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Evidence Theory based Cooperative Energy Detection under Noise Uncertainty

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Abstract—Noise power uncertainty is a major issue in detectors for spectrum sensing. Any uncertainty in the noise power leads to significant reduction in the detection performance of the energy detector and also results in a performance limitation in the form of SNR walls. In this paper, we propose an evidence theory (also called Dempster-Shafer theory (DST)) based cooperative energy detection (CED) for spectrum sensing. The noise variance is modeled as a random variable with a known distribution. The analyzed system model is similar to a distributed parallel detection network where each secondary user (SU) evaluates the energy from its received signal samples and sends it to a fusion center (FC), which makes the final decision. However, in the proposed DST-based CED scheme, the SUs send computed belief-values instead of actual energy value to the FC. Any uncertainty in the noise variance is accounted for by discounting the belief values based on the amount of uncertainty associated with each SU. Finally, the discounted belief values are combined using Dempster rule to reach at a global decision. Simulation results indicate that the proposed DST scheme significantly improves the detection probability at low average signal-to-noise ratio (ASNR) in comparison to the traditional sum fusion rule in the presence of noise uncertainty.

Index Terms—Cognitive radio, cooperative energy detection, data fusion, Dempster-Shafer theory, noise uncertainty.

I. INTRODUCTION

The technology of cognitive radio (CR) promises enhanced spectrum utilization via dynamic spectrum access provided that there are government regulations and technological standards permitting secondary use of spectrum and allowing CR devices to function within a specific paradigm [1]. Spectrum sensing is a key enabler for flexible spectrum use and CR as it spectrum awareness needed for adjusting radio operating parameters. Several spectrum sensing techniques have been proposed in the existing literature such as energy detection, autocorrelation detection, eigen-value based detection and cyclostationary feature based detection [2]. However, a single CR performing spectrum sensing in solitude suffers from path loss, multipath fading, shadowing and uncertainties present in the device, which makes identifying idle spectrum less reliable. To overcome this hurdle due to channel and device non-idealities, cooperative spectrum sensing (CSS) has been proposed to enhance spectrum sensing performance by exploiting spatial diversity [3], [4]. In CSS, a number of CR-based-SUs collaboratively sense the spectrum and arrive at

a global decision of whether a particular frequency band is occupied or not.

In this paper, our prime focus is on energy detection, which is widely used for its simplicity and ability to detect any signal [5]. However, in the presence of uncertainty in noise statistics, its performance degrades significantly and may also result in performance limitation in the form of the SNR wall phenomenon [6]. In the case of cooperative energy detection (CED), all the SUs transmit their sensing information in the form of energy of received signal samples, to the FC. At the FC, a suitable fusion rule is applied to combine these data and make a global decision. One of the commonly used fusion rules is the sum of energies from each SU. However, under noise power uncertainty, there is significant loss in detection performance [7]. The utilization of DST in such case can enhance the performance. The goal of this paper is to demonstrate the obtained performance gains.

A CSS scheme based on credibility and evidence (i.e., DST) for cognitive radios was first proposed in [8]. However, the paper did not consider any method to evaluate the credibility of different nodes before combining stage at the FC. In [9], a DST based CED with double threshold spectrum sensing method was proposed. However, such approach requires perfect knowledge of noise variance in order to set the threshold. Therefore, such methods may fail drastically under noise power fluctuation. In [10] and [11], a CED scheme based on DST is proposed where energy of the received signal is used to evaluate the basic probability assignment (BPA) or belief values. The paper also proposed a method to utilize the credibility factor to measure the reliability of each spectrum sensor. However, the optimality of scheme is not considered.

Noise power in an energy detector has been modeled using different statistical models [12]–[14]. In [6], the noise variance was modeled as an unknown constant lying between an upper and lower bound, where the bounds are determined by the uncertainty factor. In [15], we proposed a novel DST based CED scheme considering noise power as an unknown constant and showed that the proposed method is able to achieve higher gain than the sum rule based traditional CED. In this paper, similar DST model is considered with the extension that the noise variance is modeled as a random variable having a known distribution with some average noise power and

standard deviation. First, a method is proposed to evaluate the BPA for each sensor using the conditional likelihood functions of the energies evaluated from the received observations. Next, a technique to discount the credibility of each node is provided. A novel method of determining the discount rate is proposed, where the discount factor depends on the mean and standard deviation of noise variance. These discounted beliefs are then fused using a DST fusion rule at the FC. It is shown that in the absence of noise power uncertainty, the DST fusion rule will be identical to the optimal fusion rule of likelihood ratio. Later, the performance of the proposed method is compared to the traditional sum fusion rule, assuming Neyman-Pearson detection criteria.

The paper is organized as follows: In Section II we discuss the CED under noise uncertainty. Section III briefly discusses the basics of Dempster-Shafer theory of evidence. Next, in section IV, we present in detail the proposed DST based CED method. Section V presents the simulation results while Section VI concludes the paper.

