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Published in:
2024 18th International Congress on Artificial Materials for Novel Wave Phenomena, Metamaterials 2024

DOI:
10.1109/Metamaterials62190.2024.10703229

Published: 01/01/2024

Document Version
Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:
Shabanpour, J., Tretyakov, S., & Simovski, C. (2024). Applicability of Physical Optics for Predicting the Diffraction Pattern of Binary Metasurfaces. In *2024 18th International Congress on Artificial Materials for Novel Wave Phenomena, Metamaterials 2024* IEEE. https://doi.org/10.1109/Metamaterials62190.2024.10703229

18th International Congress on Artificial Materials for Novel Wave Phenomena - Metamaterials 2024
Crete, Greece, Sep. 9$^{\text{th}}$ - 14$^{\text{th}}$ 2024

Applicability of Physical Optics for Predicting the Diffraction Pattern of Binary Metasurfaces

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Abstract – **In this paper, we outline the domain of adequacy for the well-known approximation of physical optics (PO) for binary metasurfaces (MSs). First, we show that beyond the approximation of so-called reflection locality (RL) PO cannot be applied for practical reflecting MSs. Next, in the framework of RL approximation we derive a closed-form expression for the diffraction pattern of a finite-size binary MS and compare the results with full-wave simulations. We show that PO may be applied for a binary MS if and only if the incident and deflection angles are sufficiently small, i.e. the MS period is sufficiently large. In this case, the desired anomalous reflection is due to one of the two first-order diffraction lobes, whose magnitudes are adequately predicted by PO.**

I. Introduction

Many reconfigurable intelligent surfaces (RISs) reported in the recent literature are implemented as binary MSs with a controllable period (see e.g. overview [1]), and physical optics (PO) is commonly used for their initial synthesis (see e.g. in [2]). Moreover, in the absolute majority of works dedicated to the signal processing in telecommunication systems comprising RISs, the utilization of PO is adopted by default, without discussing of its applicability region (see e.g. in [3]). However, PO is a rough approximation that not only treats a periodically non-uniform MS (PNUMS) as a sheet of continuous electric and magnetic surface polarizations varying periodically, but also treats the incident wave as a set of parallel rays. In PO, as in the ray optics, each ray is reflected from the corresponding point of a non-uniform reflecting surface as if it were infinite and uniform. This approximation implies that the spatial variation of the surface polarization is slow, i.e., the local response of the reflector varies noticeably only on the scale much larger than the wavelength λ. However, the period D of a PNUMS is always comparable with λ, D may be only slightly larger. Thus, predictions obtained in the framework of this rough approximation need to be properly validated. However, for a finite PNUMS simulations are very time-consuming, and it is important to know in advance in which cases PO may work and when it cannot. In this paper, we answer this question for a binary MS – a PNUMS of alternating supercells (groups of identical reflecting unit cells) whose length is $D/2$. The unit cell is strongly subwavelength and the two supercells differ by the state of the unit cell loads, denoted as states 0 and 1, respectively.

II. Theory

The PO and reflection locality (RL) are different approximations whose applicability domains are in general different but may intersect. As we will see below, for reflecting PNUMSs the PO cannot be adequate beyond RL, and RL does not guarantee the adequacy of PO. Let us discuss RL for a non-uniform MS. This approximation means that we attribute to any unit cell of this MS the same induced electric $\mathbf{J}_e$ and magnetic $\mathbf{J}_m$ surface current densities that would exist at this point if the MS were in its fundamental (uniform) state, i.e., all its unit cells were loaded by identical loads. For a binary MS it means that we characterize the supercells loaded by loads '0' and '1' by the local reflection coefficients R_0 and R_1 that are equal to those of an infinite MS in its fundamental (uniform) states'0' and '1'. Both reflection amplitudes $|R_0|$ and $|R_1|$ in these fundamental states should be nearly equal unity whereas the reflection phases Φ_0 and Φ_1 should differ approximately by π. In fact, the electromagnetic interactions in a non-uniform MS may lead to $\Phi_0 - \Phi_1$ being very different from π. In work [6], we have shown

