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Hinkkanen, Marko; Saarakkala, Seppo E.; Awan, Hafiz Asad Ali; Mölsä, Eemeli; Tuovinen, Toni

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Observers for Sensorless Synchronous Motor Drives: Framework for Design and Analysis

Marko Hinkkanen, *Senior Member, IEEE,* Seppo E. Saarakkala, Hafiz Asad Ali Awan, Eemeli Mölsä, and Toni Tuovinen

Abstract—This paper deals with the speed and position estimation for synchronous reluctance motors (SyRMs) and interior permanent-magnet synchronous motors (IPMs). A unified design and analysis framework for a class of back-electromotive-force (back-EMF)-based observers is developed and the links between apparently different estimation methods are brought out. State observers equipped with a speed-adaptation law are shown to be mathematically equivalent to voltage-model-based flux observers equipped with a position-tracking loop. The error signal driving the adaptation law or the tracking loop is presented in a generalized form. Using the framework, a stabilizing gain design is reviewed and detailed design guidelines are given. Selected observer designs are experimentally evaluated using a 6.7-kW SyRM drive and a 2.2-kW IPM drive.

Index Terms—Control systems, observers, permanent-magnet motors, reluctance motors, stability criteria, synchronous motor drives.

I. INTRODUCTION

I N THE future, energy-efficient synchronous reluctance motors (SyRMs), with or without permanent magnets (PMs), could replace induction motors in many applications, such as pumps, fans, and conveyors. This kind of industrial drives should obviously be sensorless for cost savings. Furthermore, interior PM synchronous motors (IPMs) and other salient-rotor synchronous motors are increasingly used in electric vehicles and heavy-duty working machines. Even though the drives in electric vehicles are typically equipped with a motion sensor, a sensorless mode is beneficial for providing tolerance to sensor failures.

In this paper, we focus on back-electromotive-force (back-EMF)-based observers that estimate the rotor speed and position based on the mathematical motor model. The observer should provide (locally) stable and sufficiently fast estimationerror dynamics at all speeds and loads. It should also be robust against the measurement noise and parameter errors. Crossing the zero speed even with large load torque as well as smooth starting and stopping in a no-load condition should be possible without additional algorithms. These requirements

T. Tuovinen is with ABB Oy Drives, Helsinki, Finland (e-mail: toni.tuovinen@fi.abb.com).

are not trivial to fulfil, since the estimation-error dynamics unavoidably become nonlinear.

Already in the late 1980s, an advanced nonlinear state observer, operating in estimated rotor coordinates, has been systematically developed [1]. The speed estimation is based on the equation of motion, corrected with the current estimation error. Since this seminal work, numerous other speed and position estimation schemes have been proposed. A voltagemodel-based flux observer, operating in stator coordinates, is combined with a position-tracking loop in [2], [3]. A state observer, operating in estimated rotor coordinates, is augmented with a speed-adaptation loop in [4], [5]. In order to simplify the gain selection, a modified state variable is used to form a D-state observer in [6] and a minimum-order flux observer in [7].

Common to the observers in [1]–[7] is that their order is four, thus matching the order of the motor model. Lower-order observers [8]–[10] tend to be more sensitive to measurement noise and parameter errors, as shown in [5]. The observers proposed in [11]–[17] are similar to [1]–[7], but their order is higher and they have unnecessarily many design parameters. In addition to the current vector, the PM-flux vector is estimated in [11]–[13] and the back-EMF vector in [14]. An additional integral action is used in [15]–[17]. The methods in [11], [13], [14] are limited to surface-mounted PM machines (SPMs) and the methods in [16], [17] to SyRMs.

The magnetic saliency increases the coupling between the electrical and mechanical dynamics, which complicates designing back-EMF-based observers. In [12], a concept of the fictitious flux is used to turn the salient-rotor motor model into a more favourable nonsalient form. However, the dynamics of the d-axis current component are omitted in this analysis. For salient-rotor motors, the estimation-error dynamics are rigorously analyzed only in a few papers [1], [4], [5], [10]. In [1], a linearized model is derived for analysis purposes and an unstable region at higher speeds is found. As concluded in [1], the linearized model predicts the nonlinear error dynamics surprisingly well, even for large perturbations. In [4], a linearized model similar to [1] is developed and an unstable region in the regenerating mode at low speeds is revealed. In [5], a stabilizing observer gain is developed based on the linearized model. This gain has a unique feature of decoupling the flux observer from the speed-adaptation law.

This paper is an extended version of the conference paper [18]. We show that the observers structures [1]–[7], which have been independently developed from different starting points, are mathematically equivalent. The main contributions are:

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M. Hinkkanen, S. E. Saarakkala, H. A. A. Awan, and E. Mölsä are with the Department of Electrical Engineering and Automation, Aalto University, Espoo, Finland (e-mail: marko.hinkkanen@aalto.fi; seppo.saarakkala@aalto.fi; hafiz.awan@aalto.fi; olli.molsa@aalto.fi).

- A unified design framework for the observers similar to [1]–[7] is developed in Section III. We adopt the voltagemodel-based flux observer [2], [3] for the framework, augmented with a generalized speed-estimation law. The framework enables exploiting existing results, such as the linearized model [4] and the stabilizing gains [5].
- 2) Design guidelines based on the stabilizing observer gain [5] and pole placement are developed in Section IV. Furthermore, the stability conditions given in [5] are extended for the generalized speed-estimation law. Discrete-time implementation aspects, which are important in the high-speed operation, are also covered.
- Two selected observer designs are evaluated by means of the stability analysis and experiments in Section V. A risk of an unstable region appearing in the fieldweakening range at high torque values is discovered for the design in [2], [3].

A 6.7-kW SyRM drive and a 2.2-kW IPM drive are used in experimental evaluation. If the application requires sustained operation at very low speeds in a loaded condition, the observer should be combined with a signal-injection scheme, cf. [2]–[4], [15]–[17], [19]–[23].

II. MOTOR MODEL

Real space vectors are used. Vectors are denoted using boldface lowercase letters and matrices using boldface uppercase letters. For example, the current vector is $\mathbf{i} = [i_d, i_q]^T$, where i_d and i_q are the components of the vector. The identity matrix is $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the orthogonal rotation matrix is $\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Space vectors in stator coordinates are marked with the superscript s. No superscript is used for space vectors in estimated rotor coordinates.

The electrical rotor angle is $\vartheta_{\rm m}$ and the electrical angular rotor speed is $\omega_{\rm m} = d\vartheta_{\rm m}/dt$. The electrical radians are used throughout the paper. In rotor coordinates, the inductance matrix and the PM-flux linkage vector, respectively, are denoted by

$$\boldsymbol{L} = \begin{bmatrix} L_{\rm d} & 0\\ 0 & L_{\rm q} \end{bmatrix} \qquad \boldsymbol{\psi}_{\rm f} = \begin{bmatrix} \psi_{\rm f}\\ 0 \end{bmatrix} \tag{1}$$

where $L_{\rm d}$ is the direct-axis inductance, $L_{\rm q}$ is the quadratureaxis inductance, and $\psi_{\rm f}$ is the PM flux.

