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Uncertainty of evaluation of spectral mismatch correction factor

Y Rezazadeh[1], A Sperling[2], T Gerloff[2], U Krüger[3], A Ferrero[4], J Campos[4], O Pellegrino[5], J Dubard[6], and E Ikonen[1,7]

[1]Metrology Research Institute, Aalto University, Finland; [2]Physikalisch-Technische Bundesanstalt, Germany; [3]TechnoTeam Bildverarbeitung GmbH, Germany; [4]Instituto de Óptica, Consejo Superior de Investigaciones Científicas, Spain; [5]Instituto Português da Qualidade, Portugal; [6]Laboratoire National de Métrologie et d'Essais, France; [7]VTT MIKES, Finland

Corresponding e-mail address: yasaman.rezazadeh@aalto.fi

Abstract. Addressing the effect of partially correlated components of spectral irradiance data on spectral integrals poses a substantial challenge. This study employs Monte Carlo methods to conduct an uncertainty analysis of spectral integrals, with a focus on the uncertainty of the spectral mismatch correction factor. The investigation encompasses the primary types of uncertainty contributors of spectral irradiance, based on carefully calibrated FEL lamp measurements. A novel approach is introduced to estimate the impacts of partial correlations among spectral irradiance values at different wavelengths. The findings reveal that uncertainty components arising from partial correlations significantly outweigh those associated with full spectral correlation or complete lack of correlation at different wavelengths. This insight advances our understanding of uncertainty analysis of spectral integrals and underscores the importance of accounting for partial correlations in accurate measurements.

1. Introduction

Various properties of the light sources and detectors, including colour temperature, chromaticity coordinates and spectral mismatch correction factor, are determined by evaluating ratios of integrated spectral irradiance values. In such quantities, factors described by a constant multiplier of spectral irradiance are cancelled and do not contribute to the total uncertainty. The constant multiplier describes the case of full correlation between spectral irradiance values, which can arise, for example, from uncertainty in the aperture area of the detector. On the contrary, non-existing or partial correlations between measured irradiance values at different wavelengths influence the uncertainty of quantities defined in terms of ratios of spectral integrals [1-4]. These correlations manifest themselves in three distinct types of uncertainty components: uncorrelated components (e.g., noise), partially correlated components with known spectral structure (e.g., lamp filament temperature), and partially correlated components with unknown spectral structure. The third type poses the greatest challenge for quantitative assessment due to the unknown and potentially non-linear nature of the spectral correlations [1].

Observations indicate that in spectral data, on average, harmonic deviation amplitudes from the reference value tend to be inversely proportional to the harmonic order [5]. This feature holds the potential for enhancing the reliability of uncertainty analysis of spectral integrals by improving the analysis method by Kärhä *et al*. [1]. We conduct here an uncertainty analysis of the spectral mismatch correction factor for a carefully characterized 1000-W FEL lamp [6]. We employ two different methods to estimate the uncertainty of this quantity, taking into account partial correlations in spectral irradiance through both the initial method of [1] and an improved method based on the inverse proportionality of harmonic deviation amplitudes on the harmonic order [5].

2. Mathematical description

Spectral mismatch correction factor is used here as a model quantity of ratios of spectral integrals. The illuminance E_V of a light source at a photometer is calculated by

$$E_V = \frac{K_m}{As_0} F \cdot I, \qquad (1)$$

where I is the photocurrent of the photometer, $K_m = 683.002$ lm·W^{-1} is the maximum luminous efficacy, A is the area of the precision aperture of the photometer, s_0 is the absolute spectral responsivity of the photometer at the air wavelength of $\lambda_0 = 555$ nm, and

$$F = \frac{\int E_e(\lambda) V(\lambda) d\lambda}{\int E_e(\lambda) s_{rel}(\lambda) d\lambda} \qquad (2)$$

is the correction factor needed for spectral mismatch of $V(\lambda)$ and $s_{rel}(\lambda)$, where $V(\lambda)$ is the spectral luminous efficiency function of photopic vision and $s_{rel}(\lambda)$ is the relative spectral responsivity of the photometer, normalized to 1 at $\lambda_0 = 555$ nm. In Eq. (2), $E_e(\lambda)$ is the spectral irradiance of the light source and λ is the wavelength within the limits of integration $\lambda_1 = 360$ nm and $\lambda_2 = 830$ nm.

