

---

This is an electronic reprint of the original article.  
This reprint may differ from the original in pagination and typographic detail.

Albooyeh, M.; Kwon, D. H.; Capolino, F.; Tretyakov, S. A.

## Equivalent realizations of reciprocal metasurfaces

*Published in:*  
Physical Review B

*DOI:*  
[10.1103/PhysRevB.95.115435](https://doi.org/10.1103/PhysRevB.95.115435)

Published: 27/03/2017

*Document Version*  
Publisher's PDF, also known as Version of record

*Please cite the original version:*  
Albooyeh, M., Kwon, D. H., Capolino, F., & Tretyakov, S. A. (2017). Equivalent realizations of reciprocal metasurfaces: Role of tangential and normal polarization. *Physical Review B*, 95(11), Article 115435. <https://doi.org/10.1103/PhysRevB.95.115435>

---

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

## Equivalent realizations of reciprocal metasurfaces: Role of tangential and normal polarization

M. Albooyeh,<sup>1</sup> D.-H. Kwon,<sup>2,3</sup> F. Capolino,<sup>1</sup> and S. A. Tretyakov<sup>3</sup>

<sup>1</sup>*Department of Electrical Engineering and Computer Science, University of California, Irvine, California 92697, USA*

<sup>2</sup>*Department of Electrical and Computer Engineering, University of Massachusetts Amherst, Amherst, Massachusetts 01003, USA*

<sup>3</sup>*Department of Electronics and Nanoengineering, Aalto University, Post Office Box 15500, FI-00076 Aalto, Finland*

(Received 8 December 2016; revised manuscript received 2 March 2017; published 27 March 2017)

We discuss possible topologies of physically different metasurfaces with equivalent macroscopic electromagnetic responses. We specifically demonstrate that the presence of normal components (with respect to the metasurface plane) of the polarization currents does not add any functionality in engineering the reflection and transmission characteristics which cannot occur in metasurfaces maintaining only tangential currents. Therefore, if a certain property is achievable using general volumetric inclusions, polarizable along all three directions, the same response can be achieved using some equivalent perfectly planar elements, polarizable only in the metasurface plane. Our results clarify what physical properties are necessary for realizing desired metasurface responses and promote flat microwave and optical designs, providing possibilities for dramatic complexity reduction of metasurfaces for rather general field transformations, including asymmetric reflection and transmission.

DOI: [10.1103/PhysRevB.95.115435](https://doi.org/10.1103/PhysRevB.95.115435)

### I. INTRODUCTION

Recent years have witnessed considerable interest in new possibilities to control and transform electromagnetic waves using electrically thin composite layers, called *metasurfaces* (see review papers [1–4]). Within the metasurface paradigm, electromagnetic fields are controlled by polarization and conduction currents in a very thin layer, effectively a sheet of negligible thickness. Thus, one can understand and synthesize metasurfaces using Huygens’s principle and its generalizations.

From the Huygens principle it is generally known that the fields in a given volume are fully determined by the values of the *tangential components* of the electric and magnetic fields on the surface of this volume. This property suggests that if we want to be able to fully control the reflection and transmission coefficients of a metasurface, it is enough to properly engineer the electric and magnetic currents which flow *along the metasurface plane*. In general, a metasurface composite layer has a nonvanishing thickness, therefore currents and polarizations with a component normal to the metasurface plane can also be induced in the metasurface structure. However, it is generally expected that these normally directed polarizations and currents are not essential for metasurface functioning. For example, it can be inferred [5] that the effect of any normal component of the current on a surface can be subsumed into an equivalent tangential current component to that surface.

However, the implications of this principle in actual designs of advanced metasurfaces with volumetric, three-dimensional inclusions are far from being simple, especially in cases of metasurfaces with complex, asymmetric responses. For example, in Ref. [6] it was stated that “adding longitudinal (normal) surface currents significantly expands the scope of electromagnetic phenomena that can be engineered with reciprocal materials”. However, from the Huygens principle, it can be expected that reflected and transmitted fields are fully determined by only tangential equivalent surface currents. Thus, it appears that this issue needs a careful and deeper examination.

In this paper, we study alternative realizations of equivalent metasurfaces, that is, metasurfaces with and without induced normal (to the metasurface plane) polarizations and currents, that exhibit identical macroscopic responses. Hence, we prove that the presence of normally polarizable components is not necessary in designing metasurfaces with engineered reflection and transmission coefficients at a given frequency and angle of incidence. Our results are important because they explicitly open up possibilities to realize fully planar structures with the most general reflection and transmission properties, limited only by the reciprocity of the used constituent materials.

### II. EQUIVALENCE OF THE ACTION OF TANGENTIAL AND NORMAL POLARIZATION CURRENTS

Let us consider an electrically thin planar layer which is illuminated by an arbitrary incident plane wave at angular frequency  $\omega$  with an  $e^{-i\omega t}$  time dependence assumed and suppressed. If the layer is uniform in its plane, fields and polarization vectors, averaged over the surface of identical unit cells of the structure will all depend on the tangential coordinates as the incident plane wave with transverse (to the metasurface normal) wave vector  $\mathbf{k}_t$ . In general, upon illumination the layer is polarized both electrically and magnetically, and the two polarization density vectors  $\mathbf{P}$  and  $\mathbf{M}$  (averaged over the unit-cell area) can have arbitrary directions, and we mark their normal and tangential components by subscripts “n” and “t”, respectively.

We start from writing the vectorial forms of the generalized sheet boundary conditions [7–10]

$$\mathbf{E}_t^+ \times \hat{\mathbf{n}} - \mathbf{E}_t^- \times \hat{\mathbf{n}} = -i\omega \left( \mathbf{M}_t - \hat{\mathbf{n}} \times \frac{\mathbf{k}_t}{\omega\epsilon} P_n \right), \quad (1)$$

$$\hat{\mathbf{n}} \times \mathbf{H}_t^+ - \hat{\mathbf{n}} \times \mathbf{H}_t^- = -i\omega \left( \mathbf{P}_t + \hat{\mathbf{n}} \times \frac{\mathbf{k}_t}{\omega\mu} M_n \right), \quad (2)$$

where  $\epsilon$  and  $\mu$  are the permittivity and permeability of the surrounding medium, respectively. Moreover,  $\hat{\mathbf{n}}$  is the

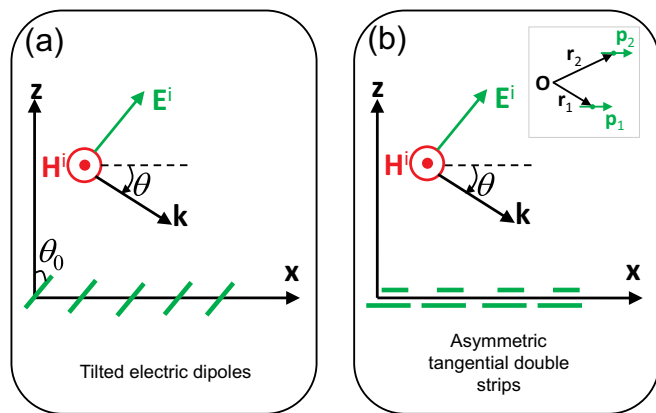


FIG. 1. Schematic of two physically different metasurfaces with equal responses to the same excitation. The nature of the induced currents is different. In (a), there are only induced electric moments with tangential and normal components while in (b) there are both electric and magnetic moments which are only tangential to the metasurface plane.

unit normal vector to the metasurface plane that in this paper coincides with the normal unit vector  $\hat{z}$  (see Fig. 1). Furthermore, the + and – superscripts refer to field quantities at  $z = 0^+$  and  $0^-$ , respectively. Equations (1) and (2) relate the jumps of the tangential electric ( $\mathbf{E}_t^\pm$ ) and magnetic ( $\mathbf{H}_t^\pm$ ) fields across the sheet through the surface electric ( $\mathbf{P}$ ) and magnetic ( $\mathbf{M}$ ) polarization densities.