The machine model is expressed in *estimated* rotor coordinates, whose d-axis is aligned at $\hat{\vartheta}_m$ with respect to the stator coordinates. The stator flux linkage is

$$\psi = L'i + \psi'_{\rm f} \tag{2}$$

where the inductance matrix and PM-flux vector, respectively,

$$\boldsymbol{L}' = \mathrm{e}^{(\vartheta_{\mathrm{m}} - \hat{\vartheta}_{\mathrm{m}})\mathbf{J}} \boldsymbol{L} \, \mathrm{e}^{-(\vartheta_{\mathrm{m}} - \hat{\vartheta}_{\mathrm{m}})\mathbf{J}} \qquad \boldsymbol{\psi}_{\mathrm{f}}' = \mathrm{e}^{(\vartheta_{\mathrm{m}} - \hat{\vartheta}_{\mathrm{m}})\mathbf{J}} \boldsymbol{\psi}_{\mathrm{f}} \quad (3)$$

depend nonlinearly on the estimation error $\vartheta_m - \vartheta_m$ of the rotor position. The stator voltage is

$$\boldsymbol{u} = R\boldsymbol{i} + \frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}t} + \hat{\omega}_{\mathrm{m}}\mathbf{J}\boldsymbol{\psi}$$
(4)

where R is the resistance and $\hat{\omega}_{m} = d\hat{\vartheta}_{m}/dt$ is the angular speed of the coordinate system. The electromagnetic torque is

$$T = \frac{3p}{2} \boldsymbol{i}^{\mathrm{T}} \mathbf{J} \boldsymbol{\psi}$$
 (5)

where p is the number of pole pairs and the superscript T marks the transpose. As special cases, this model represents the SPM if $L_{\rm d} = L_{\rm g}$ and the SyRM if $\psi_{\rm f} = 0$.

III. SPEED AND POSITION OBSERVER

Fig. 1(a) shows the block diagram of a typical sensorless control system, part of which is the speed and position observer. Other control schemes, such as flux-linkage control or direct torque control, could be used instead of current control. A speed controller is not shown in the figure for simplicity, but it uses the estimated speed as its feedback signal. The pulsewidth modulator (PWM) calculates the duty ratios based on the voltage reference u_{ref}^{s} and the DC-bus voltage u_{dc} . The observer is implemented in estimated rotor coordinates in Fig. 1(a). Alternatively, the observer could be implemented in stator coordinates, as shown in Fig. 2(a).

A. Observer Structure Used in the Framework

1) Flux Observer in Estimated Rotor Coordinates: A voltage-model-based flux observer similar to [2], [3] is adopted for the framework, since this structure leads to the simplest form of the equations and simplifies the inclusion of the magnetic saturation model in the observer. Fig. 1(b) shows the structure of the speed and position observer, operating in estimated rotor coordinates. The flux observer is defined by

$$\frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}t} = \boldsymbol{u} - R\boldsymbol{i} - \hat{\omega}_{\mathrm{m}}\mathbf{J}\hat{\boldsymbol{\psi}} + \boldsymbol{K}\left(\boldsymbol{L}\boldsymbol{i} + \boldsymbol{\psi}_{\mathrm{f}} - \hat{\boldsymbol{\psi}}\right) \qquad (6)$$

where K is a 2 × 2 observer gain matrix and estimates are marked with a hat. In sensorless drives, the actual rotor position ϑ_m is naturally unknown. Therefore, the correction vector generally differs from the real flux estimation error $\psi - \hat{\psi}$ during transients, even if the accurate model parameter estimates are assumed, as can be realized from (2) and (3). As seen later, the correction vector in (6) is equal to the difference between the measured current and the estimated current, scaled by the inductance matrix.

2) *Flux Observer in Stator Coordinates:* Alternatively, the flux observer can operate in stator coordinates according to Fig. 2(b). Eq. (6) transformed to stator coordinates is

$$\frac{\mathrm{d}\hat{\boldsymbol{\psi}}^{\mathrm{s}}}{\mathrm{d}t} = \boldsymbol{u}^{\mathrm{s}} - R\boldsymbol{i}^{\mathrm{s}} + \boldsymbol{K}^{\mathrm{s}} \left(\boldsymbol{L}^{\mathrm{s}}\boldsymbol{i}^{\mathrm{s}} + \boldsymbol{\psi}_{\mathrm{f}}^{\mathrm{s}} - \hat{\boldsymbol{\psi}}^{\mathrm{s}} \right)$$
(7)

where

$$\boldsymbol{L}^{\mathrm{s}} = \mathrm{e}^{\hat{\vartheta}_{\mathrm{m}}\mathbf{J}}\boldsymbol{L}\,\mathrm{e}^{-\hat{\vartheta}_{\mathrm{m}}\mathbf{J}} \qquad \boldsymbol{\psi}_{\mathrm{f}}^{\mathrm{s}} = \mathrm{e}^{\hat{\vartheta}_{\mathrm{m}}\mathbf{J}}\boldsymbol{\psi}_{\mathrm{f}}$$
$$\boldsymbol{K}^{\mathrm{s}} = \mathrm{e}^{\hat{\vartheta}_{\mathrm{m}}\mathbf{J}}\boldsymbol{K}\,\mathrm{e}^{-\hat{\vartheta}_{\mathrm{m}}\mathbf{J}} \qquad (8)$$

3) Generalized Speed Estimation: As seen in Figs. 1(b) and 2(b), the proportional-integral (PI) mechanism is used to drive the error signal ε to zero by adjusting the speed estimate, which is further fed to the integrator for getting the position estimate,

$$\hat{\omega}_{\rm m} = k_{\rm p}\varepsilon + \int k_{\rm i}\varepsilon \,\mathrm{d}t \qquad \hat{\vartheta}_{\rm m} = \int \hat{\omega}_{\rm m} \mathrm{d}t \qquad (9)$$

where $k_{\rm p}$ and $k_{\rm i}$ are the gains. The generalized error signal is defined by means of the scalar product

$$\varepsilon = \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{J} \left(\boldsymbol{L} \boldsymbol{i} + \boldsymbol{\psi}_{\mathrm{f}} - \hat{\boldsymbol{\psi}} \right)$$
(10)



Fig. 1. Observer in estimated rotor coordinates: (a) sensorless control system; (b) observer structure.

where the projection vector λ can be a constant vector or it may depend on $\hat{\psi}$ and *i*. The rotation matrix **J** has been included in (10) in order to simplify the expressions in the latter part of the paper. It is worth noticing that the form of the scalar product in (10) resembles that of the torque expression (5). Furthermore, the magnitude of the projection vector λ is irrelevant due to the gains k_p and k_i in (9).

The expression (10) for the error signal ε is valid in stator coordinates as well. The observers in Figs. 1(b) and 2(b) are mathematically equivalent even though they look different: the flux observer (6) in estimated coordinates depends on the speed estimate while the flux observer (7) in stator coordinates depends on the position estimate. The position-tracking mechanism in Fig. 2(b) corresponds to a phase-locked loop (PLL).