II. COOPERATIVE ENERGY DETECTION UNDER NOISE UNCERTAINTY

In this section, a brief overview of CED in a CR network is provided. First, normal CED without noise uncertainty is described. Next, noise power uncertainty in the energy detector is considered and its effects on setting threshold for decision making at the FC are explained.

A. System Model

Fig. 1 shows the traditional CED model, which consists of a primary user (PU), U number of SUs and a FC. Each SU calculates the received signal energy based on local observations, evaluates the corresponding decision statistic and sends it to the FC via reporting channel. On receiving the data from each SU the FC will employ fusion rule to combine the data and based on the detection criterion arrive at a global decision. In this paper, it is assumed that the reporting channels are error-free while the sensing channels are additive white Gaussian noise (AWGN) channels.

In a distributed detection scheme, the presence or absence of a PU signal based on local sensory measurements can be formulated as a binary hypothesis testing problem. There are two hypotheses H_0 and H_1 , where H_0 denotes the absence of PU signal and H_1 denotes the presence of PU signal. The considered signal model is

$$\begin{aligned} H_0 : x_i[n] &= w_i[n] \\ H_1 : x_i[n] &= s[n] + w_i[n] \end{aligned} \quad (1)$$

for $i = 1, 2, \dots, U$ and $n = 1, 2, \dots, N$. Here, $x_i[n]$, $w_i[n]$, and $s[n]$ are the samples of received signal, AWGN and PU signal, respectively at i^{th} SU, while N is the number of received signal samples. Note that the local observations of SUs, conditioned on either of the hypotheses are assumed to be independent of each other. The noise samples $w_i[n]$ are assumed to be zero mean with variance σ_w^2 . It is also a

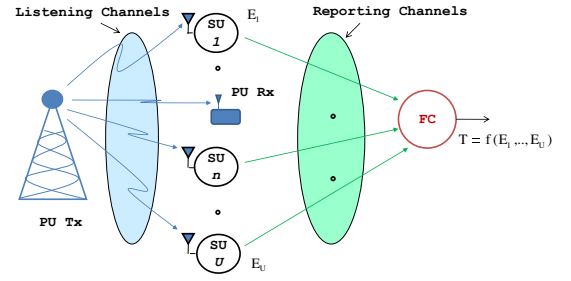


Figure 1. Cooperative energy detection model

complex circularly symmetric Gaussian random variable with a real and imaginary part $w_r[n]$ and $w_k[n]$ respectively, i.e., $w_i[n] = w_r[n] + jw_k[n]$. Therefore,

$$\begin{aligned} w(n) &\sim \mathcal{N}_c(0, \sigma_w^2), \\ w_r(n), w_k(n) &\sim \mathcal{N}(0, \sigma_w^2/2). \end{aligned}$$

Moreover, the noise samples $w_i[n]$ are also assumed to be independent from sensor to sensor. Several commonly used PU waveforms such as OFDM signals, are Gaussian signals [2]. Therefore, the PU signal $s[n]$ is also assumed to be complex circularly symmetric Gaussian random variable with zero mean and variance σ_s^2 . Consequently, $x_i[n]$ is also a complex circularly symmetric Gaussian random variable with zero mean and variance σ_x^2 , i.e., $x_i[n] \sim \mathcal{N}_c(0, \sigma_x^2)$ and thus for different hypotheses we have

$$\begin{aligned} H_0 &: x_i[n] \sim \mathcal{N}_c(0, \sigma_w^2) \\ H_1 &: x_i[n] \sim \mathcal{N}_c(0, \sigma_s^2 + \sigma_w^2). \end{aligned} \quad (2)$$

B. Cooperative Energy Detection

The received signal energy at i^{th} SU, i.e., E_i , can be evaluated from N received samples as

$$E_i = \sum_{n=1}^N |x_i[n]|^2. \quad (3)$$

Since E_i is a sum of the squares of N complex Gaussian random variables, $\frac{E_i}{\sigma_x^2/2}$ follow central chi-square (χ^2) distribution with $2N$ degrees of freedom under both the hypotheses H_0 and H_1 [16] so that

$$\begin{aligned} H_0 &: \frac{2E_i}{\sigma_w^2} \sim \chi_{2N}^2 \\ H_1 &: \frac{2E_i}{\sigma_s^2 + \sigma_w^2} \sim \chi_{2N}^2. \end{aligned} \quad (4)$$

However, according to central limit theorem [17], if the number of samples N is sufficiently large (e.g., > 100), the test statistic E_i is asymptotically Gaussian distributed and its conditional distribution at the i^{th} SU, conditioned on either of the hypotheses H_0 and H_1 , is given in [5] as

$$\begin{aligned} H_0 : E_i &\sim \mathcal{N}(\mu_{0i}, \sigma_{0i}^2), \\ H_1 : E_i &\sim \mathcal{N}(\mu_{1i}, \sigma_{1i}^2), \end{aligned} \quad (5)$$

where

$$\begin{aligned} \mu_{0i} &= N\sigma_w^2 ; & \mu_{1i} &= N\sigma_w^2(1 + \text{SNR}), \\ \sigma_{0i}^2 &= N(\sigma_w^2)^2 ; & \sigma_{1i}^2 &= N(\sigma_w^2)^2(1 + \text{SNR})^2. \end{aligned} \quad (6)$$

