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Discrete Z_4 Symmetry in Quantum Gravity

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Abstract: We consider the discrete Z_4 symmetry $\hat{\mathbf{i}}$, which takes place in the scenario of quantum gravity where the gravitational tetrads emerge as the order parameter—the vacuum expectation value of the bilinear combination of fermionic operators. Under this symmetry operation, $\hat{\mathbf{i}}$, the emerging tetrads are multiplied by the imaginary unit, $\hat{\mathbf{i}}\, e^a_\mu = -ie^a_\mu$. The existence of such symmetry and the spontaneous breaking of this symmetry are also supported by the consideration of the symmetry breaking scheme in the topological superfluid ^{3}He-B. The order parameter in ^{3}He-B is also the bilinear combination of the fermionic operators. This order parameter is the analog of the tetrad field, but it has complex values. The $\hat{\mathbf{i}}$-symmetry operation changes the phase of the complex order parameter by $\pi/2$, which corresponds to the Z_4 discrete symmetry in quantum gravity. We also considered the alternative scenario of the breaking of this Z_4 symmetry, in which the $\hat{\mathbf{i}}$-operation changes sign of the scalar curvature, $\hat{\mathbf{i}}\,\mathcal{R} = -\mathcal{R}$, and thus the Einstein–Hilbert action violates the $\hat{\mathbf{i}}$-symmetry. In the alternative scenario of symmetry breaking, the gravitational coupling $K = 1/16\pi G$ plays the role of the order parameter, which changes sign under $\hat{\mathbf{i}}$-transformation.

Keywords: quantum gravity; tetrad gravity; discrete symmetry; complex tetrads; symmetry breaking; cosmic walls

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1. Introduction

The discrete symmetries, such as P, T, and C symmetries, play an important role in particle physics, gravity, and cosmology [1], as well as in topological matter [2]. We consider the discrete Z_4 symmetry $\hat{\mathbf{i}}$, which takes place in the Akama–Diakonov–Wetterich (ADW) scenario of quantum gravity. In this scenario, the gravitational tetrads emerge as the symmetry-breaking order parameter—the vacuum expectation values of the bilinear combinations of the fermionic operators [3–10]. Under the symmetry operation $\hat{\mathbf{i}}$, the tetrads are multiplied by the imaginary unit, $\hat{\mathbf{i}}\, e^a_\mu = -ie^a_\mu$. A similar symmetry-breaking scenario characterizes the topological superfluid ^{3}He-B. But in ^{3}He-B, the order parameter, which corresponds to tetrads in quantum gravity, is the complex matrix. The $\hat{\mathbf{i}}$-symmetry operation changes the phase of this complex order parameter by $\pi/2$, which corresponds to the Z_4 discrete symmetry in quantum gravity.

In Section 2, we consider the $\hat{\mathbf{i}}$-symmetry in the ADW scenario. The consequences of the spontaneous breaking of $\hat{\mathbf{i}}$-symmetry are discussed in Section 3. Section 4 is devoted to the alternative scenario of the symmetry breaking, in which the scalar curvature is not invariant under the $\hat{\mathbf{i}}$-operation: it changes sign, $\hat{\mathbf{i}}\mathcal{R} = -\mathcal{R}$.

2. Composite Tetrads and $\hat{\mathbf{i}}$-Symmetry

In the Akama–Diakonov–Wetterich (ADW) approach, the gravitational tetrads appear as composite objects made of the more fundamental fields, the quantum fermionic fields [3–10]:

$$\hat{E}^a_\mu = \frac{1}{2}\left(\Psi^\dagger \gamma^a \partial_\mu \Psi - \Psi^\dagger \overleftarrow{\partial_\mu} \gamma^a \Psi\right). \tag{1}$$

The original action does not depend on tetrads and metric and is described solely in terms of differential forms:

$$S = \frac{1}{24} e^{\alpha\beta\mu\nu} e_{abcd} \int d^4x \, \hat{E}^a_\alpha \hat{E}^b_\beta \hat{E}^c_\mu \hat{E}^d_\nu \,. \tag{2}$$

This operator analog of the cosmological term has high symmetry. It is symmetric under coordinate transformations $x^\mu \to \tilde{x}^\mu(x)$, and thus is also scale invariant. In addition, the action is symmetric under spin rotations or under the corresponding gauge transformations when the spin connection is added to the gradients.

