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In this paper we show that the coherence properties of the radiation emanating from an aperture in the single wall of a half-space blackbody cavity can be determined by considering how free-space blackbody radiation is scattered by those apertures. That is, although the presence of the cavity wall modifies the blackbody radiation field, it has no effect on the field components that are relevant in the scattering computations. This conclusion is essential since it justifies the methodology employed in earlier papers. Furthermore, our analysis shows that this computationally simple procedure is rigorous only for the half-space geometry and it also gives indications of when the results obtained by the procedure provide good approximations of the radiated field properties. This work thereby defines the domain of validity of earlier results connected with the coherence properties of blackbody radiation outside the cavity.

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I. INTRODUCTION

The importance and abundance of thermal emitters imply that a good understanding of blackbody radiation is essential to the analysis and modeling of radiation phenomena. It was the desire to explain the early measurements of the spectrum of light (and radiated heat) obtained from heated bodies that led Planck to formulate his celebrated expression for the spectral distribution of blackbody radiation, given by [1–3]

$$4a_0(\omega) = \frac{2\hbar\omega^3}{\pi c^3} \frac{1}{\exp(\hbar\omega/k_B T) - 1}, \quad (1)$$

where T is the absolute (equilibrium) temperature of the body, k_B is the Boltzmann constant, and \hbar is the reduced Planck constant. Here the left-hand side is displayed in a form, which is consistent with the notation used in [4].

Although Planck in places applied rigorous electromagnetic theory in his analysis [1], Planck's description of the spatial behavior of the blackbody radiation field was mainly based on ray optics considerations [5]. It was only later that a full electromagnetic treatment of blackbody radiation in closed (infinitely large) cavities was established [6–10]. The far-field patterns of blackbody apertures and other surface emitters have been extensively studied in radiometry [11], but there the treatment is in terms of scalar fields. Furthermore, for blackbody radiation, in particular, the planar source distribution is deduced from the far-field properties of the radiation rather than vice versa (see for example [12] and [13]). It seems that the electromagnetic cross-spectral density of a blackbody field in the aperture was first determined in [14], where it was also shown that the radiation emanating from the aperture is Lambertian and unpolarized in every direction of the far field. These results were extended in the paraxial case to the cross-spectral density of the far field in [15]. As is shown in [4] the influence of an opening in the cavity wall on the field inside the aperture is, however, treated incompletely in these papers. The correct expressions for the cross-spectral density in the aperture as well as in the far field are also derived therein.

In the analysis of the blackbody field inside the cavity aperture as done by us and others [4,14,15], the cross-spectral density function of the field is assumed to be that of blackbody radiation in free space. The correctness of this assumption is clearly not self-evident and it is, in fact, incorrect as we will show in this work. However, as our results indicate, the deviations from the correct cross-spectral density prove to be such that the results concerning the field in the aperture and the field radiated from the aperture remain unchanged. This is shown to follow from the specific half-space cavity geometry employed in the referenced papers and does not extend rigorously in any obvious way to other cavity geometries. In this specific geometry the aperture itself is also demonstrated to have no influence on the cross-spectral density of the field inside the cavity or in the cavity aperture.

The structure of this paper is such that in Sec. II we present the main results of the fluctuation-dissipation theorem as applied to electromagnetic fluctuations in blackbody cavities, which were originally derived by Agarwal [10]. We recall these results here since the notations in his paper differ somewhat from ours. Also, we present the derivations in a form which avoids potential pitfalls related to the singularities in the electromagnetic dyadic Green's functions (see, e.g., [16] for a discussion of the singularities). In Sec. III we consider the cross-spectral density tensors of free-space and half-space cavities and discuss both their differences and their similarities. Specifically we show that in the half-space geometry considered here these tensors will always yield the same results for fields propagating away from the half-space cavity. Finally, we summarize our results and discuss their implications in Sec. IV.