Here $\text{SNR} = \sigma_s^2/\sigma_w^2$ is signal-to-noise ratio at the individual CR node. In traditional sum fusion rule, assuming conditional independence, each SU_i sends the computed local energy value E_i to the FC, which are then added so that the final decision statistic at the FC is given by

$$T_{sum} = \sum_{i=1}^U E_i \quad (7)$$

while the decision is made by comparing T_{sum} to a threshold so that

$$T_{sum} \underset{H_0}{\overset{H_1}{\geq}} \eta_{sum} \quad (8)$$

where η_{sum} is the threshold of a Neyman-Pearson detector with some false alarm constraint β . Here, the threshold η_{sum} depends on the distribution of T_{sum} under the null hypothesis H_0 and the constraint on the false alarm probability β . T_{sum} being a linear combination of U Gaussian random variables, is also Gaussian distributed under both the hypotheses H_0 and H_1 as given in [5] by

$$\begin{aligned} H_0 : T_{sum} &\sim \mathcal{N}(\mu_0, \sigma_0^2), \\ H_1 : T_{sum} &\sim \mathcal{N}(\mu_1, \sigma_1^2), \end{aligned} \quad (9)$$

where

$$\begin{aligned} \mu_0 &= UN\sigma_w^2 ; & \mu_1 &= UN\sigma_w^2(1 + \text{SNR}), \\ \sigma_0^2 &= UN(\sigma_w^2)^2 ; & \sigma_1^2 &= UN(\sigma_w^2)^2(1 + \text{SNR})^2. \end{aligned} \quad (10)$$

Note that the sum fusion rule given by (7) is an optimal fusion rule for the binary hypothesis testing problem in (5) under the assumption of conditional independence of observations at SUs, conditioned on either of the hypotheses [5]. Now when σ_w^2 is exactly known, the probability of false alarm (P_{fa}) and probability of detection (P_d) for a Neyman-Pearson detector are expressed as

$$P_{fa} = Q\left(\frac{\eta_{sum} - \mu_0}{\sigma_0}\right); \quad P_d = Q\left(\frac{\eta_{sum} - \mu_1}{\sigma_1}\right), \quad (11)$$

where $Q(\cdot)$ is the tail probability of the standard normal distribution. Therefore, for known noise power at the CR nodes, the expression for threshold η_{sum} with false alarm constraint of β is

$$\begin{aligned} \eta_{sum} &= Q^{-1}(\beta)\sigma_0 + \mu_0 \\ &= \sqrt{UN}(\sigma_w^2)Q^{-1}(\beta) + UN\sigma_w^2. \end{aligned} \quad (12)$$

However, this definition of threshold fails, when noise power is not a known fixed quantity but fluctuates and may have different values at different time instances. This is discussed in the next section.

C. Noise power uncertainty

In our work, we have assumed the noise variance to be a random variable having a known distribution. We consider two widely-used distributions for modeling the noise variance: *Gaussian* and *uniform*. In most of the scenarios, noise variance is not known and has to be estimated. Most of the estimators of noise variance (σ_w^2) for AWGN would result in the estimate to be Gaussian distributed. For example, the asymptotic distribution of the maximum likelihood estimate of σ_w^2 has Gaussian distribution [18]. The probability density function (pdf) of the noise variance σ_w^2 assuming it to be Gaussian distributed is given by

$$f(\sigma_w^2) = \frac{1}{\sqrt{2\pi}\sigma_\Delta} \exp\left(-\frac{(\sigma_w^2 - \sigma_n^2)^2}{2\sigma_\Delta^2}\right), \quad (13)$$

where σ_n^2 is the mean value and σ_Δ is the standard deviation.

When, the only prior information regarding the noise-variance is the limits within which the value lies, the uniform distribution is a natural choice. Note that uniform distribution is the least informative distribution as it does not favor any particular value of noise-variance. The uniform distribution for σ_w^2 is given as

$$f(\sigma_w^2) = \begin{cases} \frac{1}{\sigma_U^2 - \sigma_L^2}, & \sigma_L^2 \leq \sigma_w^2 \leq \sigma_U^2 \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

where σ_U^2 and σ_L^2 are the upper and the lower bounds of σ_w^2 under uniform distribution. For convenience of specifying both the distributions given in (13) and (14) in terms of the same distribution parameters, σ_U^2 and σ_L^2 are chosen in this paper as

$$\begin{aligned} \sigma_U^2 &= \sigma_n^2 + \sqrt{3}\sigma_\Delta, \\ \sigma_L^2 &= \sigma_n^2 - \sqrt{3}\sigma_\Delta. \end{aligned} \quad (15)$$