Let us note in passing that the dependence of the field jumps of the tangential field components on the wave vector does not indicate that the layer is spatially dispersive, in contrast to some statements in [6]. Indeed, boundary conditions (1) and (2) are, in fact, Maxwell’s equations averaged over the unit-cell area and simplified assuming that the layer thickness is electrically small [10]. In other words, they do not contain any information about the layer properties and only allow us to find the field jumps if the polarization vectors are known. The layer properties (including spatial dispersion) are coded into metasurface parameters, such as sheet impedance or polarizabilities of unit cells [11]. The presence of the tangential component of the wave vector  $\mathbf{k}_t$  in the right-hand side of Eqs. (1) and (2) simply reflects the fact that Maxwell’s equations contain spatial derivatives along this direction, independently on the occurrence of spatial dispersion.

One immediate physical conclusion which may be deduced from the boundary conditions is that a tangential magnetic (electric) polarization  $\mathbf{M}_t$  ( $\mathbf{P}_t$ ) together with a normal electric (magnetic) polarization  $P_n$  ( $M_n$ ) cause the discontinuity of the transversal electric (magnetic) field. Therefore, we can introduce the new vectors

$$\mathbf{M}_{te} = \mathbf{M}_t - \hat{\mathbf{n}} \times \frac{\mathbf{k}_t}{\omega\epsilon} P_n, \quad (3)$$

and

$$\mathbf{P}_{te} = \mathbf{P}_t + \hat{\mathbf{n}} \times \frac{\mathbf{k}_t}{\omega\mu} M_n, \quad (4)$$

which, respectively, represent the *total* equivalent tangential magnetic and electric surface polarization densities contributing to the discontinuity of the tangential electric and magnetic

fields. Indeed, from the boundary conditions (1) and (2) together with the definitions (3) and (4), it can be seen that the polarizations  $\mathbf{M}_t$  and  $P_n$  or  $\mathbf{P}_t$  and  $M_n$  produce similar effects on the fields. That is, conceptually, an action of normal polarization  $P_n$  ( $M_n$ ) can be replaced by an equivalent action of a tangential polarization  $\mathbf{M}_t$  ( $\mathbf{P}_t$ ) and vice versa [12].

Let us now discuss the special case of Huygens’ metasurfaces, that is, metasurfaces which produce no reflections because tangential electric and magnetic polarizations are balanced as in a Huygens’ element. In order to realize a Huygens’ metasurface, the simultaneous presence of both electric and magnetic tangential polarizations is inevitable [13–16]. Therefore, in the language of boundary conditions (1) and (2), we require both nonzero total induced electric and magnetic polarizations  $\mathbf{P}_{te}$  and  $\mathbf{M}_{te}$  [see Eqs. (3) and (4)]. Therefore, in order to preserve the simultaneous presence of total equivalent electric and magnetic polarizations we consider one of the following simplified cases: (1)  $\mathbf{P}_t \neq 0$ ,  $P_n \neq 0$ ,  $\mathbf{M}_t = \mathbf{M}_n = 0$ ; (2)  $\mathbf{P}_t \neq 0$ ,  $\mathbf{M}_t \neq 0$ ,  $P_n = M_n = 0$ ; (3)  $\mathbf{M}_t \neq 0$ ,  $M_n \neq 0$ ,  $\mathbf{P}_t = P_n = 0$ ; (4)  $P_n \neq 0$ ,  $M_n \neq 0$ ,  $\mathbf{M}_t = \mathbf{P}_t = 0$ .

Although the above combinations represent the cases for which the minimum number of polarization components are assumed to be nonzero, possible configurations are not limited to these four, since more than the minimum number of components can be nonzero. In general, based on the presence or absence of each polarization component  $\mathbf{P}_t$ ,  $\mathbf{M}_t$ ,  $P_n$ , and  $M_n$  there may exist twelve different metasurfaces with simultaneous presence of electric and magnetic polarizations  $\mathbf{P}_{te}$  and  $\mathbf{M}_{te}$ . In this paper, we only discuss the proposed minimum required polarization components for realizing a Huygens’ metasurface, that is, the above four options.

Notice, not all the above cases ensure equal reflection and transmission responses for a given incident field. For example, some of them may create polarization rotations in the reflected and/or transmitted waves while the others may not. In order to clarify how these polarizations define the reflected and transmitted fields, we need to find the reflection and transmission coefficients for a surface in terms of the total tangential polarization vectors  $\mathbf{P}_{te}$  and  $\mathbf{M}_{te}$ . It is easy to show that the transversal components of the reflected  $\mathbf{E}_t^r$  and transmitted  $\mathbf{E}_t^t$  electric fields evaluated right at the boundary of the metasurface are given by [10]

$$\mathbf{E}_t^r = \frac{i\omega}{2} \overline{\overline{Z}} \cdot [\mathbf{P}_{te} \mp \overline{\overline{Z}}^{-1} \cdot \hat{\mathbf{n}} \times \mathbf{M}_{te}], \quad (5)$$

and

$$\mathbf{E}_t^t = \mathbf{E}_t^i + \frac{i\omega}{2} \overline{\overline{Z}} \cdot [\mathbf{P}_{te} \pm \overline{\overline{Z}}^{-1} \cdot \hat{\mathbf{n}} \times \mathbf{M}_{te}]. \quad (6)$$

Here the  $\overset{\text{top}}{\text{bottom}}$  sign in the brackets corresponds to the illumination from  $\overset{\text{top}}{\text{bottom}}$  side and  $\overline{\overline{Z}}$  is a  $2 \times 2$  dyadic:

$$\overline{\overline{Z}} = \eta \left( |\sin\theta| \frac{\mathbf{k}_t \mathbf{k}_t}{k_t^2} + \frac{1}{|\sin\theta|} \frac{\hat{\mathbf{n}} \times \mathbf{k}_t \hat{\mathbf{n}} \times \mathbf{k}_t}{k_t^2} \right). \quad (7)$$

In Eq. (7),  $\eta = \sqrt{\mu/\epsilon}$  is the intrinsic wave impedance in the host medium and  $\theta$  is the angle of the incident wave vector  $\mathbf{k}$  measured from the  $+x$  axis in the clockwise direction as shown in Fig. 1(a). In this paper we assume the wave

vector lies in the  $x$ - $z$  plane, and we allow the value of  $\theta$  in the full range of  $0 < \theta < 2\pi$ . Illuminations from the top and bottom sides correspond to the ranges  $0 < \theta < \pi$  and  $\pi < \theta < 2\pi$ , respectively. Propagation directions of the incident plane wave with the  $x$  component of their electric fields directed along  $-\hat{\mathbf{x}}$  are associated with  $\pi/2 < \theta < 3\pi/2$ . From Eqs. (5) and (6) it is obvious that the reflection and transmission coefficients are not equal for different discussed cases nos. 1–4. Consider, for example, the situation when the incident electric field and vector  $\mathbf{P}_t$  are polarized along the vector  $\mathbf{k}_t$ . Equivalent tangential electric polarization  $\mathbf{P}_{te}$  is a contribution of two different polarizations  $\mathbf{P}_t$  and  $M_n$ . However, while the first polarization ( $\mathbf{P}_t$ ) is contributing to the copolar component of reflection/transmission, the second one ( $M_n$ ) is always responsible for the cross-polar component of the reflection/transmission due to the term  $\hat{\mathbf{n}} \times \mathbf{k}_t$  [see Eq. (4)]. Therefore, if we require a certain copolar operation, then we are allowed to use only the first two cases, i.e., nos. 1 and 2 with  $M_n = 0$ . However, it is still not obvious how the first two cases can be equivalent.

