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Induced interactions in a superfluid Bose-Fermi mixture

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We analyze a Bose-Einstein condensate (BEC) mixed with a superfluid two-component Fermi gas in the whole BCS-BEC crossover. Using a quasiparticle random-phase approximation combined with Beliaev theory to describe the Fermi superfluid and the BEC, respectively, we show that the single-particle and collective excitations of the Fermi gas give rise to an induced interaction between the bosons, which varies strongly with momentum and frequency. It diverges at the sound mode of the Fermi superfluid, resulting in a sharp avoided crossing feature and a corresponding sign change of the interaction energy shift in the excitation spectrum of the BEC. In addition, the excitation of quasiparticles in the Fermi superfluid leads to damping of the excitations in the BEC. Besides studying induced interactions themselves, we can use these prominent effects to systematically probe the strongly interacting Fermi gas.

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The interplay between induced interactions and superfluidity plays an important role in low-temperature physics. In metals, the phonon-mediated interaction between electrons leads to the formation of Cooper pairs [1], and induced electron-hole excitations significantly suppress the critical temperature of a BCS superconductor [2,3]. A prominent theory for high-temperature superconductivity is that it is caused by spin fluctuations leading to an attractive interaction [4], and induced interactions are important for understanding the properties of liquid-helium mixtures [5]. The systems where induced interactions are significant often consist of fermionic and bosonic degrees of freedom. In cold-atom gases, Bose-Fermi mixtures have been realized experimentally for sympathetic cooling [6–8], molecule formation [9–12], and studying few-body physics [13]. The theoretical studies have focused on mixtures where the Fermi gas is in the normal state [14–23]. Recently, an experimental breakthrough was reported with the realization of a mixture of superfluid ^{7}Li and ^{6}Li gases [24]. This opens up the exciting possibility to experimentally study the role of induced interactions in a Bose-Fermi mixture, where both components are superfluid.

Here we study a Bose-Einstein condensate (BEC) mixed with a two-component superfluid Fermi gas in the whole BCS-BEC crossover at zero temperature. Using a quasiparticle random-phase approximation (QRPA) to describe the excitations in the Fermi gas, combined with Beliaev theory for the bosons, we show how the fermions give rise to an induced frequency- or momentum-dependent Bose-Bose interaction, which diverges at the sound mode of the Fermi gas. This results in two qualitatively new effects. First, the dispersion relation of the bosons in the BEC is significantly changed at frequencies or momenta close to the sound mode of the Fermi gas. Second, bosonic excitations are damped due to dissipation, as they can excite quasiparticles in the superfluid Fermi gas [25]. These effects can be used to systematically probe the single-particle and collective properties of the strongly correlated Fermi gas.

We consider a gas of bosons with mass $m_{\rm B}$ mixed with a two-component ($\sigma = \uparrow,\downarrow$) gas of fermions with mass $m_{\rm F}$. The populations of the two fermionic states are taken to be the same. The Hamiltonian of the Bose-Fermi mixture is $H = H_{\rm B} + H_{\rm F} + H_{\rm BF}$, where

$$H_{\rm B} = \sum_{\mathbf{k}} \epsilon_k a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \frac{1}{2\mathcal{V}} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\rm B}(q) a^\dagger_{\mathbf{k}+\mathbf{q}} a^\dagger_{\mathbf{k}'-\mathbf{q}} a_{\mathbf{k}'} a_{\mathbf{k}} \tag{1}$$

is the Bose Hamiltonian with $\epsilon_k = k^2/2m_{\rm B}$,

$$H_{\rm F} = \sum_{\mathbf{k}\sigma} \frac{k^2}{2m_{\rm F}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{\mathcal{V}} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\rm F}(q) c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow} c^\dagger_{\mathbf{k}'-\mathbf{q}\downarrow} c_{\mathbf{k}'\downarrow} c_{\mathbf{k}\uparrow} \tag{2}$$

is the Fermi Hamiltonian, and

$$H_{\rm BF} = \frac{1}{\mathcal{V}} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}\sigma} V_{\rm BF}(q) c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma} a^\dagger_{\mathbf{k}'-\mathbf{q}} a_{\mathbf{k}'} \tag{3}$$

