



This is an electronic reprint of the original article. This reprint may differ from the original in pagination and typographic detail.

Semchenko, Igor V.; Khakhomov, Sergei A.; Asadchy, V. S.; Golod, S. V.; Naumova, E. V.; Prinz, V. Ya; Goncharenko, A. M.; Sinitsyn, G. V.; Lyakhnovich, A. V.; Malevich, V. L. Investigation of electromagnetic properties of a high absorptive, weakly reflective metamaterial - Substrate system with compensated chirality

Published in: Journal of Applied Physics

DOI: 10.1063/1.4973679

Published: 07/01/2017

Document Version Publisher's PDF, also known as Version of record

Please cite the original version:

Semchenko, I. V., Khakhomov, S. A., Asadchy, V. S., Golod, S. V., Naumova, E. V., Prinz, V. Y., Goncharenko, A. M., Sinitsyn, G. V., Lyakhnovich, A. V., & Malevich, V. L. (2017). Investigation of electromagnetic properties of a high absorptive, weakly reflective metamaterial - Substrate system with compensated chirality. *Journal of Applied Physics*, *121*(1), Article 015108. https://doi.org/10.1063/1.4973679

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

Investigation of electromagnetic properties of a high absorptive, weakly reflective metamaterial—substrate system with compensated chirality

I. V. Semchenko, S. A. Khakhomov, V. S. Asadchy, S. V. Golod, E. V. Naumova, V. Ya. Prinz, A. M. Goncharenko, G. V. Sinitsyn, A. V. Lyakhnovich, and V. L. Malevich

Citation: Journal of Applied Physics **121**, 015108 (2017); doi: 10.1063/1.4973679 View online: https://doi.org/10.1063/1.4973679 View Table of Contents: http://aip.scitation.org/toc/jap/121/1 Published by the American Institute of Physics

Articles you may be interested in

Asymmetric transmission of linearly polarized waves in terahertz chiral metamaterials Journal of Applied Physics **121**, 033103 (2017); 10.1063/1.4974477

Tunable quasi-monochromatic near-field radiative heat transfer in s and p polarizations by a hyperbolic metamaterial layer Journal of Applied Physics **121**, 013106 (2017); 10.1063/1.4973530

An analysis of beamed wireless power transfer in the Fresnel zone using a dynamic, metasurface aperture Journal of Applied Physics **121**, 014901 (2017); 10.1063/1.4973345

Ultrathin lightweight plate-type acoustic metamaterials with positive lumped coupling resonant Journal of Applied Physics **121**, 015102 (2017); 10.1063/1.4972839

Sensing with toroidal metamaterial Applied Physics Letters **110**, 121108 (2017); 10.1063/1.4978672

Low-frequency band gap and defect state characteristics in a multi-stub phononic crystal plate with slit structure Journal of Applied Physics **121**, 015106 (2017); 10.1063/1.4973577





Investigation of electromagnetic properties of a high absorptive, weakly reflective metamaterial—substrate system with compensated chirality

I. V. Semchenko,^{1,a)} S. A. Khakhomov,¹ V. S. Asadchy,^{1,2} S. V. Golod,³ E. V. Naumova,³ V. Ya. Prinz,³ A. M. Goncharenko,⁴ G. V. Sinitsyn,⁴ A. V. Lyakhnovich,⁴ and V. L. Malevich⁴ ¹Francisk Skorina Gomel State University, 104, Sovetskaya Str., Gomel 246019, Belarus ²Aalto University, Aalto FI-00076, Finland

 ³Rzhanov Institute of Semiconductor Physics, Siberian Branch of Russian Academy of Science, 13, Acad. Lavrent' ev Avenue, 630090 Novosibirsk, Russia
 ⁴Stepanov Institute of Physics, National Academy of Science, Belarus, 68, Nezavisimosti Avenue, 220072 Minsk, Belarus

(Received 25 September 2016; accepted 22 December 2016; published online 5 January 2017)

In the present paper, a theoretical and experimental study of a highly absorptive, weakly reflective coating designed and fabricated on the basis of 3D THz resonant elements is reported. Transmission and reflection of electromagnetic waves from the metamaterial-substrate structure involving a highly absorptive, weakly reflective array of artificial bi-anisotropic elements were analyzed. The samples contained paired right-handed and left-handed helices, due to the fact that the chirality was compensated. The parameters of helices were optimized to achieve roughly identical values of dielectric permittivity and magnetic permeability. As a result, the metamaterial exhibited weak reflectivity in the vicinity of resonance frequency. On the other hand, effective resonance properties of the helices were tuned to ensure substantial absorption of THz radiation. Analytical expressions for the coefficients of radiation reflection and transmission in the samples were derived by solving a boundary-value problem for the propagation of electromagnetic waves in the metamaterial-substrate system. Simulated properties of fabricated structures were compared with experimental data. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4973679]

I. INTRODUCTION

Artificial composite media with chiral properties in the microwave range have been extensively studied during the past several decades.^{1–13} Chirality is an asymmetry property. An object is chiral if it cannot be made coincident with its mirror-reflected image; examples here are given by righthanded and left-handed helices. Chirality may be a property pertaining not only to natural but, also, to artificial objects such as metamaterials. Metamaterials are artificial materials designed so that to make their electromagnetic properties different from those of natural materials. The properties of metamaterials are normally defined by their structure rather than composition, with effective macroscopic effects being achieved using a non-uniform or discrete structure of the medium.¹⁴ Metamaterials are sometimes defined as periodic arrays of artificial objects with a pitch value smaller than excitation wavelength.¹⁵ Because of the sub-wave periodicity of metamaterials, electromagnetic waves do not diffract in them. That is why, for incident waves a metamaterial presents a homogeneous medium, and it can be given description in terms of effective or space-average characteristics defined by the geometry of unit cells and composite elements in the material structure.