Next, we present two examples in order to clarify this issue, that is, the equivalence of the first two cases in providing equal reflection/transmission response.

### III. METASURFACE OF TILTED ELECTRIC DIPOLES: TANGENTIAL AND NORMAL ELECTRIC POLARIZATIONS

Let us consider a metasurface formed by a periodic planar array of small electric dipole scatterers tilted so that they are intersecting the array plane with an angle  $\theta_0$  [Fig. 1(a)]. The array period is assumed to be small as compared with the wavelength, so that the array can be homogenized and considered as a metasurface.

The metasurface is excited by an obliquely incident plane wave of TM polarization, as shown in Fig. 1(a) (generally,  $\theta \neq \theta_0$ ). The incident plane wave induces electric polarization, which has both normal and tangential components  $P_n$  and  $\mathbf{P}_t$  (i.e., case no. 1). In this case, Eqs. (1) and (2) reduce to

$$\begin{aligned} \mathbf{E}_t^+ - \mathbf{E}_t^- &= -i\mathbf{k}_t \frac{P_n}{\epsilon}, \\ \hat{\mathbf{n}} \times \mathbf{H}_t^+ - \hat{\mathbf{n}} \times \mathbf{H}_t^- &= -i\omega\mathbf{P}_t. \end{aligned} \quad (8)$$

Let the effective polarizability of each dipole [17] directed along  $\hat{\mathbf{d}} = \hat{\mathbf{x}} \sin \theta_0 + \hat{\mathbf{z}} \cos \theta_0$  be denoted by  $\hat{\alpha}^{ee}$  such that the relation between the  $d$  component of the induced electric dipole moment and the incident electric field is given simply by  $p_d = \hat{\alpha}^{ee} E_d^i$ . Then, the electric polarizability tensor, which accounts also for all dipole interactions, written in the  $xz$  plane is expressed as

$$\hat{\hat{\alpha}}^{ee} = \begin{pmatrix} \hat{\alpha}_{xx}^{ee} & \hat{\alpha}_{xz}^{ee} \\ \hat{\alpha}_{zx}^{ee} & \hat{\alpha}_{zz}^{ee} \end{pmatrix} = \begin{pmatrix} \hat{\alpha}^{ee} \sin^2 \theta_0 & \hat{\alpha}^{ee} \sin \theta_0 \cos \theta_0 \\ \hat{\alpha}^{ee} \sin \theta_0 \cos \theta_0 & \hat{\alpha}^{ee} \cos^2 \theta_0 \end{pmatrix}. \quad (9)$$

The tangential and normal polarization density vectors  $\mathbf{P}_t$  and  $P_n$  may be written as [10]

$$\mathbf{P}_t = [\hat{\alpha}_{xx}^{ee} \sin \theta + \hat{\alpha}_{xz}^{ee} \cos \theta] \frac{E^i}{S} \hat{\mathbf{x}}, \quad (10)$$

and

$$P_n = P_z = [\hat{\alpha}_{zx}^{ee} \sin \theta + \hat{\alpha}_{zz}^{ee} \cos \theta] \frac{E^i}{S}, \quad (11)$$

in terms of the incident field, where  $S$  is the metasurface unit-cell area in the  $xy$  plane. In Eqs. (10) and (11),  $\hat{\mathbf{x}}$  is the unit vector in the  $x$  direction and  $|E^i| = E^i$  is the amplitude of the incident electric field. In this case, magnetic moments averaged over the unit cell are  $\mathbf{M}_t = M_n = 0$ .

Next, we substitute Eqs. (10) and (11) in Eqs. (5) and (6) [using definitions (3) and (4) and keeping in mind that  $\mathbf{M}_t = M_n = 0$ ] and find the reflection and transmission coefficients for the studied case. The results read

$$r = \frac{E_x^r}{E_x^i} = i \frac{\omega\eta}{2S} \left[ \hat{\alpha}_{xx}^{ee} |\sin \theta| - \frac{\hat{\alpha}_{zz}^{ee} \cos^2 \theta}{|\sin \theta|} \right], \quad (12)$$

and

$$t = \frac{E_x^t}{E_x^i} = 1 + i \frac{\omega\eta}{2S} \left[ \hat{\alpha}_{xx}^{ee} |\sin \theta| \pm 2\hat{\alpha}_{xz}^{ee} \cos \theta + \frac{\hat{\alpha}_{zz}^{ee} \cos^2 \theta}{|\sin \theta|} \right], \quad (13)$$

where the  $\pm$  signs correspond to illuminations of the top and bottom sides of the metasurface, respectively. This difference in sign depending on the direction of illumination originates from the asymmetry of the tangential electric field on the two sides of the metasurface generated by the normal component of the electric polarization.

Let us now look for the zero reflection condition in the specular direction (full transmission in the lossless case). It can be simply observed from Eq. (12) that it is possible to realize a reflectionless metasurface using this scenario. The required condition is

$$\hat{\alpha}_{xx}^{ee} \sin^2 \theta = \hat{\alpha}_{zz}^{ee} \cos^2 \theta. \quad (14)$$

For straight wire dipoles tilted at  $\theta_0$  in Fig. 1(a) with the polarizability tensor (9), this condition leads to  $\theta = \pi/2 \mp \theta_0$ . Note that only the upper sign choice (i.e.,  $\theta = \pi/2 - \theta_0$ ) is associated with an alignment of the specular reflection direction with the dipole axis, where there is a scattering pattern null. The lower sign choice (i.e.,  $\theta = \pi/2 + \theta_0$ ) corresponds to illumination of the metasurface from the dipole axis direction, in which case there is no interaction due to the orthogonality of the incident electric field and the dipole.

The transmission coefficient in this case (i.e., for the metasurface which does not produce reflection in the specular direction) simplifies to

$$t|_{r=0} = 1 + i \frac{\omega\eta}{S} (\hat{\alpha}_{xx}^{ee} |\sin \theta| \pm \hat{\alpha}_{xz}^{ee} \cos \theta). \quad (15)$$

It is easy to see from Eq. (15) that the transmission phase is different for different illumination directions (top/bottom signs) only for dipoles with a tilt ( $\hat{\alpha}_{xz}^{ee} \neq 0$ ) under illumination at oblique incidence ( $\cos \theta \neq 0$ ). If the dipole array is illuminated from the dipole axis direction ( $\theta = \pi/2 + \theta_0$ ), the transmission coefficient becomes unity.

