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Curved boundary integral method for off-axis parabolic reflector analysis

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We recently introduced a new boundary integral method for numerical electromagnetic beam synthesis on curved surfaces. This study applies the methodology to computing gaussian beam reflection from an off-axis parabolic mirror and validates the results with physical optics simulation. Good agreement was observed.

Keywords; electromagnetism, vector beams, beam propagation

I. Introduction

The angular spectrum method (ASM) is a well utilized method for synthesizing electromagnetic and acoustic fields, offering a systematic approach to derive near- and far-field distributions from known planar field configurations. Despite its efficacy, ASM has traditionally been limited to planar symmetric scenarios, constraining its applicability to more complex environments. We have recently extended the theoretical foundations of planar ASM to a curved-boundary integral method (CBIM) [1,2], which enables the synthesis of electromagnetic fields from arbitrary electric field distributions positioned on compact and continuous surfaces. Notably, CBIM facilitates the positioning of sources within or outside the computational domain and even on sub-regions of scatterers' surfaces, enhancing beam design flexibility.

Incorporating CBIM into the synthesis framework offers a unified approach for analyzing electromagnetic forward and backward propagation between optical elements [3,4]. This capability simplifies the analytical process, allowing for a comprehensive exploration of electromagnetic interactions within intricate optical systems using a single method.

In this study, we use CBIM to calculate reflected fields of an off-axis parabolic (OAP) mirror illuminated by a Gaussian beam and compare the obtained results with those from physical optics simulations [5]. In terahertz systems research, such simulations are typical for characterizing the behavior of optical components in the design and optimization of terahertz imaging and spectroscopy setups.

II. Curved-Boundary Integral Method

CBIM synthesizes electromagnetic fields from a source electric field positioned on the compact and continuous surface $\Omega(\boldsymbol{o})$, where $\boldsymbol{o}$ is a point at this surface. The surface Ω is discretized to source points, each of which has its local coordinate system, spanned with local base vectors $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$, presenting the direction of electric field, magnetic field, and beam propagation, respectively.

The electric field synthesized with CBIM is presented as a surface integral over the source points as

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi^2}\iint_{\Omega} E_0(\boldsymbol{o})[E_1(\boldsymbol{r};\boldsymbol{o})\boldsymbol{e}_1(\boldsymbol{o}) + E_3(\boldsymbol{r};\boldsymbol{o})\boldsymbol{e}_3(\boldsymbol{o})]d\Omega$$

$$= \frac{1}{4\pi^2}\iint_{\Omega} E_0(\boldsymbol{o})\boldsymbol{E}(\boldsymbol{r};\boldsymbol{o})d\Omega, \quad (1)$$

where $\boldsymbol{r}$ is a global position vector and E_0 is the source field. The source points in Eq. (1) are obtained as

$$\boldsymbol{E}(\boldsymbol{r};\boldsymbol{o}) = \iint_{\mathbb{R}^2} e^{i(k_x x + k_y y)} f(r;|z|)\boldsymbol{e}_0\, dk_x dk_y, \quad (2)$$

where x, y, and z are coordinates of $\boldsymbol{r}-\boldsymbol{o}$ in the local coordinate system related to $\boldsymbol{o}$. The integrand is defined as

$$f(r;|z|) = \begin{cases} e^{i|z|\sqrt{k^2-k_x^2-k_y^2}}, & k_x^2+k_y^2 \le k^2 \\ e^{-|z|\sqrt{k_x^2+k_y^2-k^2}}, & k_x^2+k_y^2 > k^2, \end{cases} \quad (3)$$

where $r^2 = k_x^2 + k_y^2$ and k is a wavenumber. Without loss of generality, the electric field is chosen to be polarized at local $(\boldsymbol{e}_1, \boldsymbol{e}_3)$ − plane as

$$\boldsymbol{e}_0(k_x, k_y, k_z) = \boldsymbol{e}_1 - \operatorname{sign}(z)\frac{k_x}{k_z}\boldsymbol{e}_3. \quad (4)$$

The local magnetic field can be obtained by using Faraday´s law to a local electric field in Eq. (2) as

$$\boldsymbol{H}(\boldsymbol{r};\boldsymbol{o}) = \frac{i}{\omega\mu}[\nabla \times \boldsymbol{E}(\boldsymbol{r};\boldsymbol{o})]. \quad (5)$$

