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Torque Reconstruction for Maritime Powertrains Using Trend Filtering ⋆

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Abstract: This paper presents a convex optimization approach for the simultaneous reconstruction of unknown input torque and torsional response in the driveline of an azimuth thruster. Accurate estimates of the shaft torque responses are necessary for condition monitoring purposes. Estimating torque responses also enables flexibility in the choice of sensor location, with subsequent potential savings in installation and maintenance costs. It is shown that the unknown inputs and states can be reconstructed using batch torque measurements from a single location in the propulsion line. The estimation problem is formulated as a trend-filtering problem, enforcing the smoothness of input estimates. The performance of the proposed method is evaluated by means of simulations and experiments on a small-scale testbench of a maritime azimuthing thruster. The results show that the torsional response of the propeller shaft can be accurately reconstructed using torque measurements from sensors installed near the driving motor at the opposite end of the driveline.

Keywords: Input estimation, virtual sensor, convex optimization, marine propulsion, trend filtering, torque reconstruction, azimuthing thruster

1. INTRODUCTION

Maritime propulsion systems are affected by various excitations caused by hydrodynamic loads, ranging from excitations related to normal operation, emergency situations, and heavy sea conditions (Kozłowska, 2019). In the arctic regions, which have gained increasing interest for maritime operations (Jutterström et al., 2021; Kondratenko et al., 2023), excitations caused by propeller-ice interactions are especially of concern (Soininen, 1998; Ikonen et al., 2015; de Waal et al., 2018). Monitoring the condition of the system is crucial for preventing fatal scenarios, such as shaft or gear failures, and for implementing predictive maintenance strategies. However, condition monitoring requires knowledge of the unknown external excitations that rarely can be measured directly. Thus, this paper proposes a virtual sensor for reconstructing torque excitations and responses in maritime thruster drivetrains.

External forces due to, e.g., propeller-ice interactions have been measured with strain gauges installed directly on the propeller blades (Ikonen et al., 2015). As the propeller is subject to heavy excitations, the measurement equipment can break, requiring a reinstallation of the measurement equipment. Measuring the torsional response of the complete propulsion system is often not a viable option in practice, as this requires installing measuring equipment on multiple sections of the system, which can be limited by space and installation costs. By estimating the response of the system, more freedom is attained in the choice of physical sensor locations.

Several approaches have been proposed for online estimation of unknown inputs and maritime propulsion system states. Whereas the simultaneous input-and-state estimation methods (Gillijns and De Moor, 2007; Bitmead et al., 2019) and augmented Kalman filtering and smoothing methods (Lourens et al., 2012; Manngård et al., 2019; Lagerblad et al., 2021; Manngård et al., 2022) are considered state of the art, Manngård et al. (2022) presented a Kalman-filter approach for estimating the states and unknown inputs of an azimuthing thruster. The thruster model was augmented with excitations modeled as quasi-stationary stochastic signals with bounded spectral densities.

Input reconstruction from batch measurements can be considered an inverse problem, which has, e.g., been solved by regularization methods (Ikonen et al., 2015; de Waal et al., 2018; Nickerson and Bekker, 2021; Koker and Bekker, 2022). Ikonen et al. (2015) considered truncated singular value decomposition and Tikhonov regularization for inverse determination of propeller torque in a direct driven propulsion system. The work of Ikonen et al.

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(2015) was extended by Nickerson and Bekker (2021), where the accuracy and computational speed of different regularization methods were studied, finding Tikhonov regularization to perform the best. However, Tikhonov regularization might perform poorly for the estimation for non-stationary excitations or step-like disturbances. Thus, we propose a regularization approach based on Hodrick-Prescott filtering (Hodrick and Prescott, 1997) for reconstructing unknown input excitations.

This article studies the problem of reconstructing torque excitation and responses in maritime azimuthing-thruster drivetrains. The input-reconstruction problem is formulated as a trend filtering problem, where shaft dynamics and measurement errors are accounted for in the system model, and regularization is applied to enforce smoothness in input estimates. The proposed method has been implemented on a laboratory-scale test bench designed to emulate the real-life operating conditions of a maritime azimuth thruster. It is shown that the proposed trend-filtering method can be used to estimate the propeller torque excitations and propeller-shaft torque responses using measurements from a location close to the driving motor. The method is verified using simulations and experiments on the testbench.