As a numerical validation, reflection and transmission of a sample metasurface made of a perfectly conducting and tilted thin-wire dipole array is analyzed. Each wire is assumed to be perfectly conducting, making the metasurface lossless. Moreover, each wire is assumed to have a loading reactance

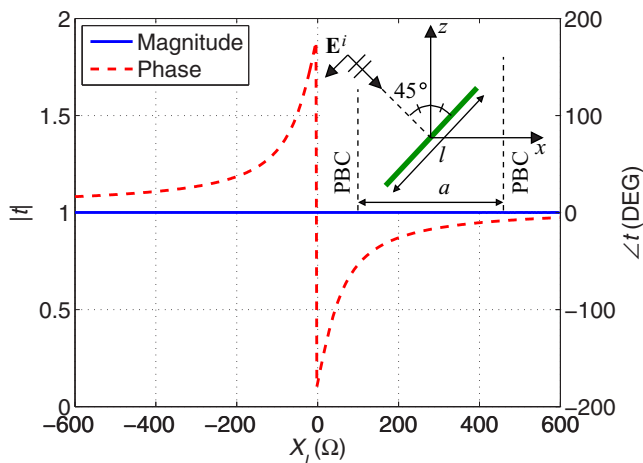


FIG. 2. Reflection and transmission from an array of perfectly conducting and tilted thin-wire dipoles with respect to a loading reactance at resonance. The inset shows a side view of the unit cell. The dipole length and the wire radius are  $l = 50.0$  mm and  $a_w = 0.25$  mm, respectively. The unit-cell dimension is  $a = 50$  mm in both  $x$ - and  $y$ -axis directions. At the middle point, each dipole is loaded with a reactance  $X_l$ . The frequency is 3.0 GHz.

$X_l$  at its center. At the frequency of self-resonance, a planar array of thin-wire dipoles with a tilt angle of  $\theta_0 = 45^\circ$  is illuminated by a TM-polarized plane wave with  $\theta = \pi/2 - \theta_0$ . The element spacing is set to a half free-space wavelength to avoid grating lobes.

Figure 2 shows the transmission coefficient  $t$  as a function of the loading reactance  $X_l$  at the frequency corresponding to resonance of the short-circuited dipole. The full-wave simulation results were obtained using FEKO 14 by Altair, which is a software tool coded based on the Method of Moments [18]. The magnitude of  $t$  is equal to unity, signifying full transmission—no power is transmitted in the specular reflection direction. By properly choosing  $X_l$ , the transmission phase can be set to an arbitrary value in the entire  $360^\circ$  range. In a sharp contrast, an illumination from the bottom side of the metasurface, from  $\theta = 225^\circ$ , would not excite the dipoles at all, resulting in  $t = 1$ .

According to the results of [6], this phenomenon is possible because the normal electric polarization is nonzero. In the next section, however, we present an equivalent metasurface which provides exactly the same transmission and reflection properties, but we use only tangential polarizations to realize such performance (i.e., case no. 2).

#### IV. METASURFACE OF TANGENTIAL ASYMMETRIC DOUBLE STRIPS: TANGENTIAL POLARIZATIONS ONLY

In the previous section we presented an example with normal and tangential electric polarizations. In this section we present a metasurface with only tangential polarization response(s). However, in order to create the same behavior (scattering properties) as the previous case we need to have both electric and magnetic polarizations. Indeed, as discussed above, tangential magnetic polarization plays an equivalent

role to the normal electric polarization in the previous scenario [see definition (3)].

An important fact to preserve the same behavior of the previous scenario is to consider the asymmetry in the tangential direction. The topology in the previous scenario was not symmetric with respect to the  $+x$  and  $-x$  directions. Therefore, we need to maintain this property while using only tangential polarizations. One possible solution is to use double dipole scatterers asymmetrically positioned in the  $xy$  plane as shown in Fig. 1(b). At first glance, this structure may be deduced to be modeled as  $\Omega$ -type bianisotropic metasurfaces as in [19,20].

However, it turns out that an array of  $\Omega$ -type inclusions cannot effectively model the effect of asymmetry of two horizontal strips in the  $x$ -axis direction. Instead, the two strips in a unit cell can be modeled as separate induced electric dipoles. The inset in Fig. 1(b) illustrates two induced dipoles with electric dipole moments  $\mathbf{p}_1 = \hat{\mathbf{x}}p_1$  and  $\mathbf{p}_2 = \hat{\mathbf{x}}p_2$  at positions  $\mathbf{r}_1$  and  $\mathbf{r}_2$  referenced at the coordinate origin  $O$ , respectively. In the homogeneous limit, this array of asymmetric strips represents two layers of horizontal electric polarizations  $\mathbf{P}_1 = \hat{\mathbf{x}}P_1 = \hat{\mathbf{x}}p_1/S$  and  $\mathbf{P}_2 = \hat{\mathbf{x}}P_2 = \hat{\mathbf{x}}p_2/S$  at their respective  $z$  coordinates.

Let us denote the individual polarizabilities of the two representative electric dipole moments of Fig. 1(b) at positions  $\mathbf{r}_1$  and  $\mathbf{r}_2$  by  $\alpha_1$  and  $\alpha_2$  and the interaction constants of each array layer (top and bottom) on its own corresponding dipole by  $\beta_1$  and  $\beta_2$ , and the interaction of each array layer on the dipole of the other layer by  $\beta$ . Then, for each dipole moment, we may write [11]

$$p_1 = \alpha_1 [E_x^i(\mathbf{r}_1) + \beta_1 p_1 + \beta p_2], \quad (16)$$

$$p_2 = \alpha_2 [E_x^i(\mathbf{r}_2) + \beta_2 p_2 + \beta p_1]. \quad (17)$$

These relations are further simplified to

$$p_1 = \hat{\alpha}_1 [E_x^i(\mathbf{r}_1) + \beta p_2], \quad (18)$$

$$p_2 = \hat{\alpha}_2 [\beta p_1 + E_x^i(\mathbf{r}_2)]. \quad (19)$$

Here,  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  are the effective polarizabilities of each dipole in the corresponding array which are shown to satisfy

$$\hat{\alpha}_1 = \frac{1}{\frac{1}{\alpha_1} - \beta_1}, \quad (20)$$

$$\hat{\alpha}_2 = \frac{1}{\frac{1}{\alpha_2} - \beta_2}. \quad (21)$$

From Eqs. (18) and (19), the solution for  $p_1$ ,  $p_2$  is found:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{1}{1 - \hat{\alpha}_1 \hat{\alpha}_2 \beta^2} \begin{pmatrix} \hat{\alpha}_1 [E_x^i(\mathbf{r}_1) + \hat{\alpha}_2 \beta E_x^i(\mathbf{r}_2)] \\ \hat{\alpha}_2 [\hat{\alpha}_1 \beta E_x^i(\mathbf{r}_1) + E_x^i(\mathbf{r}_2)] \end{pmatrix}. \quad (22)$$

As will become apparent shortly, strong mutual coupling between the two strips is critical for achieving asymmetric transmission properties. This property can be ensured with a subwavelength separation between the strips in the  $z$  coordinate, allowing the overall structure to be considered as a metasurface with both tangential electric and magnetic (due to strong coupling) polarization responses.

