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## Short-term analysis of timber-concrete composite bridges

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### Summary

Timber-concrete composite (TCC) structures have been studied intensively during the last decades and some analysis methods has been proposed, but mostly for floor construction. Applicability of different analysis methods for TCC bridge design were evaluated in a parametric study. In the evaluation, the influence of the parameters on the deformation, shear force in the connector, compressive stresses in the concrete deck and tensile stresses in the timber beam were investigated. The study was performed on a simply supported TCC beam, with span of 18 m, and with following parameters: different loads, connector arrangements, connector stiffnesses and cross-sectional dimensions. The results indicate large differences between the evaluated methods, in particular for beams with relatively low connector stiffness.

**Keywords:** timber-concrete composite, bridges, structural analysis, composite beam.

## 1. Introduction

Reliable design methods are important for ensuring safety and serviceability of bridges. However, Eurocodes currently do not present guidelines for the design of timber-concrete composite (TCC) buildings or bridges. Some rules of thumb for TCC bridges are given in national guidelines, e.g. Finnish NCCI5 [1], but they are too limited for detailed analysis. TCC bridges were researched in Finland in the 90's, producing new connector types, bridge concepts and some guidelines [2]. Similar research projects on TCC bridges have been carried out in other countries. TCC structures in general have been studied quite intensively in past few decades, thus some validated simplified analysis methods have been proposed, but mainly for floor structures [3]. Purpose of the published study was to investigate applicability of the existing analysis methods for design of TCC beams and the results were used in the paper by Jaaranen et al. [4].

## 2. Structural behavior of TCC beam

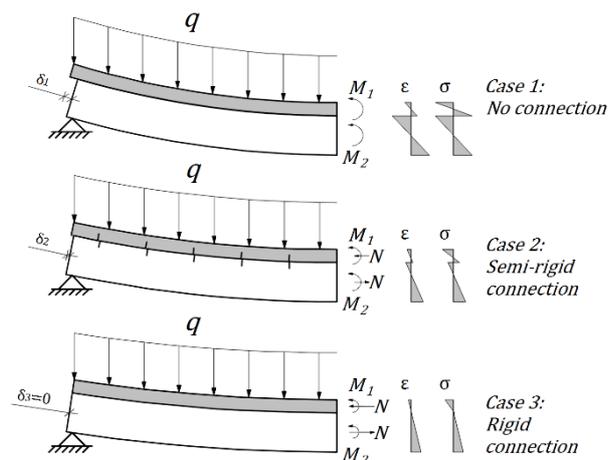


Figure 1. Deformations, internal forces, strains and stresses of a composite beam with different type of shear connection.

A typical TCC beam consists of timber section with concrete slab connected on top by shear connectors. In the case of simple supported beam, timber is in tension and concrete in compression, utilizing the best properties of both materials. Effects of shear connection stiffness on the beam behavior can be illustrated by considering three different Cases (Figure 1):

- Case 1. no shear connection
- Case 2. flexible shear connection
- Case 3. rigid shear connection.

In the Case 1, slip between the components is not resisted by the connection and shear flow will not develop. In the Case 3, the beam is fully composite, with no slip, thus maximum shear flow will develop. In the Case 2, slip can happen, but at the same time shear flow develops due to connectors resisting the slip. Since the connection

stiffness can vary between Cases 1 and 3, those Cases present bounds for the bending stiffness and stresses of the beam with flexible connection.

Degree of composite action is defined in this paper by connection efficiency factor  $\gamma$ , as in Eq. (1) [5], where  $(EI)_{eff}$ ,  $(EI)_0$  and  $(EI)_\infty$  are bending stiffnesses of: a beam with flexible connection, a beam with no shear connection and a beam with rigid connection, respectively.

$$\gamma = \frac{(EI)_{eff} - (EI)_0}{(EI)_\infty - (EI)_0}, \quad (1)$$

## 3. Analysis methods of TCC beams

In this chapter, four different analysis methods applicable for analysis of TCC beam, namely, (1) fully composite method, (2) continuous flexible connection (CFC) method, (3)  $\gamma$ -method and (4) discrete flexible connection (DFC) method, are introduced.

### 3.1 Fully composite method

The simplest way to analyze a TCC beam is to assume that the connection is smeared and rigid corresponding to the Case 3. Then the beam can be analyzed by using Euler-Bernoulli (E-B) beam theory.