For us, it is important that this action is also symmetric under the complex coordinate transformation $x^\mu \to ix^\mu(x)$. Let us denote this symmetry as $\hat{\mathbf{i}}$-symmetry:

$$\hat{\mathbf{i}}\, x^\mu = ix^\mu \,, \tag{3}$$

Since the field operator $\hat{E}^a_\mu$ in Equation (1) is linear in gradients and represents the 1-form, $\mathbf{E} = \mathbf{E}_\mu dx^\mu$, it is multiplied by $-i$ under this symmetry operation:

$$\hat{\mathbf{i}}\, \hat{E}^a_\mu = -i\hat{E}^a_\mu \,. \tag{4}$$

The operator $\hat{E}^a_\mu$ is Hermitian or anti-Hermitian, once a representation of γ-matrices is specified. Under the symmetry transformation (3) the operator $\hat{E}^a_\mu$ becomes correspondingly anti-Hermitian or Hermitian, but the action (2) remains invariant under this $\hat{\mathbf{i}}$-transformation.

The action may also contain the operator analog of the Einstein–Hilbert–Cartan term [6],

$$e^{\alpha\beta\mu\nu} e_{abcd} \int d^4x \, \hat{E}^a_\alpha \hat{E}^b_\beta \hat{F}^{cd}_{\mu\nu} \,. \tag{5}$$

Here, $\hat{F}^{cd}_{\mu\nu}$ is the operator of the Cartan curvature 2-form. As the 2-form, $\mathbf{F} = \mathbf{F}_{\mu\nu} dx^\mu \wedge dx^\nu$, it changes sign under $\hat{\mathbf{i}}$-transformation:

$$\hat{\mathbf{i}}\, \hat{F}^{cd}_{\mu\nu} = -\hat{F}^{cd}_{\mu\nu} \,, \tag{6}$$

while the action (5) is $\hat{\mathbf{i}}$-invariant.

3. Broken $\hat{\mathbf{i}}$-Symmetry

In the ADW scenario of quantum gravity, the tetrads e^a_μ emerge as the order parameter of the spontaneous symmetry breaking. These are the vacuum expectation values of the bilinear fermionic 1-form $\hat{E}^a_\mu$:

$$e^a_\mu = < \hat{E}^a_\mu > \,. \tag{7}$$

If we use the Hermitian choice of the operator $\hat{E}^a_\mu$, the emergent tetrads e^a_μ become the real functions. Since in the ADW theory, the fermionic fields are dimensionless [8], the covariant tetrads have dimension of the inverse length, $[e^a_\mu] = 1/[length]$.

The tetrad order parameter breaks the separate symmetries under orbital and spin transformations but remains invariant under the combined rotations. On the level of the Lorentz symmetries the symmetry breaking scheme is $L_L \times L_S \to L_J$. Here, L_L is the group of Lorentz transformations in the coordinates space, L_S is the group of Lorentz transformations in the spin space, and L_J is the residual symmetry. It is the symmetry group of the order parameter, which is invariant under the combined Lorentz transformations L_J. Note also that the discrete P and T symmetries of the Standard Model are also the combined symmetries since they include both the coordinate transformations and transformations of the fermionic fields.

