Saarelma, Jukka; Savioja, Lauri

**Audibility of dispersion error in room acoustic finite-difference time-domain simulation in the presence of absorption of air**

*Published in:*
Journal of the Acoustical Society of America

**DOI:** 10.1121/1.4972529

Published: 01/12/2016

**Document Version**
Publisher's PDF, also known as Version of record

*Please cite the original version:*
Audibility of dispersion error in room acoustic finite-difference time-domain simulation in the presence of absorption of air

Jukka Saarelma, and Lauri Savioja

Citation: The Journal of the Acoustical Society of America 140, EL545 (2016); doi: 10.1121/1.4972529
View online: https://doi.org/10.1121/1.4972529
View Table of Contents: http://asa.scitation.org/toc/jas/140/6
Published by the Acoustical Society of America

Articles you may be interested in

Audibility of dispersion error in room acoustic finite-difference time-domain simulation as a function of simulation distance
The Journal of the Acoustical Society of America 139, 1822 (2016); 10.1121/1.4945746

Noise control zone for a periodic ducted Helmholtz resonator system
The Journal of the Acoustical Society of America 140, EL471 (2016); 10.1121/1.4968530

Spatially separating language masker from target results in spatial and linguistic masking release
The Journal of the Acoustical Society of America 140, EL465 (2016); 10.1121/1.4968034

Talker variability effects on vocal emotion recognition in acoustic and simulated electric hearing
The Journal of the Acoustical Society of America 140, EL497 (2016); 10.1121/1.4971758

Enhanced auditory spatial performance using individualized head-related transfer functions: An event-related potential study
The Journal of the Acoustical Society of America 140, EL539 (2016); 10.1121/1.4972301

Consistent sound change between stops and affricates in Seoul Korean within and across individuals: A diachronic investigation
The Journal of the Acoustical Society of America 140, EL491 (2016); 10.1121/1.4971203
Audibility of dispersion error in room acoustic finite-difference time-domain simulation in the presence of absorption of air

Jukka Saarelma and Lauri Savioja
Department of Computer Science, Aalto University School of Science, FI-00067 Aalto, Finland
jukka.saarelma@aalto.fi, lauri.savioja@aalto.fi

Abstract: The finite-difference time-domain method has gained increasing interest for room acoustic prediction use. A well-known limitation of the method is a frequency and direction dependent dispersion error. In this study, the audibility of dispersion error in the presence of air absorption is measured. The results indicate that the dispersion error in the worst-case direction of the studied scheme gets masked by the air absorption at a phase velocity error percentage of 0.28% at the frequency of 20 kHz.
© 2016 Acoustical Society of America

1. Introduction

The finite-difference time-domain (FDTD) method has become a popular area of research in room acoustic prediction, as the method allows one to directly solve the wave equation in the time-domain. A general downside of the method is a direction and frequency dependent dispersion error. The dispersion error, which may be quantified as a frequency dependent phase velocity of the propagating waves, causes a strong group delay error that increases as a function of distance; the further the waves propagate, the more the components with less velocity get delayed. For compact explicit FDTD schemes, the relative phase velocity is unity at a temporal frequency of 0, and for the worst case direction it decreases monotonically towards the analytic cut-off frequency of each scheme, hence the group delay error increases towards the high frequencies.

The absorption of air is a similar phenomenon in this respect: it increases as a function of frequency and as a function of distance. Due to this similarity, it is likely that the air absorption masks the group delay error partially, or completely at a certain distance limit with a low-enough phase velocity error level. Even though several studies have examined the audibility of dispersion error in the context of room acoustic simulation,1–3 the perceptual threshold of the phase velocity error has not been addressed to the present authors’ knowledge. Such a threshold is highly useful due to the fact that the phase velocity error can be evaluated directly from the sampling frequency of the simulation.

In this study, the audibility of dispersion error in an audio signal is measured for different propagation distances with and without air absorption included. The threshold is measured as a function of the phase velocity error percentage. An adaptive psychophysical procedure is used for the measurements. The dispersion error represents the error of a close-cubic packed4 (CCP) scheme in the worst-case direction of error in free-field conditions. The CCP scheme is a non-staggered, compact explicit scheme that has been referred to as one of the most efficient schemes in its family.5 The decision of using a FDTD scheme that models the lossless wave equation is justified by the aim to not limit the results to a specific numerical viscous model. The attenuation of air is introduced the signal via filtering using an analytic model of air absorption in order to maintain generality of the result. The selection of a single scheme is justified with the result presented in Ref. 2, that with schemes with a similar phase velocity error contour, the audibility of the error as function of distance is similar. Therefore, in order to reduce the number of trials, only a single scheme condition is used.

