,615	19	0%	€ 0
9	5%	€ 48,829	20	0%	€ 0
Total Risk Cost			**€ 518,169**		

Table 7: Financial risk quantification using roof fall probabilities

4.2. Bayesian networks for real time risk management

Bayesian networks can work with varying amount of data which makes it a good tool to carry out real time risk assessment in mine sites. Bayesian networks can either be integrated with the existing information management system on sites or strategic instrumentations and monitoring can be implemented once the Bayesian model has been defined. The process of incorporating real time risk management using BNs is shown in Figure 13. The first step, define a causal model, is to define the best understanding of the failure mechanism. The nodes are then divided into numeric and non-numeric nodes. For all the non-numeric nodes, appropriate numbers of node states are selected and a prior probability is assigned to them. For the numeric nodes, the extent and interval of data acquisition are evaluated. If a numeric node takes intermittent data, its prior probability can either be assumed to fit one of the probability distributions with pre-defined parameters or the parameters can be learnt using induction idiom from observed variables. For example, if Barton's Q value (Barton, 1988) is used as a node to represent rock mass competency, it would be an intermittent data node with a probability distribution. A continuous data node deals with parameters which change in shorter and unexpected intervals and may therefore warrant a continuous measurement for risk management. Roof convergence measurement is one such node when dealing with roof collapse. If the mine site has installed extensometers, readings from it can be directly fed into the roof convergence node as evidence to update the overall roof collapse risk. If the data is not available in real time, subsequent cost benefit analysis needs to be carried out for type and extent of instrumentation. It is followed by defining conditional probabilities between child and parent nodes and marginal probabilities to estimate current risk levels.

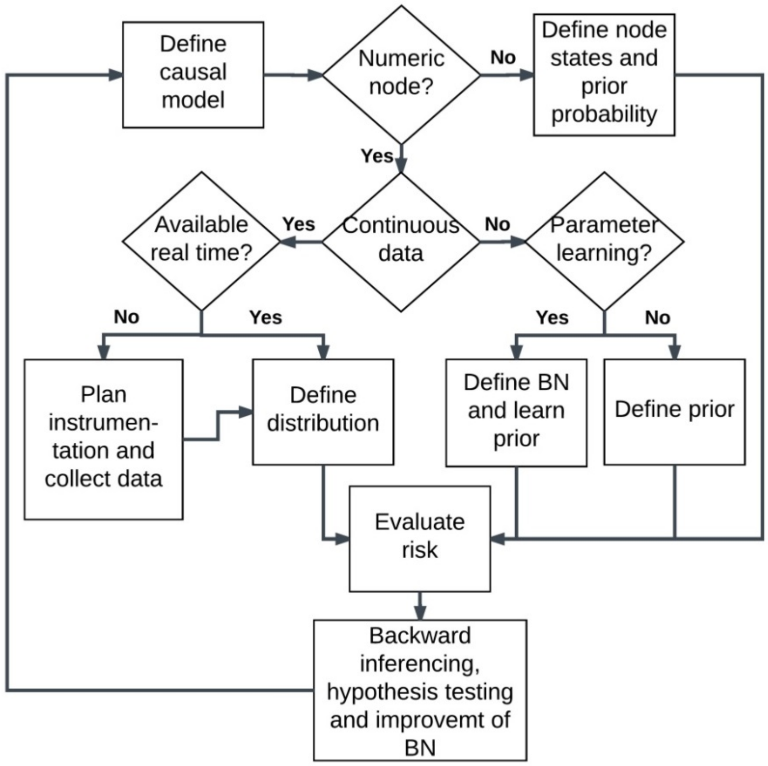

Figure 13: Real time risk management process flow using BN

Once the causal model is constructed with real time connections established to suitable nodes, it can be used to estimate and update risk levels in real time in sensitive areas. The completed model can be connected to a traffic light system of risk representation with green, orange and red showing low, moderate and high risk levels respectively. When evidence becomes available in the form of a realized event, backward inferencing can be used to update the probabilities of parent nodes. Hypothesis testing can then be carried out by comparing forecasted probabilities with actual evidence. Results from the hypothesis testing may point at missing causal factors which can then be investigated and included in the revised causal model. If the causal model is complete, prior probabilities can be updated in real time to improve the accuracy of the model.

4.3. Compatibility with the Observational Method

The Observational Method (Peck, 1969) in a modified form is one of the four allowed design methods in the Eurocode 7 for Geotechnical design. Variations of the observational method are used thorough the geotechnical field and mining (Moritz and Schubert, 2009, Miranda1a et al., 2015). Spross *et al.* (2014, 2016) have pointed out that the method lacks a society acceptable way of defining the probability of failure. Showing that the actual behavior will most likely be within the acceptable limits is difficult in most practical use cases. The Bayesian networks have the capability of addressing both these needs and they can function as the reliability framework for the observational method. Bayesian decision framework for the observational method is suggested by Spross and Johansson (2017), who used decision tree analyses. In actual use, cases with multiple influencing factors, such trees can become overly complicated. In this paper, we have presented alternative approach using Bayesian networks, which can handle complicated cases. One benefit is the use of continuous variables instead of fixed outcomes and explicit probabilities. This allows for a range of outcomes and associated probabilities, which can be integrated to obtain the expected risk level. In conclusions, the Bayesian network approach is compatible with the observational method and it is suggested to be used in a real case to gain experience of the real-life performance. Combining the Observational Method and reliability-based methods is also possible (Bjureland et al., 2017) to fulfill the requirement to analyze expected behavior.

5. Conclusions

Roof fall frequency forecasting using parameter learning was demonstrated using 1,141 roof fall data across 12 coal mines during a time interval of 19 years in the USA. A hybrid approach of combining multiple probability distribution curves from historical data with expert opinion from empirical methods is proposed along with financial quantification of risk values. The Bayesian Network (BN) approach demonstrated that the proposed normal distribution curve was twice as likely to fit the observed data compared to the initial Poisson distribution for the studied population of roof falls. The BN provides a flexible framework to carry out a risk assessment from none to scarce to abundant amount of measured data available.

A hybrid approach to include a qualitative assessment from domain experts combined with measured/observed data when available to build a robust risk management framework was demonstrated. The biggest challenge with risk assessment is the lack of suitable data to be used for the probability of failure assessment and lack of historical evidence in case of a new project. In such cases, the risk assessment often needs to be subjective and qualitative based on expert judgement. Subjective opinion of experts can be incorporated as nodes in the network and a prior belief can be an expert opinion such as the probability of strain burst given the rock type and local stress. The approach is iterative and the emphasis of expert assessment will decrease as more measured data becomes available. It can be concluded that Bayesian network forms a good real time risk assessment tool by combining expert knowledge with available data.

The parameter learning capability of Bayesian Networks was successfully applied for evaluating geohazards in a mining environment. The mean and variance parameters for the normal distribution were learned using parameter learning from the observed data for 19 years. When comparing the Bayesian distribution with the classical method, the Bayesian prediction is nearly identical for the studied mines. The difference, however, becomes more evident when data from multiple mine sites is used to evaluate the roof fall frequency for a particular region. The results for the studied case indicate that it is difficult to have a single distribution that is the best fit for roof fall data across different mines. One possible cause for this is that, failures across different mines can be triggered due to different geotechnical reasons even if the extent of human and design errors are discounted. Bayesian networks can overcome this problem by creating a hybrid distribution curve weighted on the goodness of fit test results.

6. Acknowledgement

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