B. Linearized Estimation-Error Dynamics

The nonlinear estimation-error dynamics consisting of (2)–(4), (6), (9), and (10) can be linearized for analysis purposes, as explained in [1], [4], [5]. The operating-point quantities are marked with the subscript 0. The accurate model parameters are assumed, making the operating-point estimation errors zero, e.g., $\hat{\vartheta}_{m0} = \vartheta_{m0}$, further leading to $L'_0 = L$ and $\psi'_{f0} = \psi_f$. The standard linearization procedure gives

$$\frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}t} = -(\boldsymbol{K}_0 + \omega_{\mathrm{m}0}\mathbf{J})\tilde{\boldsymbol{\psi}} + \boldsymbol{K}_0\mathbf{J}\boldsymbol{\psi}_{\mathrm{a}0}\tilde{\vartheta}_{\mathrm{m}} \qquad (11a)$$

$$\varepsilon = \boldsymbol{\lambda}_0^{\mathrm{T}} \mathbf{J} \tilde{\boldsymbol{\psi}} + \boldsymbol{\lambda}_0^{\mathrm{T}} \boldsymbol{\psi}_{\mathrm{a0}} \tilde{\vartheta}_{\mathrm{m}}$$
(11b)

where $\tilde{\psi} = \psi - \hat{\psi}$ is the flux estimation error and other errors are marked similarly. To simplify the notation, an auxiliary flux linkage vector is defined as

$$\psi_{\mathrm{a0}} = (\boldsymbol{L} + \mathbf{J}\boldsymbol{L}\mathbf{J})\boldsymbol{i}_{0} + \psi_{\mathrm{f}}$$
$$= \begin{bmatrix} (L_{\mathrm{d}} - L_{\mathrm{q}})\boldsymbol{i}_{\mathrm{d0}} + \psi_{\mathrm{f}} \\ -(L_{\mathrm{d}} - L_{\mathrm{q}})\boldsymbol{i}_{\mathrm{q0}} \end{bmatrix} = \begin{bmatrix} \psi_{\mathrm{ad0}} \\ \psi_{\mathrm{aq0}} \end{bmatrix}$$
(12)



Fig. 2. Observer in stator coordinates: (a) sensorless control system; (b) observer structure.



Fig. 3. Linearized model of the estimation-error dynamics for analysis purposes.

The linear system (11) can be represented by the transfer function

$$H(s) = \boldsymbol{\lambda}_0^{\mathrm{T}} \mathbf{J} (s \mathbf{I} + \boldsymbol{K}_0 + \omega_{\mathrm{m}0} \mathbf{J})^{-1} \boldsymbol{K}_0 \mathbf{J} \boldsymbol{\psi}_{\mathrm{a}0} + \boldsymbol{\lambda}_0^{\mathrm{T}} \boldsymbol{\psi}_{\mathrm{a}0}$$
(13)

from $\vartheta_{\rm m}(s)$ to $\varepsilon(s)$. Fig. 3 shows the block diagram of the resulting linearized model, where also the speed-estimation loop is included. According to the figure, the closed-loop transfer function from the actual speed to the estimated speed is

$$\frac{\hat{\omega}_{\rm m}(s)}{\omega_{\rm m}(s)} = \frac{(sk_{\rm p} + k_{\rm i})H(s)}{s^2 + (sk_{\rm p} + k_{\rm i})H(s)} = \frac{B(s)}{A(s)}$$
(14)

where A(s) is the fourth-order characteristic polynomial and B(s) is the third-order numerator polynomial. The closed-form expressions for these polynomials can be easily calculated using, e.g., any symbolic mathematics package. In a general case, the properties of (14) depend on the observer gain K_0 , the projection vector λ_0 , and the speed-adaptation gains k_p and k_i . All the observer designs corresponding to Figs. 1(b) and 2(b) can be analyzed by means of the linearized model.

C. Equivalent Structures and Existing Designs

1) Voltage-Model-Based Flux Observer With a Position-Tracking Loop: In [2], [3], the observer corresponding to Fig. 2(b) is developed and a simple observer gain $K^{s} = K = kI$ is used. In this special case, (7) can be represented as

$$\hat{\boldsymbol{\psi}}^{\mathrm{s}} = \frac{s}{s+k} \frac{\boldsymbol{u}^{\mathrm{s}} - R\boldsymbol{i}^{\mathrm{s}}}{s} + \frac{k}{s+k} \left(\boldsymbol{L}^{\mathrm{s}} \boldsymbol{i}^{\mathrm{s}} + \boldsymbol{\psi}_{\mathrm{f}}^{\mathrm{s}} \right) \qquad (15)$$

where s/(s+k) and k/(s+k) are the first-order high-pass and low-pass filters, respectively, and s is used as the derivative operator. This form clearly shows that the flux observer can be parametrized to behave as the voltage model at higher speeds and as the flux model at low speeds, the parameter k defining the corner frequency (typically $k = 2\pi \cdot 15 \dots 30$ rad/s).

According to [3], the position-tracking loop is driven by the error signal

$$\varepsilon = \frac{\hat{\psi}^{\mathrm{I}} \mathbf{J} \left(L i + \psi_{\mathrm{f}} \right)}{\|\hat{\psi}\|^2} \tag{16}$$

where the superscript s has been dropped from the space vectors, since the expression is valid also in estimated rotor coordinates. It can be seen¹ that this error signal equals (10) with $\lambda = \hat{\psi}/||\hat{\psi}||^2$. When the effect of the position-tracking loop is taken into account, the constant gain K = kI results in unstable operating regions, as will be shown in Section V-A.

2) Nonlinear State Observer: In [1], a nonlinear state observer is developed

$$\frac{\mathrm{d}\hat{\boldsymbol{i}}}{\mathrm{d}t} = \boldsymbol{L}^{-1}\boldsymbol{u} - \left(R\boldsymbol{L}^{-1} + \hat{\omega}_{\mathrm{m}}\boldsymbol{L}^{-1}\mathbf{J}\boldsymbol{L}\right)\hat{\boldsymbol{i}} \\ - \hat{\omega}_{\mathrm{m}}\boldsymbol{L}^{-1}\mathbf{J}\boldsymbol{\psi}_{\mathrm{f}} + \boldsymbol{G}_{\mathrm{i}}\left(\boldsymbol{i} - \hat{\boldsymbol{i}}\right)$$
(17a)

$$\frac{\mathrm{d}\hat{\omega}_{\mathrm{m}}}{\mathrm{d}t} = \frac{p}{J} \left(\hat{T} - \hat{T}_{\mathrm{L}} \right) + \boldsymbol{g}_{\omega}^{\mathrm{T}} \mathbf{J} \left(\boldsymbol{i} - \hat{\boldsymbol{i}} \right)$$
(17b)

$$\frac{\mathrm{d}\vartheta_{\mathrm{m}}}{\mathrm{d}t} = \hat{\omega}_{\mathrm{m}} \tag{17c}$$

$$\hat{T} = \frac{3p}{2}\hat{i}^{\mathrm{T}}\mathbf{J}(\boldsymbol{L}\hat{i} + \boldsymbol{\psi}_{\mathrm{f}})$$
(17d)

where J is the inertia estimate, $\hat{T}_{\rm L}$ is the load torque estimate, \hat{T} is the electromagnetic torque calculated using the estimated quantities, and $G_{\rm i}$ and g_{ω} are the observer gains. The change of the state variable, $\hat{i} = L^{-1}(\hat{\psi} - \psi_{\rm f})$, reveals that (17a) is mathematically equivalent to (6) if

$$\boldsymbol{K} = \boldsymbol{L}\boldsymbol{G}_{\mathrm{i}}\boldsymbol{L}^{-1} + \boldsymbol{R}\boldsymbol{L}^{-1}$$
(18)

The speed estimation in (17b) needs the information of the inertia and the load torque. Since the load torque is typically unknown, the term based on the mechanical dynamics is omitted in the following. In this case, the speed and position estimation in (17) is equivalent to (9) and (10), if $k_{\rm p} = 0$ is substituted in (9) and if the projection vector

$$\boldsymbol{\lambda} = -\mathbf{J}\boldsymbol{L}^{-1}\mathbf{J}\boldsymbol{g}_{\omega}/k_{\mathrm{i}} \tag{19}$$

is used in (10). In [1], the constant gains G_i and g_{ω} are used, leading to the instability at higher speeds.