II. CORRELATION TENSORS DERIVED FROM THE FLUCTUATION-DISSIPATION THEOREM

Following Agarwal [10] and for mathematical symmetry, we use Gaussian units to write the Maxwell equations at angular frequency ω in vacuum as

$$\nabla \times \mathbf{E}(\mathbf{r}, \omega) = ik_0[\mathbf{H}(\mathbf{r}, \omega) + 4\pi\mathbf{M}(\mathbf{r}, \omega)] \quad (2)$$

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and

$$\nabla \times \mathbf{H}(\mathbf{r}, \omega) = -ik_0[\mathbf{E}(\mathbf{r}, \omega) + 4\pi\mathbf{P}(\mathbf{r}, \omega)], \quad (3)$$

where $k_0 = \omega/c$ and c is the speed of light in vacuum. Here \mathbf{E} and \mathbf{H} denote the electric and magnetic fields, which are sourced by the (external) polarization and magnetization vectors \mathbf{P} and \mathbf{M} , respectively. For a unique solution to exist, these equations must also be accompanied by boundary conditions, which can be expressed in terms of the tangential components of \mathbf{E} and \mathbf{H} , and the normal components of $\mathbf{E} + 4\pi\mathbf{P}$ and $\mathbf{H} + 4\pi\mathbf{M}$ at interfaces, together with the Silver-Müller radiation conditions [17] at infinity. When blackbody radiation is studied the boundary conditions are usually due to constraints imposed by the geometry of the blackbody cavity.

It is useful to separate the effect of the boundary conditions from the rest of the solution to Eqs. (2) and (3). This can be done by noting that, for a well-posed problem, there is a unique solution \mathbf{E}' and \mathbf{H}' to the specified boundary conditions and the homogeneous system of differential equations obtained from Eqs. (2) and (3) by putting $\mathbf{P}, \mathbf{M} = \mathbf{0}$. The difference fields $\Delta\mathbf{E} = \mathbf{E} - \mathbf{E}'$ and $\Delta\mathbf{H} = \mathbf{H} - \mathbf{H}'$ then satisfy Eqs. (2) and (3), but with the boundary conditions changed so that they are everywhere replaced by the value zero. We now consider the solution to this modified system when $\mathbf{M} = \mathbf{0}$. The symmetrical roles of \mathbf{P} and \mathbf{M} means that the situation $\mathbf{P} = \mathbf{0}$ has a completely analogous treatment, and hence the simplification can be used to obtain the full solution as the superposition of these two special cases. Thereby we consider the equations

$$\nabla \times \Delta\mathbf{E}(\mathbf{r}, \omega) = ik_0\Delta\mathbf{H}(\mathbf{r}, \omega) \quad (4)$$

and

$$\nabla \times \Delta\mathbf{H}(\mathbf{r}, \omega) = -ik_0[\Delta\mathbf{E}(\mathbf{r}, \omega) + 4\pi\mathbf{P}(\mathbf{r}, \omega)]. \quad (5)$$

By taking the curl of the latter equation and by using the former equation to get rid of $\Delta\mathbf{E}$, we arrive at the wave equation

$$(\nabla \times \nabla \times - k_0^2)\Delta\mathbf{H}(\mathbf{r}, \omega) = -4\pi ik_0\nabla \times \mathbf{P}(\mathbf{r}, \omega), \quad (6)$$

which immediately implies the divergence condition

$$\nabla \cdot \Delta\mathbf{H}(\mathbf{r}, \omega) = 0. \quad (7)$$

When we apply the vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ together with this divergence condition, we can rewrite Eq. (6) as

$$(\nabla^2 + k_0^2)\Delta\mathbf{H}(\mathbf{r}, \omega) = 4\pi ik_0\nabla \times \mathbf{P}(\mathbf{r}, \omega), \quad (8)$$

which is the vector Helmholtz equation.