Rapid development of metamaterials in the past few decades was greatly stimulated by study of negative refraction (see theoretical^{16,17} and experimental^{18–21} papers, review on history of the problem,²² and earlier relevant Refs. 22–36) and such fascinating applications of metamaterials with negative index of refraction (NIMs) as perfect lenses³⁷ and an invisibility cloaking.^{38–40} The chiral route to negative refraction^{41–43} opened a novel way to all applications of NIMs and made development of fabrication of metamaterials with chiral resonators very important.

Besides, chiral NIM researchers keep investigating into the possibility of using chiral media for creation of weakly reflective screens and coatings based on single- or multi-layer chiral structures and absorptive coatings^{44,45} based on uniform chiral metal structures. Presently, in connection with the extensive development of THz devices, there is a tendency toward creation and investigation of metamaterials for this spectral range. Terahertz radiation (100 GHz–10 THz) attracts interest because of its unique potential applications in nondestructive imaging, spectroscopy, and computed tomography for industrial or security applications, molecular biological science, astronomy, and future communication systems.^{46–49} Absorptive THz metamaterials are extremely important for creating polarizers,⁵⁰ modulators,^{51,52} and filters,⁵³ antireflection coatings in radiometry,⁵⁴ and imaging.^{55,56}

To remain smaller than radiation wavelengths, in THz metamaterials, artificial elements-resonators must have typical sizes ranging from several micrometers to several ten micrometers. For producing a coherent response, all resonators in a large array must be tuned with a high accuracy. Among the widely used technologies, only traditional planar technology, which allows formation of flat elements and layers, is capable of ensuring required dimensions and accuracy. Properties of

^{a)}Author to whom correspondence should be addressed. Electronic mail: isemchenko@gsu.by

multilayer metamaterials made of flat elements cannot be freely tailored in all three dimensions. Moreover, because of limitations inherent to planar technology, in most cases experimental opportunities available to researchers are restricted just to one element layer (that is, to one metamaterial mono-layer); this circumstance seriously hampers the study of bulk electromagnetic properties of materials. Simultaneously, all metamaterial applications of interest require bulk samples with specified 3D electromagnetic properties.^{57,58}

Metamaterials made from 3D shells rolled-up from strained nanofilms^{59–63} allow one to predefine the electromagnetic response of a metamaterial in all three dimensions, which enables creation of metamaterials with fundamentally new properties. The formation principle of shells rolled up from strained films is illustrated in Fig. 1. Nowadays, it is the only one nanotechnology allowing mass fabrication of THz metamaterials based on smooth resonant 3D helices.⁶⁴ Helices are promising elements for metamaterials since both electrical and magnetic dipole moments can be simultaneously generated in them under the action of external electric and magnetic fields. In this manner, the helix combines both the properties of the rod and those of the split ring; yet, in contrast to those traditional metamaterial elements, due to the difference from its mirror image, the helix possesses magnetoelectric or chiral properties. If chirality is not required in a metamaterial, the metamaterial can be made compensated by using identical amounts of right-handed and left-handed helices.

II. OPTIMAL SHAPE OF HELICAL ELEMENTS: IDENTICAL VALUES OF DIELECTRIC, MAGNETIC, AND CHIRAL POLARIZABILITIES

Previously, analytical relations between the dielectric, magnetic, and chiral (magneto-electrical) polarizabilities of small metal helices were derived. It was shown that there exists an "optimal" relation between the helix radius and the helix pitch such that, at a certain frequency, all three polarizabilities will become identical in magnitude (this relation was introduced in Refs. 9, 65, and 66 for helices used as polarization transformers). A helix is investigated under the condition of principal resonance if the length of the straightened helix is equal approximately to half the wavelength of



FIG. 1. Schematic of the rolling-up process of a bi-layer strained film. The film is detached from the substrate by selective etching of a sacrificial layer. The freed film rolls up as a coil under the force moment M caused by the internal stresses in the stretched and compressed layers. The fastening element of negative photoresist keeps the free-standing shell on the substrate after the etching out of the underlying sacrificial layer.

the electromagnetic field. Under such conditions, the shape of the helix is uniquely defined by the pitch angle of the helix. Earlier, optimal helix pitch angles were calculated as a function of the number of helix coils.

III. ARRANGEMENT OF HELICES IN AN ARRAY

An analysis of necessary mutual arrangement and orientation of paired helices providing for compensated chirality was performed. A sample with weak reflective properties, formed by single-coil helices with a pitch angle of 13.5°, must contain identical amounts of right-handed and left-handed helices, which must be arranged in the plane of the sample both vertically and horizontally (Fig. 2). Simultaneously, the amounts of vertically and horizontally arranged helices must be identical, making the electromagnetic properties of the sample isotropic in the sample plane.

The ANSYS HFSS software was used to simulate the properties of 2D arrays formed by paired helices. Such arrays exhibit equally significant dielectric and magnetic properties defined by the optimal shape of the helices. Simultaneously, the chiral properties of artificial structures are made compensated, because, here, pairs of optimal right-handed and lefthanded helices are used. As a result, in the THz range the created metamaterial possesses a wave impedance close in magnitude to the wave impedance of free space.

IV. SOLUTION OF THE BOUNDARY-VALUE PROBLEM AND CALCULATION OF THE COEFFICIENTS OF TRANSMISSION AND REFLECTION OF ELECTROMAGNETIC WAVES FOR A HIGHLY ABSORPTIVE, WEAKLY REFLECTIVE 2D ARRAY OF HELICES ON A SUBSTRATE

We take into account the fact that, for optimal helices, the following universal relation holds: $^{67-69}$

$$m_x = -\frac{j\omega r^2 q}{2} p_x. \tag{1}$$

Here, the coordinate x is directed along the helix axis, r is the helix radius, and $|q| = 2\pi/h$ (h is the helix pitch), ω is the electric-current cyclic frequency, and j is the imaginary unit. The sign of the helix specific torsion q is defined by the handedness of helix: for a right-handed helix, we have q > 0. Relation (1) is a universal one since it holds for any distribution of the induced electric current. The latter is a fact of extreme significance for metamaterials, where, unlike in photonic crystals, the volumetric density of elements is high.