3) D-State and Minimum-Order Observers: A modified state observer with constant observer gain is proposed in [6], [7]. The implementation in estimated rotor coordinates is referred to as a D-state observer in [6] and the implementation in stator coordinates as a minimum-order state observer in [7]. Here, the variant in estimated rotor coordinates is taken as an example, defined by

$$\frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}t} = \boldsymbol{D} \left(\boldsymbol{u} - R\boldsymbol{i}\right) - \hat{\omega}_{\mathrm{m}} \mathbf{J} \hat{\boldsymbol{\phi}} + \hat{\omega}_{\mathrm{m}} \mathbf{J} (\boldsymbol{D} - \mathbf{I}) (\boldsymbol{\phi}_{\mathrm{f}} - \hat{\boldsymbol{\phi}}_{\mathrm{f}}) \quad (20a)$$
$$\hat{\boldsymbol{\phi}}_{\mathrm{f}} = \hat{\boldsymbol{\phi}} - \boldsymbol{D} \boldsymbol{L} \boldsymbol{i} \quad (20b)$$

¹Note that $\hat{\boldsymbol{\psi}}^{\mathrm{T}} \mathbf{J} \hat{\boldsymbol{\psi}} = 0$ and similarly for other vectors.

where $\hat{\phi}$ is the state variable, $\hat{\phi}_{\rm f}$ is the output variable, and $\phi_{\rm f} = D\psi_{\rm f}$ is the transformed constant PM-flux vector.² The matrix gain is

$$\boldsymbol{D} = d_1 \mathbf{I} - \operatorname{sign}(\hat{\omega}_{\mathrm{m}}) d_2 \mathbf{J}$$
(21)

with constants d_1 and $d_2 > 0$. The speed-estimation law equals (9) and the error signal is defined by

$$\varepsilon = \frac{(\hat{\boldsymbol{\phi}}_{\mathrm{f}})^{\mathrm{T}} \mathbf{J} \boldsymbol{\phi}_{\mathrm{f}}}{\|\boldsymbol{\phi}_{\mathrm{f}}\|^2}$$
(22)

Since the matrix gain D is constant, the state variable can be changed as $\hat{\phi} = D\hat{\psi}$, leading to (6) with

$$\boldsymbol{K} = \hat{\omega}_{\mathrm{m}} \mathbf{J} (\boldsymbol{D} - \mathbf{I})$$

= $|\hat{\omega}_{\mathrm{m}}| [d_2 \mathbf{I} + (d_1 - 1) \operatorname{sign}(\hat{\omega}_{\mathrm{m}}) \mathbf{J}]$ (23)

Furthermore, applying (20b) and (21), it can be shown that the error signal in (22) equals the error signal in (10), if the projection vector $\lambda = \psi_f / ||\psi_f||^2$ is chosen. In [6], [7], the gains $d_1 = 1$ and $d_2 > 0$ are used, giving $K = d_2 |\hat{\omega}_m| \mathbf{I}$. Therefore, this observer design is similar to one in [2], [3], but the observer gain is proportional to the speed. At zero speed, this observer reduces to the pure voltage model, which makes it difficult to start and stop the drive. The coupling between the state observer and the speed estimation is omitted in the analysis in [6], [7].

4) State Observer With a Speed-Adaptation Law: In [4], [5], an open-loop flux observer is augmented with an outputerror-based correction term

$$\frac{\mathrm{d}\hat{\boldsymbol{\psi}}}{\mathrm{d}t} = \boldsymbol{u} - R\hat{\boldsymbol{i}} - \hat{\omega}_{\mathrm{m}}\mathbf{J}\hat{\boldsymbol{\psi}} + \boldsymbol{G}\big(\boldsymbol{i} - \hat{\boldsymbol{i}}\big)$$
(24a)

$$\hat{\boldsymbol{i}} = \boldsymbol{L}^{-1} (\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_{\mathrm{f}})$$
 (24b)

It is easy to show that (24) is mathematically equivalent to (6) if

$$K = GL^{-1} + RL^{-1}$$
(25)

The speed-adaptation law used in [4], [5] equals (9) and (10). The error signal $\varepsilon = \hat{i}_q - i_q$ is obtained by substituting $\lambda = \begin{bmatrix} 0 \\ 1 \end{bmatrix} / L_q$ in (10).

In [4], the observer gain below the base speed is

$$\boldsymbol{G} = l|\hat{\omega}_{\rm m}| \left[\mathbf{I} + \operatorname{sign}(\hat{\omega}_{\rm m}) \mathbf{J} \right]$$
(26)

where l is constant. The interaction between the state observer and the speed-estimation loop is analyzed using the linearized model. In [4], an unstable region at low speeds in the regenerating mode for salient-pole rotors is found out. In [5], a stabilizing gain is derived and applied. However, the selection of its free design parameters is not explained in detail in [5].

²In [6], [7], zero vector $\phi_f = \mathbf{0}$ is used in the correction vector in (20a). This choice causes an undesirable speed-dependent disturbance input to the observer, which deteriorates its performance at higher speeds, unless some additional compensation is used. To avoid this problem, the constant vector $\phi_f = D\psi_f$ is included in (20a).

IV. DESIGN GUIDELINES

A. Stabilizing Gains

1) Flux Observer: The stabilizing observer gain developed in [5] is briefly reviewed here. The fourth-order closedloop system (14) is complicated and the gains can be difficult to tune. In order to simplify the tuning procedure, the flux-estimation dynamics can be decoupled from the speedestimation dynamics. From (13), it can be seen that this decoupling is achieved if and only if $K_0 J \psi_{a0} = 0$ holds or

$$\boldsymbol{K}_{0} = \begin{bmatrix} k_{1} \\ k_{2} \end{bmatrix} \frac{\boldsymbol{\psi}_{a0}^{T}}{\|\boldsymbol{\psi}_{a0}\|^{2}} = \frac{1}{\psi_{ad0}^{2} + \psi_{aq0}^{2}} \begin{bmatrix} k_{1}\psi_{ad0} & k_{1}\psi_{aq0} \\ k_{2}\psi_{ad0} & k_{2}\psi_{aq0} \end{bmatrix} (27)$$

where k_1 and k_2 are the two remaining free gains in the observer gain matrix. The gain matrix is normalized by dividing it by $\|\psi_{a0}\|^2$ in order to simplify the following equations. The condition (27) makes the transfer function (13) to reduce to the static gain $H(s) = \lambda_0^T \psi_{a0}$