### 3.2 Continuous flexible connection method

For more detailed analysis, effects of the shear slip between elements need to be taken into account. One such method, commonly found in literature, is described e.g. in [6, 7]. The model is based on the assumption that the elements are connected by close and equally spaced linearly elastic shear connectors. The connection is treated as smeared connection with constant shear stiffness per unit length. The shear flow in the interlayer is then linearly dependent of the slip between the components along the beam. Individual elements of the beam are assumed to be linearly elastic and behave according to E-B beam theory.

The mathematical model for composite beams is derived by writing static equilibrium of internal and external forces of a beam element and then applying kinematics of E-B beam theory and linear load-slip relation of the connection, leading to a system of three differential equations. The system is then solved by applying boundary and load conditions. However, closed form solution can be found only in special cases. Detailed treatment on the topic can be found in [6, 7, 8]. For this paper, solutions for simply supported beams under uniform and concentrated loads, adopted from Natterer & Hoeft [8], were used.

### 3.3 $\gamma$ -method

In the case of sinusoidal load on a simply supported beam, the CFC model has a particularly simple solution, in which bending stiffness is constant along the whole span leading to simple expressions of deformations and stresses. The  $\gamma$ -method is an approximate method, in which effects of other load types are approximated using the bending stiffness derived for the sinusoidal load [7]. Due to the simplifications, the method is exact only under sinusoidal load. In the other cases, the accuracy depends on type of the load, cross-section properties, connection stiffness and the span of the beam.

The method is adopted in EN 1995-1-1, Annex B, where relevant formulas are given. The use is restricted to simply supported, cantilever or continuous beams with components continuous over the whole length and connected by linear elastic connectors, whose spacing is constant or varies according the shear force. This method should be used only for sinusoidal and uniform loads. In the case of varying connector spacing, effective connector spacing  $s_{ef}$  according to [7] may be used.

### 3.4 Discrete flexible connection method

Composite beams can be also analyzed by considering the behavior of individual connectors as in the model proposed in [9, 10]. In both, the authors have treated only simply supported beams. In the [10] it is suggested that the approach can be extended to multi-span systems, and briefly explained how inelastic strains can be included in the analysis. A model with the suggested extensions was derived in this study.

In this method, including the extensions, the composite beam is assumed to consist of two individual beams, connected by arbitrary number of arbitrarily placed shear connectors each with individual slip modulus. Both elements are assumed to follow E-B beam theory and have equal deflections and curvatures.

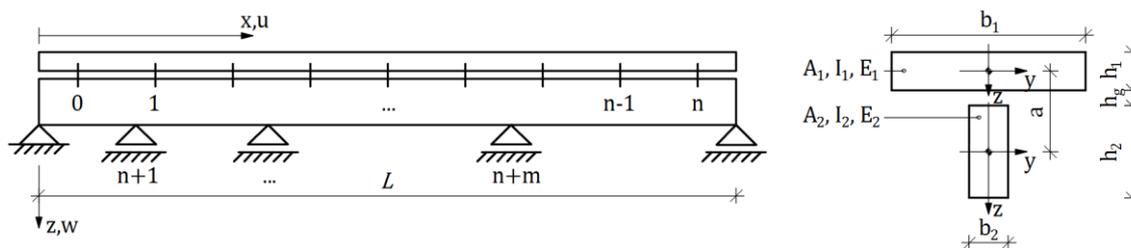


Figure 2. Model and cross-section of a composite beam in the discrete flexible connection method.

The model and the cross-section of the beam is shown in Figure 2. In the general case, the beam is an externally and internally statically indeterminate system. Response to loads is obtained by applying the force method, in which the system is separated into a statically determinate primary system (stage S-1) and a system with redundant connector forces and support reactions (stage S-2). In the stage S-1, connectors  $\{1, \dots, n\}$  and additional supports  $\{n+1, \dots, n+m\}$  are released and the beam is subjected to external loads and inelastic strains. In the stage S-2, effects of unknown redundant connector forces and support reactions are applied on the structure.