In addition, the order parameter in Equation (7) breaks the discrete $\hat{\mathbf{i}}$-symmetry and becomes anti-Hermitian under $\hat{\mathbf{i}}$-transformation:

$$\hat{\mathbf{i}}\, e^a_\mu = -ie^a_\mu\,. \tag{8}$$

We can compare this symmetry breaking with the spontaneous breaking of the PT-symmetry proposed in Ref. [11], where the special type of the PT-symmetry operation changes sign of all tetrads, $PTe^a_\mu = -e^a_\mu$. In connection to the transformation of tetrads, the $\hat{\mathbf{i}}$-symmetry corresponds to the square root of this PT-symmetry:

$$\hat{\mathbf{i}}^2\, e^a_\mu = PT\, e^a_\mu = -e^a_\mu\,. \tag{9}$$

That is why the $\hat{\mathbf{i}}$-operation belongs to the discrete Z_4 group, $(\hat{\mathbf{i}}, \hat{\mathbf{i}}^2 = PT, \hat{\mathbf{i}}^3 = -\hat{\mathbf{i}}, \hat{\mathbf{i}}^4 = 1)$.

The ADW symmetry-breaking mechanism of emergent gravity has the analog in the spin-triplet p-wave superfluids, where the effective gravitational vielbein also emerges as the bilinear fermionic 1-form [12,13]. In the B-phase of superfluid ^{3}He, the symmetry-breaking scheme is $SO(3)_L \times SO(3)_S \to SO(3)_J$, where $SO(3)_L$ and $SO(3)_S$ are correspondingly the orbital and spin rotation groups, and $SO(3)_J$ is the residual symmetry—the symmetry of the order parameter under combined rotations. Here, $\mathbf{J}$ is the total angular momentum operator $\mathbf{J} = \mathbf{L} + \mathbf{S}$. Such symmetry breaking to the diagonal subgroup is known in superfluid ^{3}He as relative symmetry breaking [14]. This means that the symmetry under the separate rotations in spin and orbital spaces is broken, while the properties of ^{3}He-B are isotropic due to the symmetry under the combined rotations.

It is important, that in ^{3}He-B, the $U(1)$ symmetry is also spontaneously broken. Under the $U(1)$ phase rotations, the order parameter (the triad analog of tetrads) is transformed according to $e^a_\mu \to e^{i\phi} e^a_\mu$. Then, the $\hat{\mathbf{i}}$-symmetry operation in Equation (8) corresponds in ^{3}He-B to the global $U(1)$ transformation with the phase $\phi = -\pi/2$. This, again, demonstrates that with respect to tetrads, the $\hat{\mathbf{i}}$-symmetry is the element of the discrete Z_4-symmetry group of quantum gravity with $\phi = \pi n/2$, which extends the symmetry group. The possible extension of this Z_4 group to the full $U(1)$ group of quantum mechanics is discussed in Section 3.6 in connection with another phase of superfluid ^{3}He, the planar phase with massless Dirac fermions.

Note that in our case, the symmetry operation $\hat{\mathbf{i}}$ does not include the transformations of the fermionic and bosonic operators and the interchanges of the Dirac variables with their Hermitian conjugates. It is the pure coordinate transformation of the operators, $\hat{\mathbf{i}}\Psi(x^\mu) = \Psi(ix^\mu)$, i.e., these operators are the world scalars under such discrete kind of diffeomorphisms.

3.1. Metric as Fermionic Quartet

In the broken symmetry state in the ADW scenario, the metric field emerges as the secondary object—the bilinear combination of the tetrad fields:

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu\,. \tag{10}$$

That is why, in this quantum gravity, the metric is the fermionic quartet. Under the $\hat{\mathbf{i}}$-transformation, the signature of the metric changes:

$$\hat{\mathbf{i}}\, g_{\mu\nu} = -g_{\mu\nu}\,. \tag{11}$$

Let us mention the connection with the scenario suggested in Refs. [15,16], where the signature of the metric is represented by the dynamic variable O_{ab}. Then, the tensor η_{ab} in Equation (10) emerges as the vacuum expectation value $\eta_{ab} =< O_{ab} >$ in the corresponding symmetry breaking phase transition.