The two remaining free gains of K_0 determine the two flux-estimation poles, which can be placed as

$$\det(s\mathbf{I} + \mathbf{K}_0 + \omega_{\rm m0}\mathbf{J}) = s^2 + bs + c \tag{28}$$

where b and c are the coefficients of the characteristic polynomial. Solving (28) under the condition (27) yields

$$k_1 = b\psi_{\rm ad0} + (\omega_{\rm m0} - c/\omega_{\rm m0})\psi_{\rm aq0}$$
 (29a)

$$k_2 = b\psi_{\rm aq0} - (\omega_{\rm m0} - c/\omega_{\rm m0})\psi_{\rm ad0}$$
 (29b)

These gain elements can be inserted into (27), leading to

$$\boldsymbol{K}_{0} = \left[b \mathbf{I} + \left(\frac{c}{\omega_{\mathrm{m0}}} - \omega_{\mathrm{m0}} \right) \mathbf{J} \right] \frac{\boldsymbol{\psi}_{\mathrm{a0}} \boldsymbol{\psi}_{\mathrm{a0}}^{\mathrm{T}}}{\|\boldsymbol{\psi}_{\mathrm{a0}}\|^{2}} \qquad (30)$$

where the design parameters b > 0 and c > 0 determine the flux-estimation error dynamics. The factor $\psi_{a0}\psi_{a0}^{T}/||\psi_{a0}||^2$ can be recognized as an orthogonal projection matrix, which takes the vector projection of the correction vector in the direction of the vector ψ_{a0} . This matrix can be expressed in different forms

$$\frac{\psi_{a0}\psi_{a0}^{T}}{\|\psi_{a0}\|^{2}} = \frac{1}{\psi_{ad0}^{2} + \psi_{aq0}^{2}} \begin{bmatrix} \psi_{ad0}^{2} & \psi_{ad0}\psi_{aq0} \\ \psi_{ad0}\psi_{aq0} & \psi_{aq0}^{2} \end{bmatrix}$$
$$= \frac{1}{1 + \beta^{2}} \begin{bmatrix} 1 & -\beta \\ -\beta & \beta^{2} \end{bmatrix}$$
(31)

where $\beta = -\psi_{aq0}/\psi_{ad0}$ is an auxiliary variable [5]. As special cases, $\beta = 0$ holds for SPMs and $\beta = i_{q0}/i_{d0}$ for SyRMs.

2) Generalized Speed Estimation: The projection vector λ can be freely chosen in the generalized form (10) of the error signal, unlike in the method in [5]. If the gains of the PI mechanism in (9) are positive, the stability condition for the speed-estimation loop is

$$H(s) = \boldsymbol{\lambda}_0^{\mathrm{T}} \boldsymbol{\psi}_{\mathrm{a0}} > 0 \tag{32}$$

The tuning of the speed-estimation loop becomes very simple, if H(s) = 1 holds. This goal is achieved by choosing, e.g., a projection vector

$$\boldsymbol{\lambda}_0 = \frac{\boldsymbol{\psi}_{\mathrm{a0}}}{\|\boldsymbol{\psi}_{\mathrm{a0}}\|^2} \tag{33}$$

Another example is

$$\boldsymbol{\lambda}_0 = \frac{1}{\psi_{\mathrm{ad0}}} \begin{bmatrix} 1\\0 \end{bmatrix} \tag{34}$$

which corresponds to the speed-adaptation law in [4], [5]. Both these projection vectors cause (14) to reduce to^3

$$\frac{\hat{\omega}_{\rm m}(s)}{\omega_{\rm m}(s)} = \frac{(s^2 + bs + c)(sk_{\rm p} + k_{\rm i})}{(s^2 + bs + c)(s^2 + sk_{\rm p} + k_{\rm i})} = \frac{sk_{\rm p} + k_{\rm i}}{s^2 + sk_{\rm p} + k_{\rm i}}$$
(35)

where the speed-adaptation gains $k_p > 0$ and $k_i > 0$ are now directly the coefficients of the characteristic polynomial. Even if the flux-observer dynamics cancel out from (35), they still are a part of the whole system and the closed-loop flux-observer poles should be properly placed. This observer design is a subset of all possible stable designs. However, it is easier to tune two second-order systems than one fourth-order system, which is a clear advantage of this design approach.

The choice of the projection vector affects the sensitivity to parameter errors. The vector (33) maximizes the signalto-noise ratio, while, based on the time-domain simulations, the vector (34) results in the better robustness against model parameter errors at low speeds. It can also be shown that the choice $\lambda_0 = \psi_0$ does not generally fulfil the stability condition (32) and, therefore, is not recommended. We consider further optimization of the choice of the projection vector as a suitable topic for future research.

3) Example: Selection of Design Parameters: The closedloop flux-observer poles can be arbitrarily placed using (30). In order to limit the observer gain and to reduce sensitivity to model parameter errors, the design parameters b and cshould be selected such that the poles remain in the vicinity of the open-loop poles. Fig. 4(a) shows open-loop poles of the 6.7-kW SyRM at $\omega_{m0} = 0...2$ p.u. It can be seen that the damping of the open-loop poles decreases as the speed increases. It is favorable to increase the damping of the closedloop poles at higher speeds.

In principle, the undamped natural frequency of the poles could be chosen to be proportional to the rotor speed and the damping ratio could be kept constant. However, this choice would locate both the poles at the origin at $\omega_{m0} = 0$, causing the pure voltage-model behavior, which is undesirable and complicates the starting and stopping of the motor as well as speed reversals. This problem can be avoided, e.g., using the design parameters

$$b = b' + \left(2\zeta - \frac{b'}{\omega_{\zeta}}\right)|\omega_{\rm m0}| \qquad c = \frac{b}{2\zeta}|\omega_{\rm m0}| \qquad (36)$$

where the constant ζ is the desired damping ratio at a given angular speed ω_{ζ} (e.g., the rated speed). The constant b' is recommended to be chosen larger than $R/L_{\rm d}$ and $R/L_{\rm q}$. At standstill, the poles are placed at s = 0 and s = -b'.

³This result can be checked by substituting (30) and either (33) or (34) in (13), giving $H(s) = (s^2 + bs + c)/(s^2 + bs + c)$ as expected. Substituting H(s) in (14) gives (35).



Fig. 4. Pole locations of the 6.7-kW SyRM at $\omega_{m0} = 0...2$ p.u.: (a) open-loop poles from det $(s\mathbf{I} - R\mathbf{L}^{-1} - \omega_{m0}\mathbf{J}) = 0$; (b) closed-loop poles for $\mathbf{K} = k\mathbf{I}$; (c) closed-loop poles for (30). The flux-observer poles are marked with the red line and the speed-estimation poles with the blue line. The diamonds, crosses, and stars mark the speeds of 0, 1, and 2 p.u., respectively. The dashed line corresponds to the damping ratio of 0.4. At each speed, the optimal current components i_{d0} and i_{q0} corresponding to the MTPA locus, current limit of 1.5. p.u., and MTPV limit are used. Note that the poles in (a) and (c) do not depend on the currents, however. The constant parameters are used: R = 0.04 p.u., $L_d = 2.2$ p.u., and $L_q = 0.33$ p.u.