Finally, the effects from both stages are superimposed. Applying compatibility conditions for each connector and an intermediate support leads to a system of equations as in Eq. (2), where  $\mathbf{B}$  is a flexibility matrix of the system,  $\mathbf{x}$  is a force vector containing unknown connector forces and support reactions and  $\mathbf{u}$  is a “loading” vector containing elastic and inelastic displacements of redundant connectors and supports in the stage S-1. After solving the redundant forces, the total response of the structure is obtained by superimposing the effects from stages S-1 and S-2. In general cases, the method requires numerical implementation, but since arbitrary loads, connector arrangement and support conditions can be easily implemented without the use of simplifications, this method is advantageous compared to analytical methods.

$$\mathbf{B}\mathbf{x} = \mathbf{u} \quad (2)$$

#### 4. Parametric comparison between the methods

The goal of the parametric comparison was to study differences of the aforementioned methods and evaluate their applicability in design of TCC bridges. Wider range of cases were studied, but only the elementary cases are presented here for clarity.

##### 4.1 Geometric setup and parameters

In the parametric study, a TCC beam with a timber section and a concrete slab connected with spaced shear connectors was considered. The cross-section of the beam is illustrated in Figure 3. Two different connector arrangements; connectors with (1) uniform spacing and (2) spacing varying according to shear force, as illustrated in Figure 3, were considered. In addition, two different load types were considered in the analysis; (a) uniform load and (b) point load at  $x = l/2$  as in Figure 4. In the parametric study, four different Cases, which are listed in Table 1, combining different connector arrangements and load types, were considered.

Table 1. Cases in the comparison study.

Case	Connector spacing	Load type
1a	Uniform	Uniform
1b	Uniform	Point
2a	Varying	Uniform
2b	Varying	Point

For each Case, 160 combinations of cross-section dimensions and connector stiffness in total were used. The range of the variables as well as material properties and the span were chosen so that they reflect practical range of values used in TCC bridges. The material properties and variables used in the analysis are shown in Table 2. Effective connector spacing  $s_{ef}$  [7] was adopted in both, the  $\gamma$ -method and the CFC method.

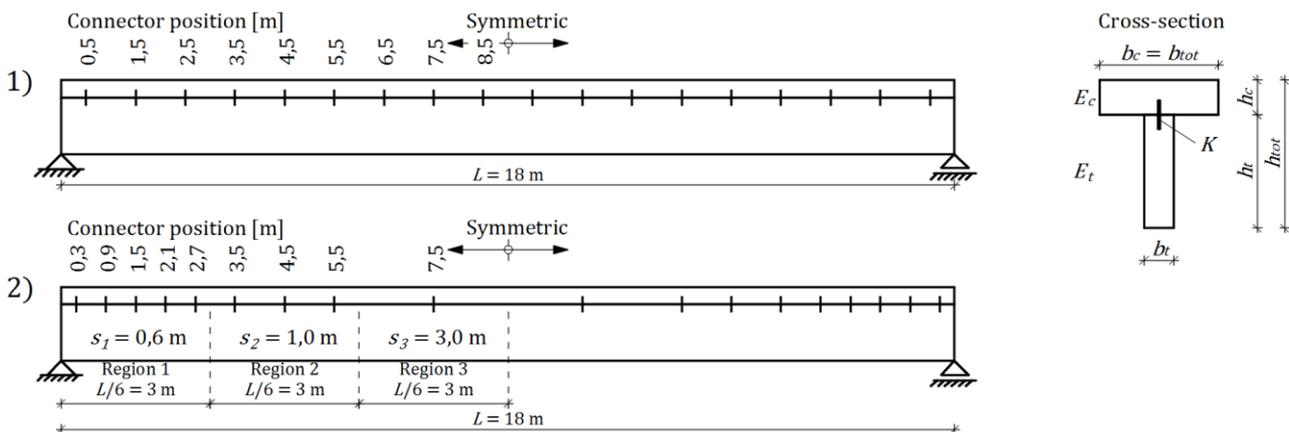


Figure 3. The composite beam and different connector arrangements, 1) uniform and 2) varying, are shown left and cross-section of the beam on right..

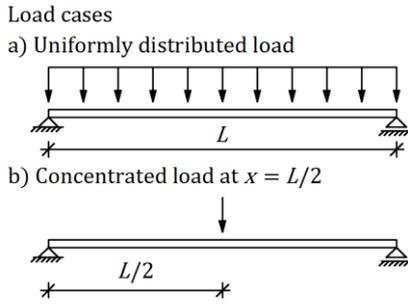


Figure 4. Different load cases.