2. MATERIALS AND METHODS

This section presents the modeling methods and problem formulation for unknown input torque estimation using trend filtering methods. The laboratory-scale maritime thruster testbench presented in Figure 1 was used to verify the method.

2.1 Dynamic model

A lumped-element model of a maritime thruster is derived in the form of a discrete-time linear time-invariant (LTI) state-space model. The discrete-time state-space model is then extended to a form allowing the use of batch input data. For torsional vibration analysis, rotating machinery, including maritime thrusters, can be modeled using the shaft-line finite element method (FEM) (Friswell et al., 2010; Genta, 2012; Murawski and Charchalis, 2014). In the shaft-line FEM, components of the system are divided into lumped-elements with inertia I_i connected by torsional springs with stiffness k_i and damping c_i (Figure 2). Torque losses due to, e.g., viscous friction, can be included in the model as internal damping of the shafts, or external damping d_i. The lumped-element model can be represented in a discrete-time state-space form

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

$$y(k) = Cx(k) + Du(k) + v(k), \tag{2}$$

where $x(k) \in \mathbb{R}^n$ are the system states. The system matrices A, B, C are assembled from the lumped mass-moment of inertia I_i, damping coefficients c_i and d_i, and stiffness k_i, cf. (Manngård et al., 2019, 2022). Differing from the standard approach (Friswell et al., 2010), to ensure that the state-space realization is minimal, the state vector is expressed in terms of angular velocities $\dot{\theta}_i$ and difference in consecutive lumped-element angles $\Delta\theta_i = (\theta_i - \theta_{i-1})$. The values $u(k) \in \mathbb{R}^m$ are unknown torque excitations, $y(k) \in \mathbb{R}^l$ are output measurements, and $v(k) \in \mathbb{R}^l$ measurement noise. The measurement noise is assumed to be a zero-mean stationary-stochastic signal with known covariance

$$\mathrm{E}\left[v(k)v(k)^\top\right] = R. \tag{3}$$

Repeated substitution of (1) into (2) allows measurements $y(0), y(1), ..., y(N-1)$ to be expressed in terms of input signals $u(k)$ and the initial state $x(0)$ as

$$y = \mathcal{O}x(0) + \Gamma u + v, \tag{4}$$

where

$$y = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix}, \ u = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}, \ v = \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix},$$

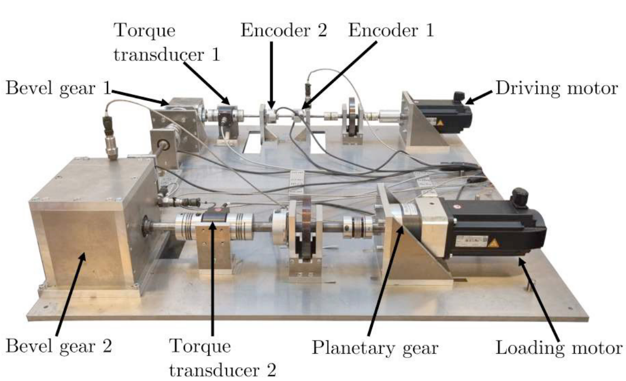

Fig. 1. Small-scale maritime thruster testbench. The testbench speed was kept constant using the driving motor. Torque excitations were applied using the loading motor. Measurements from encoder 1, encoder 2 and torque transducer 1 were used for torque reconstruction, verified using measurements from torque transducer 2.

