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Åman, Mari; Tanaka, Yuzo; Murakami, Yukitaka; Remes, Heikki; Marquis, Gary Fatigue strength evaluation of small defect at stress concentration

Published in: Procedia Structural Integrity

DOI: 10.1016/j.prostr.2017.11.099

Published: 01/01/2017

Document Version Publisher's PDF, also known as Version of record

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Please cite the original version:

Åman, M., Tanaka, Y., Murakami, Y., Remes, H., & Marquis, G. (2017). Fatigue strength evaluation of small defect at stress concentration. *Procedia Structural Integrity*, *7*, 351-358. https://doi.org/10.1016/j.prostr.2017.11.099

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Structural Integrity Procedia

Procedia Structural Integrity 7 (2017) 351-358

www.elsevier.com/locate/procedia

3rd International Symposium on Fatigue Design and Material Defects, FDMD 2017, 19-22 September 2017, Lecco, Italy

# Fatigue strength evaluation of small defect at stress concentration

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# Abstract

The effect of individual large notches on the fatigue strength of components is one of the oldest and most studied topics in the history of metal fatigue. When a small defect is present at the notch root, both the stress concentration of the main notch and the effect of the small defect interact and simultaneously influence the fatigue strength. The effect of the main notch can be evaluated from the viewpoint of stress concentration and stress gradient. Both have a strong influence on the fatigue notch factor. The  $\sqrt{area}$  parameter model has been successfully applied to fatigue limit evaluation of materials containing small defects under uniform stress condition. If a small defect is present at the notch root, the effect of stress gradient must be also considered in the application of the model. In the present study, the fatigue tests and fatigue crack growth analyses are carried out for specimens containing a small defect with the size  $\sqrt{area} = 46.3\mu$ m at the root of notch with 1mm depth and root radius of 1.0mm or 0.3mm. Fatigue limit predictions are made based on the  $\sqrt{area}$  parameter model and the stress intensity factor analyses for a small crack subject to a steep stress gradient. Existing fatigue notch effect methods are reviewed and used in fatigue limit predictions for comparison. Moreover, new fatigue notch effect method based on the  $\sqrt{area}$  parameter model is proposed. The greatest advantage of the proposed method is that it can predict fatigue limit using easily obtainable parameters and without requiring fatigue tests or troublesome analyses. Suggestions for the extension of the proposed method to practical engineering problems are also made.

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Peer-review under responsibility of the Scientific Committee of the 3rd International Symposium on Fatigue Design and Material Defects. 10.1016/j.prostr.2017.11.099

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Keywords: fatigue limit; notch;  $\sqrt{area}$ ; small defect; stress concentration; stress gradient; non-propagating crack

Nomenclature				
AS	Allowable stress			
F	Dimensionless stress intensity factor			
HV	Vickers hardness			
Kt	Stress concentration factor			
lin	Linearly changed stress condition (subscript)			
uni	Uniform stress condition (subscript)			
β	Ratio of dimensionless stress intensity factors $F_{\text{lin}}/F_{\text{uni}}$			
$\Delta K_{ m th}$	Threshold stress intensity factor range			
$\sigma_{ m w}$	Fatigue limit			
$\sigma_{ m w0}$	Fatigue limit of unnotched specimen			
$\sigma_{ m w1}$	Fatigue crack initiation limit			
$\sigma_{ m w2}$	Fatigue crack propagation limit			
$\sigma_0$	Nominal stress			
ρ	Notch root radius			
√area	Square root of defect/crack area projected normal to the maximum principal stress			

# 1. Introduction

Fatigue notch effect is one of the most studied topics in the history of metal fatigue. Typically, these studies consider an individual large notch in a component and the fatigue strength is determined in terms of stress concentration and stress gradient. It is known that the stress distribution in the close vicinity of the notch mainly influences the fatigue strength. Thus, the stress distribution can be approximated by the straight line and its gradient is used as the representing factor of the stress distribution. However, the absolute value of the stress gradient is not useful in fatigue notch effect evaluation, because it depends on the applied stress even for identical notches (Murakami (2002)). Detailed review of fatigue notch effect models has been provided by Murakami and Endo (1994). Isibasi's (1967) pioneering work on notched components pointed out the fact that two fatigue limits can be distinguished if a notch becomes sharp enough. One is the fatigue limit  $\sigma_{w1}$  as the critical stress for microscopic crack initiation and non-propagation at the notch root which is very alike the fatigue limit of unnotched specimens. The other is the fatigue limit  $\sigma_{w2}$  as the threshold stress for non-propagation of the crack around the circumference of the notch root (Fig.1).

