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Temporal Discontinuity for Splitting Polarization States of Light

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Abstract – **Recently, time-varying electromagnetic structures have been extensively investigated to unveil new physical phenomena. In this direction, one of the important and historical topics is studying temporal discontinuities in these structures. Here, we consider fast changes of bianisotropic media. Specifically, we focus on introducing a temporal interface between isotropic chiral and dielectric media. We show that due to the discontinuity in time, interestingly, a linearly polarized electromagnetic wave is decomposed into forward right-handed and forward left-handed circularly polarized waves having** *different angular frequencies* **and** *the same phase velocities*. **This salient effect allows splitting light to two different polarization states with high efficiency. Hopefully, our findings will be useful as a possibility to control polarization states of light.**

I. Introduction

During the past decade, there has been great interest in exploring electromagnetics of nonstationary media and systems (e.g. [1, 2]). As a result, new phenomena have been uncovered using temporal modulation [3]. However, most studies have mainly focused on isotropic or anisotropic media, and time-varying bianisotropic media or systems [4] have not been carefully studied. From this point of view, time modulation of bianisotropic media is an unexplored area of research while intriguing phenomena could be obtained in such media. Therefore, here, we make an initial step in this direction and contemplate a nonstationary isotropic chiral medium. In particular, we consider a temporal interface between a chiral medium and an isotropic dielectric medium.

The main property that characterizes a chiral material is that the right- and left-handed circularly polarized (RHCP/LHCP) waves that can form a linearly polarized propagating wave have different phase velocities. This property is not exclusive to chiral media, it exists also in magnetized and magneto-optical materials. In chiral media, such difference in the phase velocity arises from spatial dispersion in materials with broken mirror-inversion symmetry. Due to this important characteristic, in this paper, we show that at the temporal interface between chiral and dielectric media, the angular frequency of the RHCP and LHCP waves will be shifted to two different angular frequencies resulting in splitting the polarization states. Up to our knowledge, this is an exotic phenomenon that has not been reported before (it is worth noting that there is an analogy with the spin Hall effect of light [5]). Such phenomenon will establish a new class of wave-matter interactions, spin-temporal interactions of light. In addition, in this paper, we also calculate the forward and backward waves generated due to the temporal discontinuity, and we prove that under certain condition, the backward waves vanish meaning that only the forward waves are propagating in the dielectric medium.

II. Time-domain model of chiral media

To describe optical rotatory power, Condon suggested the following phenomenological constitutive relation which connects the electric flux density to the time derivative of the magnetic field and the magnetic flux density to the time-derivatives of the electric field [6]. In this model, suggested in 1937, the most important feature is a possibility to approximately model chirality with a nondispersive parameter g. The model is applicable at frequencies well below all resonances of chiral molecules or inclusions. In chiral media, a linearly polarized wave can be expressed as a combination of RHCP and LHCP circularly polarized waves which have the same frequency

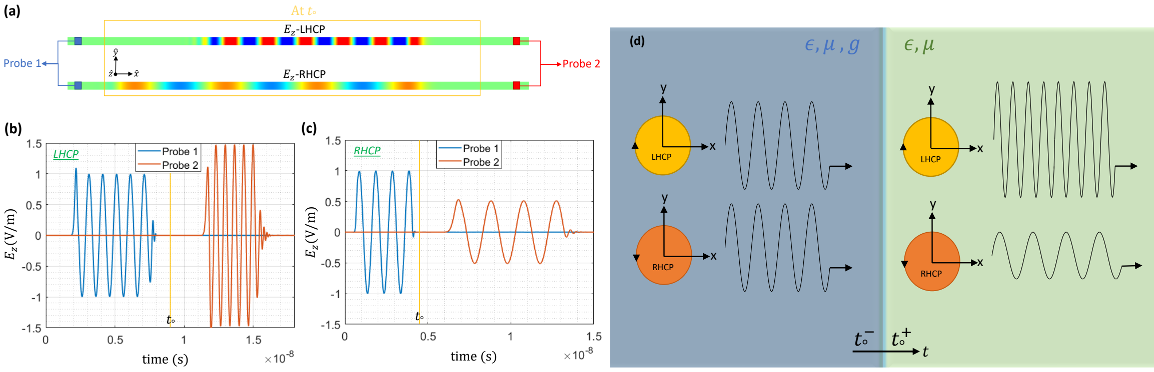

Fig. 1: Numerical simulation of a temporal interface between chiral and dielectric media using COMSOL Multiphysics.

but propagate at different phase velocities. Splitting the fields of a plane wave into RHCP and LHCP components, we write the material relations as

$$\mathbf{D} = \overbrace{\left[\epsilon\mathbf{E}^- + g\frac{\partial \mathbf{H}^-}{\partial t}\right]}^{\mathbf{D}^-} + \overbrace{\left[\epsilon\mathbf{E}^+ + g\frac{\partial \mathbf{H}^+}{\partial t}\right]}^{\mathbf{D}^+} \tag{1}$$

and

$$\mathbf{B} = \overbrace{\left[\mu\mathbf{H}^- - g\frac{\partial \mathbf{E}^-}{\partial t}\right]}^{\mathbf{B}^-} + \overbrace{\left[\mu\mathbf{H}^+ - g\frac{\partial \mathbf{E}^+}{\partial t}\right]}^{\mathbf{B}^+}, \tag{2}$$