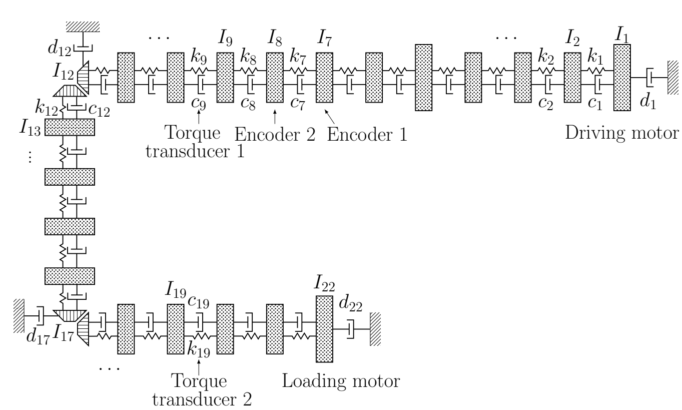

Fig. 2. Lumped-element model of the laboratory testbench. The model consists of lumped masses and gear elements connected by torsional springs and dampers. External torque is applied at the first and last lumped element positions.

$$\Gamma = \begin{bmatrix} D & 0 & 0 & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & & 0 \\ \vdots & & \ddots & \ddots & \\ CA^{N-2}B & CA^{N-3}B & \cdots & CB & D \end{bmatrix}, \ \mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{N-1} \end{bmatrix}.$$

2.2 Optimization problem

For known initial state $x(0)$ in (4), the input reconstruction problem can be posed as a weighted, regularized least-squares problem

$$\begin{aligned} \underset{u,v}{\text{minimize}} \quad & L(u) + \frac{1}{N}\sum_{k=0}^{N-1} v(k)^\top R^{-1} v(k) \\ \text{subject to} \quad & v = y - \mathcal{O}x(0) - \Gamma u \end{aligned} \tag{5}$$

where $L(u)$ is a regularization term introduced to ensure the least-squares problem is well-posed. The regularization function $L(u)$ should be determined based on known properties of the unknown input signal u.

For stationary-stochastic zero-mean input excitations with known covariance, $\mathrm{E}\left[u(k)u^\top(k)\right] = Q$, the regularized term can be set to

$$L(u) = \frac{1}{N}\sum_{k=0}^{N-1} u^\top(k) Q^{-1} u(k) \tag{6}$$

resulting in the weighted Tikhonov-regularized problem

$$\begin{aligned} \underset{u,v}{\text{minimize}} \ & \sum_{k=0}^{N-1} u(k)^\top Q^{-1} u(k) + \frac{1}{N}\sum_{k=0}^{N-1} v(k)^\top R^{-1} v(k) \\ \text{subject to} \quad & v = y - \Gamma u. \end{aligned} \tag{7}$$

Defining matrices $W_\mathrm{R} = I_N \otimes R^{-1}$ and $W_\mathrm{Q} = I_N \otimes Q^{-1}$, the problem has the explicit solution

$$u = \left(W_\mathrm{Q} + \Gamma^\top W_\mathrm{R} \Gamma\right)^{-1} \Gamma^\top W_\mathrm{R} y. \tag{8}$$

In practice, unknown input excitations are rarely zero-mean stationary stochastic. Instead, signals can be enforced to be smooth in order to constrain the feasible set of solutions in the input reconstruction problem. Similar to trend filtering, where the goal is to find a trade-off between the smoothness of estimates and the residual error (Hodrick and Prescott, 1997; Kim et al., 2009), the input reconstruction problem can be formulated to promote smoothness in input estimates by constraining the second difference of u with the regularisation function

$$L(u) = \sum_{k=1}^{N-2} \left(u(k-1) - 2u(k) + u(k+1)\right)^2 = \|\Delta_2 u\|_2^2. \tag{9}$$

In the present case, for input torques u_m from the motor and u_p from the propeller, the regularization term is

$$\Delta_2 u = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 & & & & \\ & 1 & 0 & -2 & 0 & 1 & & & \\ & & \ddots & \ddots & \ddots & \ddots & & & \\ & & & 1 & 0 & -2 & 0 & 1 & \\ & & & & 1 & 0 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_\mathrm{m}(0) \\ u_\mathrm{p}(0) \\ \vdots \\ u_\mathrm{m}(k) \\ u_\mathrm{p}(k) \end{bmatrix}. \tag{10}$$