Table 2. Material properties and dimensions in the parametric study. Number in parenthesis displays total number of parameters for the variable.

Parameter	Value
Span length $L$ [m]	18.0
Concrete elastic modulus $E_c$ [MPa]	30000
Timber elastic modulus $E_t$ [MPa]	13000
Total height $h_{tot}$ [m]	1.5
Total width $b_{tot}$ [m]	1.2
Concrete slab height $h_c$ [m]	0.15, 0.20, 0.25, 0.30 (4)
Timber section width $b_t$ [m]	0.20, 0.25, 0.30, 0.35, 0.40 (5)
Slip modulus $K$ [MN/m]	500, 695, 965, 1341, 1864, 2590, 3598, 5000 (8)

## 4.2 Presentation of the analysis results

Since the DFC method takes into account individual connector positions, it was considered as the base case for the comparison. Maximum deflection  $w_{max}$ , maximum connector force  $V_{c,max}$ , maximum tensile stress in the bottom fiber of the timber section  $\sigma_{b,max}$  and maximum compressive stress in the top fiber of the concrete slab  $\sigma_{t,max}$  were calculated for all parametric combinations. In each combination the relative differences of the results from the other methods compared to DFC results were calculated from

$$Diff_i = \frac{S_i - S_{DFC}}{S_{DFC}}, \quad (3)$$

where  $S_{DFC}$  is a general effect ( $w_{max}$ ,  $V_{c,max}$ ,  $\sigma_{b,max}$ ,  $\sigma_{t,max}$ ) using DFC method and  $S_i$  is corresponding effect using other method ( $i =$  fully composite, CFC,  $\gamma$ -method). The relative differences for each method were plotted against connection efficiency factors  $\gamma$  as in Eq. (1). Bending stiffnesses  $(EI)_0$ ,  $(EI)_{eff}$  and  $(EI)_\infty$  were determined with the  $\gamma$ -method.

## 4.3 Limitations of the study

Response of a real composite beam is affected by a number of different phenomena, such as friction between the elements, shear deformations of individual components, cracking of concrete and non-linear behavior of the materials and connectors, some of which are not included in any of the described models. Even though DFC model is assumed to best approximate behavior of TCC beams in the study, in reality the aforementioned effects might significantly affect the accuracy of the methods for individual cases, and therefore the comparison is limited to theory.

## 5. Results

The results of the comparison are presented in this chapter. The axis scales of all the graphs were unified to allow better comparability between the graphs. In two graphs, where the results were off the scale of the graph, range of the results is marked on the figure with text and arrows.