Finally, the magnetic field is obtained similarly as a surface integral as in Eq. (1)

$$\boldsymbol{H}(\boldsymbol{r}) = \frac{1}{4\pi^2}\iint_{\Omega} E_0(\boldsymbol{o})\boldsymbol{H}(\boldsymbol{r};\boldsymbol{o})d\Omega. \quad (6)$$

III. Modeling Reflection

For computing any reflection with CBIM, we use the incident field to obtain a new source field and its local base vectors on the reflector surface satisfying the boundary conditions. To do so, let's first define incident field propagation direction vector $\boldsymbol{e}_{inc}$ at the reflector's surface from the Poynting vector as

$$\boldsymbol{e}_{inc}(\boldsymbol{o}) = \frac{Re\{\boldsymbol{E}_{inc}(\boldsymbol{o})\times\boldsymbol{H}_{inc}^*(\boldsymbol{o})\}}{|Re\{\boldsymbol{E}_{inc}(\boldsymbol{o})\times\boldsymbol{H}_{inc}^*(\boldsymbol{o})\}|}. \quad (8)$$

Then, the direction vector for the incident electric field is obtained as

$$\boldsymbol{e}_{elec\_inc}(\boldsymbol{o}) = \frac{Re\{\boldsymbol{E}_{inc}(\boldsymbol{o})-(\boldsymbol{E}_{inc}(\boldsymbol{o})\cdot\boldsymbol{e}_{inc}(\boldsymbol{o}))\boldsymbol{e}_{inc}(\boldsymbol{o})\}}{\|Re\{\boldsymbol{E}_{inc}(\boldsymbol{o})-(\boldsymbol{E}_{inc}(\boldsymbol{o})\cdot\boldsymbol{e}_{inc}(\boldsymbol{o}))\boldsymbol{e}_{inc}(\boldsymbol{o})\}\|}. \quad (9)$$

Base vectors $(\boldsymbol{h}_1, \boldsymbol{h}_2, \boldsymbol{h}_3)$ at each reflection point are defined for using the reflected fields as the new source fields. The base vectors are obtained by using the reflection dyadic $\hat{\hat{\boldsymbol{R}}} = \hat{\boldsymbol{I}} - 2\hat{\boldsymbol{n}}\hat{\boldsymbol{n}}$, where $\hat{\mathbf{I}}$ is identity dyadic and $\hat{\boldsymbol{n}}$ is the mirror's surface normal, see Figure 1. Reflected base vectors are

$$\boldsymbol{h}_1 = \hat{\hat{\boldsymbol{R}}}\cdot\boldsymbol{e}_{elec\_inc},\ \boldsymbol{h}_3 = \hat{\hat{\boldsymbol{R}}}\cdot\boldsymbol{e}_{inc},\ \boldsymbol{h}_2 = \boldsymbol{h}_3\times\boldsymbol{h}_1. \quad (10)$$

The last step is to obtain the source field i.e. the reflected field on the mirror´s surface as

$$E_{ref}(\boldsymbol{o}) = \boldsymbol{e}_{elec\_inc}(\boldsymbol{o})\cdot\boldsymbol{E}_{inc}(\boldsymbol{o}). \quad (11)$$

Now the electromagnetic field can be repropagated from the reflector surface using Eqs. (1) and (6) by substituting $E_0 \rightarrow E_{ref}$, and Eq. (4) by substituting $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3) \rightarrow (\boldsymbol{h}_1, \boldsymbol{h}_2, \boldsymbol{h}_3)$.

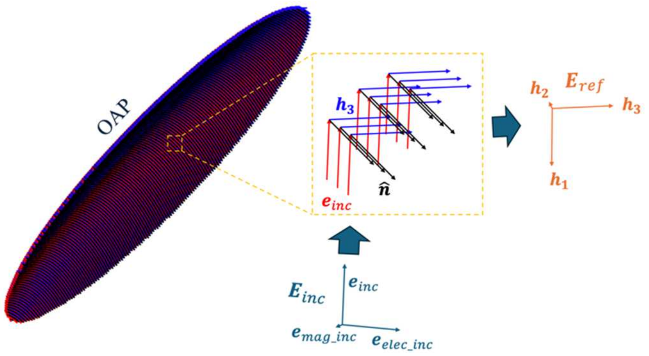

Figure 1: Base vectors for reflected fields on off-axis parabolic mirror surface, where $\boldsymbol{e}_{mag\_inc} = \boldsymbol{e}_{inc} \times \boldsymbol{e}_{elec\_inc}$.