According to Nisitani (1968), the fatigue limit tends to become constant, when notch root radius  $\rho$  is smaller than critical  $\rho (= \rho_0)$  for a material (typically 0.4-0.5mm for various materials with tensile strength less than 1000 MPa). Small  $\rho$  implies to large stress concentration and a crack initiates easily from notch root. At the fatigue limit, cracks initiate but stop propagation, thus, *fatigue limit must be determined by the threshold condition of non-propagating cracks*. If a small defect is present at the notch root the fatigue limit is determined by the threshold condition of a non-propagating crack emanating from the small defect. The  $\sqrt{area}$  parameter model, proposed by Murakami & Endo (1983), has been successfully applied to fatigue limit evaluation of materials containing small defects under uniform stress condition. However, as the stress condition is not uniform at the notch root, the model must be modified to consider the effect of stress gradient. In the present study, the fatigue tests and stress intensity factor analyses are carried out for specimens containing a small defect with the size  $\sqrt{area} = 46.3 \mu m$  at the root of notch with 1mm depth and root radius of 1.0mm or 0.3mm. Fatigue limit predictions are made based on the  $\sqrt{area}$  parameter model and the stress intensity factor analyses for a small crack subject to a steep stress gradient. In addition, new fatigue notch effect model is proposed.



Figure 1. Two fatigue limits and notch root radius  $\rho_0$ 

Figure 2. Geometries of specimen, notches and drilled hole

## 2. Experimental results

Material used in this study is common commercial low-carbon steel JIS-SS400 (Mass%: 0.05C-0.02Si-0.40Mn-0.023P-0.010S-Bal.Fe) having mechanical properties: Yield stress  $\sigma_v$ =332 MPa, Tensile strength  $\sigma_B$ =432 MPa, Elongation=36% and Vickers hardness HV=140 kgfmm<sup>-2</sup>. As suggested by Isibasi (1967), HV was measured from the notch root, because HV varies from the surface to the core and because notch machining may influence to the local hardness. Figure 2 shows the geometries of a specimen, notches and a drilled hole. The testing machine used is a special portable displacement controlled bending testing machine newly made by KMTL (Kobe Material Testing Laboratory, Co. Ltd., Kobe, Japan). Loading corresponds cantilever bending. Testing frequency was 33 Hz and stress ratio was R=-1 in all tests. All stresses in Figures and Tables are nominal ones calculated by the formula  $\sigma = M/Z = 6M/Wt^2$  where t is the thickness of the minimum section (2mm), W is the width of the specimen (10mm) and M is the bending moment. Firstly, fatigue limits of notched components without a drilled hole were determined. Secondly, fatigue limits of specimens having a small drilled hole at the bottom of the main notch were obtained. Each fatigue limit was defined as the maximum stress amplitude at which the specimen did not fail after 10 million cycles. As mentioned above, fatigue limit must be determined by the threshold condition of non-propagating cracks. Precisely, the fatigue limit of a notched component with a drilled hole at the notch root must be determined by the threshold condition of a non-propagating crack emanating from the drilled hole. In the case of  $\rho=1.0$  mm, non-propagating cracks were not observed at the fatigue limit. This is because the stress required for crack initiation is high and due to large  $\rho$ , two fatigue limits,  $\sigma_{w1}$  and  $\sigma_{w2}$ , are very difficult to distinguish.

Figure 3 shows examples of non-propagating cracks which were observed at the fatigue limit. In all other cases, similar non-propagating cracks were observed. As shown in Fig. 4., the fatigue limit of a  $\rho$ =0.3mm specimen is only 10 MPa lower than that of a  $\rho$ =1.0mm specimen, whereas the fatigue limit  $\rho$ =0.1mm specimen is significantly, almost 50%, lower. This phenomenon contradicts with Nisitani's (1968)  $\rho_0$  idea. The reason can be understood considering the differences in specimen thickness and non-propagating crack size. The thickness of the minimum section of Nisitani's specimens was 5-10mm, thus, even when a non-propagating crack exists, the thickness of the minimum section does not remarkably decrease compared to the initial thickness. In this study, the minimum section is 2 mm and considering the non-propagating crack size shown in Fig. 3(b), it can be understood that the size of the non-propagating crack has an influence on final thickness of the specimen. It should be noted that the sharper the notch, the more variation in non-propagating crack size. Thus, when  $\rho$  is as small as 0.1mm, non-propagating cracks *may* become so long that specimens with smaller thickness fail whereas specimens with larger thickness do not, which explains the result of  $\rho$ =0.1mm case in this study.