The plane waves generated by the induced polarization currents determine the transmission and reflection coefficients. Referenced at  $z = 0$ , the results are

$$t = 1 + i\omega\eta |\sin\theta| \frac{\hat{\alpha}_1 + \hat{\alpha}_2 + 2\hat{\alpha}_1\hat{\alpha}_2\beta \cos[\mathbf{k} \cdot (\mathbf{r}_2 - \mathbf{r}_1)]}{2S(1 - \hat{\alpha}_1\hat{\alpha}_2\beta^2)} \quad (23)$$

and

$$r = i\omega\eta |\sin\theta| \frac{\hat{\alpha}_1 e^{i2k_z^i z_1} + \hat{\alpha}_2 e^{i2k_z^i z_2} + 2\hat{\alpha}_1\hat{\alpha}_2\beta e^{ik_z^i(z_1+z_2)} \cos[\mathbf{k}_i^i \cdot (\mathbf{r}_2 - \mathbf{r}_1)]}{2S(1 - \hat{\alpha}_1\hat{\alpha}_2\beta^2)}, \quad (24)$$

where  $k_z^i = \hat{\mathbf{z}} \cdot \mathbf{k} = -k \sin\theta$ ,  $\mathbf{k}_i^i = \mathbf{k} - \hat{\mathbf{z}}k_z^i$ ,  $z_1 = \hat{\mathbf{z}} \cdot \mathbf{r}_1$ , and  $z_2 = \hat{\mathbf{z}} \cdot \mathbf{r}_2$ . Inspection of Eq. (23) reveals the conditions for obtaining asymmetric transmission properties from opposite illumination directions, i.e.,  $t(\theta) \neq t(2\pi - \theta)$ : First, a nonzero interaction constant  $\beta$  is required which ensures tangential magnetic polarization response due to spatial dispersion. Next, it is required that  $\cos[\mathbf{k} \cdot (\mathbf{r}_2 - \mathbf{r}_1)]$  must be different for illuminations from different sides which implies that asymmetric transmission is possible only for oblique incidence angles (i.e.,  $\theta \neq \pi/2$ ) and for asymmetric strip arrangement in the  $x$  direction (i.e.,  $\mathbf{r}_2 - \mathbf{r}_1$  not parallel with the  $z$  axis). In addition, single planar structures (i.e., with  $z_1 = z_2$ ) do not permit asymmetric transmission. The same is true for normal incidence regardless of  $\mathbf{r}_2 - \mathbf{r}_1$ , which agrees with the prediction by reciprocity.

The condition for zero reflection can be found from Eq. (24), relating the polarizabilities, relative positions, and mutual coupling of the two strips. In practice, numerical optimization can be used to design unit cells for equivalence with the tilted dipole array of Sec. III—full transmission with an arbitrary and asymmetric transmission phase.

As an example of asymmetric transmission, illumination of an array of asymmetric patches in free space by a plane wave at an oblique incidence angle of  $45^\circ$  is considered, as illustrated in Fig. 3(a). Two perfectly conducting horizontal patches of dimensions  $l_t \times w$  and  $l_b \times w$  are separated by  $d$  in the vertical direction. The bottom patch is centered at  $O$  and the center point of the top patch is located at  $(x_c, 0, d)$ . The target characteristics were chosen to be full transmission with a transmission phase of  $180^\circ$ . For the vertical patch separation and the patch width fixed at  $d = \lambda_0/10$  and  $w = \lambda_0/4$ , the values of  $l_t$ ,  $l_b$ , and  $x_c$  were numerically optimized. Full transmission requires synthesis of a scattering pattern null of the particle in the specular reflection direction. For the optimized design, Fig. 3(b) shows the normalized scattering pattern of one metasurface unit cell in the  $xz$  plane. Although not perfect, a deep null in the specular reflection direction is synthesized. The transmission coefficient is found to be  $t = 0.99 \angle 179.82^\circ$ , which is very close to the target value. This is associated with a 98.7% transmission and a 1.30% reflection of the incident power. If the metasurface is illuminated from the bottom side at the same incidence angle, the transmission coefficient is found to be  $t = 0.99 \angle 100.65^\circ$ , providing an asymmetric transmission property. For comparison, Fig. 3(b) also shows the scattering pattern of the wire dipole in Sec. III tilted at  $45^\circ$  under the same illumination. A perfect scattering pattern null in the specular reflection direction owing to the dipole orientation forces a complete suppression of the reflected plane wave, allowing all incident power to be transmitted.

It is noted that the presented optimization focused on synthesizing a specific transmission coefficient of  $t = -1$  from the top-side illumination. An additional constraint on the value of  $t$  for the bottom-side illumination may be added to ensure the metasurface exhibits the same performance regardless of the side of the illumination. In that case, a larger degree of design freedom is desirable to engineer a metasurface cell that presents a desired pair of transmission coefficients for the two different illuminations, presenting a more challenging design problem.

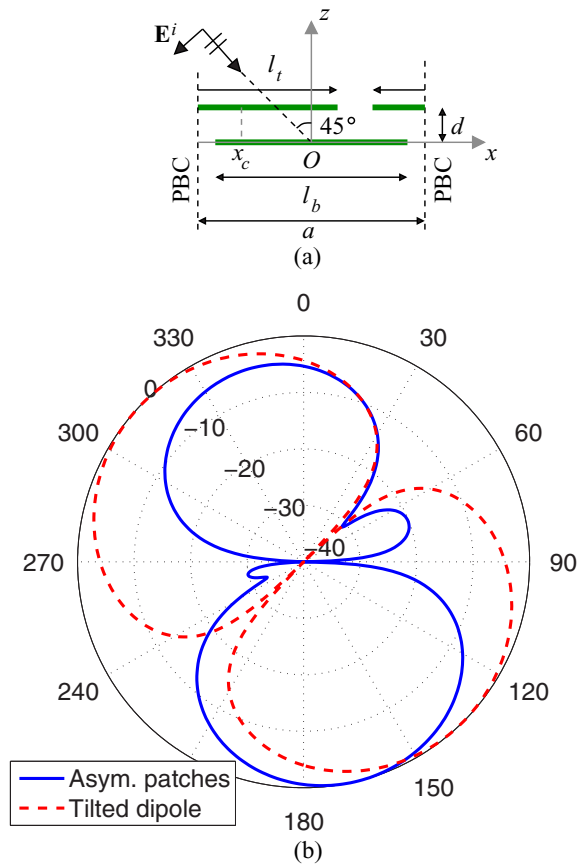


FIG. 3. Plane-wave scattering by an array of asymmetric conducting patches in free space. (a) A side view of the unit cell. (b) Comparison of the normalized single element scattering pattern in dB between the asymmetric patches and tilted dipole particles. The dimensions of the top and bottom patches are  $l_t \times w$  and  $l_b \times w$ , respectively. In terms of the free-space wavelength  $\lambda_0$ , the optimized design parameters are given by  $l_t = 0.4615\lambda_0$ ,  $l_b = 0.4633\lambda_0$ ,  $w = \lambda_0/4$ ,  $d = \lambda_0/10$ , and  $x_c = -0.0907\lambda_0$ . The unit-cell dimension is  $a = 0.5\lambda_0$  in both  $x$ - and  $y$ -axis directions.