Following Kärhä *et al*. [1] in analysing the effect of correlations by Monte Carlo simulations, the nominal spectral irradiance $E(\lambda)$ is modified according to

$$E_e(\lambda) = [1 + \delta(\lambda) u_c(\lambda)] E(\lambda) \qquad (3)$$

where $u_c(\lambda)$ is the relative combined standard uncertainty of spectral irradiance, consisting of various uncertainty components of a given type. For analysing the effects of fully uncorrelated uncertainty, the deviation function $\delta(\lambda)$ is selected from a zero-mean Gaussian probability distribution with unity variance, independently at each wavelength.

For analysing the effects of uncertainty components with unknown partial correlation, the deviation function is calculated according to [1]

$$\delta(\lambda) = \sum_{i=0}^{N} \gamma_i f_i(\lambda), \qquad (4)$$

where $N + 1$ is the number of basis functions used and γ_i are the coordinates of a random point on the surface of an (N+1)-dimensional unit sphere. The basis functions $f_i(\lambda)$ used by Kärhä *et al*. are given by [1,2]

$$f_i(\lambda) = \sqrt{2} \sin(2\pi i \, \frac{\lambda - \lambda_2}{\lambda_2 - \lambda_1} + \phi_i) \qquad (5)$$

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$$= \sqrt{2}\sin\left(\pi i\frac{2\lambda-\lambda_1-\lambda_2}{\lambda_2-\lambda_1}\right)\cos\phi_i + \sqrt{2}\cos\left(\pi i\frac{2\lambda-\lambda_1-\lambda_2}{\lambda_2-\lambda_1}\right)\sin\phi_i$$

where the phases ϕ_i are random variables uniformly distributed within the interval $[-\pi, \pi]$ when $i \geq 1$. The zeroth order basis function $f_0(\lambda) = 1$ is used to account for full correlation and it is considered as a special case as compared to other basis functions. The basis functions of Eq. (5) are orthogonal and normalised to unity variance over the wavelength interval from λ_1 to λ_2. This implies that in the limit of large N the relative error function $\delta(\lambda)u_c(\lambda)$ in the Monte Carlo simulation using Eqs. (4) and (5) can represent all spectral shapes compatible with the combined standard uncertainty of spectral irradiance.

In this work, we take advantage of the results of a recent spectral analysis of deviations from the key comparison reference value in seven comparisons of radiometric quantities [5]. The results reveal an approximate outcome that, on the average, each harmonic deviation amplitude, i.e., parameters γ_i in Eq. (4) with $i \geq 1$, is inversely proportional to the order i of the harmonic in all studied key comparisons. We thus improve Eq. (4) to the form [5]

$$\delta(\lambda) = \sin\theta + \frac{\cos\theta}{\sqrt{\sum_{i=1}^{N}\frac{1}{i^2}}}\sum_{i=1}^{N}\frac{f_i(\lambda)}{i}, \qquad (6)$$

where the square-root denominator comes from the normalization requirement that the deviation function $\delta(\lambda)$ has unity variance when integrated from λ_1 to λ_2. Phase θ is a random variable, uniformly distributed within the interval $[-\pi, \pi]$. Furthermore, instead of the sinusoidal basis functions of Eq. (5), Legendre polynomials P_i are used in Eq. (6),

$$f_i(\lambda) = g_{2i-1}(\lambda)\cos\phi_i + g_{2i}(\lambda)\sin\phi_i \qquad (7)$$

where

$$g_i(\lambda) = P_i\left(\frac{2\lambda-\lambda_1-\lambda_2}{\lambda_2-\lambda_1}\right)/\sigma_i \qquad (8)$$

and σ_i is the standard deviation of P_i when integrated from λ_1 to λ_2. Legendre polynomials are defined within the interval [-1, 1]. The argument of the polynomial is thus scaled in the same way as the argument in the latter form of Eq. (5). Factors $\cos\phi_i$ and $\sin\phi_i$ correspond to random weights of the odd and even contributions, respectively, of the sinusoidal and polynomial basis functions of Eqs. (5) and (7). Legendre polynomials are used to construct the basis functions because they are orthogonal with a weighting function equal to 1 when integrated from λ_1 to λ_2 [5].