IV. Simulation results

We use CBIM to compute Gaussian beam reflection from an OAP mirror with 90° offset angle at 330 GHz. The incident Gaussian beam, polarized along xz-plane, is created at the source plane, with a beam waist of $\omega_0 = 1.34$ mm. The source plane is located at a distance $d_1 = 76.2$ mm from OAP, which is twice the OAP's focal length $f = 38.1$ mm. The target plane is located at the distance $d_2 = 152.4$ mm from the OAP. Both the OAP and the target plane have diameter of 50 mm, see and Figure 2.

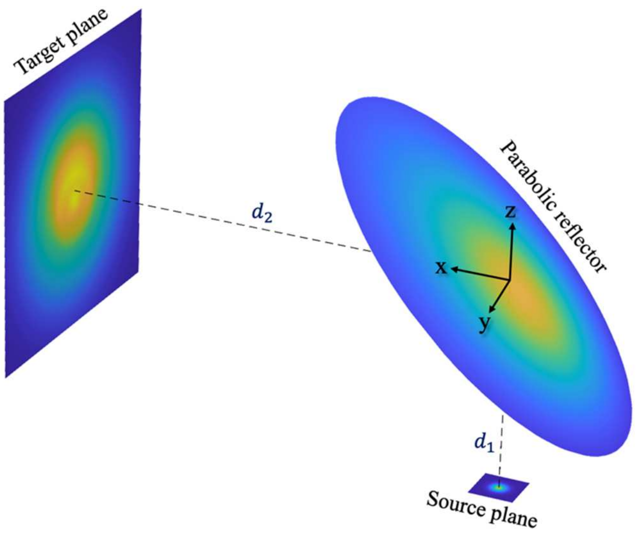

Figure 2: Quasi-optical arrangement with CBIM simulated surface fields. First, the Gaussian beam is propagated from the source plane to the OAP. Then, the beam is repropagated from the OAP to the target plane.

The polarization components, in Cartesian coordinates, of the electromagnetic field at the target plane are illustrated in Figure 3. These field components are compared to the physical-optics simulations with a high agreement down to less than –45 dB average difference in amplitude and less than 1° in phase (not shown). This result validates the accuracy of CBIM for electromagnetic reflector simulations.

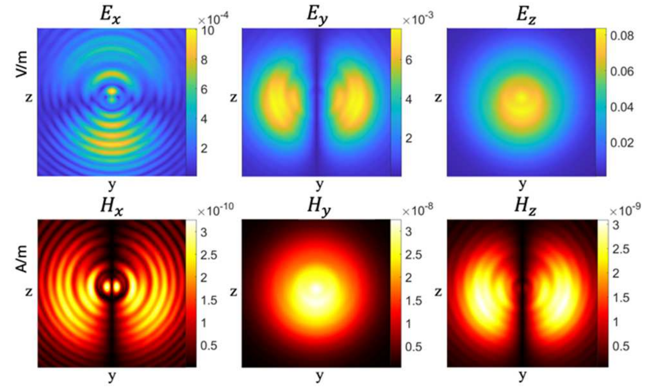

Figure 3: Cartesian electric and magnetic field components at the target plane simulated with CBIM, illustrated in linear scale.

The amplitude difference between the CBIM and physical-optics components are computed as

$$E_{diff\_i} = \left| \left| E_{CBIM\_i} \right| - \left| E_{PO\_i} \right| \right|, \quad i = x, y, z, \quad (12)$$

and the results are presented in Figure 4.

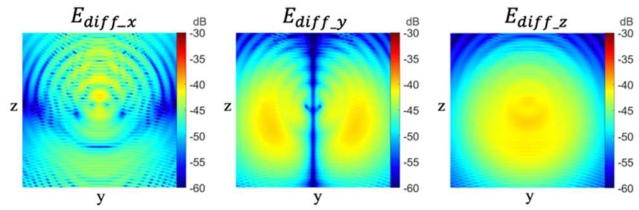

Figure 4: Difference in normalized field amplitudes between CBIM and physical optics, illustrated in logarithmic scale.

V. Conclusions

Recently, we introduced a curved boundary integral method for synthesizing electromagnetic beams. In this study, we demonstrate utility via simulations of Gaussian beam reflection from an off-axis parabolic mirror. Comparison with physical-optics simulations revealed a high degree of agreement, validating the accuracy of the proposed method. This work contributes to advancing computational electromagnetic techniques and offers a reliable tool for analyzing and optimizing optical systems involving curved surfaces in terahertz region and beyond.

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