Tunable subwavelength photonic lattices and solitons in periodically patterned graphene monolayer

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Abstract: We study linear and nonlinear mode properties in a periodically patterned graphene sheet. We demonstrate that a subwavelength one-dimensional photonic lattice can be defined across the graphene monolayer, with its modulation depth and correspondingly the associated photonic band structures being controlled rapidly, by an external gate voltage. We find the existences of graphene lattice solitons at the deep-subwavelength scales in both dimensions, thanks to the combination of graphene intrinsic self-focusing nonlinearity and the graphene plasmonic confinement effects.

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References and links
1. Introduction

Graphene, a two-dimensional form of carbon in which the atoms are arranged in a honeycomb lattice, possesses various unique optical, electronic, and mechanical properties, which have important scientific and technological implications [1, 2]. It has already created a significant impact on the fields of photonics and optoelectronics [3], where applications such as light sources [4, 5], photodetection [6, 7] and light modulation [8] have been identified. Graphene is experimentally observed to support surface plasmonic polaritons (SPPs) [9, 10]. Compared with the traditional SPPs in noble metals, graphene plasmons are highly confined to a much smaller regions, and have much longer propagation distances [11, 12], which render the graphene as a promising material to be used as a deeply-subwavelength and lower-lossy waveguide, in the spectrum of infrared and THz regime. More importantly, the surface conductivity of graphene...
depends on chemical potential and can be tuned by external means, such as chemical doping or gate voltage. Therefore, graphene provides a unique opportunity in the implementations of tunable ultra-thin waveguides.

A single-atomic layer of graphene confines plasmonic oscillations to the graphene plane but leaving unbounded in graphene plane. Reliable light waveguiding however necessarily deals with systems that are bounded in two or three dimensions, and for that, graphene with finite-extent like graphene ribbons have recently drawn much attentions [13–15]. Graphene waveguide can also be defined by properly patterning the surface conductivity across a flake of doped graphene [16]. Very recently, it was suggested that the lateral confinement of plasmonic oscillations is also possible in extended graphene layer(s), thanks to the arrest of the in-plane diffraction by the intrinsic optical nonlinearity of graphene [17–19]. Graphene is a promising nonlinear optical material, whose third-order nonlinear coefficient has been theoretically predicted [20] and experimentally verified [21, 22] to be very large. The nonlinear optical response of graphene can be further enhanced by the ultra-tight bounding of the optical fields onto the graphene sheet [23, 24]. Thus there have been increasing research interests in the exploitation of graphene to boost various nonlinear optical processes and to decrease the power requirement for novel nonlinear photonic devices [25–27]. In this context, the spatial self-phase modulation (SPM) of the graphene-based nanostructures have been exploited to counteract the beam diffraction leading to the formation of the self-guided beams, namely, the spatial solitons [17–19]. Compared with the usual spatial optical solitons [28] in dielectrics and their counterparts in metallic structures [29, 30], graphene-based spatial solitons are of much tighter confinements. They also have much longer propagation distance than their metallic counterparts. Moreover, due to the strong nonlinear responses of graphene, one may expect that their experimental observations at relatively lower optical powers are possible.

As mentioned above, a very appealing optical property of graphene lies in the fact that the graphene conductivity can be be tuned by means of external ways, such as chemical doping and gate voltage $V_{bias}$. A practical way to implement that is to use an uneven ground plane underneath the graphene layer[see an example in Fig. 1(a)], so that a spatially inhomogeneous distance is introduced between the flat graphene and the ground. Being applied a dc biasing voltage, the graphene layer will experience different values of local biasing dc electric fields at the position with different distance from the ground plane. As a result, a spatially inhomogeneous carrier density across the graphene plane will be accumulated and that creates a desirable spatial distributions of chemical potentials. Graphene patterned in this way has been suggested to be a one-atom-thick platform for infrared metamaterials and transformation optics [16].

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Relating to the present study, it is therefore understandable that gating the graphene monolayer over a periodically corrugated ground will pattern the graphene with a periodic conductivity lattice. Thus, the doped and patterned graphene provides a unique and practical opportunity to realize photonic lattice at the deep-subwavelength scale.

In this paper, we address the formation of a conductivity lattice across the graphene monolayer by gating a doped graphene over a periodically corrugated ground plane. The aim of the paper is twofold. First, we reveal that the modulation depth of the conductivity lattice can be controlled externally by the gate voltage, basing on which we demonstrate a tunable photonic band structure whose bandgap size and even the closure/opening of the bandgaps can be electrically controlled, dynamically and rapidly. Second, in both the semi-infinite and the finite gaps of such tunable photonic band structures, we predict the existences of graphene lattice solitons localized in both dimensions, where the in-plane localization is achieved through the balance of coupling between lattices by graphene intrinsic nonlinearity, while the out-of-plane localizations are achieved mainly by the graphene plasmonic effects. We calculate the transverse localization lengths of graphene lattice solitons, and find that they feature deeply-subwavelength
scales in both directions, at a relatively lower power level.