Let us also mention the complexification of the tetrads [5] and the complexification of the Lorentz group in Refs. [17,18] and references therein. The scenario of the emergence of gravity from the breaking of gauge symmetry in Refs. [17,18] is similar to the scenario with the so-called translational gauge fields in crystals, where the gravitational tetrads emerge from the elasticity tetrads describing the elasticity theory in crystals [19–23]. In the elasticity theory, an arbitrarily deformed crystal structure is described as a system of the crystallographic surfaces of constant phase $X^a(x) = 2\pi n^a$, and the elasticity tetrads are $e^a_\mu = \partial_\mu X^a$. Note that the elasticity tetrads change sign under the conventional PT transformation of coordinates, and thus the $\hat{\mathbf{i}}$-symmetry is the square root of this PT symmetry.

In principle, the so-called vestigial gravity is possible in quantum gravity, where the tetrad order parameter is absent, $e^a_\mu =< \hat{E}^a_\mu >= 0$, while the metric emerges as the vacuum expectation value of the bilinear combination of tetrad operators, $g_{\mu\nu} = \eta_{ab} < \hat{E}^a_\mu \hat{E}^b_\nu >$ [24]. In the vestigial gravity, the Equivalence Principle is violated, since such gravity acts with different strength on fermions and bosons, and they do not follow the same trajectories in a given gravitational field.

3.2. Interval and Scalar Field

While the metric in Equation (11) changes sign under the $\hat{\mathbf{i}}$-transformation, the interval remains $\hat{\mathbf{i}}$-invariant due to Equations (3) and (11):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \ , \ \ \hat{\mathbf{i}}\, ds^2 = ds^2 \,. \tag{12}$$

This makes $\hat{\mathbf{i}}$-invariant the classical action $S = M \int ds$ for massive particle, $\hat{\mathbf{i}}\, S = S$.

The emergence of the metric gives rise to the quadratic action for the scalar field Φ:

$$S = \int d^4x \, \sqrt{-g}\, g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi \,, \tag{13}$$

which is also $\hat{\mathbf{i}}$-invariant. This is because the coordinate transformation of the gradients of the scalar field is compensated by the change of the sign of the metric: $\hat{\mathbf{i}}\, \nabla_\mu = -i\nabla_\mu$, $\hat{\mathbf{i}}\, g^{\mu\nu} = -g^{\mu\nu}$ and $\hat{\mathbf{i}}\, g = g$.

3.3. Gauge Fields

The gauge potential is the 1-form field, $A = A_\mu dx^\mu$, and thus it transforms in the same way as the gradient:

$$\hat{\mathbf{i}}\, A_\mu = -i A_\mu \,. \tag{14}$$

The zero-form action describing the interaction of a charged point particle with the $U(1)$ gauge field remains invariant under the $\hat{\mathbf{i}}$-transformation:

$$S = q \int dx^\mu A_\mu \ , \ \hat{\mathbf{i}}\, S = S \,, \tag{15}$$

where q is the dimensionless electric charge. On the other hand, the field strength, being the 2-form, $F = F_{\mu\nu} dx^\mu \wedge dx^\nu$, changes sign under the $\hat{\mathbf{i}}$-transformation:

$$\hat{\mathbf{i}}\, F_{\mu\nu} = -F_{\mu\nu} \,. \tag{16}$$

The quadratic action for the gauge field, which appears after the emergence of the metric, is quadratic in $F_{\mu\nu}$ and is quadratic in $g^{\mu\nu}$:

$$S \propto \int d^4x \, \sqrt{-g}\, F_{\mu\nu} F^{\mu\nu} \,, \tag{17}$$

and thus, it is $\hat{\mathbf{i}}$-invariant.