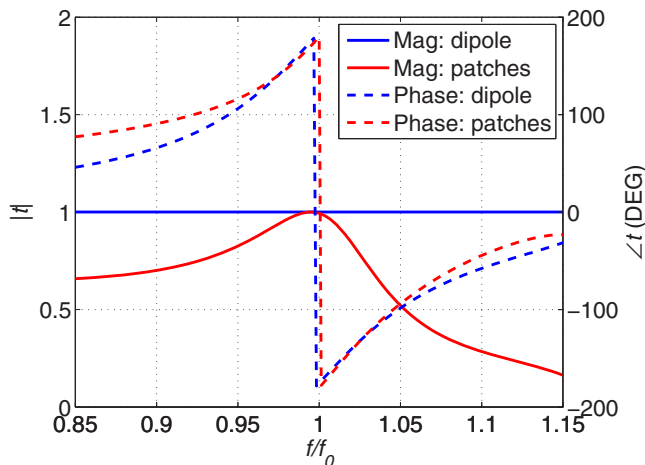


FIG. 4. Comparison between transmission coefficients of the tilted dipole array of Fig. 2 and the asymmetric patch array of Fig. 3 vs frequency. In each design, the frequency is normalized by the respective design frequency  $f_0$ . For the dipole array, the element port is short-circuited, i.e.,  $X_l = 0$ .

For the tilted dipole array in Sec. III, zero reflection is achieved by properly orienting the dipoles, owing to a simple relation between their geometry and pattern. In contrast, a null has to be synthesized based on destructive interference between the two patches comprising the unit cell for the asymmetric patches. Hence, the asymmetric patch metasurface is expected to be narrow band in achieving full transmission. This is well demonstrated in Fig. 4, where the transmission coefficients  $t$  for the two metasurfaces under  $45^\circ$  incidence in Figs. 2 and 3 are compared versus frequency. For both metasurfaces, the transmission phase undergoes frequency dispersion of the standard Lorentz-type resonance. The tilted dipole array features a dispersionless perfect transmission. In comparison, the transmission magnitude of the asymmetric patch array exhibits a narrow-band resonance with 100% transmission realized near the design frequency. The fractional bandwidth for 90% transmission power is found to be 3.2%.

As a result, we can conclude that a set of two strongly coupled and properly positioned *horizontal* electric dipoles provides the same asymmetric reflection and transmission properties as tilted dipoles, polarizable both in the array plane and in the normal direction. From the fundamental point of view this is possible because both structures can be considered as equivalent electric and magnetic dipoles polarized in the array plane. As expected from the Huygens principle, the presence of both tangential polarizations enables the most general reciprocal response of metasurfaces. With one type of tangential polarization in the case of lossless particles, full transmission is achievable only with vanishing polarization, resulting in  $t = 1$ ; full reflection necessarily requires  $|r| = 1$ , which reduces to  $r = \mp 1$  for electric/magnetic polarization. As follows from the results of [14] and [15] (see also the review paper [21]) for normal incidence cases, a metasurface which supports both electric and magnetic surface currents allows full control of the transmission phase without reflections. Another interesting example of similar equivalency can be seen considering an array of lossy tilted electric dipoles [22].

It is well known that if the electric dipoles are polarized along the array plane (no tilt), a maximum of one-half of the incident plane-wave power can be absorbed [13,23]: both electric and magnetic tangential polarizations are necessary to allow full absorption. However, if the electric dipoles are tilted, full power absorption becomes possible [22]. The reason is the same as for the considered equivalency of lossless arrays: additional normal polarization of tilted electric dipoles plays the same role as the equivalent *tangential* magnetic polarization.

As a concluding remark regarding the equivalence of the examples considered in Secs. III and IV we state that although the two examples provide identical results at a single design frequency and incidence angle, on one hand, the metasurface of titled dipoles is more robust with respect to frequency and load variations as the null in the scattering pattern is fixed by the dipole axis direction. On the other hand, the metasurface of asymmetric patches requires a careful synthesis to achieve the desired performance at a single frequency and its manufacturing is easier. Therefore, in this case there is a tradeoff between easy manufacturing process and robustness of the response.

In the previous examples, the same polarization of the incident field was maintained in the reflected and transmitted fields. Next, we consider two metasurfaces, with and without normal polarizations, that allow polarization rotation in the reflected and transmitted fields. These examples further demonstrate the conclusion that the transverse currents are enough for engineering wave polarization manipulation at transmission and reflection.

## V. METASURFACE OF CHIRAL INCLUSIONS WITH ONLY NORMAL POLARIZATIONS

In order to transform polarization, canonical chiral particles may be used to construct a metasurface. The goal here is to show that two metasurfaces with differently oriented chiral inclusions and hence creating different polarization currents can behave equally. In the first example, we consider a metasurface of chiral inclusions as is depicted in Fig. 5(a).

The excitation is considered to be similar to the two previous cases (TM incidence, with the  $xz$  excitation plane— $\mathbf{E}^i$  and  $\mathbf{k}$  are in the excitation plane). In this case the only excited polarizations are  $P_n$  and  $M_n$  (i.e., case no. 4). Due to the symmetric excitation with respect to the  $xz$  plane [see Fig. 5(a)], the tangential magnetic component of the incident field does not excite the particles. Moreover, we assume that tangential electric polarization of the loop induced by  $E_x^i$  can be neglected as a nonresonant effect in the operational frequency band. As a result, the normal component of the electric field is the only responsible field component for the excitation of metasurface.

Considering the above assumptions, it is easy to relate these polarizations to the incident field through the polarizabilities  $\hat{\alpha}_{zz}^{ee}$  and  $\hat{\alpha}_{zz}^{me}$ . The result reads [10]

$$P_n = P_z = \frac{\hat{\alpha}_{zz}^{ee}}{S} \cos \theta E^i, \quad (25)$$

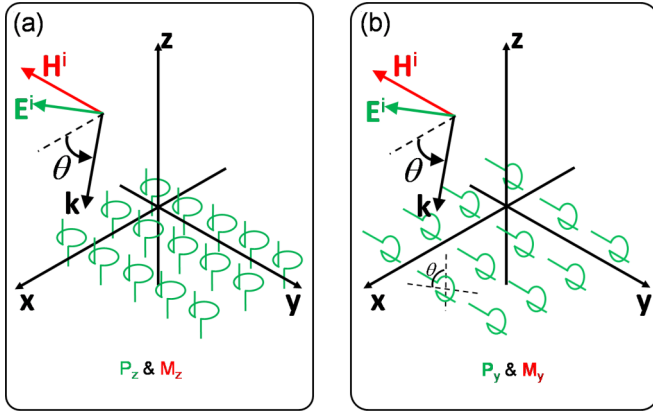


FIG. 5. Schematic of the two physically different metasurfaces with equal responses to the same excitation. The natures of the induced currents are different. In (a), there are only normal (with respect to the metasurface plane) induced electric and magnetic moments while in (b) there are only tangential electric and magnetic moments. The plane of incidence is  $xz$  and  $\mathbf{H}^i$  is in the  $y$  direction.