2. Theoretical model

We start the analysis by considering the graphene structure as sketched in Fig. 1(a): A monolayer graphene which is infinite in the yz plane is held by a dielectric spacer. The ground plane underneath the spacer is assumed to be periodically corrugated along the y direction, while the uniformity of the ground in the z direction is maintained (z is the beam propagation direction). As described above, if a biased voltage is applied between graphene and ground, the surface conductivity will be periodically modulated across the graphene plane. Following we use a classical simplified capacitor model to understand the dependence of the resulting conductivity lattice on the corrugation geometry and external voltages.

![Graphene structure](image)

Fig. 1. (a) Surface corrugated ground plane for achieving a conductivity modulation. (b, c) The band structure of the patterned graphene at chemical potential $\mu_0 = 0.3 \text{ ev}$ (b) and 0.5 ev (c). (d) Evolution of the band edges versus chemical potential. In (b, c, d), solid lines stand for $\beta_r$, and dashed for $\beta_i$. (e, f) The Bloch mode profiles at the first (e) and second band (f), corresponding to the points labelled in (b). $D = 400 \text{ nm, } \lambda = 10 \mu \text{ m}$. 
We thus assume the distance between graphene and ground plane is given by, \( d(y) = d_0 + af(y) \), where \( d_0 \) is their separation before corrugation and the normalized function \( f(y) \) \((f(y)_{\text{max}} = 1)\) describes the corrugation profile of the ground, with \( a \) being the amplitude of the corrugation. For simplicity we assume \( f(y) \) is a smooth function with a period \( D \), and as a specific example, we choose \( f(y) = \cos(2\pi y/D) \). We also assume that the surface is only weakly corrugated \((a \ll d_0)\), therefore the use of a perturbative analysis is justified as detailed below.

As in [1], we treat a doped graphene as a parallel-plate capacitor, with a capacitance given by, \( C_s = \frac{\varepsilon_0 d}{d_0} \). Here \( \varepsilon_i \) is the permittivity of the dielectric spacer, \( A \) is the area of the capacitor and \( d \) is the distance of plates. For simplicity we assume \( \varepsilon_i = 1.0 \). With the capacitor model, the structure composed of a graphene and a surface corrugated ground can be looked as a series of infinite narrow parallel capacitors whose capacitance varies continuously with coordinate \( y \). This determines a spatially varying density of surface charges, \( n(y) = \varepsilon_0 V_{\text{bias}}/[ed(y)] \). Using this expression, one obtains the spatial distribution of chemical potential, \( \mu_c(y) = \sqrt{n(y)\pi \hbar V_F} \) [12, 14], as following

\[
\mu_c(y) = \hbar V_F \sqrt{\pi n(y)} \\
= \hbar V_F \sqrt{\frac{\pi \varepsilon_0 V_{\text{bias}}}{ed}} \\
= \hbar V_F \sqrt{\frac{\pi \varepsilon_0 V_{\text{bias}}}{ed_0[1 + \frac{a}{d_0} \cos(2\pi y/D)]}} \\
\approx \mu_c^0 [1 - \frac{a}{2d_0} \cos(2\pi y/D)] \\
= \mu_c^0 - \frac{\mu_c^0 a}{2d_0} \cos(2\pi y/D),
\]

where \( \mu_c^0 = \hbar V_F \sqrt{\frac{\pi \varepsilon_0 V_{\text{bias}}}{ed_0}} \). Obviously, equation (1) describes a \( y \)-periodic modulation on chemical potential around the mean value \( \mu_c^0 \), with a modulation amplitude \( \mu_c^0 a/(2d_0) \). Thus, in addition to the dependence of modulation amplitude on the corrugation geometry \((a \text{ and } d_0)\) that might be expected straightforwardly, Equation 1 shows that the modulation amplitude also have a linear dependence on \( \mu_c^0 \).