### 5.1 Uniform connector spacing - Cases 1a and 1b

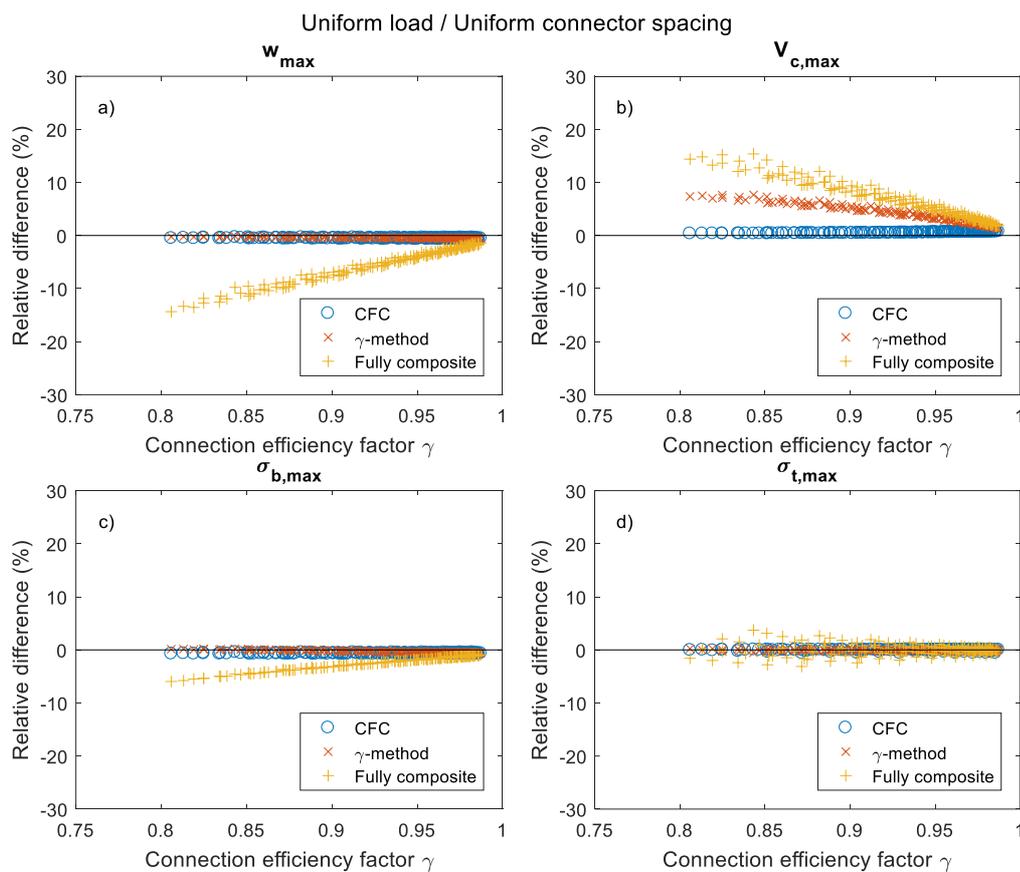


Figure 5. Differences in the Case with uniform connector spacing and uniform load (Case 1a).

The differences in the Cases 1a and 1b were plotted in Figures 5 and 6 functions of connection efficiency factors  $\gamma$  with a range of values,  $\gamma = 0.81 \dots 0.98$ .

The CFC method displayed very close agreement with DFC method in both Cases, 1a and 1b. Calculated differences were less than  $\pm 1\%$  over the whole range of parameters, thus CFC and DFC methods produce practically identical results.

The  $\gamma$ -method displayed fairly good agreement with DFC method with differences less than  $\pm 10\%$ . Differences in deflection were small in the both cases. The connector forces were overestimated in the Case 1a and underestimated in the Case 1b. In the Case 1a the stresses were in close agreement, but in the Case 1b they showed larger differences. Difference of the stress  $\sigma_{t,max}$  depended on the ratio  $h_c/h_t$ , so that with high  $h_c/h_t$  the stress was underestimated and with low  $h_c/h_t$  it was overestimated.

The fully composite method displayed largest differences of the three methods overall. The largest differences are around  $\pm 15\%$ . The deflection of the beam is generally underestimated, while connector forces are overestimated in the Case 1a and in close agreement in the Case 1b. The stresses had smaller differences in the Case 1a than in the Case 1b. Differences of the stress  $\sigma_{t,max}$  depended on the ratio  $h_c/h_t$  similarly as with  $\gamma$ -method.