Note that the P_{fa} and P_d at the FC expressed using (11) are for a known value of σ_w^2 and cannot be used for the case when σ_w^2 is a random variable. Here, we need to instead use the average probability of false alarm (P'_{fa}) and average probability of detection (P'_d) at the FC, which are given by

$$P'_{fa}(\eta'_{sum}, \sigma_\Delta^2) = \int_0^\infty Q\left(\frac{\eta'_{sum} - \mu_0}{\sigma_0}\right) f(\sigma_w^2) d\sigma_w^2, \quad (16)$$

$$P'_d(\eta'_{sum}, \sigma_\Delta^2) = \int_0^\infty Q\left(\frac{\eta'_{sum} - \mu_1}{\sigma_1}\right) f(\sigma_w^2) d\sigma_w^2. \quad (17)$$

Therefore, under noise uncertainty the threshold η'_{sum} at the FC can be determined using equation (16) for a desirable probability of false alarm level. Moreover, we can see that the two average probabilities, i.e, P'_{fa} and P'_d are functions of only the distribution parameter σ_Δ^2 as the noise variance σ_w^2 is integrated out. Furthermore, in this case we define average signal-to-noise ratio as, $\text{ASNR} = \sigma_s^2/\sigma_n^2$. In dB scale, ASNR is expressed as $\text{ASNR (dB)} = 10\log_{10}(\sigma_s^2/\sigma_n^2)$.

III. DEMPSTER-SHAFFER THEORY (DST) OF EVIDENCE

This section gives a basic overview of the DST or the evidence theory and highlights the fundamental difference between the DST and the Bayesian approach. Interested readers are referred to read [19] for a thorough description and performance analysis of DST.

In DST, we first define a set of mutually exclusive hypotheses that the event under observation can take. This initial set of hypotheses is called *frame of discernment*. For example, let us define a frame of discernment say $\Theta = \{\theta_1, \theta_2\}$. The power set of Θ is given as $2^\Theta = \{\phi, \theta_1, \theta_2, \{\theta_1, \theta_2\}\}$, which basically represents all the possible subsets of Θ , including Θ itself. Then a function $m : 2^\Theta \rightarrow [0, 1]$ is called *basic probability assignment* (BPA) whenever [19]

$$m(\phi) = 0, \\ \sum_{A \in 2^\Theta} m(A) = 1.$$

Here, ϕ denotes the empty set and $m(\phi) = 0$ always because at least one of the hypotheses has to be true. The quantity $m(A)$ is the measure of belief that is committed exactly to A and not to any subset of A . The value of $m(A)$ always lies between 0 and 1. Here, it is important to note that the basic probabilities are assigned not to the elements of Θ , but to the power set of Θ which is $(\phi, \theta_1, \theta_2, \{\theta_1, \theta_2\})$. This is a key difference between the Bayesian theory and the DST. Bayesian theory is more concerned with evidence that supports single conclusions, e.g., evidence for each outcome θ_i in Θ . On the other hand DST is concerned with evidences which support subsets of outcomes in Θ , e.g., $\{\theta_1, \theta_2\}$. To elaborate this point, we take an example. Let's say a colour blind person is asked to evaluate the colour of an object as either red (R) or Blue (B). So $\Theta = \{R, B\}$. Considering Bayesian philosophy, the colour blind person will assign his support values individually to R and B as $\{0.4, 0.6\}$ say. But from the perspective of DS theory, he can assign support values or weight to each of the four possibilities $\{\phi, R, B, \{R, B\}\}$ as $\{0, 0.3, 0.5, 0.2\}$ say. Here, $m(R, B) = 0.2$ signifies the amount of uncertainty or ignorance that the colour blind person is experiencing in deciding whether the colour of the object is R or B . This grants DS theory more flexibility and allows for the inclusion of *unquantified uncertainty*, which is the most significant advantage of DST over Bayesian.

Dempster rule of combination enables us to compute the orthogonal sum of several belief functions over the same frame of reference but based on distinct bodies of evidence. If there are n independent sources having a common frame of reference with BPA $m_1(\cdot), m_2(\cdot), \dots, m_n(\cdot)$, then the combined BPA for an element A based on the Dempster rule of combination is given as in [19]

$$m(A) = [m_1 \oplus m_2 \oplus \dots \oplus m_n](A) \\ = \frac{\sum_{\substack{A_1, A_2, \dots, A_n \in 2^\Theta \\ A_1 \cap A_2 \cap \dots \cap A_n = A}} m_1(A_1) \dots m_n(A_n)}{K}, \quad (18)$$

where K is given as

$$K = \sum_{\substack{A_1, A_2, \dots, A_n \in 2^\Theta \\ A_1 \cap A_2 \cap \dots \cap A_n \neq \phi}} m_1(A_1) \dots m_n(A_n). \quad (19)$$

Here the symbol \oplus denotes the orthogonal sum and K is the re-normalisation factor.