Now we consider the linear conductivity of graphene, \( \sigma_t \). Generally, \( \sigma_t \) contains both contributions from intra- and interband transitions. However, in the range of THz and far-infrared spectrum, the intraband transition dominates [11] and the conductivity is simplified to \( \sigma_t = \frac{i e^2 \mu_c^0}{\pi \hbar^2 (\omega + i\tau^{-1})} \) for a highly doped graphene \((\mu_c \gg k_B T)\), where \( \tau \) is the momentum relaxation time and \( k_B \) is Boltzmann’s constant. In the present study, we use the excitation wavelength (in vacuum) \( \lambda = 10\mu m \) and \( \tau = 0.3\text{ps} \) [14]. Further, as a frequently used technique in numerical simulations, graphene is modelled as an ultra-thin metallic layer with a thickness \( \Delta \) (we assumed \( \Delta = 0.50\text{nm} \) in the simulation) and endowed with an equivalent dielectric permittivity \( \varepsilon_1 = 1 + \frac{\varepsilon_0 \eta_0}{k_0 \Delta} \), which, after the submission of Eq. (1), yields

\[
\varepsilon_1 = 1 - \frac{\eta_0}{k_0 \Delta} \frac{e^2 \mu_c^0}{\pi \hbar^2 (\omega + i\tau^{-1})} [1 - \frac{a}{2d_0} \cos(2\pi y/D)] \\
= 1 - \frac{\eta_0}{k_0 \Delta} \frac{e^2 \mu_c^0}{\pi \hbar^2 (\omega + i\tau^{-1})} + \frac{\eta_0}{k_0 \Delta} \frac{e^2 \mu_c^0}{\pi \hbar^2 (\omega + i\tau^{-1})} \frac{a}{2d_0} \cos(2\pi y/D).
\]

Equation (2) explicitly shows that a one-dimensional(1D) photonic lattice is defined across the graphene monolayer, and the modulation depth of the lattice can be tuned through the
external gate voltage, $\mu_c^0(V_{\text{bias}})$, dynamically and quickly. Finally, it should be noted that Eq. (2) only gives the permittivity distribution within the graphene region, and outside that region the permittivity is assumed to be always 1.

3. Results and discussions

3.1. Tunable bandstructures

As described in the previous section, the periodically patterned graphene defines a tunable in-plane photonic lattice, and thus a tunable photonic band structures can be expected. To demonstrate this, we search the eigenmodes (Bloch modes) of the quasi-1D photonic lattice by substituting the electromagnetic field of the form $(E, H) = [E_0(x, y), H_0(x, y)]e^{i[k_x x + k_y y]}$ into the full sets of Maxwell equations (MEs) and solving the resulting eigenproblem numerically. Here $E_0$ and $H_0$ are the electric and magnetic components of the Bloch waves, respectively, and $\beta = \beta_r + i\beta_i$ is the complex propagation constants. The results are shown in Fig. 1, where both the propagation constants[Fig. 1(b)–1(d)] and some particular Bloch waves [Fig. 1(e) and 1(f)] are presented.

A notable property observed from Fig. 1 is that, graphene photonic lattice, although by nature supporting plasmonic Bloch waves, their associated band structures exhibit a similar scenario as that of all-dielectric periodic lattice: $\beta_r(k_y = 0) > \beta_r(k_y = \pi/D)$ for the first band, and $\beta_r(k_y = 0) < \beta_r(k_y = \pi/D)$ for the second band, and so on. This is in sharp contrast to the cases for periodic metallic layers or nanowires where the photonic band structure are inverted [29, 30]. The restoration of band structures in graphene photonic lattices should be related to the fact that, despite modulations, the real part of the lattice permittivity defined in Eq. (2) remains negative in the whole graphene region.

As emphasized above, one appealing property of the graphene lattice lies in its highly tunability, and accordingly the tunable band structures. As shown in Fig. 1(b), when $\mu_c^0 = 0.3$ ev that corresponds to $\varepsilon_{bg} = -214.01 + 4.25i + 10 \cos(2\pi y/D)$ (note that the modulation depth is much smaller than the mean value, a condition that was used in the derivation of Eq. (1)), the first and second band exhibits a clear bandgap in-between them. However, by increasing chemical potential to 0.5 ev that now corresponds to $\varepsilon_1 = -367.79 + 6.73i + 17 \cos(2\pi y/D)$, the bandgap closes [Fig. 1(c)]. The control of the bandgap-size with chemical potential is further illustrated in Fig. 1(d), where the dependence of propagation constants at the two bandedges on $\mu_c^0$ is plotted. Figure 1(d) also shows the lowering of both the first and second bands with the increase of $\mu_c^0$, indicating overall the band structure are shifting down along the vertical axis. As a result, after $\mu_c^0$ increases to some critical value, the second band starts immersing below the horizontal axis [see an example in Fig. 1(c)], a similar phenomena reported in 1D dielectric nanostructures too [31, 32].