FIG. 2. Geometry of the layer with helical inclusions (R and L are right-handed and left-handed helices, respectively).

The interaction between elements can be rather strong, and the electric current in a helix can undergo variations under the action of neighbor helices. However, here again relation (1) is not violated, that is, it remains valid also for metamaterials with the highest possible element packing density.

With the aim of solving the boundary-value problem about propagation of an electromagnetic wave in the metamaterial-substrate system, consider a sample made up by many paired helices with optimal parameters. We introduce the following notations (see Fig. 3):

 $\mathbf{E}^{i} = E_{0}^{i} \mathbf{x}_{0} e^{j\frac{\partial c}{c}z - j\omega t}$ is the incident wave in free space, $\mathbf{E}^{r} = E_{0}^{r} \mathbf{x}_{0} e^{-j\frac{\partial c}{c}z - j\omega t}$ is the reflected wave, and $\mathbf{E}^{h} = E_{01}^{+}$ $\mathbf{x}_{0} e^{jk_{1}z - j\omega t} + E_{01}^{-} \mathbf{x}_{0} e^{-jk_{1}z - j\omega t}$ is the wave in the structure made up by the helices.

The structure possesses some dielectric and magnetic properties yet no chiral ones since, here, identical amounts of paired right-handed and left-handed helices are used to diminish the chirality parameter (Fig. 2). The densities of helices oriented in the vertical and horizontal directions in the plane of sample are identical. That is why the metamaterial is isotropic in the plane XOY and, while solving the boundary-layer problem, we may restrict our consideration to the waves polarized along the X axis only. Also, for the waves in the substrate we have $\mathbf{E}^s = E_{02}^+ \mathbf{x}_0 e^{jk_2 z - j\omega t} + E_{02}^- \mathbf{x}_0 e^{-jk_2 z - j\omega t}$. In addition, $\mathbf{E}^{\tau} = E_0^{\tau} \mathbf{x}_0 e^{jk_2 z - j\omega t}$ is the wave in air having passed the whole sample (the helix layer plus the substrate).

The condition of continuity for the electric-field vector at Z = 0 is

$$E_0^i + E_0^r = E_{01}^+ + E_{01}^-.$$
⁽²⁾

A similar condition at $Z = L_h = L_1$ (designations L_1 , L_2 , $L_{h(elix)}$, and $L_{s(ubstrate)}$ are clarified by Fig. 3) is

$$E_{01}^{+}e^{jk_{1}L_{1}} + E_{01}^{-}e^{-jk_{1}L_{1}} = E_{02}^{+}e^{jk_{2}L_{1}} + E_{02}^{-}e^{-jk_{2}L_{1}}.$$
 (3)

The condition at $Z = L_s + L_h = L_2$ is

$$E_{02}^{+}e^{jk_{2}L_{2}} + E_{02}^{-}e^{-jk_{2}L_{2}} = E_{0}^{\tau}e^{j\frac{\omega}{c}L_{2}}.$$
(4)

We calculate the magnetic field intensity in the case of a plane wave. The magnetic field in the medium is $\mathbf{H} = \frac{k}{\omega\mu_0\mu}$ [$\mathbf{z}_0\mathbf{E}$], the magnetic fields of the incident and transmitted waves are $\mathbf{H} = \frac{1}{c\mu_0}[\mathbf{z}_0\mathbf{E}]$, and the magnetic field of the



FIG. 3. Geometry of the boundary-value problem.

reflected wave is $\mathbf{H} = -\frac{1}{c\mu_0} [\mathbf{z}_0 \mathbf{E}]$. Hence, for the incident wave we have $\mathbf{H}^i = \frac{1}{c\mu_0} E_0^i \mathbf{y}_0 e^{j\frac{\omega}{c}z - j\omega t}$, and for the reflected wave, $\mathbf{H}^r = -\frac{1}{c\mu_0} E_0^r \mathbf{y}_0 e^{-j\frac{\omega}{c}z - j\omega t}$. The wavenumber in the helix layer is $k_1 = \frac{\omega}{c} \sqrt{\varepsilon_1 \mu_1}$. As a result, we obtain: $\mathbf{H}^{(1)} = \frac{1}{c\mu_0} \sqrt{\frac{\varepsilon_1}{\mu_1}} E_{01}^{-1} \mathbf{y}_0 e^{jk_1 z - j\omega t} - \frac{1}{c\mu_0} \sqrt{\frac{\varepsilon_1}{\mu_1}} E_{01}^{-1} \mathbf{y}_0 e^{-jk_1 z - j\omega t}$ in the wave propagating through the helix layer, $\mathbf{H}^{(2)} = \frac{1}{c\mu_0} \sqrt{\frac{\varepsilon_2}{\mu_2}} E_{02}^{-1} \mathbf{y}_0 e^{-jk_2 z - j\omega t}$ in the wave propagating in the substrate, and $\mathbf{H}^{\tau} = \frac{1}{c\mu_0} E_0^{\tau} \mathbf{y}_0 e^{j\frac{\omega}{c}z - j\omega t}$ in the transmitted wave.