A fundamental solution to the vector Helmholtz equation (8) is provided by the tensor Green's function $G(\mathbf{r}, \mathbf{r}', \omega)\bar{\mathbf{I}}$, where $\bar{\mathbf{I}}$ is the unit tensor and $G(\mathbf{r}, \mathbf{r}')$ is the scalar Green's function

$$G(\mathbf{r}, \mathbf{r}', \omega) = \frac{\exp(ik_0|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|}, \quad (9)$$

which satisfies the scalar Helmholtz equation

$$(\nabla^2 + k_0^2)G(\mathbf{r}, \mathbf{r}', \omega) = -4\pi\delta(\mathbf{r} - \mathbf{r}'). \quad (10)$$

The tensor $G(\mathbf{r}, \mathbf{r}', \omega)\bar{\mathbf{I}}$ does not in general satisfy the zero boundary conditions that accompany Eqs. (4) and (5). We can,

however, add to this tensor a regular tensor, which satisfies the homogeneous counterpart to the vector Helmholtz equation (8) for all \mathbf{r}' , such that the resulting tensor $\bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega)$ satisfies the zero boundary conditions as well as the equation

$$(\nabla^2 + k_0^2)\bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) = -4\pi\delta(\mathbf{r} - \mathbf{r}')\bar{\mathbf{I}}. \quad (11)$$

Indeed, the solution to the vector Helmholtz equation (8) and thus via Eq. (5) to the system consisting of the equations (4) and (5), can be written as

$$\Delta\mathbf{H}(\mathbf{r}, \omega) = -ik_0 \int \bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{P}(\mathbf{r}', \omega) d\mathbf{r}', \quad (12)$$

$$\Delta\mathbf{E}(\mathbf{r}, \omega) = \int \nabla \times \bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{P}(\mathbf{r}', \omega) d\mathbf{r}' - 4\pi\mathbf{P}(\mathbf{r}, \omega). \quad (13)$$

Hence the zero boundary conditions are satisfied by $\Delta\mathbf{E}(\mathbf{r}, \omega)$ and $\Delta\mathbf{H}(\mathbf{r}, \omega)$ when they are satisfied by $\bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega)$ and $\nabla \times \bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega)$ for all \mathbf{r}' .

With the solution given by Eqs. (12) and (13), together with the analogous solution for the situation when $\mathbf{P} = \mathbf{0}$ and the homogeneous solutions given by \mathbf{E}' and \mathbf{H}' , we can now write the solution to the system consisting of Eqs. (2) and (3), and the accompanying boundary conditions as

$$\begin{aligned} \mathbf{E}(\mathbf{r}, \omega) &= \mathbf{E}'(\mathbf{r}, \omega) - 4\pi\mathbf{P}(\mathbf{r}, \omega) \\ &+ \int \nabla \times \bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{P}(\mathbf{r}', \omega) d\mathbf{r}' \\ &+ ik_0 \int \bar{\mathbf{G}}_E(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{M}(\mathbf{r}', \omega) d\mathbf{r}' \end{aligned} \quad (14)$$

and

$$\begin{aligned} \mathbf{H}(\mathbf{r}, \omega) &= \mathbf{H}'(\mathbf{r}, \omega) - 4\pi\mathbf{M}(\mathbf{r}, \omega) \\ &+ \int \nabla \times \bar{\mathbf{G}}_E(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{M}(\mathbf{r}', \omega) d\mathbf{r}' \\ &- ik_0 \int \bar{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \cdot \nabla' \times \mathbf{P}(\mathbf{r}', \omega) d\mathbf{r}', \end{aligned} \quad (15)$$

where $\bar{\mathbf{G}}_E$ differs from $\bar{\mathbf{G}}_H$ in the boundary conditions it satisfies.

Next we will consider the linear response of the electric and magnetic fields to point sources. For this purpose we will consider the special case when the magnetization vanishes and the polarization is of the form

$$\mathbf{P}_n(\mathbf{r}, \omega) = \delta_n(\mathbf{r} - \mathbf{r}'_0)\mathbf{P}(\omega), \quad (16)$$

where $\{\delta_n\}$ is a sequence of functions that converges to the δ function when $n \rightarrow \infty$ [18] and $\mathbf{P}(\omega)$ is a constant (in position) polarization vector. To ensure convergence of the expressions in what follows, we take the limit $n \rightarrow \infty$, which makes $\lim_{n \rightarrow \infty} \mathbf{P}_n(\mathbf{r}, \omega)$ a point source at \mathbf{r}_0 , only at the end of our calculations. This referral of limit taking reflects the point of view that the point source (δ function) is a mathematical construct corresponding to very small real sources. As before, the case with no polarization and a nonvanishing magnetization is completely analogous.