For example, continuing with the previous example, assume two colour blind persons are considering the proposition that an object is one of the two colours $\{R, B\}$. Let $m_1(R) = 0.3, m_1(B) = 0.5, m_1(R, B) = 0.2$ and $m_2(R) = 0.8, m_2(B) = 0.1, m_2(R, B) = 0.1$ be the BPAs of person 1 and 2 respectively, then the combined BPA for colour R and B are given as

$$m_{12}(R) = \frac{1}{K} \left\{ \begin{array}{l} m_1(R)m_2(R) + m_1(R)m_2(R, B) + \\ m_2(R)m_1(R, B) \end{array} \right\} \\ m_{12}(B) = \frac{1}{K} \left\{ \begin{array}{l} m_1(B)m_2(B) + m_1(B)m_2(R, B) + \\ m_2(B)m_1(R, B) \end{array} \right\}$$

where K is computed as

$$K = m_1(R)m_2(R) + m_1(B)m_2(B) + m_1(R)m_2(\{R, B\}) \\ + m_2(R)m_1(\{R, B\}) + m_1(B)m_2(\{R, B\}) + \\ m_2(B)m_1(\{R, B\}) + m_1(\{R, B\})m_2(\{R, B\}).$$

Using the above formulation, we get $K = 0.57$ and combined BPA for R and B as $m_{12}(R) = 0.754$ and $m_{12}(B) = 0.21$. At this point a decision can be made by simple comparison of the combined BPAs. Since $m_{12}(R) > m_{12}(B)$ we can infer that the object is red in colour.

IV. PROPOSED DST BASED CED

Fig. 2 shows the framework for the proposed DST based CED. First step in this approach is estimating BPA values based on the energy of the received signal for different hypotheses. Next, BPA values of each SU are discounted based on the noise power uncertainty associated with it. In the final step, these discounted BPA values are used in the DST fusion rule at the FC.

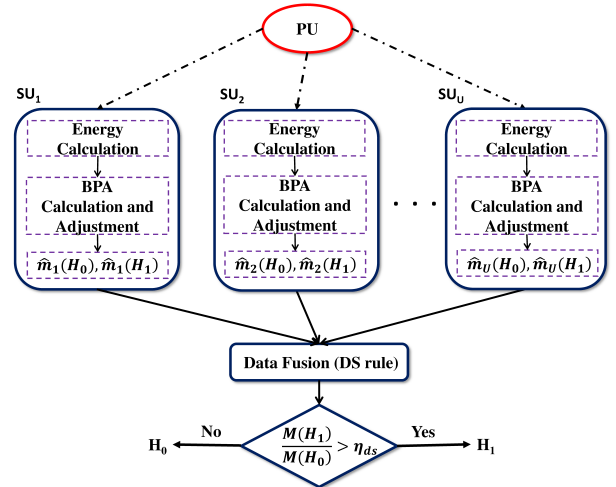


Figure 2. Framework for proposed DST based CED

A. BPA calculation

In DST based CED scheme, assigning the basic probability to the different hypotheses is a vital part as the end decision relies upon the accuracy of how the BPAs are allotted. First, let us define a frame of reference for a SU performing local sensing as $\Theta = \{H_0, H_1\}$, where H_0 denotes the hypothesis that the PU is absent and H_1 hypothesis denotes its presence. Then the power set of Θ is given as $\{\phi, H_0, H_1, \{H_0, H_1\}\}$. Here ϕ is a null set and $m(\phi) = 0$. The set $\omega = \{H_0, H_1\}$ represents the uncertainty/ignorance or the fact “don’t know”.

In [15], treating noise-variance σ_w^2 as an unknown constant, we proposed the following BPAs for hypotheses H_0 and H_1

$$\begin{aligned} m_i(H_0) &= \frac{P(E_i|H_0; \sigma_w^2)}{P(E_i|H_0; \sigma_w^2) + P(E_i|H_1; \sigma_w^2)}, \\ m_i(H_1) &= \frac{P(E_i|H_1; \sigma_w^2)}{P(E_i|H_0; \sigma_w^2) + P(E_i|H_1; \sigma_w^2)}, \end{aligned} \quad (20)$$

where $P(E_i|H_j; \sigma_w^2)$ are likelihood functions conditioned on H_j for $j = 0, 1$ while being parametrized by unknown constant σ_w^2 [18]. The likelihood functions $P(E_i|H_j; \sigma_w^2)$ for $j = 0, 1$ are expressed as

$$P(E_i|H_0; \sigma_w^2) = \frac{1}{\sqrt{2\pi}\sigma_{0i}} \exp\left(-\frac{(E_i - \mu_{0i})^2}{2\sigma_{0i}^2}\right), \quad (21)$$

$$P(E_i|H_1; \sigma_w^2) = \frac{1}{\sqrt{2\pi}\sigma_{1i}} \exp\left(-\frac{(E_i - \mu_{1i})^2}{2\sigma_{1i}^2}\right). \quad (22)$$