The condition of continuity for the magnetic-field vector at Z = 0 is

$$E_0^i - E_0^r = \sqrt{\frac{\varepsilon_1}{\mu_1}} \left(E_{01}^+ - E_{01}^- \right).$$
 (5)

A similar condition at $Z = L_1$ is

$$\sqrt{\frac{\varepsilon_1}{\mu_1}} \left(E_{01}^+ e^{jk_1L_1} - E_{01}^- e^{-jk_1L_1} \right) = \sqrt{\frac{\varepsilon_2}{\mu_2}} \left(E_{02}^+ e^{jk_2L_1} - E_{02}^- e^{-jk_2L_1} \right).$$
(6)

A condition at $Z = L_2$ is

$$\sqrt{\frac{\varepsilon_2}{\mu_2}} \left(E_{02}^+ e^{jk_2L_2} - E_{02}^- e^{-jk_2L_2} \right) = E_0^\tau e^{j\frac{\omega}{c}L_2}.$$
 (7)

By solving the boundary-value problem represented by systems (2)–(7), we derive the following expression for the ratio between the amplitudes of the reflected and incident waves:

$$\frac{E_0^r}{E_0^i} = \frac{r_{01} + r_{12}e^{2jk_1L_1} + r_{20}e^{2jk_2(L_2 - L_1)}(r_{01}r_{12} + e^{2jk_1L_1})}{1 + r_{01}r_{12}e^{2jk_1L_1} + r_{20}e^{2jk_2(L_2 - L_1)}(r_{12} + r_{01}e^{2jk_1L_1})}.$$
 (8)

In expression (8), the following designations are used:

 $\eta = \sqrt{\frac{\mu}{\epsilon}}$ is the impedance; $r_{01} = \frac{\eta_1 - 1}{\eta_1 + 1}$ is the coefficient of reflection from the air-metamaterial layer interface; $r_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ is the coefficient of reflection from the metamaterial layer-substrate interface; $r_{20} = \frac{1 - \eta_2}{1 + \eta_2}$ is the coefficient of reflection from the substrate-air interface; $t_{01} = \frac{2\eta_1}{\eta_1 + 1}$ is the transmission coefficient for the air-metamaterial interface; $t_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}$ is the transmission coefficient for the air-metamaterial interface; $t_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}$ is the transmission coefficient for the metamaterial interface; to the metamaterial interface; $t_{20} = \frac{2}{\eta_2 + 1}$ is the transmission coefficient for the metamaterial interface; to the substrate interface; to the transmission coefficient for the metamaterial interface; to the substrate interface; to the transmission coefficient for the substrate interface.

In some cases, the contribution due to the wave reflected from the substrate-air interface can be neglected. That is the case under the following conditions:

- 1. the substrate impedance is equal to the impedance of air, that is, $\eta_2 = 1$ or $r_{20} = 0$;
- in the substrate, the wave undergoes absorption and, on condition that the substrate is sufficiently thick, we have L₂≫ L₁ and e^{2jk₂(L₂-L₁)} → 0;
- 3. provided that measurements are conducted in pulsed mode and the substrate is sufficiently thick, then the

second reflected pulse follows at a substantial delay with respect to the primary pulse reflected both from the metamaterial and from the first substrate boundary. In our experiment, this additional reflected pulse can be easily separated from the primary pulse.

In all three cases, expression (8) yields the formula

$$R = \frac{E_0^r}{E_0^i} = \frac{r_{01} + r_{12}e^{2jk_1L_1}}{1 + r_{01}r_{12}e^{2jk_1L_1}},$$
(9)

with R being the amplitude coefficient of reflection from the whole metamaterial-substrate system. On introduction of the refraction index and absorption coefficient of the metamaterial, expression (9) transforms into

$$R = \frac{E_0^r}{E_0^i} = \frac{r_{01} + r_{12}e^{\left(2j\frac{\omega}{c}n_1' - a\right)L_1}}{1 + r_{01}r_{12}e^{\left(2j\frac{\omega}{c}n_1' - a\right)L_1}}.$$
 (10)

Here, n'_1 is the refraction index of the metamaterial, $2k''_1 = a$ is the absorption coefficient of the metamaterial, and k''_1 is the imaginary part of the wavenumber for the metamaterial.

Similarly, we introduce the designation $\frac{E_0^*}{E_0^*} = T$ for the amplitude transmission coefficient of the whole sample. Systems (2)–(7) yield the following expression for the amplitude transmission coefficient:

$$T = \frac{t_{01}t_{12}t_{20}e^{-j\frac{\omega}{c}L_2 + jk_1L_1 + jk_2(L_2 - L_1)}}{1 + r_{01}r_{12}e^{2jk_1L_1} + r_{20}e^{2jk_2(L_2 - L_1)}(r_{12} + r_{01}e^{2jk_1L_1})}.$$
(11)

The metamaterial possesses weak reflecting properties because at some frequency ω_0 its wave impedance is equal to the wave impedance of free space and $r_{01} = 0$ at this frequency. Then, we have $t_{01} = 1$, and formula (11) simplifies to

$$T(\omega_0) = \frac{t_{12}t_{20}e^{-j\frac{\omega}{c}L_2 + jk_1L_1 + jk_2(L_2 - L_1)}}{1 + r_{12}r_{20}e^{2jk_2(L_2 - L_1)}}.$$
 (12)

In the THz range, the substrate is transparent to radiation and, then, $k_2 = k_2^*$ is a real number. The quantities t_{20} and t_{12} have real values as well since, at the frequency ω_0 , we have: $\eta_1 = 1$.

Now, we find the power transmission coefficient at frequency ω_0

$$|T(\omega_0)|^2 = \frac{t_{12}^2 t_{20}^2 e^{-2\frac{\omega_0}{c}} c_1'' L_1}{1 + 2r_{12}r_{20}\cos(2k_2(L_2 - L_1)) + r_{12}^2 r_{20}^2}.$$
 (13)

At frequency ω_0 , we have: $\varepsilon_1 = \mu_1$, so that at this frequency the metamaterial has identically significant dielectric and magnetic properties. Hence

$$k_1 = \frac{\omega_0}{c} \sqrt{\varepsilon_1 \mu_1} = \frac{\omega_0}{c} \varepsilon_1$$
 and $k_1'' = \frac{\omega_0}{c} \varepsilon_1''$. (14)

Formulas (14) show that, for such a metamaterial, the wavenumber depends on dielectric permittivity linearly and not quadratically, whose dependence is known from the electrodynamics of ordinary dielectric media. Hence, on condition that $\varepsilon'_1 > 1$, the metamaterial has a greater refractive index than ordinary dielectrics. Simultaneously, in the case of $\varepsilon''_1 < 1$ the absorption coefficient of such a metamaterial has a lower value in comparison with ordinary absorptive dielectrics.