When we introduce the polarization (16) and set the magnetization to zero in Eqs. (14) and (15), we obtain after

rearranging the results the representations

$$\begin{aligned} \mathbf{E}_n(\mathbf{r}, \omega) = & \mathbf{E}'(\mathbf{r}, \omega) + \left\{ \int \nabla \times \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \right. \\ & \left. \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}' - 4\pi \overline{\mathbf{I}} \delta_n(\mathbf{r} - \mathbf{r}_0) \right\} \cdot \mathbf{P}(\omega) \end{aligned} \quad (17)$$

and

$$\begin{aligned} \mathbf{H}_n(\mathbf{r}, \omega) = & \mathbf{H}'(\mathbf{r}, \omega) + \left\{ -ik_0 \int \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \right. \\ & \left. \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}' \right\} \cdot \mathbf{P}(\omega) \end{aligned} \quad (18)$$

for the electric and magnetic fields, respectively. Linear response theory now lets us connect the perturbations of the field with respect to $\mathbf{P}(\omega)$ to the susceptibility tensors of the system as [10]

$$\begin{aligned} \overline{\chi}_{ee,n}(\mathbf{r}, \mathbf{r}_0, \omega) = & \frac{\delta \langle \mathbf{E}_n(\mathbf{r}, \omega) \rangle}{\delta \mathbf{P}(\omega)} \\ = & \int \nabla \times \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}' \\ & - 4\pi \overline{\mathbf{I}} \delta_n(\mathbf{r} - \mathbf{r}_0) \end{aligned} \quad (19)$$

and

$$\begin{aligned} \overline{\chi}_{he,n}(\mathbf{r}, \mathbf{r}_0, \omega) = & \frac{\delta \langle \mathbf{H}_n(\mathbf{r}, \omega) \rangle}{\delta \mathbf{P}(\omega)} \\ = & -ik_0 \int \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}', \omega) \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}', \end{aligned} \quad (20)$$

where the argument \mathbf{r}_0 refers to the location of the to-be point source. Accordingly, the tensors defined here are strictly speaking the susceptibility tensors between the source polarization $\mathbf{P}(\omega)$ around the point \mathbf{r}_0 and the fields at the point \mathbf{r} . Only at the limit $n \rightarrow \infty$ do these tensors yield the susceptibility tensors between the field point \mathbf{r} and the source point \mathbf{r}_0 .

The fluctuation-dissipation theory connects the real and imaginary parts of the susceptibility tensors (of point sources) to the correlation tensors of the electric and magnetic field components at different spatial locations [10]. In particular, we then have

$$\begin{aligned} \overline{\mathbf{W}}_{ee}(\mathbf{r}, \mathbf{r}_0, \omega) = & \langle \mathbf{E}(\mathbf{r}, \omega) \mathbf{E}^*(\mathbf{r}_0, \omega) \rangle \\ = & \lim_{n \rightarrow \infty} \frac{4\pi a_0(\omega)}{k_0^3} \text{Im} \{ \overline{\chi}_{ee,n}(\mathbf{r}, \mathbf{r}_0, \omega) \} \\ = & \lim_{n \rightarrow \infty} \frac{4\pi a_0(\omega)}{k_0^3} \int \nabla \times \text{Im} \{ \overline{\mathbf{G}}_H^{(R)}(\mathbf{r}, \mathbf{r}', \omega) \} \\ & \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}' \\ = & \frac{4\pi a_0(\omega)}{k_0} \text{Im} \{ \overline{\mathbf{G}}_H^{(R)}(\mathbf{r}, \mathbf{r}_0, \omega) \} \\ = & \frac{4\pi a_0(\omega)}{k_0} \text{Im} \{ \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}_0, \omega) \}, \end{aligned} \quad (21)$$