## 3. Analysis and Discussion

#### 3.1 Review of existing fatigue notch methods

Prior to experiments, several existing fatigue notch methods were compared to estimate the fatigue limit of a notched component without a drilled hole. It was found that the method of Siebel and Stieler (1955) gave the most accurate prediction for the case of  $\rho$ =1.0mm, none of the methods gave good prediction for  $\rho$ =0.3mm and Isibasi's

400

300

250

200

150

50µm

 $\sigma_{a}$  (MPa) 350

(1967) and Taylor's (1999) methods gave the most accurate predictions for the case  $\rho=0.1$ mm. Peterson's (1959) method was conservative for two largest  $\rho$ 's and unconservative for the smallest  $\rho$ . Necessary parameters for each method and analysis results are listed in Table 1. When comparing the existing notch methods, it is worth to shortly describe them and discuss their convenience, advantages and disadvantages.



Figure 3. Non-propagating cracks (a)  $\rho$ =1.0mm,  $\sigma_w$ =210 MPa, (b)  $\rho=0.3$  mm,  $\sigma_w=200$  MPa (c)  $\rho=1.0$  mm,  $(d,h)=(50,50)\mu$ m,  $\sigma_{\rm w}$ =175 MPa (d)  $\rho$ =0.3mm, (d,h)=(50,50) $\mu$ m,  $\sigma_{\rm w}$ =130 MPa.





the same as dashed mild steel curve.

Figure 5. AS curves from Siebel & Stieler (1955)

400

Figure 6. Combined linear and uniform stress distributions

=

Z.

0

 $\rho = 1.0$ mm (notch)

a=0.3mm (notch)  $\rho$ =0.1mm (notch)

p=1.0mm (notch + hole)

=0.3mm (notch + hole

 $\sigma_{lin}$ 

z

Isibasi's so-called  $\varepsilon_0$  concept states that the fatigue limit of a notched specimen can be determined under the condition when the stress at a distance  $\varepsilon_0$  from the notch root is equal to the fatigue limit of an unnotched specimen. Isibasi's model is simple in theory, but on the contrary,  $\varepsilon_0$  is a material constant which must be determined experimentally. However, it has been found that  $\varepsilon_0$  is typically the order of grain size of a material, thus, grain size is used in the prediction in this study. Peterson's model determines fatigue limit of a notched component using fatigue notch factor  $K_{\rm f}$  which is a function of  $\rho$ ,  $K_{\rm t}$  and material characteristic length a which can be determined if tensile strength is known. Using  $K_{\rm f}$  is very convenient as it can be determined by a single simple formula. However, tensile strength used in a model is a bulk material property and fails to describe possible variations in *local* strength characteristics (HV variations) which determine the fatigue strength. In addition,  $K_t$  may be difficult to determine accurately in some practical problems. Siebel and Stieler proposed a method where stress gradient is used to determine the fatigue limit of notched specimen. Based on their extensive experimental data, they obtained "allowable stress curves" (AS curves thereafter) for several materials as well as approximations for stress gradients for common loading conditions (Fig.5). The disadvantages of Siebel & Stieler method are that it also employs  $K_t$  and if the exact AS curve for a material is not included in Fig.5, it must be estimated by the users. Prior to experiments in this study, the AS curve for the material in question was estimated from Fig.5. After the experimental data for  $\rho=1.0$ mm case was obtained, the curve estimation was revised. Taylors theory of critical distances is a group of methods, the simplest of which is a *point method* which is essentially equal to Isibasi's model. The material parameter critical distance L is a function of  $\Delta K_{\rm th}$  and the fatigue limit of an unnotched specimen. However, care must be exercised when employing ambiguous parameters such as  $\Delta K_{\text{th}}$  in a model, because even a small difference in testing conditions can result in very different  $\Delta K_{\text{th}}$ . In addition, careless consideration of  $\Delta K_{\text{th}}$  as a material constant results in large prediction error if the

σ<sub>max</sub>/σ<sub>w0</sub> (allowable stress, AS)

model is applied to a small crack problem, because  $\Delta K_{\text{th}}$  has a crack size dependency in small crack regime. Anyway, we need some experimental data on every material for the application of Taylor's method.