The input reconstruction problem then becomes

$$\begin{aligned} \underset{u,v}{\text{minimize}} \quad & \lambda\|\Delta_2 u\|_2^2 + \sum_{k=0}^{N-1} v(k)^\top R^{-1} v(k) \\ \text{subject to} \quad & v = y - \mathcal{O}x(0) - \Gamma u. \end{aligned} \tag{11}$$

The regularization parameter $\lambda \geq 0$ was introduced to control the trade-off between the smoothness of u and the output error $y - \Gamma u$. The problem has an explicit solution

$$u = \left(\lambda \Delta_2^\top \Delta_2 + \Gamma^\top W_\mathrm{R} \Gamma\right)^{-1} \Gamma^\top W_\mathrm{R} y. \tag{12}$$

The proposed method resembles the Hodrick-Prescott trend filter (Hodrick and Prescott, 1997; Kim et al., 2009). However, the crucial difference is that, here, the known dynamics of the system and measurement errors have been properly accounted for.

2.3 Measurements on a small-scale testbench

A lumped-element model of the testbench was created and formulated to a discrete-time LTI state-space model as described in Section 2.1. Theoretical analysis of the method was conducted using simulated excitations. The method was verified experimentally using shaft torque (Torque transducer 1) and rotational speed (Encoder 1 & 2) measurements near the driving motor of the testbench (Figure 1). The torsional response reconstructed using estimated motor and propeller input torques was compared to shaft torque measurements from the propeller shaft (Torque transducer 2).

The testbench represents a down-scaled maritime thruster. It has been designed to have dynamical properties comparable to a full-scale thruster, for example similar torsional natural frequencies and angular displacement of shafts at nominal load. The testbench consists of shafts, couplings and two bevel gears. The bevel gears, annotated in Figure 1, have gear ratios 3:1 and 4:1 respectively. The testbench includes two identical 2.63 kW servomotors for driving and loading the testbench. The loading motor is coupled with a planetary gear with a gear ratio 1:8. The used measurement equipment include two rotary encoders and a torque transducer on the driving motor shaft. Another torque transducer is located on the propeller shaft for verification measurements. For a thorough description of the testbench, the reader is referred to (Haikonen et al., 2022).

3. RESULTS

3.1 Simulated experiments

Three simulated excitation cases were considered for theoretical analysis. The considered cases were an impulse, step, and periodic excitation (Figure 3). All of these input torques act on the propeller inertia in the system model. The length of the simulated excitations were 500 timesteps, where the step size $\Delta t = 0.001$s. In all of the simulated excitation cases, the driving motor torque was set to a constant 2.7 Nm to mimic real operating conditions of the testbench. Zero-mean Gaussian white noise with a standard deviation $\sigma = 0.1$ was added to the simulated excitations and the response to include process and measurement noise in the simulation experiments.

The regularization parameter λ affects how strongly regularization is enforced in the input reconstruction problem, and its value is defined by the user. One way to determine values λ is to use the L-curve method. The L-curve criterion states that a pareto-optimal λ can be found at

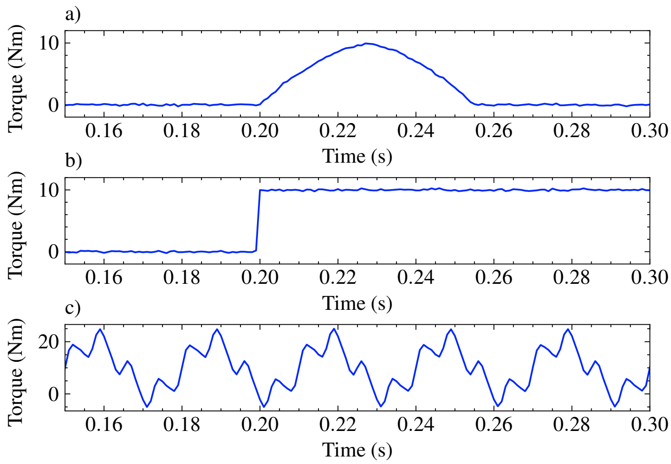

Fig. 3. Simulated excitations, where a) is a single impulse with an amplitude of 10 Nm, b) is a 10 Nm step and c) is a sum of three sine waves with 10 Nm DC-offset, amplitudes of 10, 5 and 1 Nm and frequencies corresponding to 1, 4 and 8 times 2000 rpm without phase difference.