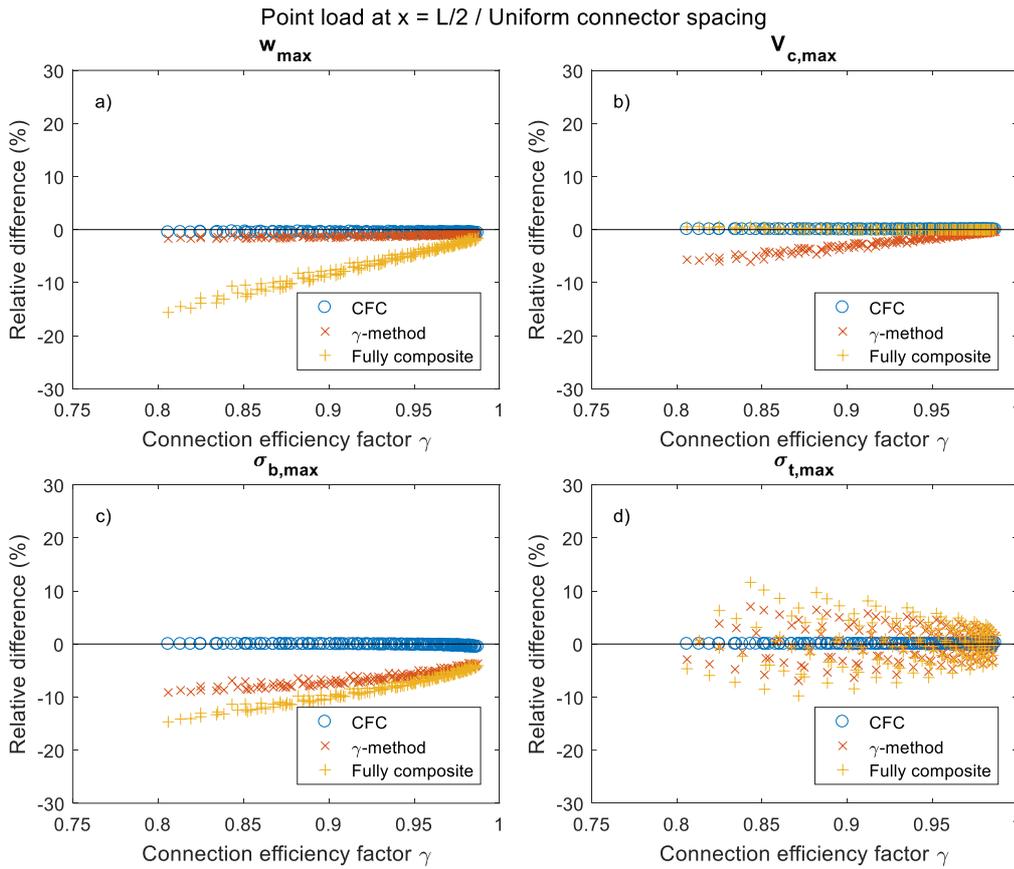


Figure 6. Differences in the Case with uniform connector spacing and point load (Case 1b).

## 5.2 Variable connector spacing - Cases 2a and 2b

The differences in the Cases 2a and 2b were plotted in Figures 7 and 8 as functions of connection efficiency factors  $\gamma$  with a range of values,  $\gamma = 0.78 \dots 0.98$ .

The CFC method had very close agreement in the Cases 1a and 1b, but in the Cases 2a and 2b, the differences were larger. The deflection had a close agreement with DFC, while connector forces displayed very large differences, up to 100% in the Case 2a and around -25...15% in the Case 2b. In the latter Case, connector forces were overestimated with lower  $\gamma$  values and overestimated with higher  $\gamma$  values. The stresses agreed reasonably well with DFC in general, the differences were less than  $\pm 5\%$  in all the Cases.

The  $\gamma$ -method had a fair agreement with largest differences around  $\pm 10\%$ , excluding the connector forces in the Case 2b, where differences up to 170% could be found. The differences in deflections were less than  $\pm 5\%$  in general, while the stresses display larger differences up to  $\pm 10\%$ . The stress  $\sigma_{c,max}$  is generally underestimated, while difference of  $\sigma_{t,max}$  depends on the ratio  $h_c/h_t$  as in the Cases 1a and 1b.

The fully composite method displayed largest differences, up to  $\pm 20\%$ , excluding connector forces in the Case 2b where differences up to 190% could be found. The deflections were underestimated up to 15%.  $\sigma_{c,max}$  is generally underestimated, while difference of  $\sigma_{t,max}$  depends on the ratio  $h_c/h_t$  as in the Cases 1a and 1b.