3.4. Fermions

The action for the massive Dirac particles

$$S = \int d^4x\, e\, (ie^\mu_a \bar{\Psi}\gamma^a \nabla_\mu \Psi - M\bar{\Psi}\Psi)\,. \tag{18}$$

is also $\hat{\mathbf{i}}$-invariant. This is because of the following transformations of the contravariant tetrads, gradients, and the tetrad determinant:

$$\hat{\mathbf{i}}\, e^\mu_a = ie^\mu_a \ , \ \hat{\mathbf{i}}\, \nabla_\mu = -i\nabla_\mu \ , \ \hat{\mathbf{i}}\, e = e\,. \tag{19}$$

Let us recall that the symmetry operation $\hat{\mathbf{i}}$ is the pure coordinate transformation, which does not involve the γ-matrices. Note also that in the ADW theory, the contravariant tetrads have dimension of the length, $[e^\mu_a] = [length]$, while the fermionic fields and the mass M are dimensionless [8,25]. That is why the action (18) is dimensionless.

3.5. Gravity from Bosons

Till now, we considered gravity emerging in the background of the fermionic vacuum. The more restricted gravity emerges in the bosonic background, see e.g., Ref. [26]. Now, instead of tetrads, the metric field is the emerging gravitational variable. An example of the corresponding order parameter as the vacuum expectation value of the bosonic fields is:

$$g_{\mu\nu} \propto \left\langle \nabla_\mu \Phi^\dagger \nabla_\nu \Phi + \nabla_\nu \Phi^\dagger \nabla_\mu \Phi \right\rangle. \tag{20}$$

Under $\hat{\mathbf{i}}$-transformation, the effective metric changes sign in the same way as in the fermionic vacuum, $\hat{\mathbf{i}} g_{\mu\nu} = -g_{\mu\nu}$. That is why the quadratic action for the scalar field in Equation (13) remains $\hat{\mathbf{i}}$-invariant.

3.6. Discrete Z_4 Symmetry and Imaginary Unit in Quantum Mechanics

The complexification of the coordinate transformations using $x^\mu \to ix^\mu$ becomes more transparent when complex numbers are expressed in terms of real numbers. The latter also explains why only the elements of the discrete subgroup Z_4 of the $U(1)$ symmetry group participate in the coordinate transformations.

This may also have some connection to the fundamental problem of the role of complex numbers in quantum mechanics. As is known, Schrödinger strongly resisted the introduction of such a product of the human mind as $\sqrt{-1}$ into the wave equations. The possible solution to the problem [27] is to express the effective imaginary unit in terms of the real 2×2 matrix:

$$a + ib \equiv \begin{pmatrix} a \\ b \end{pmatrix} \equiv a\hat{\mathbf{I}} + b\hat{\mathbf{i}}\,, \tag{21}$$

where

$$\hat{\mathbf{I}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \ \hat{\mathbf{i}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} , \ \hat{\mathbf{i}}^2 = -1\,. \tag{22}$$

The possible topological origin of the emergence of these real matrices in quantum mechanics is discussed in Refs. [28,29]. The main role in this scenario, which gives rise to the effective imaginary unit, is played by the topology of exceptional points of the level crossing in the fermionic spectrum—the so-called conical, diabolic, Dirac, and Weyl points.

Another possible origin of the effective imaginary unit $i_{\rm eff}$ is discussed by Adler in the theory of the trace dynamics [30]. According to Adler [30], the emergent quantum theory may have two sectors, one with imaginary unit i and one with imaginary unit $-i$. This corresponds to the Z_2 symmetry between the states with:

$$\hat{\mathbf{i}} = \pm \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{23}$$

This can be responsible for the dark matter arising in the hidden sector of the standard model with the opposite i [31].

The full Z_4 group $(\hat{\mathbf{1}}, \hat{\mathbf{i}}, \hat{\mathbf{i}}^2, \hat{\mathbf{i}}^3)$ for the considered coordinate transformations comes from the symmetry between the Hermitian and anti-Hermitian presentations of the coordinate and momentum operators, which are similar to the Hermitian and anti-Hermitiian representations of γ-matrices in Section 2. The Hermitian matrices for the real-valued coordinates and momenta are:

$$\hat{\mathbf{x}} = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} , \ \hat{\mathbf{p}} = \begin{pmatrix} 0 & -\partial_x \\ \partial_x & 0 \end{pmatrix} , \ \hat{\mathbf{x}}\hat{\mathbf{p}} - \hat{\mathbf{p}}\hat{\mathbf{x}} = \hat{\mathbf{i}} \,. \tag{24}$$