For the metamaterial without a substrate, we have $r_{12} = 0$ and $r_{20} = 0$; then

$$\left|T_0(\omega_0)\right|^2 = e^{-2k_1''L_1} = e^{-2\frac{\omega_0}{c}\varepsilon_1''L_1}.$$
(15)

By using formula (13) and measuring the dependence $T(\omega_0)$, given the values of substrate parameters, we can calculate the imaginary part of the dielectric permittivity of the metamaterial $\varepsilon_1''(\omega_0)$, because at this frequency the impedance of the metamaterial is unity. Since at frequency ω_0 , the metamaterial is matched to free space, then

$$\eta_1 = 1$$
 $r_{12} = \frac{\eta_2 - 1}{\eta_2 + 1}$ $r_{20} = \frac{1 - \eta_2}{1 + \eta_2}$

Hence, at frequency ω_0 we have $r_{12} = -r_{20}$, and now we obtain

$$|T(\omega_0)|^2 = \frac{\left(1 - r_{20}^2\right)^2 e^{-\frac{2q_0}{c}} \varepsilon_1'' L_1}{1 - r_{20}^2 \cos(2k_2(L_2 - L_1)) + r_{20}^4}.$$
 (16)

The latter coefficient has a lower value, in comparison with the case in which waves propagate across the bare substrate, due to the exponential term in the nominator, which takes into account the propagation of waves in the metamaterial.

Consider formula (9), which is applicable in pulsed mode at frequency ω_0 . Then, we have $r_{01} = 0$ and $r_{12} = -r_{20}$ and find the reflection coefficient in terms of the wave power

$$R(\omega_0)|^2 = r_{20}^2 e^{-4\frac{\omega_0}{c}\varepsilon_1''L_1}, \text{ since } k_1 = \frac{\omega_0}{c}\sqrt{\varepsilon_1\mu_1} = \frac{\omega_0}{c}\varepsilon_1.$$
(17)

This coefficient is smaller in value than the coefficient of reflection just from the substrate because the wave undergoes absorption in the metamaterial.

From an experiment performed individually for the bare substrate (without the metamaterial), we can determine the coefficients of reflection and transmission of the substrate for THz waves. Also, those coefficients can be calculated by solving the boundary-value problem on the assumption of $L_1 = 0$. Then, for the bare substrate we have

$$|R_s|^2 = r_{20}^2; \quad |T_s|^2 = \frac{(1 - r_{20})^2}{1 - r_{20}^2 2\cos(2k_2L_s) + r_{20}^4}.$$
 (18)

In (18), $L_2 - L_1 = L_s$ is the thickness of the substrate.

For making a comparison between experimental and calculated data, we introduce the reflection and transmission coefficients

$$R_n(\omega) = |R(\omega)|/|R_s(\omega)|$$
 and $T_n(\omega) = |T(\omega)|/|T_s(\omega)|,$
(19)

normalized by the reflection and transmission coefficients of the bare substrate in terms of wave amplitude. Then, on the condition that no reflection of electromagnetic waves occurs at the air-metamaterial interface, for the resonance frequency we obtain the following simple relation:

$$R_n(\omega_0) = T_n^2(\omega_0). \tag{20}$$

Formula (20) arises as a consequence of the fact that the waves undergo reflection at the metamaterial-substrate interface only. As a result, in pulsed mode the reflected wave traverses the absorptive layer with helices two times whereas the transmitted wave, just one time.

The proposed metamaterial, which is based on the array of paired helices, reveals equally strong dielectric and magnetic properties originating from the optimal shape of the helices. The chiral properties of the artificial structure are compensated due to the use of the paired optimal left-handed helices. As a result, the proposed metamaterial has the wave impedance that is close to the free space impedance. In the vicinity of the resonant frequency, the sample shows weak reflecting properties. At the same time, although the substrate is transparent in the frequency range, substantial absorption of waves near the resonant frequency takes place. The absorptive properties of the metamaterial result from resonant excitation of currents in well-conducting (goldplated) helices.

V. COMPARISON OF EXPERIMENTAL WITH SIMULATED DATA

Experimental implementation of the metamaterials based on THz helices has become possible with the advent of the precise 3D nanostructuring technique based on the rolling-up of strained nanofilms.^{59–63} Samples in the form of 2D arrays of right-handed and left-handed helices were fabricated from strained metal–semiconductor films and experimentally studied. The pairs of helices were aligned horizontally and vertically in the plane of the sample