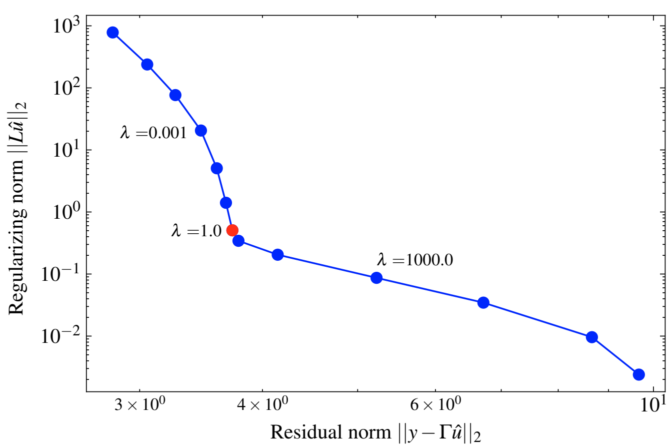

Fig. 4. Residual norm and the regularizing norm, resulting from the estimation of the simulated impulse excitation with different λ values, forms the L-curve. A suitable regularization parameter $\lambda = 1$ is annotated in red near the corner of the L-curve.

the corner of the L-curve. The corner can be found by calculating the curvature of the L-curve and the pareto-optimal λ is found where the curvature is the largest (Hansen, 1998). This method includes a challenge that the regularization parameter can be too large, thus the regularization is enforced too strongly (Hansen, 2001). In the present application, this means that smoothness could be enforced more than is desirable. Another challenge regarding the pareto-optimal λ is that it most likely varies between different data batches. To find a precise value for λ, the L-curve should be calculated for each batch of data used in the estimation, increasing computational cost significantly. Thus, it would be beneficial for the present application to predetermine a value for λ using simulations, which yields satisfactory results in practice.

Instead of calculating the L-curve and its maximum curvature for every individual data batch, λ was determined from L-curve using simulation results. This way, the opti-

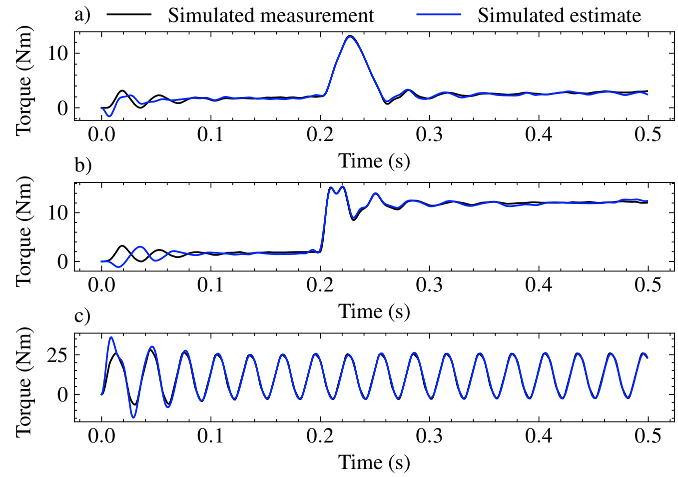

Fig. 5. Propeller shaft torque reconstructed using the simulated unit excitations. In a) is the response to the impulse, in b) the step excitation and in c) the periodic excitation. Regularization parameter $\lambda = 1$ was used in all three simulations, determined from L-curves of each case respectively.

mality of λ is not guaranteed, however, a sufficient value λ can readily be found. Furthermore, the user may want to vary how strongly the smoothness in the estimate is enforced, thus, a reasonable range for values λ could be determined using the L-curve. The testbench model and the simulated excitations were used to form an L-curve for each simulation case. The L-curves were used to determine a suitable regularization parameter λ to be used in the input estimation problem. In Figure 4 is presented the L-curve produced using the simulated impulse excitation.