The two remaining poles of (35) can be placed at $s = -\omega_0$, leading to the critically damped speed-estimation dynamics. This choice corresponds to the speed-estimation gains

$$k_{\rm p} = 2\omega_{\rm o} \qquad k_{\rm i} = \omega_{\rm o}^2 \tag{37}$$

where ω_o can be considered as an approximate speedestimation bandwidth.

B. Implementation Aspects

1) Operating-Point Quantities in the Gains: In the implementation, the operating-point quantities in (12), (30), (34), and (36) are replaced with the instantaneous estimated quantities

$$\omega_{\rm m0} \leftarrow \hat{\omega}_{\rm m} \qquad \boldsymbol{i}_0 \leftarrow \boldsymbol{L}^{-1}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_{\rm f})$$
 (38)

Under the assumption of accurate model parameters, the linearized model is still valid after these replacements. Alternatively, the measured current could be used, i.e. $i_0 \leftarrow i$.

2) Discrete-Time Implementation: The continuous-time observer (6) can be discretized in various ways. The simplest option would be to apply the Euler approximation, but it leads to severely distorted pole locations and instability issues at higher speeds, unless a comparatively high sampling frequency is used.

The state-observer form (24) is chosen as a starting point for discretization, making it possible to apply an exact holdequivalent motor model (or its series approximation) for the open-loop observer part. Hence, the structure of the discretetime observer is analogous to (24),

$$\hat{\boldsymbol{\psi}}(k+1) = \boldsymbol{\Phi}\hat{\boldsymbol{\psi}}(k) + \boldsymbol{\Gamma}_{\mathrm{f}}\boldsymbol{\psi}_{\mathrm{f}} + \boldsymbol{\Gamma}\boldsymbol{u}(k)$$

$$+ \boldsymbol{G}_{\mathrm{d}} [\boldsymbol{i}(k) - \hat{\boldsymbol{i}}(k)]$$
 (39a)

$$\hat{\boldsymbol{i}}(k) = \boldsymbol{L}^{-1} \big[\hat{\boldsymbol{\psi}}(k) - \boldsymbol{\psi}_{\mathrm{f}} \big]$$
(39b)

where k is the discrete-time index, G_d is the observer gain matrix, and the system matrices Φ , Γ_f , and Γ are defined in Appendix A. The system matrices depend on the speed estimate, e.g., $\Phi = \Phi[\hat{\omega}_m(k)]$. The same system matrices are used in the discrete-time controller [24]. If the control system has a typical computational delay of one sampling period $T_{\rm s}$, the voltage in (39) is

$$\boldsymbol{u}(k) = \mathrm{e}^{-T_{\mathrm{s}}\hat{\omega}_{\mathrm{m}}\mathbf{J}}\boldsymbol{u}_{\mathrm{ref}}(k-1)$$
(40)

where u_{ref} is the reference voltage to the PWM in rotor coordinates, cf. Fig. 1(a). The matrix Γ inherently takes the PWM delay into account and no additional compensation should be used. The observer gain K is mapped to the discrete-time domain as, cf. (25),

$$\boldsymbol{G}_{\mathrm{d}} = T_{\mathrm{s}} \left(\boldsymbol{K} \boldsymbol{L} - R \mathbf{I} \right) \tag{41}$$

If the exact system matrices (or their second-order series approximations) are used, the estimation-error dynamics remain stable at very low ratios between the sampling frequency and fundamental frequency (much below ten).

The speed-adaptation law (9) is discretized as

$$\hat{\omega}_{\rm mi}(k+1) = \hat{\omega}_{\rm mi}(k) + T_{\rm s}k_{\rm i}\varepsilon(k)$$
 (42a)

$$\hat{\omega}_{\rm m}(k) = k_{\rm p}\varepsilon(k) + \hat{\omega}_{\rm mi}(k) \tag{42b}$$

where $\hat{\omega}_{mi}$ is the integral state. The sampled error signal is

$$\varepsilon(k) = \boldsymbol{\lambda}(k)^{\mathrm{T}} \mathbf{J} \big[\boldsymbol{L} \boldsymbol{i}(k) + \boldsymbol{\psi}_{\mathrm{f}} - \hat{\boldsymbol{\psi}}(k) \big]$$
(43)

The estimate $\hat{\omega}_{m}$ is used in the observer (39), while the integral state $\hat{\omega}_{mi}$ is used as a feedback signal for the speed controller.

3) Magnetic Model: The inductances may vary significantly due to the magnetic saturation, especially in the case of SyRMs. Look-up tables could be used to model the saturation characteristics as a function of the current, $L_d = L_d(i_d, i_q)$ and $L_q = L_q(i_d, i_q)$, as in [2], [3]. In the implementation of this paper, an algebraic magnetic model described in Appendix B is used and the inductances depend on the flux estimates, $L_d = L_d(\hat{\psi}_d, \hat{\psi}_q)$ and $L_q = L_q(\hat{\psi}_d, \hat{\psi}_q)$. This same magnetic model is consistently used everywhere in the sensorless control system.

V. RESULTS

A. Stability Analysis

The 6.7-kW SyRM is considered in the stability analysis. Here, the constant parameters corresponding to the rated



Fig. 5. Experimental results for the 6.7-kW SyRM showing torque reference steps at 1.2-p.u. speed: (a) observer gain K = kI; (b) observer gain (30). First subplot: reference torque and estimated torque $\hat{T} = (3/2)pi^{T}J\hat{\psi}$. Second subplot: actual speed and estimated speed. Third subplot: estimated flux components in estimated rotor coordinates. Last subplot: measured current components in estimated rotor coordinates.

operating point are used: R = 0.04 p.u., $L_d = 2.2$ p.u., and $L_q = 0.33$ p.u. Two different observer designs are compared:

- 1) $\mathbf{K} = k\mathbf{I}$ with $k = 2\pi \cdot 20$ rad/s;
- 2) K is defined by (30) and (36) with $b' = 2\pi \cdot 20$ rad/s, $\zeta = 0.4$, and $\omega_{\zeta} = 1$ p.u.

In both designs, the projection vector (34) is used and the speed-estimation gains are given by (37) with $\omega_0 = 2\pi \cdot 100$ rad/s. The local stability of the two observer designs is analyzed by calculating the poles of (14) in the speed range of $\omega_{m0} = 0...2$ p.u. at the maximum positive torque, when the current magnitude is limited to 1.5 p.u. At each speed, the optimal current components i_{d0} and i_{q0} corresponding to the maximum-torque-per-ampere (MTPA) locus, current limit, and maximum-torque-per-volt (MTPV) limit are used.

Fig. 4(b) shows the pole locations of *Design 1*. It can be seen that the system is unstable at higher speeds. The unstable region could be decreased by limiting the current (and the torque) at higher speeds. Since the flux-observer dynamics are coupled with the speed-estimation dynamics, also the speed-estimation gains strongly affect the stability. Under the same conditions, Fig. 4(c) shows the pole locations of *Design 2*. The system is stable and the pole locations match the designed values (and they are independent of the current). It can be seen that the flux-observer dynamics are decoupled from the speed-estimation dynamics.