Fable 1. Necessa	ry parameters	for existing me	thods, parameter	ers used in this stu	ly and anal	ysis results
		(7)			-	

Method	Necessary parameters	Parameters in this study*	Predictions (experimental results in parenthesis)
Isibasi (1967)	$\sigma_{ m w0}$	1.6HV=224 MPa	σ <sub>w</sub> (ρ=0.1mm)=100 MPa (110 MPa)
	Material parameter $\varepsilon_0$	Average grain size = $30 \mu m$	$\sigma_{\rm w}(\rho=0.3{\rm mm})=122{\rm MPa}~(200{\rm MPa})$
	Stress distribution	See Fig.8	$\sigma_{\rm w}(\rho=1.0{\rm mm})=165{\rm MPa}(210{\rm MPa})$
	$K_{\rm t}$ (for notches)	See Fig.8	
Peterson (1959)	$\sigma_{ m w0}$	1.6HV=224 MPa	$\sigma_{\rm w}(\rho=0.1 {\rm mm})=150 {\rm MPa} (110 {\rm MPa})$
	$K_{ m t}$ and $ ho$	See Fig.8	$\sigma_{\rm w}(\rho=0.3{\rm mm})=148{\rm MPa}(200{\rm MPa})$
	$\sigma_{\rm b}$ (or $HV$ )	See Chapter 2	$\sigma_{\rm w}( ho=1.0{\rm mm})=169{ m MPa}$ (210 MPa)
Siebel & Stieler	$\sigma_{ m w0}$	1.6 <i>HV</i> =224 MPa	$\sigma_{\rm w}(\rho=0.1{\rm mm})$ cannot be determined (Fig.5)
(1955)	$K_{ m t}$ and $ ho$	See Fig.8	$\sigma_{w}(\rho=0.3 \text{ mm})=148 \text{ MPa} (200 \text{ MPa})$
	$\sigma_{\rm y}$ or $\sigma_{\rm b}$ (cast steels)	See Chapter 2	$\sigma_{\rm w}(\rho=1.0{\rm mm})=198{\rm MPa}(210{\rm MPa})$
	AS curve	Fig.5	
D. Taylor (1999)	$\sigma_{ m w0}$	1.6 <i>HV</i> =224 MPa	$\sigma_{w}(\rho=0.1 \text{mm})=103 \text{ MPa} (110 \text{ MPa})$
	$\Delta K_{ m th}$	6.37 MPa√m**	$\sigma_{w}(\rho=0.3 \text{ mm})=124 \text{ MPa} (200 \text{ MPa})$
	Stress distribution	See Fig.8	σ <sub>w</sub> (ρ=1.0mm)=166 MPa (210 MPa)

\* *HV* was used for the estimation of fatigue limit of unnotched specimen, because all the models request the fatigue data. \*\*Estimated from Dowling (2007)

#### 3.2 New method

Engineering materials include various kind of natural defects, which may act as crack initiation sites and thus reduce fatigue strength. However, it is known that there exists a critical defect size for a material and defects smaller than that can be considered harmless. The critical defect size tends to decrease with increasing hardness of the material. In this study, the critical defect size is estimated using an empirical equation for unnotched specimen fatigue limit  $\sigma_{w0}=1.6HV$  and the  $\sqrt{area}$  parameter model, where  $\sqrt{area}$  is an area of a defect normal to the maximum principal stress.

$$\sigma_{\rm w} = \frac{1.43(HV+120)}{(\sqrt{area})^{1/6}}$$
(1)

$$1.6HV = \frac{1.43(HV + 120)}{\left(\sqrt{area}\right)^{V^6}}$$
(2)

Thus, solving Eq.(2) for  $\sqrt{area}$  we obtain  $\sqrt{area} = 21 \mu m$  for a material used in this study (*HV*=140). Now, let us assume that 21 µm size defect is harmless also if it exists at the notch root. Eq. (1) assumes that the loading condition is uniform and that the defect is at the surface of a smooth specimen. Once the defect is assumed to exist at the notch root, Eq.(1) must be modified to consider *stress concentration, stress gradient and stress intensity factor* in the case of *combined uniform and linear loading*. Stress intensity factors for any linear stress distribution can be modelled by dividing elastic stress distribution into linear and uniform parts and superposing the solutions together, as shown in Fig.6. It is noted that the stress distribution ahead of the notch may not be exactly linear in most cases, but it can be assumed to be linear with reasonable accuracy in the close vicinity of the notch root. In order to modify Eq.(1), it must be disassembled into two equations:

$$\Delta K_{\rm th} = 3.3 \times 10^{-3} \left( HV + 120 \right) \left( \sqrt{area} \right)^{1/3} \qquad (\sqrt{area} \text{ is in } \mu\text{m}, HV \text{ is in } \text{kgf/mm}^2) \tag{3}$$