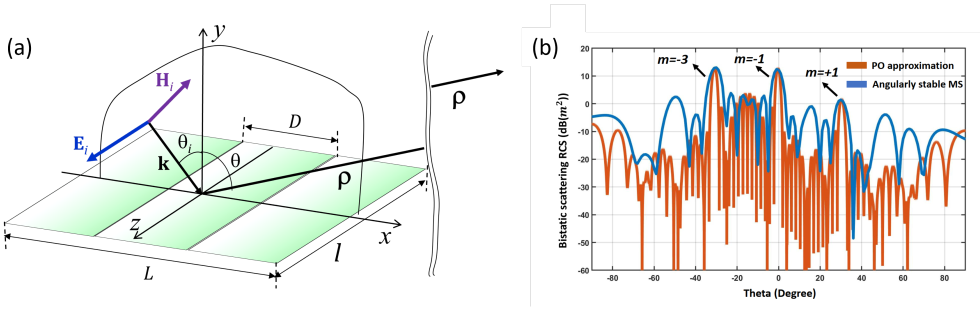

Fig. 1: (a) The far-zone reflection problem for a finite PNUMS (color illustrates variations of the local reflection coefficient). (b) The diffraction pattern of a binary MS from [7] – PO versus full-wave simulations (in the numerically simulated pattern small-scale angular oscillations corresponding to zeros of F are smoothed).

the following: though the electromagnetic interaction of unit cells is never negligible, under certain conditions it may allow $\Phi_0 - \Phi_1 \approx \pi$ for supercells of properly designed binary MSs. Then RL is adequate at least for an infinite PNUMS. Meanwhile, PO demands that RL holds for the whole finite-size MS.

In work [5] the general formula (18) was derived that expressed the approximation of the PO to a rectangular reflecting MS of area S illuminated by a plane wave incident on it under the angle θ_i. In that derivation, it was assumed that the observation point is located at a distance from the MS center that is much larger than its size L. In our own notations, this formula is as follows:

$$\mathbf{E}_s = \frac{jke^{-jk\rho}}{4\pi\rho}\int_S e^{jk\boldsymbol{\rho}_0\cdot\mathbf{r}}\boldsymbol{\rho}_0\times[\eta\boldsymbol{\rho}_0\times\mathbf{J}_e(\mathbf{r})+\mathbf{J}_m(\mathbf{r})]dxdz. \tag{1}$$

Here, $\boldsymbol{\rho}_0$ is the unit vector in the direction of the vector $\boldsymbol{\rho}$ connecting the far-zone observation point with the center of the MS, $\mathbf{r}=(x,z)$ is the radius-vector of the integration point, $\eta=\sqrt{\mu_0/\varepsilon_0}$, and all other notations are shown in Fig. 1. In the framework of RL, the electric and magnetic surface currents can be expressed through the incident electric field $\mathbf{E}_i(\mathbf{r})$ at the illuminated side and the local reflection coefficient $R(\mathbf{r})$. In the case of transverse electric (TE) polarization with $\mathbf{E}_i(\mathbf{r})=\mathbf{z}_0E_0\exp(-jkx\sin\theta_i)$, we have

$$\mathbf{J}_m = \mathbf{z}_0E_0e^{-jkx\sin\theta_i}[1+R(\mathbf{r})], \quad \mathbf{J}_e=\mathbf{x}_0\frac{E_0e^{-jkx\sin\theta_i}}{\eta}[1-R(\mathbf{r})]\cos\theta_i.currents \tag{2}$$

For a binary MS the approximation of RL gives

$$R(x)=R_0, \quad nD-\frac{D}{2}<x<nD, \qquad R(x)=R_1, \quad nD<x<nD+\frac{D}{2}, \tag{3}$$

where $n=1,2...L/D$ is the number of the spatial period. Assuming that relations (3) are valid for the whole finite MS implies that in addition to the approximation of RL we adopt the approximation of PO.

From (1), (**??**), and (3) we have found the scattered field for the case of the TE polarization of the incident wave:

$$E_s(\theta,\rho)=\frac{jkE_0lLe^{-jk\rho}}{4\pi\rho}\left[(\cos\theta-\cos\theta_i)\mathrm{sinc}(kb_0)+\frac{jkD}{2}e^{jkb}\mathrm{sinc}(kb)\left(R_0e^{-jkb}+R_1e^{jkb}\right)F\right], \tag{4}$$

where $F(L,D,\theta)\equiv\sin\left(\frac{kL}{2}\sin\theta-\frac{kL}{2}\sin\theta_i\right)/\sin\left(\frac{kD}{2}\sin\theta-\frac{kD}{2}\sin\theta_i\right)$ is the well-known array factor, and $b=D(\sin\theta-\sin\theta_i)/4$, $b_0=L(\sin\theta-\sin\theta_i)/4$. In the TM-case a similar formula was derived for the magnetic field amplitude H_s. Since in the far zone $E_s=\eta H_s$, the right-hand side of (4) describes the diffraction pattern in both cases of the incident wave polarization.