where we have replaced the tensor $\overline{\mathbf{G}}_H$ by its regular form $\overline{\mathbf{G}}_H^{(R)}$ during the computation since the singularity stemming from the δ function in Eq. (11) is fully contained in the real part of that tensor. We have then used the definition of the δ sequence $\{\delta_n\}$ as well as the fact that the regular tensor satisfies the homogeneous version of the Helmholtz equation (11) to obtain the final result. In Eq. (21) the angular brackets denote ensemble averaging and the asterisk denotes complex conjugation. Observe that the convention in recent papers on coherence theory is to take the complex conjugate of the first vector in the angular brackets instead of the second, and hence our results here are the complex conjugates of results presented in such papers (including our own). The function $a_0(\omega)$ is the spectral distribution of blackbody radiation as given by Planck's radiation law (1).

Similarly to the derivation in Eq. (21) we have

$$\begin{aligned} \overline{\mathbf{W}}_{he}(\mathbf{r}, \mathbf{r}_0, \omega) = & \langle \mathbf{H}(\mathbf{r}, \omega) \mathbf{E}^*(\mathbf{r}_0, \omega) \rangle \\ = & \lim_{n \rightarrow \infty} (-i) \frac{4\pi a_0(\omega)}{k_0^3} \text{Re} \{ \overline{\chi}_{he,n}(\mathbf{r}, \mathbf{r}_0, \omega) \} \\ = & \lim_{n \rightarrow \infty} (-i) \frac{4\pi a_0(\omega)}{k_0^2} \int \text{Im} \{ \overline{\mathbf{G}}_H^{(R)}(\mathbf{r}, \mathbf{r}', \omega) \} \\ & \times \nabla' \delta_n(\mathbf{r}' - \mathbf{r}_0) d\mathbf{r}' \\ = & i \frac{4\pi a_0(\omega)}{k_0^2} \text{Im} \{ \nabla \times \overline{\mathbf{G}}_H(\mathbf{r}, \mathbf{r}_0, \omega) \}. \end{aligned} \quad (22)$$

Furthermore, because of the symmetry between the polarization and magnetization vectors, we completely analogously have

$$\begin{aligned} \overline{\mathbf{W}}_{hh}(\mathbf{r}, \mathbf{r}_0, \omega) = & \langle \mathbf{H}(\mathbf{r}, \omega) \mathbf{H}^*(\mathbf{r}_0, \omega) \rangle \\ = & \frac{4\pi a_0(\omega)}{k_0} \text{Im} \{ \overline{\mathbf{G}}_E(\mathbf{r}, \mathbf{r}_0, \omega) \} \end{aligned} \quad (23)$$

and

$$\begin{aligned} \overline{\mathbf{W}}_{eh}(\mathbf{r}, \mathbf{r}_0, \omega) = & \langle \mathbf{E}(\mathbf{r}, \omega) \mathbf{H}^*(\mathbf{r}_0, \omega) \rangle \\ = & -i \frac{4\pi a_0(\omega)}{k_0^2} \text{Im} \{ \nabla \times \overline{\mathbf{G}}_E(\mathbf{r}, \mathbf{r}_0, \omega) \}. \end{aligned} \quad (24)$$

We note that in obtaining these representations we have sidestepped the singularities in the tensor Green's functions by applying their regular counterparts at the critical points of the derivations. This is made possible by the fact that the imaginary parts of the original and the regularized tensors are the same.