$$\Delta K_{\rm Imax} = F \Delta \sigma_{\rm max} \sqrt{\pi \sqrt{area}} \qquad (\sqrt{area} \text{ is in m, } \sigma \text{ is in MPa}) \tag{4}$$

where *F* is a *dimensionless stress intensity factor*. Dimensionless stress intensity factors for 3D semi-elliptical surface cracks with various aspect ratios under linear and uniform loading are shown in Fig.7., where *F*-values are determined using a half crack length instead of  $\sqrt{area}$ . This is because the fundamental solutions in Fig.7 have no size dependency, i.e. the solutions can be used in the case of long cracks as well as in the case of small cracks, whereas the  $\sqrt{area}$  parameter model is a small crack model. Therefore, *F*-values of Fig.7 must be adjusted to correspond  $\sqrt{area}$  instead of half crack length by introducing a parameter  $\beta = F_{lin}/F_{uni}$  which does not depend on the geometrical terms. In this study, it is assumed that the harmless  $\sqrt{area} = 21 \mu m$  defect is geometrically similar as a drilled hole having size  $(d,h)=(50,50)\mu m$ , which results dimensions  $d=h=22.7\mu m$  for harmless defect and  $\beta=0.7678/0.8725=0.88$  (Fig.7).





Figure 8. Stress distributions of notched components and  $K_t$ 's

Moreover, stress term in Eq.(4) should not only consider stress concentration but also stress gradient. The gradient effect is automatically considered when uniform and linear loadings are separately determined from stress distributions (Fig.8). Stress concentration factors for uniform and linear parts,  $K_{t,uni}$  and  $K_{t,lin}$ , respectively, are obtained as shown in Fig.8;  $K_{t,lin}$  is  $K_t$  of the notch and  $K_{t,uni}$  depends on the depth of the drilled hole or defect. Considering necessary modifications described above, Eq.(4) is re-written as

$$\Delta K_{1,\max} = 2(K_{\text{uni}} + K_{\text{lin}})$$

$$\Delta K_{1,\max} = 2(\sigma_{\text{uni}}F_{\text{uni}}\sqrt{\pi d/2} + \sigma_{\text{lin}}F_{\text{lin}}\sqrt{\pi d/2})$$

$$\Delta K_{1,\max} = 2(\sigma_0 K_{1,\min}F_{\text{uni}}\sqrt{\pi d/2} + \beta\sigma_0 F_{\text{uni}}\left(K_{1,\text{lin}} - K_{1,\min}\right)\sqrt{\pi d/2})$$

$$\Delta K_{1,\max} = 2\sigma_0 F_{\text{uni}}\sqrt{\frac{\pi d}{2}}\left(K_{1,\min} + \beta\left(K_{1,\min} - K_{1,\min}\right)\right)$$
(5)

where F-values are dimensionless stress intensity factors from Fig.7 and  $K_t$ 's are stress concentration factors from Fig.8. In a similar manner, we can write

$$\Delta K_{\rm L,max} = 2F^* \sigma_0 \sqrt{\pi \sqrt{area}} \left( K_{\rm t,uni} + \beta \left( K_{\rm t,lin} - K_{\rm t,uni} \right) \right) \tag{6}$$

where  $F^*$  is *F*-value adjusted to correspond  $\sqrt{area}$  instead of half crack length. Combining equations (5), (6), (3) and (4) we get

$$\sigma_{\rm w} = \frac{3.3(HV + 120)}{2\sqrt{\pi}F^* \left(K_{\rm t,uni} + \beta \left(K_{\rm t,lin} - K_{\rm t,uni}\right)\right) \left(\sqrt{area}\right)^{1/6}}$$
(7)

#### 3.3 Verification of the method and discussion

As mentioned earlier, the fatigue limit for microscopic non-propagating cracks  $\sigma_{wl}$  can be considered fairly similar to the fatigue limit for a plain specimen. This curve can be obtained using Eq.(7), assuming  $\sqrt{area}=21\mu m$ . The parameters required are listed in Table 2. The case of  $\rho=0.1mm$  specimen with drilled hole was left out from experimental study because introducing a drilled hole precisely into such small notch root is practically very difficult. The analytical results are compared with experimental data to verify the validity of the proposed method.