3. Experimental

The uncertainty budget of the spectral irradiance of the studied FEL lamp at 500 nm wavelength is shown in Table 1 [6]. Uncertainty components due to constant multiplicative factors are not included, because factors independent of λ are cancelled out in Eq. (2) and do not produce any residual uncertainty. Each of the uncertainty components in Table 1 have specific spectral correlation features. Row (i) describes noise in the calibration which is uncorrelated at different measurement wavelengths. Uncertainty of the lamp filament temperature is 0.7 K on row (ii) of Table 1. Variation of the nominal temperature of 3100 K of the lamp filament within the Gaussian probability distribution of $(0.7\ \text{K})^2$ variance causes changes of the irradiance spectrum which can be calculated by the Planck radiation law. The third component (iii), related to drift and repeatability of the lamp irradiance, is assumed to have unknown partial correlations between the spectral irradiance values measured at different wavelengths. This component is the main interest of the present work.

Table 1. Main uncertainty components (k=1) of the spectral irradiance of a 1000-W FEL lamp [6], for the uncertainty analysis of F in Eq. (2).

	Component	**Relative uncertainty of the spectral irradiance at 500 nm (as an example)**
(i)	Noise in the calibration	0.05 %
(ii)	Temperature deviation of the lamp	0.2 %
(iii)	Uncorrected drift and repeatability of the lamp	0.2 %

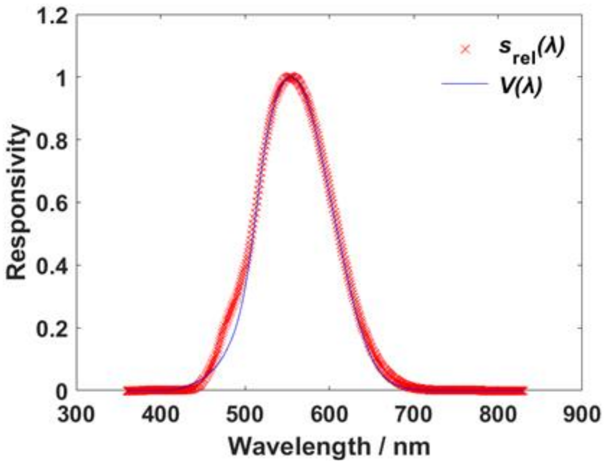

Figure 1. Relative spectral responsivity $s_{rel}(\lambda)$ of the studied photometer and $V(\lambda)$ curve in the wavelength range between 360 nm and 830 nm. The photometer quality index f_1' is 7.3 %.

Figure 1 shows the relative spectral responsivity of the studied photometer. It corresponds to a real photometer selected from the PhotoLED database [7] in such a way that there is a large deviation relative to the $V(\lambda)$ function in the blue spectral region. According to estimates in [6], it is assumed that the relative uncertainty in $s_{\mathrm{rel}}(\lambda)$ is negligible as compared with the relative uncertainty of spectral irradiance.

4. Simulation Results

As determined from Eq. (2), the value of the spectral mismatch correction factor F is 0.95414 for the photometer of Fig. 1 and a Planckian radiator of temperature 3100 K. Monte Carlo simulations were then carried out corresponding to the uncertainty components of Table 1. The noise in calibration is entirely uncorrelated across different measurement wavelengths of spectral irradiance. To evaluate the effect of this uncertainty component, independent random numbers $\delta(\lambda)$ from a Gaussian distribution at each wavelength were generated for the modified spectral irradiance of Eq. (3). The standard deviation of F was found to be 0.2×10^{-5} due to noise in the calibration.

The variation in temperature, specifically at 3100 K, of the FEL lamp filament follows a Gaussian distribution with $(0.7\ \mathrm{K})^2$ variance. This temperature uncertainty leads to alterations in the irradiance spectrum, which are computed using the Planck radiation law. The standard deviation of *F* was found to be 0.07 × 10^{-5} due to uncertainty in the lamp filament temperature.

For analysis of the effects of partial correlations, the drift and repeatability component of Table 1 was used for $u_c(\lambda)$ in Eq. (3). Figure 2 shows the resulting standard deviation of *F* calculated by Eqs. (3), (4) and (6) as a function of *N*. At each value of *N*, 10000 Monte Carlo cycles were repeated. The standard deviation obtained using Eqs. (3) and (6) does not depend on *N* with large values of *N*. This convenient feature allows to assign a standard deviation of 2.3 × 10^{-5} of *F* due to the drift and repeatability uncertainty component.