If we substitute R_1 in formula (4) as $R_1=\psi R_0\exp(j\alpha)$, the case $\psi=1$ and $\alpha=0$ corresponds to the standard far-field pattern of a uniform reflector of length $L=ND$, where $N\gg1$. To engineer domination of the needed diffraction lobe with a binary MS, one needs $|R_0|\approx|R_1|$ and $\alpha\approx\pi$. If $\psi=1$ and $\alpha=\pi$, all diffraction lobes corresponding to spatial harmonics $m=0,\pm2,\pm4,\ldots$ vanish in the right-hand side of (4). There are no even spatial harmonic in the diffraction pattern, and the specular reflection is also absent. This observation means that for a binary PNUMS the approximation of PO adequately describes only diffraction lobes of the order $m = \pm 1$. All spatial harmonics except these ones appear to be either suppressed (if the binary PNUMS is ideal) or very small and comparable with the parasitic sidelobes (harmonics with $|m| \geq 3$). However, the PNUMS having $D < \lambda$ do not create anomalous reflection, only the specular one which is in this case the only scattered mode that can propagate in space. Of course, practical PNUMSs have $D > \lambda$, but an approximation which gives a weird result for slightly smaller D cannot become suddenly adequate at a frequency when $D = \lambda$. Indeed, when D is only slightly larger than λ, the approximation of PO cannot be applicable. Below we compare the diffraction patterns calculated using formula (4) with the results of full-wave (COMSOL) simulations.

III. Numerical results and conclusions

We performed a comparison of the PO prediction of the diffraction pattern in the incidence plane with that of full-wave simulations for three types of binary MSs: 1) ideal MS of alternating PEC/PMC strips, 2) realistic MS for which the approximation of RL is adequate [7], 3) well-known *mushroom* MS by Sievenpiper. The PEC/PMC array exhibits ideal RL for any angle of incidence and any period. The second type of PNUMS possesses RL with acceptable accuracy in a broad angular range. For the normal incidence RL is applicable for that MS if the required deflection angle (the angle between the specular reflection and the direction of the engineered diffraction lobes of the order $m = \pm 1$) does not exceed $75°$. If θ_i is as large as $75°$, RL is adequate if the deflection angle (of the lobe $m = -1$) is below $75°$, i.e., $\theta_{-1} = 0$. This is so for both TE and TM polarizations. Finally, the third MS demonstrates the applicability of RL in the same limits for TE-polarization and does not show RL for incidence or deflection angles exceeding $(30 - 40)°$ if the incident wave is TM-polarized. All three MSs have the same size $L = 11\lambda$. The 2d and 3d MSs comprised 40 unit cells along x and were designed for the same operation frequency, where the reflection phase difference $\Phi_0 - \Phi_1$ for the two fundamental states of the uniform MS is maximally close to π. We performed extensive simulations for both normal and oblique incidences (both TE and TM polarizations) and for periods varying from $D = 1.1\lambda$ ($L = 10D$) to $D = 3.66\lambda$ ($L = 3D$).

The results are as follows. First, the accuracy of the PO approximation does not depend on L/λ, and it is determined by two factors: the adequacy of RL and the value D/λ. Second, for both ideally local binary MS and realistic MS from [7] possessing RL approximately, PO adequately predicts the diffraction pattern if $D > 3\lambda$. Then the error in the predictions of the intensity in the main lobes is below 1 dB, as one can see in Fig. 1(b) (normal incidence on the second MS). Meanwhile, if $D < 2\lambda$, PO does not work. E.g., for $D = 1.1\lambda$ the error in the intensity of the main lobe is about 20 dB. Third, the same result holds for the mushroom MS illuminated by TE waves, whereas for the TM-polarization the PO gives erroneous predictions even for D as large as $(3 \ldots 3.66)\lambda$.

To conclude: the applicability of RL is a prerequisite of the applicability of PO for binary PNUMSs, but the application of PO also demands sufficiently large periods. Since large values of D/λ correspond to modest deflection angles, it means that for very strongly anomalous reflections the PO is not applicable even if the concept of local reflection is adequate.

Acknowledgement

This work has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie grant agreement No 956256.

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