Finally, Figure 1 also plots the imaginary parts of the propagation constants, $\beta_i$, that representing the modal loss upon propagation. As the modal loss is given by the spatially weighted average of the imaginary part of the complex permittivity over the modal field profile, $\beta_i$ is found to be a function of the Bloch wavevectors $k_y$.

3.2. Graphene lattice solitons

After identifying the band structures and Bloch waves, we proceed with the lattice solitons by taking into account the nonlinear correction to the conductivity of graphene, $\sigma_3$, so that the total conductivity now takes as $\sigma = \sigma_1 + \sigma_3|E_\parallel|^2$, where $E_\parallel$ is the in-plane component of the electric field and the nonlinear conductivity coefficient is [20, 21]

$$\sigma_3 = -i\frac{3}{32\pi\mu_r^0}\frac{e^4V_F^2}{\omega^3}.$$  (3)
which means the total dielectric permittivity is

\[ \varepsilon_g = 1 + \frac{i \sigma_g \eta_0}{k_0 \Delta} = 1 + \frac{i \eta_0}{k_0 \Delta} (\sigma_1 + \sigma_3 |E_\parallel|^2) = \varepsilon_1 + \chi_g^{(3)} |E_\parallel|^2, \]

(4)

where, \( \chi_g^{(3)} = \frac{i \eta_0 \sigma_3}{k_0 \Delta} \). We should mention that, due to their inversely proportional to the chemical potential, \( \sigma_3 \) or \( \chi_g^{(3)} \) is also periodically modulated, but out-of-phase with the linear modulation: wherever \( \varepsilon_1 \) achieves maximum (minimum) the nonlinear permittivity coefficient achieves minimum (maximum). Such out-of-phase modulation in linear and nonlinear coefficients may induce new properties in the light dynamics like in dielectric waveguides \([33, 34]\). However, in the present study, the nonlinear corrections, \( \delta \varepsilon_g = \chi_g^{(3)} |E_\parallel|^2 \), are kept much less than their linear coefficients (as shown below), as such the weakly nonlinearity modulations do not lead to visible modifications in soliton profiles.

![Profiles of a typical soliton at the semi-infinite gap.](image)

Fig. 2. Profiles of a typical soliton at the semi-infinite gap. (a) and (b) show the transverse distributions of the electric amplitude \(|E|\) (a), and imaginary part of \(E_z\) (b) of the soliton solution, respectively. (c) and (d) show the 1D profile of soliton along \(x\) axis (at \(y = 0\)) and \(y\) axis (at \(x = 0\)), respectively. \(D = 400 \text{ nm}, \lambda = 10 \mu\text{m}, \delta \varepsilon_g = 4\).

In order to find lattice solitons we adopt a self-consistent method as described in \([35]\). Both soliton solutions with propagation constants residing in the semi-infinite and the first-finite photonic gaps are found. Figure 2 presents a typical soliton profile from the semi-infinite photonic gap. Such solitons origin from the Bloch modes at the top of the first bands (see mode labelled \(A\) in Fig. 1(e)), and thus can be recognized as unstaggered lattice solitons as the phase difference of the longitudinal electric component, \(E_z\), between adjacent lattice is zero [Fig. 2(b)]. Please note that this is in sharp contrast to the previously reported plasmonic lattice solitons where the self-focusing nonlinearity supports only staggered soliton states in the semi-infinite gaps \([30, 36]\), a property being determined by the band structure as described above. Such 2D soliton states have different localization mechanisms at its two dimensions: in the perpendicular direction (with respect to the graphene plane), the localization is mainly due to the SPPs effect therefore soliton intensity features exponentially decaying away from the both sides of the graphene, see Fig. 2(c). The intensity dip in Fig. 2 (c) indicates the coupling nature of the graphene SPPs between the two interfaces of the graphene layer (recall that we treat graphene layer as a ultra-thin metallic layer). In contrast, the wave localization in the in-plane direction is of nonlinearity origin, where the self-focusing nonlinearity arrests the EM tunneling among ad-
jacent lattices [Fig. 2(d)]. As the conventional lattice solitons, the in-plane profile of the soliton features field modulations and on-site peaks.

![Graphs showing propagation constants, effective width, and power for solitons in the semi-infinite gap and the first finite gap.](image)

**Fig. 3.** Propagation constants (a,d), effective width (b,e) and power (c,f) for solitons in the semi-infinite gap (left column) and the first finite gap (right column). $D = 400 \text{ nm}$, $\lambda = 10 \mu\text{m}$.