B. Experiments

The two observer designs are experimentally evaluated using the 6.7-kW SyRM drive (also used in the previous stability analysis) and and the 2.2-kW IPM drive. The data of these machines are given in Appendix B. The observer has been implemented according to Section IV-B, using the exact discrete-time system matrices.

1) Sensorless Control System: A sensorless control system was implemented on a dSPACE DS1006 processor board. The stator currents and the DC-link voltage are sampled in the beginning of each PWM period; both the switching and sampling frequencies are 5 kHz. The inverter nonlinearities are compensated for using a simple current feedforward method. The actual rotor speed ω_m is measured using an incremental encoder only for monitoring purposes.

The control scheme shown in Fig. 1(a) was augmented with a speed controller, which provides the torque reference $T_{\rm ref}$ based on the speed reference $\omega_{\rm m,ref}$ and the estimated speed $\hat{\omega}_{\rm m}$. Instead of controlling the measured current *i*, the estimated flux linkage $\hat{\psi}$ is controlled. This choice makes it easy to take the magnetic saturation effects into account, since the incremental inductances are not needed at all in the control system. The flux-linkage controller and its tuning is based on the discrete-time controller presented in [24]; only the state variable to be controlled has been changed from the current to the flux. The flux reference $\psi_{\rm ref}$ is determined from $T_{\rm ref}$ and $\hat{\omega}_{\rm m}$ using the optimal torque control scheme [25], which includes the MTPA locus, field weakening, and MTPV limit.

2) Constant-Speed Tests: Fig. 5 shows the results when the load drive of the test bench regulates the speed at 1.2 p.u. and the sensorless SyRM drive under test is driven in the torquecontrol mode. The torque reference is stepped from zero to the rated torque with increments of 20% of the rated torque. Fig. 5(a) shows the results for *Design 1*. The system becomes



Fig. 6. Experimental results for the 6.7-kW SyRM showing fast acceleration to the 2-p.u. speed, with the current limit of 1.5 p.u.: (a) observer gain $K = k\mathbf{I}$; (b) observer gain (30). First subplot: reference speed, actual speed, and estimated speed. Second subplot: estimated flux components in estimated rotor coordinates. Last subplot: measured current components in estimated rotor coordinates.



Fig. 7. Experimental results for the 2.2-kW IPM showing fast acceleration to the 2-p.u. speed: (a) observer gain K = kI; (b) observer gain (30).

unstable at high torque values, which is also evident from the stability analysis results shown in Fig. 4(b). Fig. 5(b) shows the results for *Design 2*. It can be seen that the system is stable, which is in line with the stability analysis in Fig. 4(c).

3) Fast Acceleration Tests: Fig. 6 shows the results when the sensorless SyRM drive under test is driven in the speedcontrol mode and the load drive is disabled. The initial rotor position was set simply by supplying the DC-current vector to the direction of the a-phase magnetic axis, causing the rotor to rotate into this direction, and then resetting the position estimate $\hat{\vartheta}_m$ to zero. Alternatively, the initial rotor position could be obtained by using a signal-injection method, without causing the rotor to move.

The motor is accelerated from zero to 2 p.u., with the stator current magnitude limited to 1.5 p.u. Fig. 6(a) shows the results for *Design 1*. The system becomes unstable soon above the rated speed. This result agrees well with the stability

analysis in Fig. 4(b). As mentioned, the selection of k and the speed-estimation gains affects the stability. If the current limit is decreased, the system becomes stable, but acceleration time naturally increases. Fig. 6(b) shows the results for *Design 2*. It can be seen that the system is stable. The estimated speed $\hat{\omega}_{m}$ behaves smoothly and follows the measured speed ω_{m} .

Finally, Fig. 7 shows the corresponding results for the sensorless IPM drive. For this motor, the responses for *Design 1* and *Design 2* are quite similar. This is an expected result based on the stability analysis, which was carried out for the IPM as well. As compared to the SyRM, the observer-based sensorless control of the IPM is easier due to its lower saliency ratio ($L_q/L_d = 1.4$). Furthermore, lesser magnetic saturation of the IPM makes it possible to use constant inductances in the control system. It is also to be noted that both *Designs 1* and 2 are able to operate at zero speed in a no-load condition (as can been seen in Figs. 6 and 7 at t = 0...0.5 s), since

they avoid the pure voltage-model behavior.

VI. CONCLUSIONS

We have developed a design and analysis framework for a class of observers and brought out the links between apparently different observer structures. The voltage-model-based flux observer structure was adopted as a basis. A previously proposed stabilizing gain design was extended to different error signals of the speed-estimation loop. The stabilizing gain simplifies the observer design procedure and helps to avoid unstable regions. Detailed design guidelines were given. We also discovered a risk of an unstable region in the classical gain design. The observer designs were experimentally evaluated using the 6.7-kW SyRM drive and the 2.2-kW IPM drive.

Appendix A

EXACT DISCRETE-TIME MOTOR MODEL

The exact discrete-time model of (2) and (4) in rotor coordinates (i.e. $\hat{\vartheta}_{\rm m} = \vartheta_{\rm m}$) is briefly reviewed in the following. The input voltage in (4) is assumed to be constant between the sampling instants in *stator* coordinates, which matches the physical reality. The PM-flux vector $\psi_{\rm f}$ is constant in rotor coordinates. Further, the rotor speed $\omega_{\rm m}$ is assumed to be quasi-constant, since it varies slowly as compared to the electrical dynamics. These assumptions lead to the discrete-time model [24]

$$\boldsymbol{\psi}(k+1) = \boldsymbol{\Phi}\boldsymbol{\psi}(k) + \boldsymbol{\Gamma}_{\mathrm{f}}\boldsymbol{\psi}_{\mathrm{f}} + \boldsymbol{\Gamma}\boldsymbol{u}(k) \tag{44}$$

with the system matrices

$$\boldsymbol{\Phi} = e^{T_{s}\boldsymbol{A}} \qquad \boldsymbol{\Gamma}_{f} = \int_{0}^{T_{s}} e^{\tau\boldsymbol{A}} d\tau \cdot \boldsymbol{R} \boldsymbol{L}^{-1}$$
$$\boldsymbol{\Gamma} = \int_{0}^{T_{s}} e^{\tau\boldsymbol{A}} e^{\tau\omega_{m}\boldsymbol{J}} d\tau \cdot e^{-T_{s}\omega_{m}\boldsymbol{J}}$$
(45)

where $\mathbf{A} = -R\mathbf{L}^{-1} - \omega_{\rm m}\mathbf{J}$ is the continuous-time system matrix. The closed-form expressions for the elements of the system matrices are given in [24].