and

$$M_n = M_z = \frac{\hat{\alpha}_{zz}^{me}}{S} \cos \theta E^i. \quad (26)$$

Then, by considering the definitions (3) and (4), and by using the expressions (5) and (6) for the reflected and transmitted fields, one obtains

$$\frac{\mathbf{E}_t^r}{E_x^i} = r_{co} \hat{\mathbf{x}} + r_{cr} \hat{\mathbf{y}} = i \frac{\omega \eta}{2S} \cos^2 \theta \left[ -\frac{\hat{\alpha}_{zz}^{ee}}{|\sin \theta|} \hat{\mathbf{x}} \pm \frac{\hat{\alpha}_{zz}^{me}}{\eta \sin^2 \theta} \hat{\mathbf{y}} \right], \quad (27)$$

and

$$\frac{\mathbf{E}_t^t}{E^i} = t_{co} \hat{\mathbf{x}} + t_{cr} \hat{\mathbf{y}} = \hat{\mathbf{x}} + i \frac{\omega \eta}{2S} \cos^2 \theta \left[ \frac{\hat{\alpha}_{zz}^{ee}}{|\sin \theta|} \hat{\mathbf{x}} \pm \frac{\hat{\alpha}_{zz}^{me}}{\eta \sin^2 \theta} \hat{\mathbf{y}} \right]. \quad (28)$$

Here, the co- and cross-polar components of the reflected and transmitted fields are represented by subscripts ‘‘co’’ and ‘‘cr’’, respectively. As is clear from Eqs. (27) and (28), the magnetoelectric polarizability component (chirality parameter)  $\hat{\alpha}_{zz}^{me}$  is responsible for the creation of the cross-polarized reflected/transmitted field. Notice from Eqs. (4)–(6) that the presence of a normal magnetic polarization  $M_n$  results always in the creation of the cross-polar reflected/transmitted field. Moreover, Eqs. (27) and (28) show that only a part of the incident field is rotated in the reflected/transmitted regime. Furthermore, they show that decreasing the angle  $\theta$  results in increasing the rotated part of the reflected/transmitted field.

Next, we present an equivalent metasurface which possesses only tangential (with respect the metasurface plane) currents (i.e., case no. 2) in contrast to the present example which includes only normal currents. However, we demonstrate that although physically different, they can provide the same reflected/transmitted field as the case with normal currents, assuming the same incident aspect.

## VI. METASURFACE OF CHIRAL INCLUSIONS WITH ONLY TANGENTIAL POLARIZATIONS

In this example we use the same inclusions as in the previous case. However, we orient the inclusions so that only tangential currents are excited [see Fig. 5(b)]. The loops’ axes as well as the dipoles are oriented along the  $y$  axis. In this case one may obtain the tangential polarizations as [10]

$$\mathbf{P}_t = -\frac{\hat{\alpha}_{yy}^{em}}{\eta S} E^i \hat{\mathbf{y}} \quad (29)$$

and

$$\mathbf{M}_t = -\frac{\hat{\alpha}_{yy}^{mm}}{\eta S} E^i \hat{\mathbf{y}}. \quad (30)$$

Notice that with the specific alignment of chiral particles with respect to the incident field shown in Fig. 4(b), the polarization components  $P_x$  and  $P_z$ , corresponding to the effective polarizabilities  $\hat{\alpha}_{xx}^{ee}$  and  $\hat{\alpha}_{zz}^{ee}$ , are not excited. Now again, by considering the definitions (3) and (4), and by using the expressions (5) and (6) for the reflected and transmitted fields, it is easy to show that

$$\frac{\mathbf{E}_t^r}{E_x^i} = r_{co} \hat{\mathbf{x}} + r_{cr} \hat{\mathbf{y}} = i \frac{\omega \eta}{2S} \left[ -\frac{\hat{\alpha}_{yy}^{mm}}{\eta^2 |\sin \theta|} \hat{\mathbf{x}} \mp \frac{\hat{\alpha}_{yy}^{em}}{\eta \sin^2 \theta} \hat{\mathbf{y}} \right] \quad (31)$$

and

$$\frac{\mathbf{E}_t^t}{E_x^i} = t_{co} \hat{\mathbf{x}} + t_{cr} \hat{\mathbf{y}} = \hat{\mathbf{x}} + i \frac{\omega \eta}{2S} \left[ \frac{\hat{\alpha}_{yy}^{mm}}{\eta^2 |\sin \theta|} \hat{\mathbf{x}} \mp \frac{\hat{\alpha}_{yy}^{em}}{\eta \sin^2 \theta} \hat{\mathbf{y}} \right]. \quad (32)$$

These equations represent exactly the same type of response as Eqs. (27) and (28), that is, the rotated incident field through a magnetoelectric coupling coefficient  $\hat{\alpha}^{me}$ . Indeed, by comparing the equivalent currents Eq. (25) with Eq. (30) and Eq. (26) with Eq. (29) and by using the definitions (3) and (4) (or equivalently by comparing the reflection/transmission coefficients of these two cases), one may conclude that the metasurface of this type is equivalent to the previous case if

$$\frac{\hat{\alpha}_{yy}^{mm}}{\eta^2} = \hat{\alpha}_{zz}^{ee} \cos^2 \theta \quad (33)$$

and

$$\hat{\alpha}_{yy}^{em} = -\hat{\alpha}_{zz}^{me} \cos^2 \theta. \quad (34)$$

In contrast to the topology of Fig. 5(a), in the equivalent realization of Fig. 5(b) both electric and magnetic polarization vectors are in the array plane. Although the canonical-helix example shown here contains three-dimensional chiral particles, this fact suggests that it is possible to realize all the necessary polarizability components using only purely planar conductive (or otherwise polarizable) elements. Indeed, it is known that the required chiral effects are possible in arrays of properly shaped metal elements printed on one side of a dielectric substrate [24–26]. Moreover, the chiral effect can be created by using two [27,28] or more [29] parallel twisted arrays of planar elements. Furthermore, for particular directions of oblique illumination, even a single layer of planar elements can be sufficient [30,31] to provide chiral effects.



## VII. CONCLUSION

We have demonstrated that physically different metasurfaces, either having or not having normal induced polarization components, provide equal far-field electromagnetic responses. The demonstration has been carried out with attention to the case of metasurfaces with complex asymmetric responses. We have investigated the role of different components of the surface polarizations induced by the incident field, that is, normal and tangential with respect to the metasurface plane. We have demonstrated how the response of a surface with induced electric polarization with both tangential and normal components can be emulated by the response of a metasurface with only tangential induced current components for the case when transmitted and reflected fields have the same polarization as the incident field. We have also proven that for TM polarized incident waves the presence of normal magnetic polarization will always result in transmitted and reflected waves with a polarization rotation with respect to the incident wave. Moreover, for this latter case, we have demonstrated how the response of a metasurface with only normal induced polarization currents (both electric and magnetic) can be emulated with only tangential induced polarization currents. We have found the analytic conditions which make these

metasurfaces equivalent. We also state that based on Eqs. (1) and (2) a dual situation is valid for the TE waves, i.e., the presence of a normal electric current will always result in polarization rotation of the transmitted and reflected waves with respect to that of the incident wave.