is the Bose-Fermi interaction. The operators $a_{\mathbf{k}}$ ($c_{\mathbf{k}\sigma}$) remove a boson (spin-σ fermion) with momentum $\mathbf{k}$, $\mathcal{V}$ is the volume of the system, and we work in units where $\hbar = k_{\rm B} = 1$. In the following, we replace the interactions with the corresponding low-energy scattering matrices: $V_{\rm B}(q) \to \mathcal{T}_{\rm B} = 4\pi a_{\rm B}/m_{\rm B}$, $V_{\rm F}(q) \to \mathcal{T}_{\rm F} = 4\pi a_{\rm F}/m_{\rm F}$, and $V_{\rm BF}(q) \to \mathcal{T}_{\rm BF} = 2\pi a_{\rm BF}/m_{\rm r}$, where $a_{\rm B}$, $a_{\rm BF}$, and $a_{\rm F}$ are the Bose-Bose, Bose-Fermi, and Fermi-Fermi scattering lengths, respectively, and $m_{\rm r} = m_{\rm B} m_{\rm F}/(m_{\rm B} + m_{\rm F})$ is the reduced mass. As usual, this corresponds to summing all ladder diagrams in a vacuum.

The presence of the Fermi gas induces an effective interaction between the bosons since one boson tends to attract or repel fermions, giving rise to a local change in the fermion density, which is felt by the second boson. Combined with the direct Bose-Bose interaction, this results in total interaction

$$V(q,\omega) = \mathcal{T}_{\rm B} + \mathcal{T}^2_{\rm BF}\chi(q,\omega). \tag{4}$$

Here $\chi(q,\omega)$ is the density-density response function for the fermions with momentum $\mathbf{q}$ and frequency ω. The corresponding Feynman diagram for $V(q,\omega)$ is given in Fig. 1(a). The momentum dependence reflects the long range of the interaction, as density perturbations propagate in the Fermi gas. Similarly, the frequency dependence of the interaction

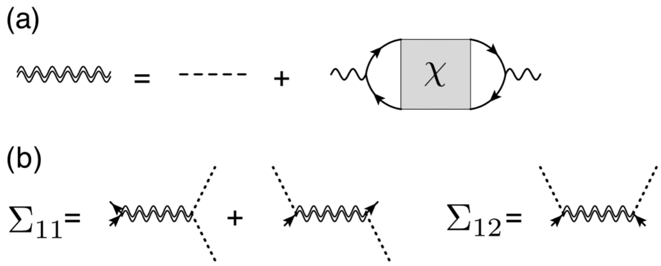

FIG. 1. (a) The effective interaction $V(q,\omega)$ (double wavy line) between the bosons. The dashed line is the bare Bose-Bose interaction $\mathcal{T}_{\rm B}$, single wavy lines are the Bose-Fermi interaction $\mathcal{T}_{\rm BF}$, and the solid lines are the Fermi Green's function. (b) The Bose self-energies $\Sigma_{11}(q,\omega)$ and $\Sigma_{12}(q,\omega)$. The dotted lines are excitations in and out of the BEC.

is due to the fact that it is not instantaneous since these perturbations have a finite speed.

In the weak-coupling BCS limit, $k_{\rm F}a_{\rm F} \to 0_-$, with $k_{\rm F}$ being the Fermi momentum of the Fermi gas, the density-density response function is given by

$$\chi(q,\omega) = \left(\frac{v_{\rm F}}{c_{\rm s}}\right)^2 \frac{\mathcal{N}(\epsilon_{\rm F})}{3\left[\left(\frac{\omega}{c_{\rm s}q}\right)^2 - 1\right]} \tag{5}$$

for frequency or momenta close to the Anderson-Bogoliubov sound mode $\omega = c_{\rm s}q$. The velocity is $c_{\rm s} = v_{\rm F}\sqrt{1+2k_{\rm F}a_{\rm F}/\pi}/\sqrt{3}$ [26–28], the density of states at the Fermi level $\epsilon_{\rm F} = k_{\rm F}^2/2m_{\rm F}$ is $\mathcal{N}(\epsilon_{\rm F}) = m_{\rm F}k_{\rm F}/\pi^2$, and $v_{\rm F} = k_{\rm F}/m$. In the BEC regime, $k_{\rm F}a_{\rm F} \to 0_+$, the Fermi gas becomes a BEC consisting of diatomic molecules (dimers) with mass $2m_{\rm F}$ and density $n_{\rm F}/2$, where $n_{\rm F} = k_{\rm F}^3/3\pi^2$ is the total density of the fermions. The density-density response function is then, from Bogoliubov theory, given by [29]