The anti-Hermitian presentation of the coordinate and momentum operators, which is obtained by the $\hat{\mathbf{i}}$ element of the Z_4 symmetry group, $x^\mu \to ix^\mu$, can be written also in terms of real numbers:

$$\hat{\mathbf{x}}_{\rm anti} = \hat{\mathbf{i}}\,\hat{\mathbf{x}} = \begin{pmatrix} 0 & x \\ -x & 0 \end{pmatrix} , \ \hat{\mathbf{p}}_{\rm anti} = -\hat{\mathbf{i}}\,\hat{\mathbf{p}} = -\begin{pmatrix} \partial_x & 0 \\ 0 & \partial_x \end{pmatrix} , \tag{25}$$

with the same canonical commutations relation as in Equation (24).

It is important that in some cases the discrete symmetry can be automatically extended to the continuous symmetry, as it happens for the planar phase of superfluid ^{3}He, where Z_2 symmetry is extended to $U(1)$ [32]. In the planar phase, the Z_2 symmetry C is the combination of the spin π rotation about the z axis and the phase rotation by $\pi/2$. In the linear approximation, the single-particle Hamiltonian and Green function commute not only with C but also with the full $SO(2) \equiv U(1)$ group of transformations $\exp(i\alpha C)$ generated by C. Since quantum mechanics is the linear theory, one may also expect that in the same way the $U(1)$ transformation $e^{i\alpha}$ of the wave function in quantum mechanics emerges as the extension of the Z_4 symmetry group with its discrete phases, $\alpha = n\pi/2$.

4. Alternative Broken Symmetry

It can be interesting to consider the other possible scenarios of quantum gravity with different schemes of the breaking of the $\hat{\mathbf{i}}$-symmetry. Let us consider as an example the symmetry breaking scheme in which the metric remains invariant under the $\hat{\mathbf{i}}$-transformation of coordinates, $x^\mu \to ix^\mu$:

$$\hat{\mathbf{i}}\, g_{\mu\nu} = g_{\mu\nu} \,. \tag{26}$$

Such gravity does not include fermions and concerns only the bosonic scalar and gauge fields interacting with the gravitational field.

4.1. Scalar Field

The action for the scalar field is modified since the Equation (13) is quadratic in gradients and thus is not $\hat{\mathbf{i}}$-invariant. But the fourth-order gradient terms in action are invariant under $\hat{\mathbf{i}}$-transformation, examples of such terms are:

$$S = \int d^4x \, \sqrt{-g} \left(a_4 g^{\mu\nu} g^{\alpha\beta} \nabla_\mu \nabla_\nu \Phi^* \nabla_\alpha \nabla_\beta \Phi + b_4 \left(g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi \right)^2 \right) . \tag{27}$$

The second-order gradient term, which violates the $\hat{\mathbf{i}}$-symmetry, may appear only in the state with the spontaneously broken $\hat{\mathbf{i}}$-symmetry. The corresponding order parameter can be, for example, the following vacuum expectation value of the bosonic operators:

$$\lambda = < g^{\mu\nu} \nabla_\mu \Phi^\dagger \nabla_\nu \Phi > \,. \tag{28}$$

This gives the following second-order gradient term:

$$S_2 = 2b_4 \lambda \int d^4x \, \sqrt{-g} \, g^{\mu\nu} \nabla_\mu \Phi^* \nabla_\nu \Phi \,. \tag{29}$$

The main difference from Equation (13) is that Equation (29) contains the parameter λ, which changes sign under $\hat{\mathbf{i}}$-transformation, $\hat{\mathbf{i}}\,\lambda = -\lambda$, and thus, the scalar field is transformed into the ghost field.