Simulated measurements were used for the estimation of input torques. The simulated measurements were produced using the testbench model and the excitations shown in Figure 3. The simulated input torques were estimated using Equation (12). Gaussian white noise was added to the simulated measurements, thus, the measurement noise covariance matrix $R = I$, where I is the identity matrix, was used. The states of the testbench were reconstructed using the discrete-time state-space model and the estimated input torques with

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + B\hat{u}(k) \\ \hat{y}(k) &= C\hat{x}(k), \end{aligned} \tag{13}$$

where $\hat{u}(k)$ is the estimated input torque and $\hat{y}(k)$ is the resulting estimated response. The reconstructed torsional response at the propeller shaft is presented in Figure 5.

3.2 Experimental verification

Three unit excitation cases similar to the simulated excitations were considered for the experimental verification. The testbench was excited using the loading motor and the driving motor speed was kept constant at 2000 rpm. Shaft torque and rotational speed measurements near the driving motor were used in the estimation of the input torques. Shaft torque measurements on the propeller shaft were used for verifying the shaft torque reconstructed using the estimated input torques. The measurement noise covariance matrix $R = \mathrm{diag}(0.05, 0.10, 0.20)$ was deter-

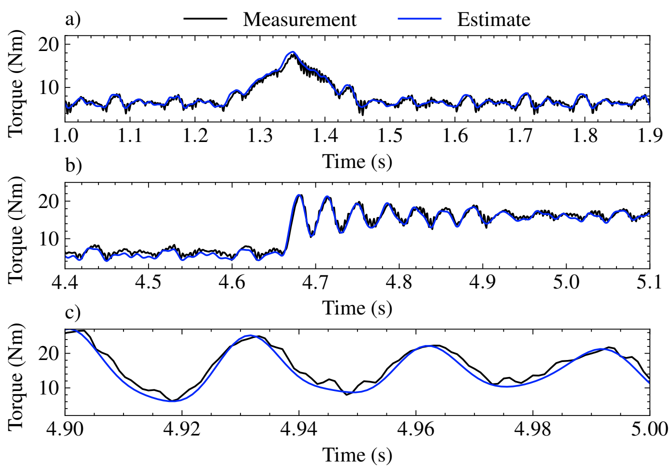

Fig. 6. Propeller shaft torque measurement from the testbench compared with the shaft torque reconstructed using the trend filter input torque estimates. In a) is the response to the impulse, in b) the step excitation and in c) the periodic excitation. Regularization parameter $\lambda = 1$ was used in all three experiments, determined from L-curves of the simulated cases.

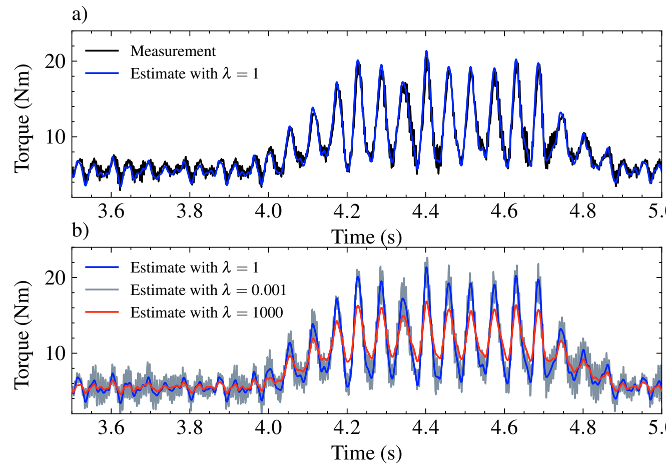

Fig. 7. a) Propeller shaft torque transducer measurement compared with the shaft torque reconstructed using the trend filter input torque estimates with $\lambda = 1$. b) Estimates with $\lambda = 0.001$, $\lambda = 1$ and $\lambda = 1000$ demonstrate the smoothing enforced by regularization, and sensitivity of the trend filter with respect to different values λ.

mined using the testbench measurement data. The diagonal elements of R correspond to the measurement noise covariance of encoder 1, encoder 2 and torque transducer 1 respectively.