$$\chi(q,\omega) = \frac{n_{\rm F}q^2}{4m_{\rm F}(\omega^2 - \omega_q^2)} \simeq \left(\frac{v_{\rm F}}{c_{\rm s}}\right)^2 \frac{\mathcal{N}(\epsilon_{\rm F})}{12\left[\left(\frac{\omega}{c_{\rm s}q}\right)^2 - 1\right]}. \tag{6}$$

Here $\omega_q^2 = q^2(4m_{\rm F})^{-1}(q^2/4m_{\rm F} + 0.6\mathcal{T}_{\rm F}n_{\rm F}/2)$ is the Bogoliubov spectrum of the dimer BEC, where we have used the fact that the scattering length between the dimers is $0.6a_{\rm F}$ in the BEC limit [30]. The second equality in (6) follows from the fact that $\omega_q \simeq c_{\rm s}q$ for small momenta, where $c_{\rm s} = \sqrt{0.6a_{\rm F}n_{\rm F}\pi/2m_{\rm F}^2}$ is the Bogoliubov sound speed.

In general, the density-density correlation function of the Fermi gas has a pole at $\omega = c_{\rm s}q$ in the whole BCS-BEC crossover, where $c_{\rm s}$ is the velocity of sound for a given scattering length $-\infty < a_{\rm F} < \infty$. It follows from (4) that the induced interaction between the bosons has the same pole structure: it is attractive for $\omega \leqslant c_{\rm s}q$, repulsive for $\omega \geqslant c_{\rm s}q$, and *diverges* when $\omega = c_{\rm s}q$. In addition, it has a nonzero imaginary part for frequency or momenta inside the quasiparticle continuum of the Fermi gas. It also follows from (4)–(6) that the strength κ of the induced interaction scales as

$$\kappa = \mathcal{T}_{\rm BF}^2\mathcal{N}(\epsilon_{\rm F})\frac{v_{\rm F}^2}{c_{\rm s}^2}, \tag{7}$$

which should be compared with the strength $\mathcal{T}_{\rm B}$ of the direct Bose-Bose interaction.

We now examine the effects of the induced interaction on the excitation spectrum of the Bose gas. To this end, we need to calculate the density-density response function of the Fermi gas in the whole BCS-BEC regime. The density response function $\chi(1,2)$ is defined as a measure for how much the density of the Fermi gas changes at point (and time) 1 when a potential perturbation $\delta\phi$ is applied at point 2:

$$\chi(1,2) = -\frac{\delta\langle n(1)\rangle}{\delta\phi(2)}. \tag{8}$$

We apply a QRPA for calculating the Fourier transform of $\chi(1,2)$ in the superfluid state [28,31–34]. This is the simplest microscopic scheme which recovers the Anderson-Bogoliubov mode in the BCS regime and the Bogoliubov mode in the BEC regime. It yields a response function of the form

$$\chi(q,\omega) = \frac{\chi_0(q,\omega)}{1 - \mathcal{T}_{\rm F}L(q,\omega)}, \tag{9}$$

where $\chi_0(q,\omega)$ is a four-dimensional vector giving response due to quasiparticle excitations in the superfluid and L is a 4×4 matrix describing the couplings of the densities and the order parameter field. The collective modes manifest themselves as poles of the density response $\chi(q,\omega)$, i.e., as the zeros of the determinant,

$$\det\left[1 - \mathcal{T}_{\rm F}L(q,\omega)\right] = 0. \tag{10}$$

The input parameters needed for the QRPA are the chemical potential μ and the pairing gap Δ of the Fermi superfluid, which are obtained self-consistently from BCS theory. We have for convergence added a small imaginary part $i\eta = i10^{-3}\,\epsilon_{\rm F}$ to the frequencies and checked that the final numerical results do not depend on η, as long as $\eta \ll \epsilon_{\rm F}$. The details of this QRPA calculation can be found, for example, in Refs. [28,31].