4.2. Gauge Field

In this scenario of symmetry breaking, Equation (16) for the $\hat{\mathbf{i}}$-transformation of the gauge field remains the same since the 2-form field does not depend on metric:

$$\hat{\mathbf{i}}\,F_{\mu\nu} = -F_{\mu\nu}\,. \tag{30}$$

The action for the gauge field in Equation (17) is quadratic in the metric field and thus remains $\hat{\mathbf{i}}$-invariant. This actually follows from the scale invariance of this action.

4.3. Gravity

In the scenario of the broken symmetry in Section 3, where the tetrads serve as the order parameter of the symmetry breaking, the term in Equation (5) is reduced to the conventional Einstein–Hilbert action $\int d^4x\sqrt{-g}K\mathcal{R}$. It is $\hat{\mathbf{i}}$-invariant, since $\hat{\mathbf{i}}\,\mathcal{R} = \mathcal{R}$. In the alternative symmetry breaking scenario discussed here, the curvature $\mathcal{R}$ is not $\hat{\mathbf{i}}$-invariant since the transformation $x^\mu \to ix^\mu$ is not compensated by the transformation of the metric, and one obtains $\hat{\mathbf{i}}\,\mathcal{R} = -\mathcal{R}$. As a result, the Einstein–Hilbert action is not $\hat{\mathbf{i}}$-invariant, and the $\hat{\mathbf{i}}$-invariant gravity is the gravity, which is quadratic in the spacetime curvature.

The $\hat{\mathbf{i}}$-invariance of the Einstein–Hilbert action is restored if the gravitational coupling changes sign under this transformation, $\hat{\mathbf{i}}\,K = -K$. This means that, in this scenario, the gravitational coupling K either serves as the order parameter of such symmetry breaking or is proportional to the order parameter λ in Equation (28), $K \propto \lambda$. In this sense, the $\hat{\mathbf{i}}$-symmetry has much in common with the scale invariance.

5. Discussion

In topological superfluid ^{3}He-B, the symmetry-breaking scheme includes the reduction $SO(3)_L \times SO(3)_S \to SO(3)_J$. Here, $SO(3)_L$ is the group of orbital rotations, $SO(3)_S$ is the group of spin rotations, and $SO(3)_J$ is the residual symmetry—the symmetry under combined rotations.

In the ADW scenario of quantum gravity, there is a similar symmetry-breaking scheme, but in the form of the Lorentz symmetries: $L_L \times L_S \to L_J$. Here, $L_L = S(3,1)_L$ is the group of Lorentz transformations in the coordinates space, $L_S = S(3,1)_S$ is the group of Lorentz transformations in the spin space, and $L_J = S(3,1)_J$ is the residual symmetry—the symmetry group of the order parameter. The order parameter here is the tetrad field e^a_μ, which is not invariant under separate Lorentz transformations, but is invariant under the residual symmetry group—the combined Lorentz transformations L_J. Two Lorentz symmetry groups as independent transformations of coordinates and spins have been also considered in Ref. [33].

The difference between the ^{3}He-B scenario and the ADW scenario of quantum gravity is not only in the different dimensions: we have 3D space dimension in ^{3}He-B and (3+1) dimension of space–time in the ADW gravity, and as a result, the order parameter in ^{3}He-B represents the triad field instead of tetrads. There is another important difference: in ^{3}He-B, in addition to the broken relative symmetry, the global $U(1)$ symmetry group is also broken, i.e., the symmetry-breaking scheme is $U(1) \times SO(3)_L \times SO(3)_S \to SO(3)_J$. As a result, the triads in ^{3}He-B become complex. This suggests the possible consideration of the extended symmetry also in the ADW scenario. Indeed, one can see that the original fermionic action in the ADW theory is invariant under the coordinate transformation $x^\mu \to ix^\mu$, where i is the imaginary unit. We called this additional symmetry the $\hat{\mathbf{i}}$-symmetry. This Z_4 symmetry is the discrete analog of the $U(1)$ symmetry in the symmetric state of liquid ^{3}He. In the ADW scenario, this $\hat{\mathbf{i}}$-operation leads to the following transformation of the emerging tetrad fields: $e^a_\mu \to -ie^a_\mu$.