III. BLACKBODY RADIATION IN A HALF-SPACE CAVITY

Here we consider only the electric correlation tensor of $\overline{\mathbf{W}}_{ee}$ in a blackbody cavity, but similar considerations apply also for the other tensors discussed in the previous section. In general the functional form of the correlation tensor is closely connected with the geometry of the blackbody cavity via the Green's function, as can be seen from the expression (21). Indeed, as the Green's function fully encompasses the electromagnetically significant geometry of the system it can be expected to change in general with changes in the geometry. Nevertheless, in some recent papers [4,14,15] the free-space blackbody correlation tensor is applied when

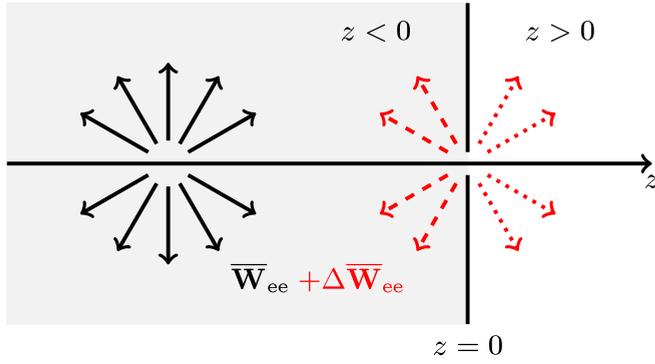


FIG. 1. (Color online) Illustration of the $z < 0$ half-space blackbody cavity geometry. Here the free-space blackbody field ($\overline{\mathbf{W}}_{ee}$) plane waves are denoted by solid (black) arrows and the half-space correction ($\Delta\overline{\mathbf{W}}_{ee}$) plane waves that are scattered from the wall at $z = 0$ are denoted by dashed (red) arrows. In addition, the radiated field plane waves in the half space $z > 0$ are represented by dotted (red) arrows.

considering the coherence properties of radiation emanating from a half-space blackbody cavity that is separated from vacuum by a (conducting) screen or wall with an aperture (see Fig. 1). In the following we will explore the justification for this choice and show that, even though the free-space correlation tensor is not equal to the correlation tensor of the half-space cavity, it still leads to the correct expressions for the radiated field.

A. Cross-spectral density tensors

To begin with we note that the electric cross-spectral density tensor (21) is for blackbody radiation in free space given by [9,10,19,20]

$$\overline{\mathbf{W}}_{ee}(\mathbf{r}, \mathbf{r}', \omega) = 4\pi a_0(\omega) \left(\overline{\mathbf{I}} + \frac{1}{k_0^2} \nabla \nabla \right) \frac{\sin(k_0 |\mathbf{r} - \mathbf{r}'|)}{k_0 |\mathbf{r} - \mathbf{r}'|}. \quad (25)$$

This cross-spectral density tensor is what is used in several papers [4,14,15] as the cross-spectral density tensor of the blackbody field in the half-space cavity depicted in Fig. 1, with no reference made to the possible effects of the wall at $z = 0$. To exactly analyze these effects on the cavity radiation we must know the material and build of this wall. Although the wall material is not specified in the referenced papers, a Planckian analysis of blackbody radiation suggests that it is sufficient to require that the wall is impermeable to radiation [1]. Here we follow Planck and take the cavity walls to be perfect conductors (but as becomes clear later, the choice is inessential), which are also perfect reflectors, whereby they cannot be permeable to radiation. Actually, it seems that the perfect conductor is the only “simple” material that fulfills the impermeability assumption. Furthermore, the cross-spectral density of the radiation in a blackbody cavity occupying the half space $z < 0$, with a perfectly conducting wall at $z = 0$, has already been determined by Agarwal [10], who obtained the additive correction (which we display here in our notation)

$$\Delta\overline{\mathbf{W}}_{ee}(\mathbf{r}, \mathbf{r}', \omega) = -\overline{\mathbf{W}}_{ee}(\mathbf{r}, \overline{\mathbf{R}} \cdot \mathbf{r}', \omega) \cdot \overline{\mathbf{R}} \quad (26)$$

to the cross-spectral density tensor $\overline{\mathbf{W}}_{ee}$ of Eq. (25). Here $\overline{\mathbf{R}} = \overline{\mathbf{I}} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}$ is a tensor that reflects the vector it operates on in the plane $z = 0$.