(Fig. 4). Fig. 4(a) presents a schematic of the initial plane element in the form of V-like InGaAs/GaAs/Ti/Au-strips united by InGaAs/GaAs film crosspieces and stiffeners made of negative photoresist. To form an array of such elements, we exposed the epitaxial AlAs/InGaAs/GaAs film on a GaAs substrate to the three photolithographic operations, etching and metal deposition. The semiconductor film mesastructure (InGaAs/GaAs) was defined by the first lithography, the pattern of V-like metal strips (Ti/Au) was formed by the second lithography, and the third lithography was used to make fastening elements and stiffeners from negative photoresist. Being detached from the substrate (Fig. 1), such a V-like plane element rolled up as InGaAs/GaAs/Ti/Au left- and right-handed helices (Fig. 4(b)) under action of internal strains between layers, the upper and bottom ends of V roll up towards each other in opposite directions as schematically shown in Fig. 4(a). Additional semiconductor crosspieces and stiffeners prevent the film from the rolling-up in undesirable directions. The central fastening elements provide the rigid fixation and positioning of the 3D nanofilm elements on the substrate (Fig. 4(b)). The rolling-up method allows formation of regular arrays of 3D nanofilm elements of a large area (Fig. 4(c)). The arrays of paired helices with the surface density of helices $N_h = 6.8 \times 10^4 \text{ cm}^{-2}$ were formed over areas of several square centimeters. The implemented helix pitch angle value, 13.5°, was optimal for reaching identical values of the dielectric and magnetic polarizabilities of the helices (see Refs. 9 and 65). Transmission and reflection spectra of fabricated metamaterial samples were studied by the time-domain spectroscopy (TDS) method. For performing the latter study, we have developed a THz laser-optical setup in which, as a source of radiation, a KYW:Yb femtosecond laser was used. The laser emitted radiation at wavelength 1030 nm. An optical diagram of the setup is shown in Fig. 5.

In the THz-radiation emitter and detectors, the photoconducting antennas were excited by laser pulses (pulse width ~ 100 fs) at 15-mW mean radiation power. Optimal



FIG. 4. (a) Schematic of the initial plane element made of strained multilayer nanofilm. Dark-yellow V-like strips with length of 77 μ m and width of 3 μ m are prepared from the In_{0.2}Ga_{0.8}As/GaAs/GiAs/Ti/Au (16/40/4/40 nm) film. The orange crosspieces between the strips are made of the semiconductor InGaAs/GaAs film. The purple fastening element of negative photoresist keeps the helices on the substrate after the etching out of the underlying sacrificial layer. (b) and (c) SEM images of an array of right-handed and left-handed single-coil metal-semiconductor helices in top and oblique (inset) views. The pitch angle of the helix α was 13.5°, and the helix radius, 12.4 μ m.



FIG. 5. Experimental setup: 1—delay line; 2—THz emitter; 3, 5, 12—off-axis parabolic mirrors (OAPMs); 4, 8, 14, 15—optical mirrors; 6—metamaterial sample; 7, 11—THz detectors; 9, 10—beam splitters; 13—laser source.



FIG. 6. Normalized coefficient of reflection of electromagnetic waves from the metamaterial-substrate system versus frequency in pulsed mode.

excitation power values were implemented using an attenuator (not shown in the figures) and light splitters developed for use with laser pulses of femtosecond width. Phase distortions in femtosecond-duration radiation pulses which arose due to the dispersion of group velocities upon reflection were minimized using, in the delay line, metal mirrors M-1–M-4 and a three-mirror corner reflector with Au coating. Simulation data for the normalized values of reflection and transmission coefficients (19) in comparison with the experimental data are shown in Figs. 6 and 7. The normalization was carried out using the substrate only; thus the measured



FIG. 7. Normalized coefficient of transmission of electromagnetic waves propagating through the metamaterial-substrate system versus frequency.



FIG. 8. Theoretically calculated absorption coefficient versus frequency.

normalized coefficients may exceed 1. Fig. 7 shows oscillations due to the Fabry-Perot resonance inside the plate taking into account its finite thickness. There is no oscillation in Fig. 6 as in this case the reflectance from the second border of the substrate was not taken into account. The graph of the theoretical coefficient (absolute) of absorption of the structure is shown in Fig. 8. The theoretical coefficient of absorption of the substrate is equal to zero as it is supposed that the substrate is lossless. In the simulations, the following values of structure parameters complying with the parameter values of the actual structure were adopted:

L = 77 μ m, α = 13.5°, ω_0 = 12.6 × 10¹² rad/s, resistivity ρ = 2.42 × 10⁻⁸ Ω m, and N_h = 6.8 × 10⁴ cm⁻².

There are several types of chiral metamaterials with 3D conductive elements for the terahertz range (0.2-3.5 THz) known from the literature. The paper⁷⁰ demonstrated a chiral NIM metamaterial based on separate metal microresonators in the form of two parallel strips with a diagonal bridge on pillars. Authors of Refs. 71 and 72 suggested to form chiral NIM metamaterials and circular polarizers as continuous free-standing metal meta-foils made of conjugated 3D elements. The polarization rotator made of pseudo-planar metal swastikas on a silicon substrate was demonstrated in Ref. 73. A tunable chiral metamaterial⁷⁴ with resonators in the form of reconfigurable suspended planar spirals is the most similar to our technological approach. In contrary to the all above mentioned structures, our metamaterial is based on smooth resonant elements of strictly helicoidal shape and has compensated chirality. The last feature can be of use for practical applications, where high absorption is required without polarization transformation.

VI. CONCLUSIONS

We have designed and fabricated a highly absorptive metamaterial with compensated chirality based on smooth paired helices with optimized parameters. A boundary-value problem for the propagation of electromagnetic waves in a bi-layer structure in air was solved to determine the properties of the metamaterial-substrate system. The obtained data were compared with the experimental values of transmission and reflection coefficients for THz radiation.

In the obtained material, based on an array of paired helices, equally significant dielectric and magnetic properties were manifested due to the optimized shape of helices. Simultaneously, the chiral properties of the artificial structure were compensated due to the fact that pairs of optimal right-handed and left-handed helices were used in this structure. As a result, in the THz range the obtained metamaterial possessed a wave-impedance value close to that of free space.

Calculations based on the solved boundary-layer problem were performed to determine the values of the transmission and reflection coefficients for electromagnetic waves versus sample parameters. The solution of the boundary-layer problem has confirmed the fact that the sample exhibited weak reflective properties in the vicinity of the previously identified resonance frequency. Simultaneously, substantial wave absorption was observed around this frequency although the substrate was transparent in this frequency range. Such absorptive properties emerge as a consequence of resonant excitation of induced currents in the conducting helices with the Au layer.