The physical meaning of the spacetime coordinate transformation $x^\mu \to ix^\mu$ is discussed in Section 3.6. This transformation corresponds to the transition between two equivalent descriptions of the quantum fields and gravity, Hermitian and anti-Hermitian.

We also considered the alternative scenario of the breaking of the Z_4 symmetry. In this scenario, the $\hat{\mathbf{i}}$-operation changes sign of the scalar curvature, $\mathcal{R} \to -\mathcal{R}$, and thus, the Einstein–Hilbert action violates the $\hat{\mathbf{i}}$-symmetry. This means that in the alternative scenario of symmetry breaking, the gravitational coupling $K = 1/16\pi G$ plays the role of the order parameter, with $K \to -K$ under the $\hat{\mathbf{i}}$ symmetry operation. In this scenario, the scalar field is transformed to the ghost field under $\hat{\mathbf{i}}$-operation; the massive particles transform to the tachyons with imaginary mass, and the de Sitter state is transformed to the anti-de Sitter state [34]. The latter is different from the time reversal operation, which transforms the expanding de Sitter state with the Hubble parameter $H > 0$ to the contracting de Sitter state with $H < 0$.

The discrete Z_4-symmetry and its breaking can be important in cosmology. In particular, due to spontaneously broken discrete symmetry in the gravitational sector, gravity can be a 'player' in the problem of the baryon asymmetry of the Universe [35,36]. Also, the breaking of discrete symmetry leads to the formation of the cosmological domain walls [37,38], see review [39]. In Ref. [35], the domain wall emerging due to the breaking of the Z_2 symmetry and the PT-symmetry was considered. It is the wall separating the states with e^a_μ and $-e^a_\mu$. In the case of the Z_4 symmetry breaking, one has, in addition, the H-antiH domain wall separating the quantum vacua with Hermitian and anti-Hermitian tetrads. Each of the two degenerate states can be described in the frame of Hermitian physics by the redefinition of the γ-matrices, then its partner behind the wall is viewed as anti-Hermitians. Within the domain wall, the $\hat{\mathbf{i}}$-symmetry can be restored, which means that the tetrads cross the zero values, $e^a_\mu = 0$. However, the symmetry of the topological objects can be also broken in their cores [40,41]. The broken symmetry of the H-antiH domain wall may lead to the complex values of the tetrads within the domain wall. This would correspond to the Neel or Bloch domain walls in ferromagnets, where the magnetization does not cross zero value.

It would be interesting to extend the consideration to the extra dimensions. Diakonov suggested the $SO(16)$ symmetry group with 16×16 components of the vielbein, and these 256 degrees of freedom come from the bilinear combinations of the Standard Mode fermions in four generations [6]. See also the compactification of the higher-dimensional spacetime in the recent paper [42] and references therein.

6. Conclusions

The Akama–Diakonov–Wetterich quantum gravity is symmetric under the complex coordinate transformation $x^\mu \to ix^\mu$. In this paper, we discussed the physical meaning of such transformation and its physical consequences. The physical meaning becomes clear when the imaginary unit i is expressed in terms of the real-valued antisymmetric matrix. Then, the transformation $x^\mu \to ix^\mu$ describes the transition between the Hermitian and anti-Hermitian descriptions of the quantum fields and gravity, and corresponds to the discrete element of the Z_4 group.

The spontaneous breaking of this symmetry leads to the formation of the "H-anti-H" walls—the cosmological domain walls separating the quantum vacua with Hermitian and anti-Hermitian tetrads. However, each of the two degenerate states can be described in the frame of conventional Hermitian physics by the redefinition of the Dirac γ-matrices.

This consideration is supported by the condensed matter analogs—the B-phase and the planar phase of superfluid ^{3}He with correspondingly massive and massless Dirac fermions.

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