Furthermore, we have proven that an arbitrary wave manipulation by the metasurface, which is possible with normal induced polarization currents, can be obtained also by using only equivalent tangential induced currents, hence removing the need for induced normal polarizations in a metasurface. In summary, we specifically show that adding normally directed currents does not enable new functionality but it could bring advantages in design and performance of particular devices. In other words, a response of a metasurface can be obtained by alternative realizations, either having or not having normal polarizations. We hope that these fundamental results will attract interest in the study of planar two-dimensional engineered resonant surfaces with optimized and application-required properties.

## ACKNOWLEDGMENT

M.A. and F.C. would like to acknowledge partial support by the W. M. Keck Foundation.

- 
- [1] S. B. Glybovski, S. A. Tretyakov, P. A. Belov, Y. S. Kivshar, and C. R. Simovski, *Metasurfaces: From microwaves to visible*, *Phys. Rep.* **634**, 1 (2016).
  - [2] C. L. Holloway, E. F. Kuester, J. A. Gordon, J. O'Hara, J. Booth, and D. R. Smith, An overview of the theory and applications of metasurfaces: The two-dimensional equivalents of metamaterials, *IEEE Antennas Propag. Mag.* **54**, 10 (2012).
  - [3] A. V. Kildishev, A. Boltasseva, and V. M. Shalaev, Planar photonics with metasurfaces, *Science* **339**, 1232009 (2013).
  - [4] N. Yu and F. Capasso, Flat optics with designer metasurfaces, *Nat. Mater.* **13**, 139 (2014).
  - [5] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves* (Wiley, New York, 1994), pp. 185–187.
  - [6] C. Pfeiffer and A. Grbic, Emulating Nonreciprocity with Spatially Dispersive Metasurfaces Excited at Oblique Incidence, *Phys. Rev. Lett.* **117**, 077401 (2016).
  - [7] M. Idemen and H. Serbest, Boundary conditions of the electromagnetic field, *Electron. Lett.* **23**, 704 (1987).
  - [8] M. M. Idemen, *Discontinuities in the Electromagnetic Field* (Wiley, New York, 2011).
  - [9] E. F. Kuester, M. A. Mohamed, M. Picket-May, and C. L. Holloway, Averaged transition conditions for electromagnetic fields at a metafilm, *IEEE Trans. Antennas Propag.* **51**, 2641 (2003).
  - [10] M. Albooyeh, S. Tretyakov, and C. Simovski, Electromagnetic characterization of bianisotropic metasurfaces on refractive substrates: General theoretical framework, *Ann. Phys. (Berlin)* **528**, 721 (2016).
  - [11] Sergei Tretyakov, *Analytical Modeling in Applied Electromagnetics* (Artech House, Boston, London, 2003).
  - [12] Of course, if the metasurface is excited by a plane wave at normal incidence ( $\mathbf{k}_t = 0$ ), normal polarizations produce no secondary plane waves at all, and it is obvious that we do not need them in order to engineer normal-incidence response.
  - [13] D.-H. Kwon and D. M. Pozar, Optimal characteristics of an arbitrary receive antenna, *IEEE Trans. Antennas Propag.* **57**, 3720 (2009).
  - [14] C. Pfeiffer and A. Grbic, Metamaterial Huygens' Surfaces: Tailoring Wave Fronts with Reflectionless Sheets, *Phys. Rev. Lett.* **110**, 197401 (2013).
  - [15] T. Niemi, A. O. Karilainen, and S. A. Tretyakov, Synthesis of polarization transformers, *IEEE Trans. Antennas Propag.* **61**, 3102 (2013).
  - [16] F. Capolino, A. Vallecchi, and M. Albani, Equivalent transmission line model with a lumped x-circuit for a metalayer made of pairs of planar conductors, *IEEE Trans. Antennas Propag.* **61**, 852 (2013).
  - [17] Effective polarizability of the dipole refers to the polarizability of the dipole considering the interaction effects of other dipoles in the metasurface.
  - [18] Roger F. Harrington, *Field Computation by Moment Methods* (Wiley, New York, 1993).
  - [19] R. Alaei, M. Albooyeh, M. Yazdi, N. Komjani, C. Simovski, F. Lederer, and C. Rockstuhl, Magnetolectric coupling in nonidentical plasmonic nanoparticles: Theory and applications, *Phys. Rev. B* **91**, 115119 (2015).
  - [20] M. Yazdi, M. Albooyeh, R. Alaei, V. Asadchy, N. Komjani, C. Rockstuhl, C. R. Simovski, and S. Tretyakov, A bianisotropic metasurface with resonant asymmetric absorption, *IEEE Trans. Antennas Propag.* **63**, 3004 (2015).
  - [21] S. A. Tretyakov, Metasurfaces for general transformations of electromagnetic fields, *Phil. Trans. R. Soc. A* **373**, 20140362 (2015).

- [22] D.-H. Kwon, Received voltage and power for an arbitrary element of infinite planar arrays, in *IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting 2015* (IEEE, 2015), pp. 2505–2506.
- [23] Y. Ra'di, C. R. Simovski, and S. A. Tretyakov, Thin perfect absorbers for electromagnetic waves: Theory, design, and realizations, *Phys. Rev. Applied* **3**, 037001 (2015).
- [24] A. Papakostas, A. Potts, D. M. Bagnall, S. L. Prosvirnin, H. J. Coles, and N. I. Zheludev, Optical Manifestations of Planar Chirality, *Phys. Rev. Lett.* **90**, 107404 (2003).
- [25] M. Kuwata-Gonokami, N. Saito, Y. Ino, M. Kauranen, K. Jefimovs, T. Vallius, J. Turunen, and Y. Svirko, Giant Optical Activity in Quasi-Two-Dimensional Planar Nanostructures, *Phys. Rev. Lett.* **95**, 227401 (2005).
- [26] S. I. Maslovski, D. K. Morits, and S. A. Tretyakov, Symmetry and reciprocity constraints on diffraction by gratings of quasi-planar particles, *J. Opt. A: Pure Appl. Opt.* **11**, 074004 (2009).
- [27] A. V. Rogacheva, V. A. Fedotov, A. S. Schwanecke, and N. I. Zheludev, Giant Gyrotropy due to Electromagnetic-Field Coupling in a Bilayered Chiral Structure, *Phys. Rev. Lett.* **97**, 177401 (2006).
- [28] M. Decker, M. W. Klein, M. Wegener, and S. Linden, Circular dichroism of planar chiral magnetic metamaterials, *Opt. Lett.* **32**, 856 (2007).
- [29] Y. Zhao, M. A. Belkin, and A. Alù, Twisted optical metamaterials for planarized ultrathin broadband circular polarizers, *Nat. Commun.* **3**, 870 (2012).
- [30] A. A. Sochava, C. R. Simovski, and S. A. Tretyakov, Chiral effects and eigenwaves in bi-anisotropic omega structures, in *Advances in Complex Electromagnetic Materials, NATO ASI Series High Technology*, edited by A. Priou, A. Sihvola, S. Tretyakov, and A. Vinogradov (Kluwer Academic, Dordrecht, 1997), Vol. 28, pp. 85–102.
- [31] E. Plum, V. A. Fedotov, and N. I. Zheludev, Extrinsic electromagnetic chirality in metamaterials, *J. Opt. A: Pure Appl. Opt.* **11**, 074009 (2009).