It was shown in [15] that these choices of BPA for H_0 and H_1 along with DST fusion at the FC, results in optimal detection performance similar to using likelihood ratio test (LRT) statistics for the binary hypothesis testing problem in (5) under no noise power uncertainty condition. However, when noise power is not an unknown deterministic quantity but a random variable, we have to calculate the average of $P(E_i|H_0, \sigma_w^2)$ and $P(E_i|H_1, \sigma_w^2)$, in order to compute the BPA values, which can be evaluated as

$$P(E_i|H_0) = \int_{-\infty}^{\infty} P(E_i|H_0, \sigma_w^2) f(\sigma_w^2) d\sigma_w^2, \quad (23)$$

$$P(E_i|H_1) = \int_{-\infty}^{\infty} P(E_i|H_1, \sigma_w^2) f(\sigma_w^2) d\sigma_w^2. \quad (24)$$

Here, $P(E_i|H_j, \sigma_w^2)$ for $j = 0, 1$ are likelihood functions conditioned on H_j and σ_w^2 . This is because σ_w^2 is no longer a unknown constant but a random variable. By substituting $P(E_i|H_j; \sigma_w^2)$ with $P'(E_i|H_j)$ for $j = 0, 1$ in (20), we get

$$m'_i(H_0) = \frac{P'(E_i|H_0)}{P'(E_i|H_0) + P'(E_i|H_1)} \quad (25)$$

and

$$m'_i(H_1) = \frac{P'(E_i|H_1)}{P'(E_i|H_0) + P'(E_i|H_1)} \quad (26)$$

while the BPA for ω is given by

$$m'_i(\omega) = 1 - m'_i(H_0) - m'_i(H_1). \quad (27)$$

Note that when there is no uncertainty in the noise variance i.e., $\sigma_w^2 = \sigma_n^2$, we have $f(\sigma_w^2) = \delta(\sigma_w^2 - \sigma_n^2) = \delta(\sigma_w^2 - \sigma_n^2)$ so that $P'(E_i|H_0) = P(E_i|H_0; \sigma_w^2 = \sigma_n^2)$ and $P'(E_i|H_1) = P(E_i|H_1; \sigma_w^2 = \sigma_n^2)$. Since BPAs based on $P(E_i|H_0; \sigma_w^2)$ and $P(E_i|H_1; \sigma_w^2)$ are shown to be optimal test statistics when noise variance σ_w^2 is exactly known for the binary hypothesis testing problem in (5) [15], BPAs based on $P'(E_i|H_0)$ and $P'(E_i|H_1)$ are also optimal for the same problem.

B. BPA adjustment under noise power uncertainty

The BPA functions $m'_i(\cdot)$ formulated above, do not take into account the amount of noise uncertainty associated with the SU. As a result, the sum of $m'_i(H_0)$ and $m'_i(H_1)$ will always be one i.e. $m'_i(H_0) + m'_i(H_1) = 1$ and $m'_i(\omega)$ will always be equal to 0. To incorporate noise uncertainty data or specifically determining the support value for $m'_i(\omega)$, the BPAs of each SU are discounted before sending them to FC by using the DST discounting rule. The discounting rule states that if we have a degree of trust of $1 - \alpha$ in the evidence as a whole, where $0 \leq \alpha \leq 1$, then α is adopted as a discount rate and reduce the degree of support for each proper subset A of Θ from $m(A)$ to $(1 - \alpha)m(A)$. Therefore, under noise uncertainty conditions the new BPAs for each SU will be,

$$\hat{m}_i(H_0) = (1 - \alpha_i)m'_i(H_0), \quad (28)$$

$$\hat{m}_i(H_1) = (1 - \alpha_i)m'_i(H_1), \quad (29)$$

where α_i denotes discount rate for i^{th} SU. In this case $\hat{m}_i(H_0) + \hat{m}_i(H_1) < 1$, therefore the support for $\hat{m}_i(\omega)$ is obtained as

$$\begin{aligned} \hat{m}_i(\omega) &= 1 - \hat{m}_i(H_0) - \hat{m}_i(H_1) \\ &= 1 - (1 - \alpha_i)[m'_i(H_0) + m'_i(H_1)] \\ &= 1 - (1 - \alpha_i) \\ \hat{m}_i(\omega) &= \alpha_i. \end{aligned} \quad (30)$$

Thus we find that the support for uncertainty set $\hat{m}_i(\omega)$ is same as the discount rate α_i .