Since $\Delta\overline{\mathbf{W}}_{ee}$ does not vanish identically, it confirms our previous notion that the cavity geometry in general affects the cross-spectral density operator of blackbody radiation. To better understand how this additive correction might influence the field at the (yet to be opened) aperture, we turn our attention to the plane-wave representations of the cross-spectral tensor Eqs. (25) and (26).

B. Plane-wave representations

The cross-spectral density tensor in Eq. (25) also describes a uniform distribution of uncorrelated, unpolarized plane waves [19,21] and hence it can be expressed in the form (see, for example, [4])

$$\overline{\mathbf{W}}_{ee}(\mathbf{r}, \mathbf{r}', \omega) = a_0(\omega) \int_{\alpha} \int_{\alpha} \Delta(\hat{\mathbf{u}}' - \hat{\mathbf{u}}) (\overline{\mathbf{I}} - \hat{\mathbf{u}}\hat{\mathbf{u}}') \times \exp[ik_0(\hat{\mathbf{u}} \cdot \mathbf{r} - \hat{\mathbf{u}}' \cdot \mathbf{r}')] d\hat{\mathbf{u}} d\hat{\mathbf{u}}', \quad (27)$$

where the function Δ is the spherical delta function [22] and α denotes the region of solid angles of interest, which here is the complete spherical shell \mathcal{S} . When we use the representation (27) in Eq. (26) we get

$$\Delta\overline{\mathbf{W}}_{ee}(\mathbf{r}, \mathbf{r}', \omega) = a_0(\omega) \int_{\alpha} \int_{\alpha} \Delta(\overline{\mathbf{R}} \cdot \hat{\mathbf{u}}' - \hat{\mathbf{u}}) (\hat{\mathbf{u}}\hat{\mathbf{u}}' - \overline{\mathbf{R}}) \times \exp[ik_0(\hat{\mathbf{u}} \cdot \mathbf{r} - \hat{\mathbf{u}}' \cdot \mathbf{r}')] d\hat{\mathbf{u}} d\hat{\mathbf{u}}', \quad (28)$$

where we have made the change of integration variables $\hat{\mathbf{u}}' \rightarrow \overline{\mathbf{R}} \cdot \hat{\mathbf{u}}'$.

Let us now divide the plane waves in the expressions (27) and (28) into two groups, those propagating in the negative z direction and those propagating in the positive z direction in Fig. 1. In the free-space blackbody correlation tensor in Eq. (27) all the fields are mutually uncorrelated regardless of their propagation directions, which is reflected in the fact that the Δ -function argument in that representation vanishes only when $\hat{\mathbf{u}}' = \hat{\mathbf{u}}$. The additive correction given by Eq. (28), however, introduces correlations between fields propagating in directions that are reflections of each other in the z direction as mediated by the tensor $\overline{\mathbf{R}}$. Hence for the half-space cavity in Fig. 1 there are with respect to the free-space blackbody correlations (or lack thereof) additional correlations between the fields scattered in the negative z direction by the wall and the fields propagating in the positive z direction. Crucially, however, the transition into a half-space cavity by introducing the wall at $z = 0$ does not introduce any new correlations between plane-wave components propagating toward the $z > 0$ half space with respect to $z = 0$. Thereby, in particular, the cross-spectral density tensor of the fields that eventually propagate through the aperture that is placed in the wall and radiate outside the cavity is the same for both the free-space and the half-space blackbody cavities. This suggests that the results obtained when the free-space cross-spectral density tensor is used to represent the blackbody radiation in the half-space cavity [4,14,15] are actually valid. Indeed, as we discuss next, this property is unique to the half-space geometry, where the

effects of the aperture on the field inside the cavity can also be ignored.

C. Comparison of free-space and half-space blackbody fields

Next we compare the free-space blackbody field with the half-space blackbody field, represented by the solid, and by the solid and dashed arrows in Fig. 1, respectively. We consider the general situation where the composition of the wall at $z = 0$ is not specified. Thereby the (symmetry) properties specifically relating to the situation when the wall is a perfect conductor (perfect reflector) and contained in the representation (28) will not be addressed specifically since they are not of particular interest here.