The measurement data was divided into batches of 700 consecutive timesteps. Consecutive batches were overlapped by 100 timesteps from the beginning and the end to remove transients in the estimate, which would be caused by setting the initial state to zero in the input reconstruction problem (Equation (11)), when it is not actually zero. The same λ values as in the simulated cases were used in the estimation of the unit excitations. The overlapping input torque estimates were dropped and the propeller shaft torque was reconstructed using Equation (13), similar to the simulated experiments. Results of the unit excitation experiment are presented in Figure 6.

In addition to the unit excitations, an ice excitation experiment was conducted. The propeller-ice excitation was defined as described in maritime rules and regulations (DNV, 2011; TRAFI, 2021). The excitation was applied using the loading motor while the driving motor speed was kept constant at 2000 rpm. The estimated propeller shaft torque is shown in Figure 7. The regularization parameter $\lambda = 1$ was again used. The trend filter input estimates resulted in a smooth reconstructed propeller shaft torque. Additionally, the estimation was carried out with $\lambda = 0.001$ and $\lambda = 1000$ to analyze, in a practical manner, the sensitivity of the trend filtering method with respect to different values λ.

Figure 8 presents the error distributions of the estimated propeller shaft torque in the four excitation cases. The error distributions appear Gaussian with a negative bias. The bias could be due to too large external damping values in the testbench model, causing a DC offset in the reconstructed propeller torque when compared to the measurements.

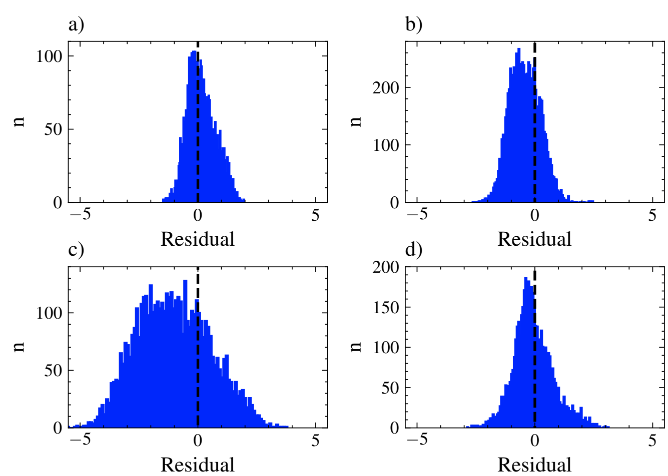

Fig. 8. Error distribution of the propeller shaft torque estimates compared to measured shaft torques in the a) impulse, b) step, c) periodic and d) ice excitation cases.

4. DISCUSSION AND CONCLUSIONS

The reconstruction of the input torques and states of a small-scale thruster testbench was studied. A trend-filtering method was proposed for reconstructing torque excitations and shaft torque responses. Experiment results show that the proposed method can accurately reconstruct torque signals from velocity measurements and a single torque sensor placed on the driving motor shaft.

In contrast to conventional Kalman-filtering approaches, the trend-filtering formulation simplifies the imposition of additional constraints, such as ensuring smoothness in the estimates. This makes the design of virtual sensors more convenient and flexible. The application of constraints to the estimation process aids in ensuring that the reconstructed signals adhere to physical principles.

The presented method contributes to the improving condition monitoring procedures of maritime propulsion systems. Knowledge of large torque variations in the propulsion system allows proper maintenance scheduling and avoiding fatal scenarios, such as failure of mechanical components. Using industry-standard modeling methods makes applying the torque estimation procedure straightforward for different systems. Predetermination of the regularization parameter with simulations decreases computational cost compared to calculating it online for every measurement batch, and the analytical solution of the trend-filtering problem can make real-time estimation viable. The results on the laboratory testbench support the viability of the torque estimation method in full-scale applications, and the method could be readily taken into use. However, verification using measurements from a full-scale thruster should be done. In addition to full-scale tests, future work includes studying other regularized least-squares problems for torque estimation, different approaches for real-time torque estimation, and including an electric motor model in the thruster model.

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