2. Air absorption

The absorption of air is caused by two different effects: classical and relaxation losses. The classical effects are caused by the transport processes: internal friction, heat conduction, and molecular diffusion. The relaxation effects are related to the energy stored in the molecule’s internal degrees of freedom as a result of compression. Following
Bass et al.,\textsuperscript{6} the absorption of air that takes into account both classical and relaxation losses is given by

\[
\tilde{\alpha}(f) = f^2 \frac{p_s}{p_{s0}} \left\{ 1.84 \times 10^{-11} \left( \frac{T}{T_0} \right)^{1/2} + \left( \frac{T}{T_0} \right)^{-5/2} \right. \\
\left. \times \left[ 0.0178 \frac{e^{-2239.1/T}}{f_{rO} + f^2/f_{rO}} + 0.1068 \frac{e^{-3352/T}}{f_{rN} + f^2/f_{rN}} \right] \right\}, \tag{1}
\]

where \( f \) is frequency, \( p_{s0} \) is the reference value of the atmospheric pressure, \( p_s \) is the atmospheric pressure, \( T \) is atmospheric temperature in K, \( T_0 = 293.15 \) K is the reference atmospheric temperature, and \( f_{rO} \) and \( f_{rN} \) are scaled relaxation frequencies for oxygen and nitrogen, respectively. Both \( f_{rO} \) and \( f_{rN} \) are humidity dependent. Detailed derivation of the variables is presented in Ref. 6. In this work, it is assumed that \( p_{s0} = p_s = 101325 \text{ Pa} \). The result, \( \tilde{\alpha} \), has the unit nepers per m. The absorption of air per 100 m of propagation in different relative humidity values using Eq. (1) is illustrated in Fig. 1(a). Temperature and relative humidity are set to be 20°C, and 50% throughout this study, respectively.

The filter representing the air absorption is evaluated using

\[
H(f) = e^{-df(f)} \times e^{-j2\pi ft}, \tag{2}
\]

where \( d \) is the distance between the source and the receiver and \( t = 0 \). A finite impulse response (FIR) filter representing the absorption of air is achieved from Eq. (2) by evaluating it at 2\textsuperscript{17} uniformly spaced frequencies, and using an inverse Fourier transform. The length of the air absorption filter is chosen to be 128 taps, which is achieved by truncating the response and windowing it using a Hanning window. The magnitude responses of the evaluated filters for the distance conditions used in this work are presented in Fig. 1(b). The filter response deviates slightly from the ideal response due to truncation, but it is considered here so small that it can be neglected. Possible phase effects related to sound propagating through air are not taken into account as they are considered small with the chosen atmospheric, and distance conditions (less than 0.04% sound speed difference between zero frequency and 20 kHz).\textsuperscript{7,8}

3. Numerical dispersion

A simplified model for propagation of sound in free-field can be described with the wave equation

\[
\frac{\partial^2 p}{\partial t^2} = c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right), \tag{3}
\]

where \( p \) is the acoustical pressure and \( c \) the speed of sound (taken 344 m/s throughout this work). Equation (3) can be discretized by substituting the partial derivatives with...
finite differences. The general form for a family of compact explicit FDTD schemes following Kowalczyk et al. is given as
\[
\delta^2_{\mu} \theta^u = \lambda^2 \left[ (\delta^2_{\mu} + \delta^2_{\nu} + \delta^2_{\zeta}) + a(\delta^2_{\mu} \delta^2_{\nu} + \delta^2_{\nu} \delta^2_{\zeta} + \delta^2_{\mu} \delta^2_{\zeta}) + b(\delta^2_{\mu} \delta^2_{\nu} \delta^2_{\zeta}) \right],
\]
where \(\delta_{\mu}, \delta_{\nu}, \delta_{\zeta}\), and \(\delta_z\) are difference operators, \(a\) and \(b\) free variables specific for each scheme, and \(\lambda\) the Courant number determining the ratio of the temporal and spatial discretization. In the case of the studied CCP scheme, the variables \(a\) and \(b\) get the values \(\frac{1}{3}\) and 0, respectively. The Courant number is limited to its stability limit in free-field time stepping for the studied scheme (\(\lambda = 1\)). The dispersion error can be quantified using the numerical wavenumber as a function of temporal frequency. As presented by van Walstijn et al., the following expression for numerical wavenumber in the diagonal direction of the compact explicit FDTD schemes can be derived:
\[
\hat{k}_d = \frac{6}{\sqrt{3} \Delta x} \arcsin \left( \frac{\hat{s}_d}{\Delta x^2} \right),
\]
where \(\Delta x\) is the spatial discretization step size and \(\hat{s}_d\) an expression indicating the frequency dependence of the wavenumber. For the CCP scheme, \(\hat{s}_d = \frac{1}{2} (3 - \sqrt{9 - 12 \tan^2})\), where \(s_n = \sin^2 (\omega \Delta t / 2)\). From the numerical wavenumber \(k_d\) the dispersion waveform can be evaluated using a plane wave solution
\[
\hat{F} = e^{-j \omega t} e^{-jk_d(\omega)d}
\]
for distance \(d\) for this case in the diagonal direction of the CCP scheme. Such a waveform corresponds to a wave introduced by a planar hard source. A hard source is a valid option due to flat magnitude response. A planar source represents a single propagation direction, in this case the worst-case error direction. A more elaborate derivation of the dispersion filter may be found from Ref. 2.