in which ϵ is the permittivity of the medium, μ denotes the permeability of the medium, and g represents the chirality parameter (or rotatory parameter as Condon called it). The $\pm$ superscripts mark the RHCP and LHCP wave components, respectively. Let us write the electric and magnetic fields of a linearly polarized plane wave propagating in the x-direction as $\mathbf{E}^\pm = \frac{a}{2}\left[\mathbf{a}_y \mp j\mathbf{a}_z\right]\exp(-j\beta^\pm x)\exp(j\omega t)$ and $\mathbf{H}^\pm = \frac{a}{2\eta}\left[\mathbf{a}_z \pm j\mathbf{a}_y\right]\exp(-j\beta^\pm x)\exp(j\omega t)$, where η is the medium intrinsic impedance, and a is the complex amplitude of the electric field. We use the electrical engineering convention for time-harmonic oscillations (i.e., $\exp(j\omega t)$). In addition, the wavenumbers are equal to $\beta^\pm = \omega(\sqrt{\mu\epsilon} \mp \omega g)$ [7]. By substituting the fields into Eqs. (1) and (2), and considering that $c = 1/\sqrt{\mu\epsilon}$, we obtain

$$\mathbf{D}^\pm = \epsilon\Big(1 \mp \omega g c\Big)\mathbf{E}^\pm, \quad \mathbf{B}^\pm = \mu\Big(1 \mp \omega g c\Big)\mathbf{H}^\pm. \tag{3}$$

III. Temporal Interface between Chiral and Dielectric Media

In his paper published in 1958, Morgenthaler showed that the electric and magnetic flux densities are continuous at a temporal interface [8]. Using that property, we write that $\mathbf{D}_1^\pm = \mathbf{D}_2^\pm$ and $\mathbf{B}_1^\pm = \mathbf{B}_2^\pm$, where the subscripts $1, 2$ correspond to the fields before $(t = t_\circ^-)$ and after $(t = t_\circ^+)$ the temporal discontinuity, respectively ($t_\circ$ is the switching moment). Let us consider a temporal interface between chiral and dielectric media having the same permittivity and permeability. However, one can solve the problem from a general point of view meaning that the two media have different permittivity and permeability. According to Morgenthaler, after the jump, there are forward and backward waves in analogy with a spatial interface at which we have transmitted and reflected waves. Keeping this in mind, and after some mathematical manipulations, we find that

$$\epsilon(1 \mp \omega_1 g c)\mathbf{E}_1^\pm = \epsilon(T^\pm + R^\pm)\mathbf{E}_1^\pm, \qquad \mu(1 \mp \omega_1 g c)\mathbf{H}_1^\pm = \mu(T^\pm - R^\pm)\mathbf{H}_1^\pm. \tag{4}$$

The above expressions indicate that the polarization states and the phase constants are conserved along the temporal interface. Due to the conservation of the phase constant (β) and by knowing that $\beta_1^\pm = \omega_1(\sqrt{\mu\epsilon} \mp \omega_1 g)$ and $\beta_2^\pm = \omega_2^\pm\sqrt{\mu\epsilon}$, we arrive to the following important relation:

$$\omega_2^\pm = \omega_1\Big(1 \mp \omega_1 g c\Big). \tag{5}$$

This result means that the RHCP and LHCP components have different angular frequencies after the temporal jump, and the polarization states are separated. In addition, from Eq. (4), we derive that $R^{\pm} = 0$ and $T^{\pm} = 1 \mp \omega g c$. In other words, when we jump from a chiral medium to a dielectric one (while keeping the permittivity and permeability the same), no backward waves are generated.

The above analytical results are verified numerically using the time-domain solver of the commercial software COMSOL Multiphysics®. Consider RHCP and LHCP incident plane waves propagating in a chiral medium, where $a = 2$ [V/m], $\omega_1/(2\pi) = 1$ [GHz], $g = 0.037 \times 10^{-17}$ [s^2/m], $\mu = \mu_0$ [H/m], $\epsilon = \epsilon_r \epsilon_0$ [F/m], and $\epsilon_r = 2$. At a temporal discontinuity, the medium properties change in time from those of this chiral medium to a simple dielectric medium having $\mu = \mu_0$ [H/m], $\epsilon = \epsilon_r \epsilon_0$ [F/m], and $g = 0$, meaning that the medium has the same permittivity and permeability before and after the time discontinuity. According to the theoretical results presented above, there should be no generated backward waves ($R^{\pm} = 0$), and the forward propagating waves should have the transmission coefficients and angular frequencies given by $T^{\pm} = 1 \mp 0.49$ and $\frac{\omega_2^{\pm}}{2\pi} = 1 \mp 0.49$ [GHz].

To simplify the data analysis, we simulate the RHCP and LHCP waves separately. The simulation domain is shown in Fig. 1(a), where periodic boundary conditions are applied at the top and bottom boundaries, to emulate plane waves. The boundary on the left is assigned to a scattering boundary behaving as a source, and the right boundary is assigned to a scattering boundary behaving as a perfect absorber. Two probes are used to measure the fields. Before the temporal interface, probe 1 measures the incident fields. On the other hand, after the temporal interface probe 1 measures the backward propagating waves, and probe 2 measures the forward propagating waves. Figures 1(b) and 1(c) show the z-component of electric field for the incident, backward, and forward propagating waves. It can be seen that there are no backward propagating waves, while the amplitudes and frequencies for RHCP and LHCP forward propagating waves are different. In addition, the amplitudes and frequencies are in agreement with the theoretical predictions given above. Similar results are obtained for the y- component of the electric field and for the magnetic field components.

IV. Conclusion

We have shown that by introducing a temporal interface between chiral and dielectric media, a linearly polarized wave propagating in a chiral medium is split into two polarization states (RHCP and LHCP) propagating in the dielectric medium. These waves propagate in the forward direction at different angular frequencies, while interestingly they have the same phase velocity $v_\mathrm{p} = 1/\sqrt{\mu\epsilon}$. Extended theoretical and numerical results will be shown in the conference presentation.

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