On the basis of the obtained results, THz-range metamaterials with optimal helical elements can be developed. Also possible is the fabrication of new NIMs suitable for making planar «lenses» in the THz range.

A comparative analysis of experimental graphs and the results of modeling, based on theoretical derivations, shows that the proposed model satisfactorily describes properties of the artificial structure metamaterial-substrate. The frequency dependences of transmittance and reflectance, computed using the principles of the proposed model, are in good agreement with experimental data. The difference between the modeling and experiment can be explained by heterogeneity of metal layers. The difference leads to deviations in some characteristics of helical elements from the estimated values.

ACKNOWLEDGMENTS

This work was supported by the Byelorussian Republican Foundation for Basic Research (Grant No. F15SO-047) and by the Russian Foundation for Basic Research (Grant No. 15-02-99696).

- ²B. V. Bokut' and A. N. Serdyukov, J. Exp. Theor. Phys. **34**, 962 (1972).
- ³S. Zouhdi and A. Sihvola, in *Advances in Electromagnetics of Complex Media and Metamaterials*, edited by M. Arsalane (Kluwer Academic Publishers, 2002).
- ⁴A. Priou, A. Sihvola, and S. A. Tretyakov, in *Advances in Complex Electromagnetic Materials*, edited by A. Vinogradov (Kluwer Academic Publishers, 1997).
- ⁵S. A. Tretyakov, A. A. Sochava, and C. R. Simovski, Electromagnetics 16, 113 (1996).
- ⁶I. V. Semchenko, S. A. Khakhomov, S. A. Tretyakov, A. H. Sihvola, and E. A. Fedosenko, J. Phys. D: Appl. Phys. **31**, 2458 (1998).
- ⁷S. A. Tretyakov, A. H. Sihvola, I. V. Semchenko, and S. A. Khakhomov, J. Phys. D: Appl. Phys. **32**, 2705 (1999).
- ⁸I. V. Semchenko and S. A. Khakhomov, Electromagnetics **22**, 71 (2002).
- ⁹I. V. Semchenko, S. A. Khakhomov, and A. L. Samofalov, Russ. Phys. J. **52**, 472 (2009).
- ¹⁰C. F. Bohren, R. Luebers, H. S. Langdon, and F. Hunsberger, Appl. Opt. 31, 6403 (1992).
- ¹¹M. H. Umari, V. V. Varadan, and V. K. Varadan, Radio Sci. 26, 1327, doi:10.1029/91RS01370 (1991).

- ¹²B. Wang, J. Zhou, T. Koschny, M. Kafesaki, and C. M. Soukoulis, J. Opt. A: Pure Appl. Opt. **11**, 114003 (2009).
- ¹³E. Plum, Appl. Phys. Lett. **108**, 241905 (2016).
- ¹⁴E. Plum, *Chirality and Metamaterials* (University of Southampton, 2010).
- ¹⁵See http://en.wikipedia.org/wiki/Metamaterial for metamaterial definition.
 ¹⁶V. G. Veselago, Sov. Phys.-Usp. 10, 509 (1968).
- ¹⁷J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Tech. 47, 2075 (1999).
- ¹⁸D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. 84, 4184 (2000).
- ¹⁹R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
- ²⁰G. Dolling, C. Enkrich, M. Wegener, C. M. Soukoulis, and S. Linden, Science **312**, 892 (2006).
- ²¹G. Dolling, M. Wegener, C. M. Soukoulis, and S. Linden, Opt. Lett. **32**, 53 (2007).
- ²²K. T. McDonald, Am. J. Phys. **69**, 607 (2001).
- ²³S. A. Schelkunoff and H. T. Friis, *Antennas: Theory and Practice* (John Willey & Sons, NY, 1952).
- ²⁴W. N. Hardy and L. A. Whitehead, Rev. Sci. Instrum. 52, 213 (1981).
- ²⁵D. V. Sivukhin, Opt. Spektrosk. **3**, 308 (1957).
- ²⁶V. E. Pafomov, J. Exp. Theor. Phys. 9, 1321 (1959).
- ²⁷V. E. Pafomov, J. Exp. Theor. Phys. **3**, 597 (1956).
- ²⁸L. I. Mandelshtam, Zh. Eksp. Teor. Fiz. **15**, 475 (1945).
- ²⁹L. I. Mandelshtam, Complete Collection of Publications (Izdatelstvo ANSSSR, Moscow, 1950), p. 5.
- ³⁰L. I. Mandelshtam, Lectures on Optics, Relativity Theory and Quantum Mechanics (Nauka, Moscow, 1972).
- ³¹V. M. Agranovich and V. L. Ginzburg, Crystal Optics with Spatial Dispersion, and Excitons (Springer-Verlag, Berlin, Heidelberg, 1984).
- ³²H. Lamb, Proc. London Math. Soc. s2-1, 473 (1904).
- ³³M. Laue, Ann. Phys. **323**, 523 (1905).
- ³⁴V. G. Veselago, Phys.-Usp. **54**, 1161 (2011).
- ³⁵A. Schuster, *An Introduction to the Theory of Optics* (Edward Arnold and Co, London, 1928).
- ³⁶H. C. Pocklington, Nature **71**, 607 (1905).
- ³⁷J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
- ³⁸J. B. Pendry, D. Schurig, and D. R. Smith, Science **312**, 1780 (2006).
- ³⁹U. Leonhard, Science **312**, 1777 (2006).
- ⁴⁰D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, Science **314**, 977 (2006).
- ⁴¹B. V. Bokut', V. V. Gvozdev, and A. N. Serdyukov, J. Appl. Spectrosc. 34, 460 (1981).
- ⁴²A. N. Lagarkov, V. N. Semenenko, V. A. Chistyaev, D. E. Ryabova, S. A. Tretyakova, and C. R. Simovski, Electromagnetics **17**, 213 (1997).
- ⁴³J. B. Pendry, Science **306**, 1353 (2004).
- ⁴⁴A. N. Lagar'kov, V. N. Kisel', and V. N. Semenenko, J. Commun. Technol. Electron. 57, 1122 (2012).
- ⁴⁵K. Song, X. P. Zhao, Q. H. Fu, Y. H. Liu, and W. R. Zhu, J. Electromagn. Waves Appl. 26, 1967 (2012).
- ⁴⁶B. Ferguson and X. C. Zhang, Nat. Mater. 1, 26 (2002).
- ⁴⁷C. Jansen, S. Wietzke, O. Peters, M. Scheller, N. Vieweg, M. Salhi, N. Krumbholz, C. Jördens, T. Hochrein, and M. Koch, Appl. Opt. **49**, E48 (2010).
- ⁴⁸H. Tao, W. J. Padilla, X. Zhang, and R. D. Averitt, IEEE J. Sel. Top. Quantum Electron. 17, 92 (2011).
- ⁴⁹J. Federici and L. Moeller, J. Appl. Phys. **107**, 111101 (2010).
- ⁵⁰N. K. Grady, J. E. Heyes, D. R. Chowdhury, Y. Zeng, M. T. Reiten, A. K. Azad, A. J. Taylor, D. A. Dalvit, and H. T. Chen, Science **340**, 1304 (2013).
- ⁵¹H. Chen, W. J. Padilla, M. J. Cich, A. K. Azad, R. D. Averitt, and A. J. Taylor, Nat. Photonics 3, 148 (2009).
- ⁵²C. P. Ho, P. Pitchappa, and C. Lee, J. Appl. Phys. **119**, 153104 (2016).
- ⁵³D. M. Wu, N. Fang, C. Sun, X. Zhang, W. J. Padilla, D. N. Basov, D. R. Smith, and S. Schultz, Appl. Phys. Lett. 83, 201 (2003).
- ⁵⁴L. Werner, H.-W. Hübers, P. Meindl, R. Müller, H. Richter, and A. Steiger, Metrologia 46, S160 (2009).
- ⁵⁵N. I. Landy, C. M. Bingham, T. Tyler, N. Jokerst, D. R. Smith, and W. J. Padilla, Phys. Rev. B **79**, 125104 (2009).
- ⁵⁶L. Ding, Q. Y. S. Wu, and J. H. Teng, Laser Photonics Rev. 8, 941 (2014).
- ⁵⁷H. O. Moser and C. Rockstuhl, Laser Photonics Rev. 6, 219 (2012).
- ⁵⁸C. M. Soukoulis and M. Wegener, Nat. Photonics **5**, 523 (2011).
- ⁵⁹V. Ya. Prinz, V. A. Seleznev, A. K. Gutakovsky, A. V. Chehovskiy, V. V. Preobrazenskii, M. A. Putyato, and T. A. Gavrilova, *Physica E* 6, 828 (2000).