4. Experimental setup

The dependent variable in the experiment is the threshold of the audibility of the dispersion error. The error level is adjusted by adjusting the phase velocity error percentage at the chosen bandwidth limit of 20 kHz. The independent variables are the distance between the source and the receiver in the simulation domain, and the inclusion or exclusion of air absorption. The experiment has a within-subjects repeated-measures design containing eight conditions that consist of the combinations of four distance conditions (10, 50, 100, 344 m), and two absorption conditions (air absorption included, air absorption excluded). The decision of limiting the distance condition to 344 m was made due to the rare occurrence of free-field propagation of larger or equal distance in room acoustic context. This distance corresponds to 1 s of propagation with the assumptions used in this study. It is assumed here that other effects than air absorption will affect the audibility of dispersion error in room acoustic context in the majority of cases, and therefore the limitation is acceptable. The hypothesis for the experiment is that for some low-enough phase velocity error percentage, the air absorption will mask the dispersion error introduced by the studied FDTD scheme at all distances. In other words, the total attenuation at high frequencies will mask the total group delay error that results from the phase velocity error.

4.1 Stimuli

The sound sample used in the test is a click-like signal that consists of a single 21 μs impulse (a single value of 1 followed by zeros at a sampling frequency of 48 kHz). The stimulus in each trial is the sound sample processed with the dispersion filter. The dispersion filter is evaluated according to the error level of each trial; the sampling frequency is set so that the normalized frequency at which the phase velocity error level of the given trial occurs correspond to 20 kHz, the waveform is evaluated accordingly using Eq. (6), and then re-sampled to 48 kHz. In the trials where the air absorption is included, both the reference and the stimulus are filtered with the air absorption filter corresponding to the distance condition of the trial.

4.2 Subjects

Seven test subjects including the present author participated in the experiment. All of the subjects worked in the field of acoustics, and had previously participated in listening tests.
4.3 Procedure

An adaptive staircase test using the QUEST method\(^\text{11}\) was used with three alternative forced choice (3AFC) procedure. The probability level of \(P = 0.82\) for correct discrimination was measured. A similar procedure to that in Ref. \(\text{2}\) is used; three test samples, consisting of two reference samples and one stimulus sample, were presented in random order with 1000 ms pause between each sample. The subject was then asked to specify which of the presented samples contained an audible dispersion error.

Before the actual experiments, each subject attended a training session. The training session consisted of ten 3AFC trials starting from an easily noticeable error level and progressing towards a low error level. After each trial the participant was given feedback whether the choice was correct or not.

The two experiment sessions consisted of four interleaved staircase routines per session. Each staircase routine corresponded to one condition \(\text{distance, air absorption included/excluded}\). The order of the conditions in the two sessions was randomized. Subsequent discrimination tasks were randomly picked from the staircase routines selected for the session. The number of trials in each staircase routine was limited to 30. Feedback for correct discrimination was not given to the subjects in the experiment sessions. An “easy” discrimination task was assigned after every 15th trial in order to aid the attention of the subject.

All of the experiments were completed using Sennheiser HD 650 headphones connected to a Motu UltraLite-mk3 Hybrid audio interface on a laptop computer in an isolated room specifically designed for psychoacoustic experiments. The level of the reference samples was calibrated using a B&K 4153 artificial ear connected to a B&K 2250 sound level meter. The levels of the reference sample was set to \(LC_{\text{peak}} = 90\) dB. A peak measure was used because the reference sample contains most of its energy in a short transient. The signals were played back diotically. An open source library for psychophysical experiments was used to implement the test.\(^\text{12}\)

5. Experiment

5.1 Results

The descriptive statistics of the measured error percentage thresholds are presented in Table 1, and visualized by group in Fig. 2(a).

A Shapiro-Wilk test for normality was performed for each group of observations. It is found that for the group \(50\) m \(\text{air absorption excluded}\), the null hypothesis of the normal distribution of the observations should be rejected.