Figure 9. Experimental and analytical results. The reason why  $\rho$  is used in abscissa instead of  $K_t$  is that  $K_t$  is same for geometrically similar specimens regardless of their size, but larger specimens may not have non-propagating cracks whereas geometrically similar but smaller specimens have non-propagating cracks.  $\rho$  is the quantity related to stress gradient which controls notch effect and size effect ((Siebel & Stieler (1955)).

When a small drilled hole is introduced to the bottom of the notch, the fatigue limit is determined by the threshold condition of a non-propagating crack emanating from a small drilled hole. In this study, a drilled hole having size  $(d,h) = (50,50)\mu m$  ( $\sqrt{area}$  of 46.3 $\mu m$ ) was introduced to the bottom of a notch. It is obvious that the fatigue limit of a specimen having a drilled hole should be lower than the fatigue limit of a plain specimen obtained analytically from Eq.(7) assuming  $\sqrt{area}=21\mu m$ . However, the fatigue limits should be very close to each other, because the difference in  $\sqrt{area}$ 's is relatively small. Similar analysis was made for drilled hole  $(d,h)=(50,50)\mu m$  at the notch root. Figure 9 compares the results of the analyses and experiments. The stress intensity factor solutions for semi-elliptical crack (Fig.7) can be applied to the drilled hole case, because drilled hole can be approximated as semi-elliptical shown in Fig.9. The unnotched specimen fatigue limit ( $\rho=\infty$ ) was calculated using formula 1.6HV.

The predicted fatigue limits are a little conservative. In reality, it is likely that there exists bi-axial stress state i.e. the stress component in specimen's width direction is not actually zero as is shown in Fig. 10. In such a case the stress concentration factors at the edge of the hole are smaller than the ones indicated by 2D FEM analysis (Fig. 8). The stress state at the center of the notch is approximately close to plane strain and by applying Hooke's law, we obtain  $\sigma_v = v \sigma_x$ . If a crack-like sharp notch is used instead of a drilled hole at the notch root,  $\sigma_v$  has less influence and the prediction should become less conservative. However, in general, the stress concentration of a small defect or drilled hole is not the crucial factor which controls fatigue limit. This is because their influence to fatigue limit is mechanically equivalent as that of small crack having same  $\sqrt{area}$  (Murakami (2002)). Thus, it is proposed that the  $\sqrt{area}$  parameter model can be also applied to fatigue notch effect evaluation. In general, the greatest advantage of the  $\sqrt{area}$  parameter model is that fatigue tests are not required, since it uses only two parameters;  $\sqrt{area}$  and HV of a material. However, when the loading condition is not uniform, the additional parameters must be included in the model. To obtain the necessary parameters, stress distribution, stress concentration factors and dimensionless stress intensity factors for both uniform and linear loading must be known. As a future work, more experiments should be carried out using different materials, notch root radiuses, specimen geometries and drilled hole geometries. The proposed fatigue notch effect method provides accurate tool to evaluate the fatigue strength in case of more complicated problems, where stress concentrations, steep stress gradients and small defects are present. An example of such practical problem is a



Figure 10. Stress state at the center of notch root is close to plane strain condition. Bi-axial tensile stress around the drilled hole decreases stress concentration at the edge of hole. Therefore, 2D FEM analysis overestimates  $K_t$ 's at some extent.

welded joint, where stress concentrations are unavoidable. In addition, welded joints typically include gaps, pores and surface irregularities which are considered mechanically equivalent to small cracks from fatigue limit point of view. The important factors in weldment fatigue, such as residual stress and microstructural characteristics, can be considered easily in the application of the proposed model. Residual stress is regarded mechanically equivalent to local mean stress and it can be taken into consideration by changing stress ratio. Microstructural changes can be considered by measuring HV from the prospective crack initiation site of the weldment. Using HV also in weldment problem as material characteristics is not only fast and practical, but also rational, as there is evident relationship between HV and fatigue strength (Murakami (2002)). Thus, the proposed model requires neither fatigue tests nor complicated analyses and provides accurate and useful tool to solve and simplify various kind of practical engineering problems.

# 4. Conclusions

Bending fatigue tests on notched specimens having different notch root radiuses were carried out as well as tests on notched specimens having *a small drilled hole at the notch root*. The experimental results were compared with the analytical results of existing fatigue notch effect models. The  $\sqrt{area}$  parameter model was extendedly applied to consider the effects of *stress concentration*, *stress gradient* and *stress intensity factors* in *combined linear and uniform loading*. The new method gave a conservative prediction to the problem of small drilled hole at notch root. The new method can be applied to other practical problems of small defects existing at stress concentration.

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