¹F. I. Fedorov, *Theory of Gyrotropy* (Nauka i Tekhnika, Minsk, 1976).

⁶⁰V. Ya. Prinz and S. V. Golod, J. Appl. Mech. Tech. Phys. 47, 867 (2006).

- ⁶¹E. V. Naumova, V. Ya. Prinz, S. V. Golod, V. A. Seleznev, R. A. Soots, and V. V. Kubarev, J. Opt. A: Pure Appl. Opt. **11**, 074010 (2009).
- ⁶²L. Zhang, E. Ruh, D. Grutzmacher, L. Dong, D. J. Bell, B. J. Nelson, and C. Schonenberger, Nano Lett. 6, 1311 (2006).
- ⁶³A. Rottler, M. Broell, N. Gerken, D. Heitmann, and S. Mendach, Opt. Lett. **36**, 4797 (2011).
- ⁶⁴G. Huang and Y. Mei, J. Materiomics 1, 296 (2015).
- ⁶⁵I. V. Semchenko, S. A. Khakhomov, and A. L. Samofalov, J. Commun. Technol. Electron. **52**, 850 (2007).
- ⁶⁶I. V. Semchenko, S. A. Khakhomov, and S. A. Tretyakov, Eur. Phys. J.: Appl. Phys. 46, 32607 (2009).
- ⁶⁷I. V. Semchenko, S. A. Khakhomov, S. A. Tretyakov, and A. H. Sihvola, Electromagnetics **21**, 401 (2001).

- ⁶⁸I. V. Semchenko, S. A. Khakhomov, and A. L. Samofalov, Eur. Phys. J.: Appl. Phys. 49, 33002 (2010).
- ⁶⁹I. V. Semchenko, S. A. Khakhomov, E. V. Naumova, V. Ya. Prinz, S. V. Golod, and V. V. Kubarev, Crystallogr. Rep. 56, 366 (2011).
- ⁷⁰S. Zhang, Y. S. Park, J. Li, X. Lu, W. Zhang, and X. Zhang, Phys. Rev. Lett. **102**, 023901 (2009).
- ⁷¹J. Wu, B. Ng, S. P. Turaga, M. B. H. Breese, S. A. Maier, M. Hong, A. A. Bettiol, and H. O. Moser, Appl. Phys. Lett. **103**, 141106 (2013).
- ⁷²J. Wu, B. Ng, H. Liang, M. B. H. Breese, M. Hong, S. A. Maier, H. O. Moser, and O. Hess, Phys. Rev. Appl. 2, 014005 (2014).
- ⁷³W. Zhang, W. M. Zhu, E. E. M. Chia, Z. X. Shen, H. Cai, Y. D. Gu, W. Ser, and A. Q. Liu, Opt. Express **22**, 10446 (2014).
- ⁷⁴T. Kan, A. Isozaki, N. Kanda, N. Nemoto, K. Konishi, M. Kuwata-Gonokami, K. Matsumoto, and I. Shimoyama, Appl. Phys. Lett. **102**, 221906 (2013).