When we consider the fields in the two geometries in terms of their plane-wave representations, we observe that in the free-space geometry fields are sourced in both the $z < 0$ (negative) and the $z > 0$ (positive) half spaces, whereas in the half-space geometry there are sources only in the negative half space, but with the wall at $z = 0$ scattering the fields propagating in the positive z direction into fields propagating in the negative z direction. This means that in general the fields propagating in the negative z direction inside the negative half space will have different correlation properties in the two geometries. In contrast, the fields propagating in the positive z direction in this half space will have exactly the same sources in both geometries and hence their correlation properties are also identical. This conclusion follows when we note that in neither geometry are the fields propagating in the negative z direction reflected into fields propagating in the positive z direction. Thus in terms of only the correlations between fields propagating in the positive z direction, the free-field cross-spectral density tensor and the half-space blackbody cross-spectral density tensor are identical, whatever the structure of the wall at $z = 0$. Particularly, the wall can include an arbitrary number of arbitrarily shaped apertures.

The fact that apertures in the wall $z = 0$ do not change the correlation properties of the fields propagating in the positive z direction means that the aperture can be treated as simply diffracting the fields from the negative half space into the positive half space. Therefore the blackbody field radiated (into the positive half space) can be determined from the free-space cross-spectral density function of the field by any method that can be used to compute the diffracted field. For example, in [4] we used the Rayleigh diffraction formula for this purpose with the assumption that the wall was impenetrable to radiation away from the aperture.

Finally, we note that the result presented here concerning the validity of the use of the free-space cross-spectral density tensor in place of the actual half-space cross-spectral density tensor for outgoing fields strictly holds only for the specific geometry of a half-space cavity, where space is divided by one perfectly planar wall of infinite extent. If the cavity wall is curved or the blackbody cavity has other wall segments at finite distances, the shielding effect of these walls and the

fields scattered from them will in general change the field at the aperture(s) and thereby cause it to be different from the free-space blackbody field therein.

IV. CONCLUSIONS

In this work we have shown that the free-space cross-spectral density function can successfully be used to determine the radiation emanating from the aperture of a half-space blackbody cavity. Specifically, we have shown that for fields propagating in the positive z direction the free-space cross-spectral density tensor exactly matches the cross-spectral density tensor of the half-space cavity. Furthermore, the fields propagating in the negative z direction in the half-space cavity will not be converted into fields propagating in the positive z direction, and hence in the half-space cavity geometry, when only the properties of the field radiated into the half-space $z > 0$ are of interest, any aperture in the cavity wall can be successfully modeled as only diffracting the radiated field, having no other effects.

Our results validate this procedure, which has been employed without comment by us and by others in previous papers, where the radiation from blackbody cavities has been studied. However, we also show that the cross-spectral density tensors are in general dependent on the cavity geometry, which means that the free-space approach is not generally valid.

Finally, it is of interest to note that no evanescent waves are present in either of the plane-wave representations (27) or (28). That is, neither the all-space (free-field) blackbody radiation nor the blackbody radiation due to a half-space cavity with a perfectly conducting wall has evanescent contributions at locations inside the cavity $z < 0$. For the all-space case this can be explained by the fact that the field at any point in the cavity consists of contributions from every spatial location in all space and these contributions together overwhelm the evanescent contribution when the total field is normalized to finite intensity (energy) in the limit of no absorption (see, for example, [20]). This explanation remains valid in the half-space case as well. The lack of evanescent waves, however, implies that no local sources contribute to the field anywhere in the cavity, which is of course somewhat bizarre. In view of the aforementioned normalization of the electromagnetic field we must conclude, that the sources are present, but that the energy related to any one source location is vanishingly small in the fluctuation-dissipation formalism when it is applied to infinite cavities. Indeed, that evanescent waves are present in finite cavity geometries can be seen, for example, by perusing the results Agarwal has obtained for cavities consisting of two parallel plates at a finite distance from each other [10].

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