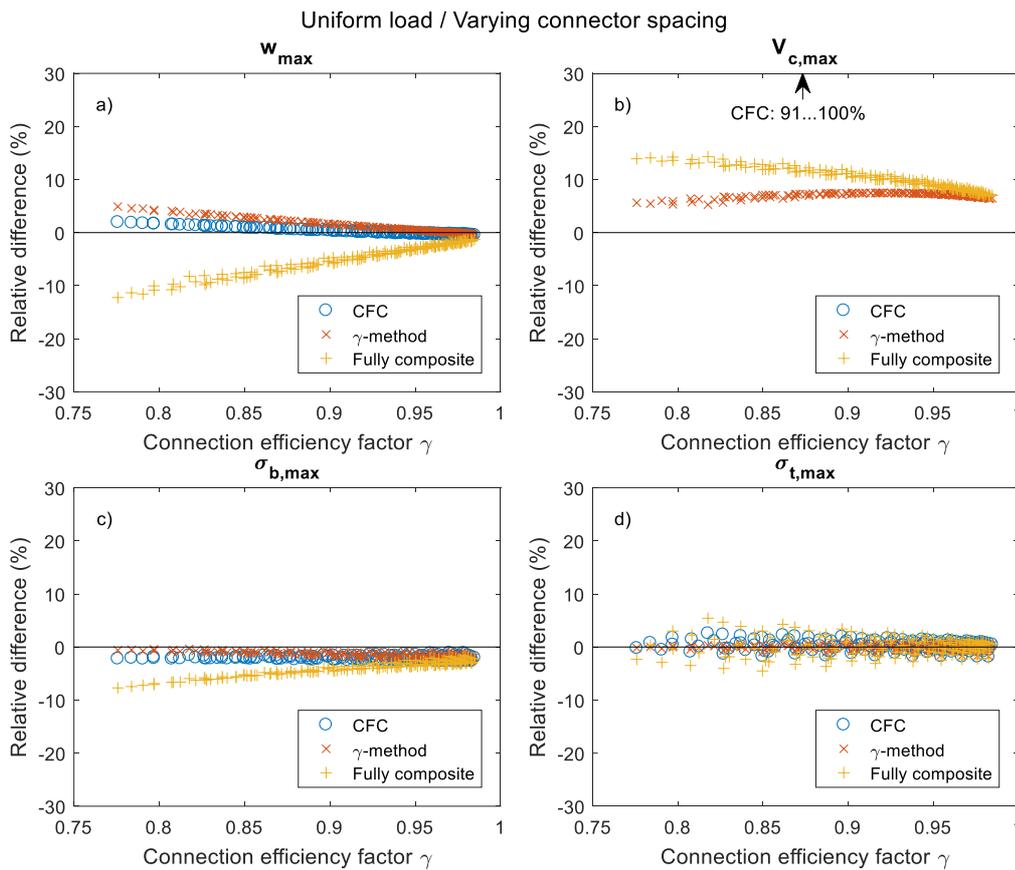


Figure 7. Differences in the Case with varying connector spacing and uniform load (Case 2a).

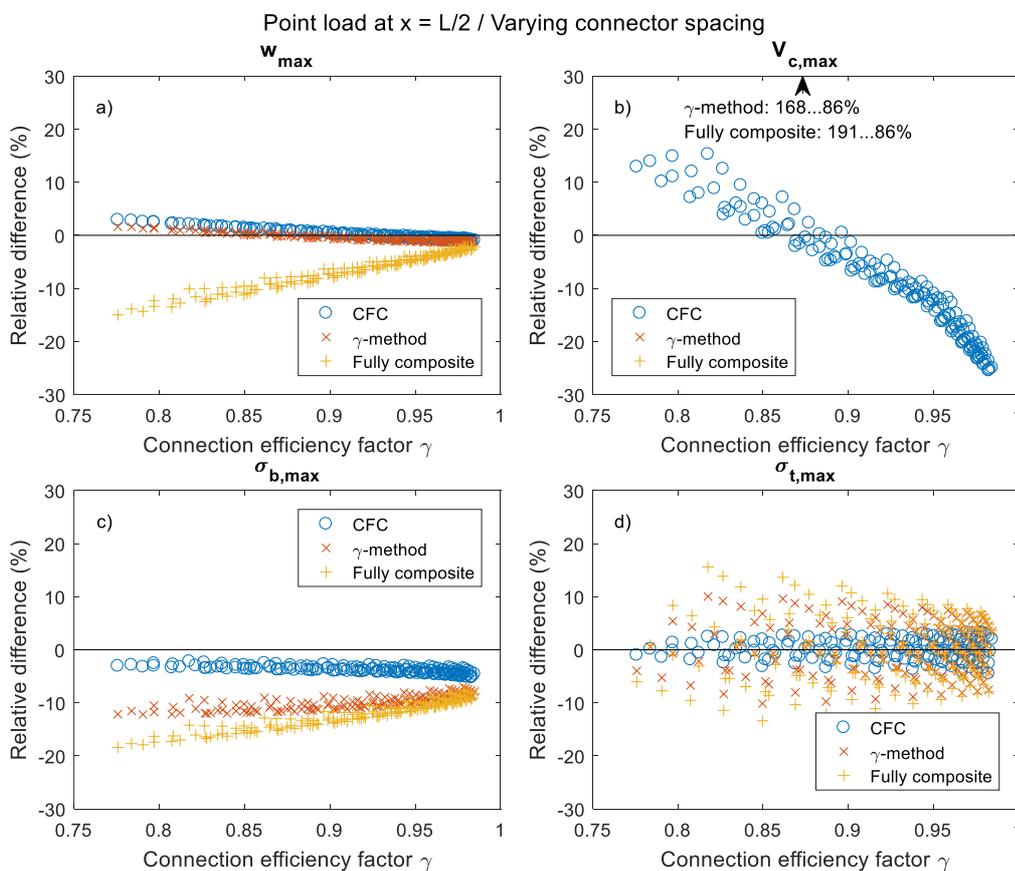


Figure 8. Differences in the Case with varying connector spacing and point load (Case 2b).