In the case of Eqs. (3) and (4), the standard deviation of *F* depends on *N* and no unambiguous standard deviation value can be selected on the basis of Fig. 2 alone. Thus Kärhä *et al*. [1] made an additional assumption that fully correlated, fully uncorrelated and severely partially correlated uncertainty contributions have equal weights. Severe partial correlation refers here to the maximum of the cumulative standard deviation curve of *F* in Fig. 2, calculated by Eq. (4). The green line in Fig. 2 indicates the standard deviation of 2.0 × 10^{-5} of *F* obtained using the assumption of equal weights of different correlation types. The value is close to the result calculated according to more reliable Eqs. (3) and (6).

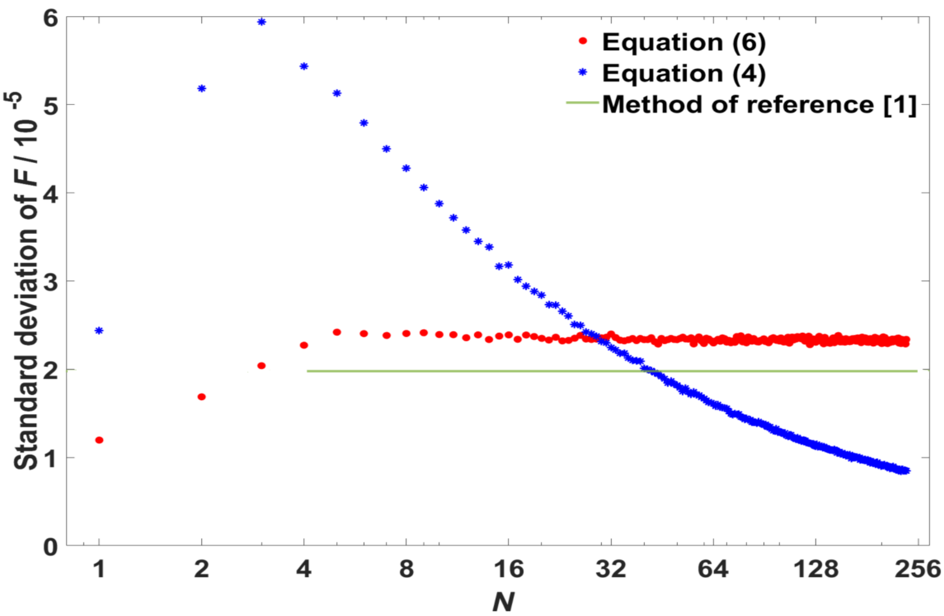

Figure 2. Standard deviation of F due to unknown partial correlations. N+1 is the number of basis functions used. The green line indicates the standard uncertainty when fully correlated, fully uncorrelated, and partially correlated uncertainty components are given equal weight.

Table 2. Main uncertainty components of the spectral irradiance of a 1000-W FEL lamp [6] and their contribution to the standard uncertainty of spectral mismatch correction factor F.

	Component	**Contribution to uncertainty of *F***
(i)	Noise in the calibration	0.2 × 10^{-5}
(ii)	Temperature deviation of the lamp	0.07 × 10^{-5}
(iii)	Uncorrected drift and repeatability of the lamp	2.3 × 10^{-5}

The uncertainty due to each type of uncertainty components is summarized in Table 2. It is seen that the uncertainty component due to unknown partial correlations is by far the largest. The final standard uncertainty of 2.3×10^{-5} of F is obtained as the quadratic sum of the components (i) to (iii) of Table 2.

5. Conclusions

In conclusion, uncertainty components of real experimental data were considered in the case of ratio of spectral integrals. The uncertainty components were categorized into three types, namely (i) fully uncorrelated components, (ii) partially correlated components with known spectral structure, and (iii) partially correlated components with unknown spectral structure. To reach reliable results, a new method was introduced for analysis of unknown partial correlations. If the partial correlations (iii) are not properly considered, the uncertainty of quantities defined in terms of ratio of spectral integrals will become severely underestimated.

The agreement found between the initial uncertainty evaluation method of [1] and the new method based on Eq. (6) is good in this case. However, that cannot be taken as a general rule and thus use of Eq. (6) for Monte Carlo simulations produces more reliable uncertainties than the initial method of [1].

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