The properties of the unstaggered lattice solitons are summarized in Fig. 3(a)–3(c), where we show the dependence of propagation constants, effective width and soliton power on the non-linear change of permittivity. As expected, with the increase of $\delta \varepsilon_g$, the real part of the propagation constant ($\beta_r$) increases continuously, indicating the substantially penetrating of soliton solutions into the semi-infinite gap [Fig. 3(a)]. The modal losses ($\beta_i$) increase with $\delta \varepsilon_g$ too, but at a rate much slower than that of $\beta_r$. The deeply subwavelengthed confinement of solitons are evident from Fig. 3(b), where the localization length of solitons in both dimensions, $w_{\text{eff}}$...
and \( w_{\text{yeff}} \) are plotted. The definition of the effective widths are: 
\[
w_{\text{xeff}} = \sqrt{\int_{-\infty}^{\infty} |E(x,y)|^2 dy} \quad \text{and} \quad w_{\text{yeff}} = \sqrt{\int_{-\infty}^{\infty} |E(x,y)|^2 dx}
\]
The slowly decreasing characteristics of \( w_{\text{xeff}}(\delta \varepsilon_g) \) indicate that, although the wave localization in x direction is dominated by graphene plasmonic effect, the nonlinearity further enhances the localization in this direction. The latter means more and more EM energy is pushed towards the graphene layer, thus we see a slowly increase of modal loss in Fig. 3(a). On the other hand, the in-plane localization \( w_{\text{yeff}} \) shows initially a strongly dependence on the nonlinearity, and then quickly saturates to a value that is nearly amount to a single lattice period \((D = 400 \text{ nm})\). Note that, although deeply subwavelengthed in both dimensions, such lattice solitons require optical power only a few hundred \( \mu \text{W} \) to form [Fig. 3(c)]. In terms of the light intensity, a change of \( \delta \varepsilon_g = 4.0 \) (corresponds to soliton solution in Fig.2) requires a peak intensity \( I_{\text{peak}} = 6.15 \times 10^{14} \text{ V}^2/\text{m}^2 \), or \( 2.33 \times 10^9 \text{ W/cm}^2 \), which is well below the damage threshold of graphene for femtosecond pulses [37].

We also find gap soliton solutions in the first-finite gap (Fig.4). Under a self-focusing nonlinearity, gap solitons are essentially the nonlinear localized version of the Bloch modes from the edges of the second band (see the Bloch mode labeled as D in Fig. 1(f)). Therefore, gap solitons can be termed as staggered modes since the phase difference of \( E_z \) component between adjacent lattices is \( \pi \) [see Fig. 4 (b)]. As conventional gap solitons, graphene gap solitons feature amplitude modulations where the local minimums are achieved at the lattice minimum. Therefore, in contrast to the lattice solitons in the semi-infinite gap that always have a single intensity peak, gap solitons feature pairs of peaks of equal intensity. The properties of gap solitons are summarized in Fig. 3(d)–3(f). Under the same strength of nonlinearity, gap solitons have larger \( w_{\text{yeff}} \) than solitons in the semi-infinite gap, while their \( w_{\text{xeff}} \) are almost the same. At stronger enough nonlinearity, \( w_{\text{yeff}} \) saturates to almost 100 nm which is slight larger than \( 2D \). Due to the same reason, the excitation of the gap solitons requires higher power[Fig.2 (f)].

Fig. 4. Profiles of a typical soliton in the first-finite gap. (a) and (b) show the transverse distributions of the amplitude \( |E| \) (a), and imaginary part of \( E_z \) (b) of the soliton solution, respectively. (c) and (d) show the 1D profile of soliton along x axis (at \( y = 0 \)) and y axis (at \( x = 0 \)), respectively. \( D = 400 \text{ nm}, \lambda = 10 \mu \text{m}, \delta \varepsilon_g = 6 \).
4. Conclusions

In conclusion, we suggested to use a periodically patterned graphenen monolayer to achieve a quasi-one-dimensional photonic lattice at the deep-subwavelength scales. We demonstrated that the modulation depth of such photonic lattice and their associated band structures can be controlled dynamically and quickly, through an external gate voltage. We also predicted the existence of the graphene lattice solitons whose propagation constants residing in the semi-infinite or the finite photonic gaps. Graphene lattice solitons are deeply-subwavelengthed localized in both dimensions due to the combination of the intrinsic nonlinearity of graphene and its plasmonic characteristics.

It is straightforward to expect that, using the same scheme as in our work but corrugating the surface of the underground plane with other topologies (instead of the periodic pattern), various novel in-plane photonic lattices can be created, such as chirped lattices, defected lattices, binary lattices, and yet their properties can be electrically controlled quickly. Therefore, the underground-mediated patterned-graphene provides an appealing platform for the implementation of the tunable nanophotonics and tunable nonlinear nanophotonics.

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