Alternatively, the system matrices in (45) can be expressed using series expansions. The matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Gamma}_{\rm f}$ are

$$\boldsymbol{\Phi} = \mathbf{I} + T_{\mathrm{s}} \boldsymbol{\Psi} \boldsymbol{A} \qquad \boldsymbol{\Gamma}_{\mathrm{f}} = T_{\mathrm{s}} \boldsymbol{\Psi} \left(R \boldsymbol{L}^{-1} \right)$$
(46)

where

$$\boldsymbol{\Psi} = \mathbf{I} + \frac{T_{\mathrm{s}}\boldsymbol{A}}{2!} + \frac{T_{\mathrm{s}}^{2}\boldsymbol{A}^{2}}{3!} + \dots \qquad (47)$$

The matrix Γ cannot be easily expressed in the exact form using a series expansion, but it can be approximated as

$$\boldsymbol{\Gamma} \approx T_{\rm s} \boldsymbol{\Psi} \frac{T_{\rm s} \omega_{\rm m}/2}{\sin(T_{\rm s} \omega_{\rm m}/2)} \mathrm{e}^{-(T_{\rm s} \omega_{\rm m}/2) \mathbf{J}}$$
(48)

where the last factors take the PWM delay into account [26]. If used in the observer or controller, the system matrices can typically be approximated using only the first two terms of (47). Choosing $\Psi = \mathbf{I}$ yields the Euler approximation. In other words, the model (44) covers the whole range of discrete-time models from the Euler approximation to the exact model. The model variant can be chosen based on the minimum sampling frequency, maximum rotor speed, and

TABLE I RATED VALUES OF EXAMPLE MACHINES

	SyRM	IPM
Power	6.7 kW	2.2 kW
Speed	3 175 r/min	1 500 r/min
Torque	20.1 Nm	14.0 Nm
Frequency	105.8 Hz (1 p.u.)	75 Hz (1 p.u.)
Voltage	$\sqrt{2/3} \cdot 370$ V (1 p.u.)	$\sqrt{2/3} \cdot 370$ V (1 p.u.)
Current	$\sqrt{2} \cdot 15.5 \text{ A} (1 \text{ p.u.})$	$\sqrt{2} \cdot 4.3$ A (1 p.u.)
$ \begin{array}{c} 1.5 \\ \overrightarrow{\text{rrd}} \\ \overrightarrow{\text{p}} \\ \overrightarrow{\text{o}} \\ 0.5 \\ 0 \\ 2 \\ 0 \\ i_q \\ (\text{p.u.}) \end{array} $	(1, 1, 1, 2, 2, 3, 3, 3, 4, 5, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5,	$0.5 \\ 0 \\ 0.5 \\ 2 \\ 0 \\ i_q (p.u.) -2 \\ 0 \\ i_d (p.u.)$
	(a)	(b)

Fig. 8. Flux linkages of the 6.7-kW SyRM: (a) $\psi_d = \psi_d(i_d, i_q)$; (b) $\psi_q = \psi_q(i_d, i_q)$. The surfaces are calculated using (49) with given parameters.

available computational power and memory. When applied in a sensorless control system, as in this paper, the actual speed appearing in the system matrices is replaced with the speed estimate, $\omega_{\rm m} \leftarrow \hat{\omega}_{\rm m}$.

Appendix B

DATA OF THE 6.7-KW SYRM AND THE 2.2-KW IPM

The 6.7-kW SyRM and the 2.2-kW IPM are used in the experiments. Their rating is given in Table I and the parameters used in the control system are given in the following.

The saturation characteristics of the SyRM are modeled using [27]

$$L_{\rm d}(\psi_{\rm d},\psi_{\rm q}) = \frac{1}{a_{\rm d0} + a_{\rm dd} |\psi_{\rm d}|^{\alpha} + \frac{a_{\rm dq}}{\delta + 2} |\psi_{\rm d}|^{\gamma} |\psi_{\rm q}|^{\delta + 2}} \quad (49a)$$

$$L_{\rm q}(\psi_{\rm d},\psi_{\rm q}) = \frac{1}{a_{\rm q0} + a_{\rm qq}|\psi_{\rm q}|^{\beta} + \frac{a_{\rm dq}}{\gamma + 2}|\psi_{\rm d}|^{\gamma + 2}|\psi_{\rm q}|^{\delta}} \quad (49b)$$

where the exponents are $\alpha = 5$, $\beta = 1$, $\gamma = 1$, and $\delta = 0$. The coefficient $a_{d0} = 0.36$ p.u. is the inverse of the unsaturated d-axis inductance and the coefficient $a_{q0} = 1.08$ p.u. is the inverse of the unsaturated q-axis inductance. The coefficients $a_{dd} = 0.15$ p.u. and $a_{qq} = 6.20$ p.u. take the self-axis saturation characteristics into account, while $a_{dq} = 2.18$ p.u. takes the cross-saturation into account. These parameters have been obtained by fitting the model (49) to the measured data. As can be seen in Fig. 8, both axes saturate significantly. The stator resistance is R = 0.04 p.u.

The saturation effects of the IPM are omitted and constant parameters are used in the control system: $L_{\rm d} = 0.34$ p.u., $L_{\rm q} = 0.48$ p.u., $\psi_{\rm f} = 0.85$ p.u., and R = 0.07 p.u.

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Marko Hinkkanen (M'06–SM'13) received the M.Sc.(Eng.) and D.Sc.(Tech.) degrees in electrical engineering from the Helsinki University of Technology, Espoo, Finland, in 2000 and 2004, respectively.

He is an Associate Professor with the School of Electrical Engineering, Aalto University, Espoo. His research interests include control systems, electric drives, and power converters.

Dr. Hinkkanen was the corecipient of the 2016 International Conference on Electrical Machines

(ICEM) Brian J. Chalmers Best Paper Award and the 2016 IEEE Industry Applications Society Industrial Drives Committee Best Paper Award. He is an Editorial Board Member of *IET Electric Power Applications*.



Seppo E. Saarakkala received the M.Sc.(Eng.) degree in electrical engineering from the Lappeenranta University of Technology, Lappeenranta, Finland, in 2008, and the D.Sc.(Tech.) degree in electrical engineering from Aalto University, Espoo, Finland, in 2014.

He has been with the School of Electrical Engineering, Aalto University, since 2010, where he is currently a Post-Doctoral Research Scientist. His main research interests include control systems and electric drives.



Hafiz Asad Ali Awan received the B.Sc. degree in electrical engineering from the University of Engineering and Technology, Lahore, Pakistan, in 2012, and the M.Sc.(Tech.) degree in electrical engineering from Aalto University, Espoo, Finland, in 2015.

He is currently working toward the D.Sc.(Tech.) degree at Aalto University. His main research interest include control of electric drives.



Eemeli Mölsä received the B.Eng. degree from the Metropolia University of Applied Sciences, Helsinki, Finland, in 2012, and the M.Sc.(Tech.) degree from Aalto University, Espoo, Finland, in 2015, both in electrical engineering. He is currently working toward the D.Sc.(Tech.)

He is currently working toward the D.Sc.(Tech.) degree at Aalto University. His main research interests include the control of electrical drives.



Toni Tuovinen received the M.Sc. degree from the University of Helsinki, Helsinki, Finland, in 2005; the M.Sc.(Eng.) degree in electrical engineering from the Helsinki University of Technology, Espoo, Finland, in 2009; and the D.Sc.(Tech.) degree in electrical engineering from Aalto University, Espoo, in 2014.

He is currently a Senior Design Engineer with ABB Oy Drives, Helsinki. His main research interests include the control of electric drives.