### 5.3 Discussion

Connector forces in the Cases 2a and 2b displayed extremely large differences, so the distributions of the forces were investigated in more detail to understand the reason behind the differences.

In the Case 2a, CFC method displayed up to 100% higher connector forces than DFC method. The connector force distributions in the Case 2a are shown in Figure 9. Increasing connector density towards the supports effectively limits the slip, thus limiting also connector forces, which are linearly dependent of the slip, as may be seen in Figure 9 (DFC curve). In contrast, in CFC method, where varying connector spacing is not taken into account, connector forces are steadily increasing towards the supports. In the fully composite method and the  $\gamma$ -method connector forces are not related to slip, but are estimated fairly well since the connector spacing is accounted for in calculating the forces.

In the Case 2b (Figure 10), connector force distribution of CFC is closer to DCF, but the fully composite method and the  $\gamma$ -method displayed extremely high connector forces near the mid-span. In the DFC method, the slip, as well as the connector forces, develop gradually towards the support. In contrast, the connector forces in the fully composite method and the  $\gamma$ -method were largely overestimated where the connector spacing is wide, since the connector force is proportional to product of total shear force and connector spacing.

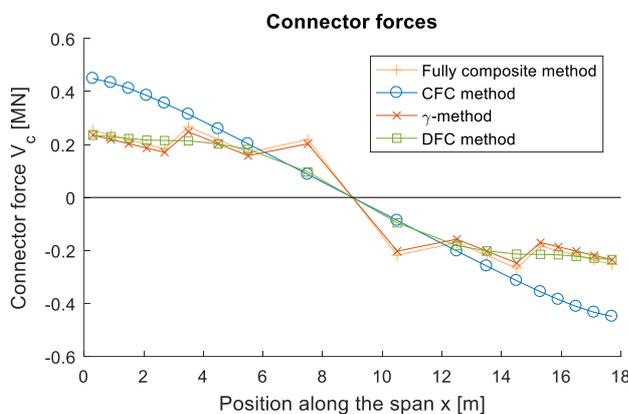


Figure 9. Shear connector force distributions in the Case 2a using different analysis methods.

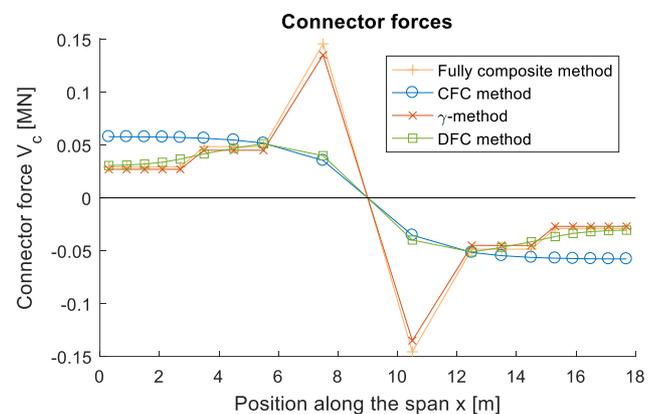


Figure 10. Shear connector force distributions in the Case 2b using different analysis methods.

## 6. Conclusions

This study showed that within the range of this parametric study, CFC and DFC methods have close agreement, when constant connector spacing is used. Thus, CFC method can be recommended as simpler alternative to DFC method, which generally requires programming a numerical tool for analysis.

Using the CFC method to analyze composite beams with varying connector spacing is not reasonable due to the high differences to DFC in connector forces.

The  $\gamma$ -method displayed lowest differences on average, even though in some cases the differences are higher than by using CFC method. Due to simplicity, the  $\gamma$ -method is best suited for preliminary design stages, when rough estimates are needed, as well as for comparison of effects of different design options.

Using the fully composite method for the analysis is not recommended, unless with very high  $\gamma$ -values, when the results are close the results of the  $\gamma$ -method.

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