C. Determining discount rate α_i

In this section, we propose a method for determining discount rate α_i . Each SU will have its own unique discount rate, depending on the noise uncertainty associated with it. To determine α_i , we take the help of receiver operating characteristics (ROC) curves for normal energy detection at the i^{th} SU based on Neyman-Pearson criterion. The key idea is that each SU has its own ROC curve when σ_w^2 is exactly known, i.e., when there is no noise power uncertainty. We denote this ROC as true P_{di} . Also, it has a ROC curve when σ_w^2 is a random quantity, which is termed as average P_{di} and denoted by P'_{di} . Both these ROC curves can be obtained via theoretical calculations. The discount rate, α_i of the i^{th} SU is evaluated as the difference between true P_{di} and average P_{di} , i.e., P'_{di} , for a particular false alarm rate, β_i . For convenience, we have assumed the value of β_i to be same as β at the FC. Therefore, α_i can be expressed as

$$\alpha_i(\beta, \text{ASNR}) = P_{di}(\beta, \text{ASNR}) - P'_{di}(\beta, \sigma_w^2, \text{ASNR}), \quad (31)$$

where the true P_{di} is calculated using equation (11) and the average P_{di} is calculated using equation (17). Fig.3 shows the discounting rate α_i as a function of constraint on the false alarm probability at ASNR (dB) = -10 dB. The noise variance σ_w^2 in this case is assumed to be Gaussian distributed with mean, $\sigma_n^2 = 1$ and variance $\sigma_\Delta^2 = 0.01$.

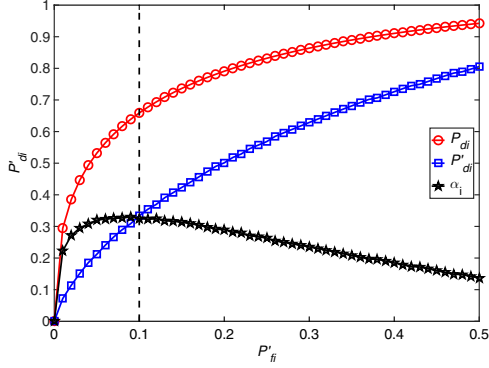


Figure 3. The discounting rate α_i , as a function of constraint on the false alarm probability for ASNR (dB) = -10 dB. For $\beta_i = \beta = 0.1$, the discount rate α_i is the point where the dotted line intersects the α_i curve.

D. Data fusion at the FC

The discounting method described above is performed at the individual SU. The BPA values $m'_i(H_0)$ and $m'_i(H_1)$ of each SU_i are adjusted with the corresponding discount rate α_i associated with it. For identical CR nodes we can assume $\alpha_1 = \alpha_2 = \dots = \alpha_U = \alpha$. However, if the SUs' have different noise variance distributions and distribution parameters, then in that case, the discount rates of each SU will also vary accordingly. The BPA adjustment is done according to (28) and (29). These discounted BPAs are then send to the FC for fusion. The DS combination rule is used to fuse the BPAs from the U SUs, which gives us the total support $M(H_0)$ and $M(H_1)$ for hypothesis H_0 and H_1 respectively as

$$M(H_0) = \frac{\sum_{A_1 \cap A_2 \cap \dots \cap A_U = H_0} \prod_{i=1}^U \hat{m}_i(A_i)}{K}, \quad (32)$$

$$M(H_1) = \frac{\sum_{A_1 \cap A_2 \cap \dots \cap A_U = H_1} \prod_{i=1}^U \hat{m}_i(A_i)}{K}.$$

Finally, test statistic at the FC is taken as

$$T_{ds} = \frac{M(H_1)}{M(H_0)} \underset{H_0}{\overset{H_1}{\geq}} \eta_{ds}, \quad (33)$$

where η_{ds} is the threshold at FC under DST scheme.

V. SIMULATION RESULTS

The key objective in this section is to compare the performance of proposed DST based CED scheme with that of the sum based CED under random noise-variance. For our simulations, we assume that the PU signal is a complex and

circularly symmetric Gaussian signal. At the FC Neyman-Pearson detection criterion is used. The number of cooperating SUs is $U = 5$, the number of received observations used for evaluating received signal energy is $N = 300$ while the number of realizations used for estimating the probability of detection is 10,000. The nominal noise variance is chosen as $\sigma_n^2 = 1$. Two different values of σ_Δ^2 are taken into account, $\sigma_\Delta^2 = 0.01$ and $\sigma_\Delta^2 = 0.02$ for performance evaluation. For P'_d vs ASNR (dB) simulations, the false alarm constraint at FC, i.e., β , is chosen as $\beta = 0.1$. For ROC curves simulation ASNR (dB) of -10 dB is fixed. As deriving the distribution of T_{ds} under both the hypotheses is non-trivial and tedious, the threshold η_{ds} is evaluated empirically under H_0 for each value of ASNR (dB) such that the P'_{fa} constraint of $\beta = 0.1$ is always satisfied at FC. Note that the empirical evaluation of threshold can be done off-line as it depends on all the known parameters such as the pdf of the noise variance, $\beta = 0.1$ and ASNR (dB).