A paired t-test between the \(\text{air absorption excluded}\) and \(\text{air absorption included}\) condition was carried out independently for each distance condition. As no significant evidence on the normality of the observations in all of the groups was found, a non-parametric Wilcoxon signed-rank test was similarly done between the absorption conditions independently for each distance group. Both tests show that with the distance condition of \(344\) m, the absorption conditions effect on the audibility of the dispersion error is highly significant \([t\text{-test: } t(6) = 6.635, P = 5.654 e^{-4}. \text{ Wilcoxon: } V = 28, P = 0.01563]\).

The corresponding maximum group delay error values for each measured threshold of phase velocity error percentage are presented in Fig. 2(b). A Friedman rank-sum test was performed for the group \(\text{air absorption excluded}\) with the distance as a factor, and the participant as a blocking variable to test the similarity of the observed maximum group delay error values. The null hypothesis that the observations in each distance category with the air absorption excluded originate from the same

<table>
<thead>
<tr>
<th>Dist. (m)</th>
<th>Absorption</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Max</th>
<th>Mean Gd. err. (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Excluded</td>
<td>2.42</td>
<td>1.3</td>
<td>1.52</td>
<td>5.23</td>
<td>2.68</td>
</tr>
<tr>
<td>50</td>
<td>Excluded</td>
<td>0.51</td>
<td>0.21</td>
<td>0.33</td>
<td>0.95</td>
<td>2.34</td>
</tr>
<tr>
<td>100</td>
<td>Excluded</td>
<td>0.24</td>
<td>0.12</td>
<td>0.13</td>
<td>0.48</td>
<td>2.11</td>
</tr>
<tr>
<td>344</td>
<td>Excluded</td>
<td>0.06</td>
<td>0.04</td>
<td>0.02</td>
<td>0.11</td>
<td>1.70</td>
</tr>
<tr>
<td>10</td>
<td>Included</td>
<td>2.45</td>
<td>1.25</td>
<td>1.42</td>
<td>4.89</td>
<td>2.70</td>
</tr>
<tr>
<td>50</td>
<td>Included</td>
<td>0.46</td>
<td>0.19</td>
<td>0.34</td>
<td>0.84</td>
<td>2.10</td>
</tr>
<tr>
<td>100</td>
<td>Included</td>
<td>0.28</td>
<td>0.14</td>
<td>0.16</td>
<td>0.55</td>
<td>2.53</td>
</tr>
<tr>
<td>344</td>
<td>Included</td>
<td>0.29</td>
<td>0.11</td>
<td>0.19</td>
<td>0.45</td>
<td>8.81</td>
</tr>
</tbody>
</table>

\(Jukka Saarelma and Lauri Savioja: JASA Express Letters [http://dx.doi.org/10.1121/1.4972529] Published Online 23 December 2016\)
distribution when the effect of the participant is taken into account, should not be rejected \( \chi^2(3) = 4.886, P = 0.1804 \).

5.2 Discussion

The main result that may be deduced from the observations is that the air absorption in the given conditions does mask the dispersion error with a low-enough phase velocity error level. None of the threshold measurements converged to a lower phase velocity error level than 0.16\% with the condition *air absorption included*, the lowest mean threshold being 0.28\%. With the condition *air absorption excluded*, the phase velocity error percentage threshold was significantly lower in the distance group 344 m. The decision of using a linear phase filter to represent the air absorption may have a small effect on the measured thresholds within the condition *air absorption included*, but the main effect that may be observed between the conditions *air absorption included* and *air absorption excluded* is clear.

The maximum group delay values for each distance condition in the group *air absorption excluded* are relatively similar. There is no significant difference between the distributions of the observations in the different distance groups. The maximum group delay values agree with the findings of the previous threshold study,\(^2\) where the mean value of the measured maximum group delay value was found to be 1.8 ms.

The measured thresholds for all conditions in this study correspond to free-field propagation in the worst case direction of dispersion error. Therefore, the audibility of the dispersion error in full room responses may not be exhaustively concluded; more error may be tolerated in the presence of, for example, surface reflections. The measured threshold may be used as a lower bound of phase velocity error in the presented scheme, and possibly in several other compact explicit schemes based on the evidence of Ref. 2.

6. Conclusions

The audibility of dispersion error was measured as a function of phase velocity error level at 20 kHz in four different distance conditions with air absorption included, and excluded to the test signals. It was found that for a phase velocity error level of 0.28\% or lower, the dispersion error becomes inaudible when the air absorption is included. This phase velocity error limit results in a sampling frequency of 283 350 Hz in the studied CCP scheme. The result suggests that at this phase velocity error limit, and with the given conditions (temperature = 20 \( ^\circ \)C, relative humidity = 50\%, pressure = 101 325 Pa) FDTD simulation may be used in auralization applications, and the dispersion error due free-field propagation will go un-noticed.

Acknowledgments

Jonathan Botts, Sebastian Prepelita, and Pierre Chobeau are thanked for commenting the manuscript. This work has received funding from Academy of Finland, Project No. 265824.
References and links