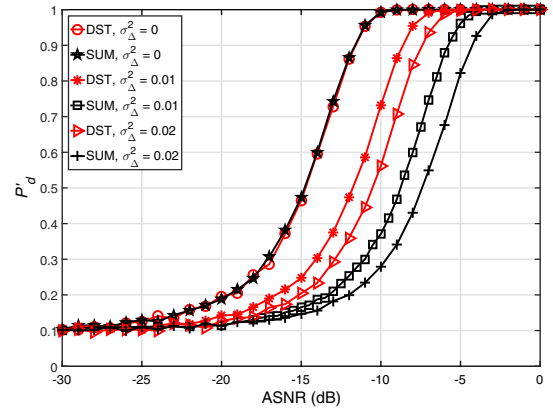


Figure 4. P'_d as a function of ASNR (dB) for $\beta = 0.1$ considering σ_w^2 as a Gaussian distributed random variable.

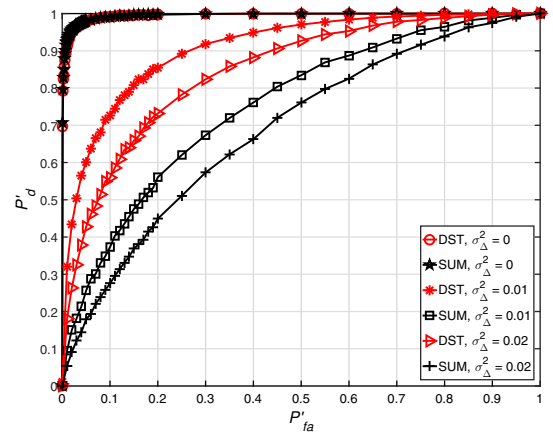


Figure 5. ROC curves comparison at ASNR (dB) = -10 dB considering σ_w^2 as a Gaussian distributed

Fig. 4 presents P'_d vs ASNR (dB) while Fig. 5 shows the ROC curves comparison of DST and sum based CED schemes for Gaussian distributed noise variance. From Figs 4 and 5, it can be clearly observed that in the presence of no noise

uncertainty, the performance of both schemes overlap. On the other hand, in the presence of noise uncertainty, there is degradation in the performance of both CED schemes. However, the degradation in the performance of the sum fusion rule is significantly higher as compared to that of proposed DST based fusion even for small noise uncertainty values of $\sigma_{\Delta}^2 = 0.01$ and $\sigma_{\Delta}^2 = 0.02$.

Similarly, Figs. 6 and 7 show P'_d vs ASNR (dB) and ROC comparison of the two schemes where noise variance is modeled as a uniformly distributed random variable. Even for this case, the proposed DST scheme outperforms sum for both values of $\sigma_{\Delta}^2 = 0.01$ and $\sigma_{\Delta}^2 = 0.02$.

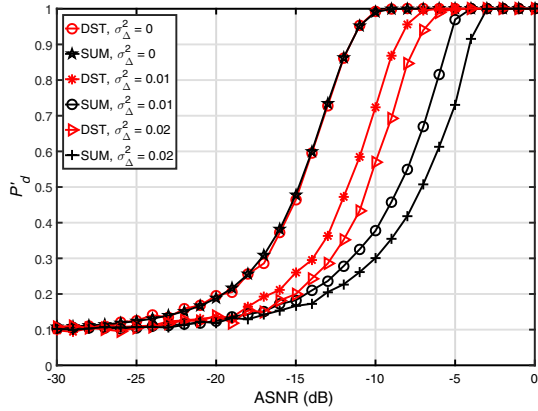


Figure 6. P'_d as a function of ASNR (dB) for $\beta = 0.1$ considering σ_w^2 as a uniformly distributed random variable.

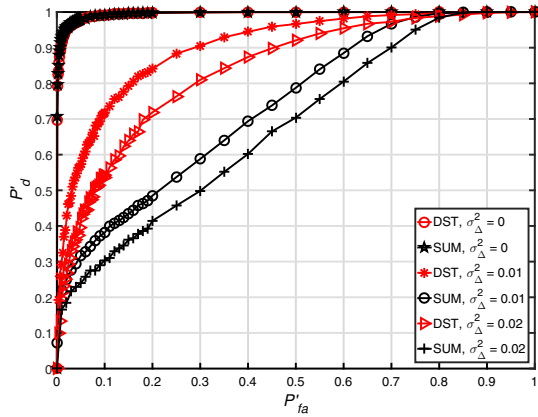


Figure 7. ROC curves comparison at ASNR (dB) = -10 dB considering σ_w^2 as a uniformly distributed random variable

VI. CONCLUSION

In this paper, we have proposed an evidence-theory based fusion rule for CED in the presence of noise power uncertainty. The uncertainty in the noise variance is modeled as a random variable with known probability model. Next, each SU first evaluates the BPA based on observations. These BPA are discounted based on the uncertainty in the noise power at each SU. In the absence of uncertainty in the noise variance, it is shown that the performance of the proposed scheme

and the sum fusion rule are same as that of optimal fusion rule (likelihood ratio test). However, even in the presence of minor noise uncertainty, the proposed scheme achieves higher detection probability than the sum fusion rule, while maintaining the same false alarm constraint at the FC.

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