Figure 2 shows the speed of the Anderson-Bogoliubov mode as a function of $1/k_Fa$, determined by finding the frequency ω at which the imaginary part of $\chi(q,\omega)$ is maximal for a given momentum q. The value of the momentum q needs to be chosen small enough so that it probes the linear part of the collective mode branch. The speed of sound is then the slope

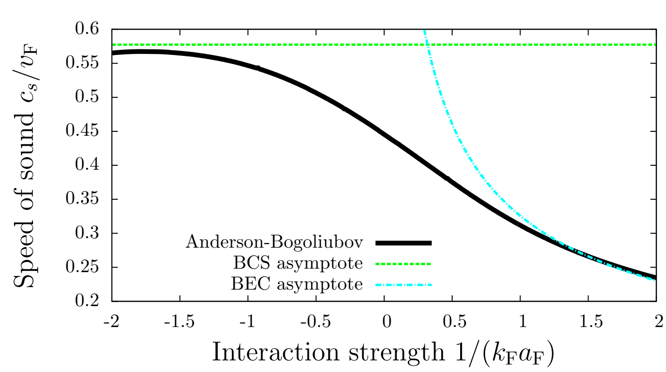

FIG. 2. (Color online) Speed of sound c_s in the two-component Fermi superfluid as calculated from the pole of the density response function $\chi(\mathbf{q},\omega)$. It approaches $c_s = v_{\rm F}/\sqrt{3}$ in the BCS limit (green dotted line) and the Bogoliubov result $c_s = v_{\rm F}\sqrt{k_{\rm F}a_{\rm F}/3\pi}$ in the BEC limit (blue dash-dotted curve).

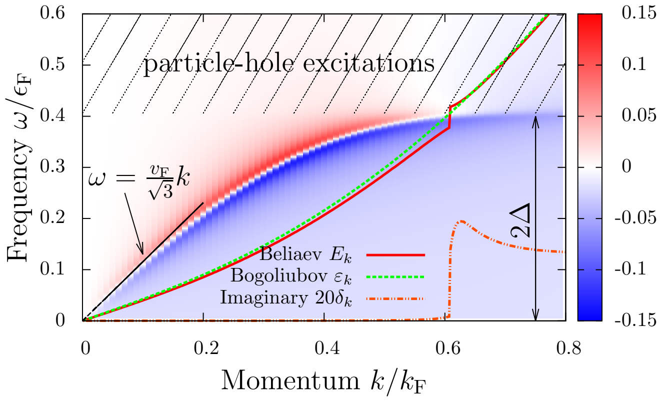

FIG. 3. (Color online) The blue (dark gray) and red (light gray) regions show Re $\chi(q,\omega)$ for $k_Fa_F = -1$. The black solid line is the weak-coupling Anderson-Bogoliubov mode, and the quasiparticle continuum for $\omega > 2\Delta$ is indicated by a dashed region. The green dashed line is the Bogoliubov spectrum ε_k of the atomic BEC, and the red solid line is the Beliaev spectrum E_k for the coupled Bose-Fermi mixture. The damping δ_k of the Beliaev excitations is shown as a red dash-dotted line.

$c_s = \omega/q$. In the BCS limit, the speed of sound approaches the weakly interacting limit $v_F/\sqrt{3}$. The numerically calculated speed of sound deviates slightly from this in the very weakly interacting regime due to the difficulty of determining the slope when the pairing gap is very small. Our numerics reproduce to an excellent accuracy the speed of sound results in Ref. [35]. Note that this theory is, of course, not quantitatively correct in the whole BCS-BEC crossover. For instance, the speed of sound approaches $c_s = v_F\sqrt{k_Fa_F/3\pi}$ in the BEC limit (see Fig. 2). This corresponds to a molecular BEC with a scattering length $2a_F$ instead of the correct value $0.6a_F$. We emphasize, however, that the effects discussed below are completely general and do not depend on which approximate theory we apply to describe the strongly correlated system.

Figure 3 depicts the real part of the calculated density-density response for $k_Fa = -1$. At low frequency or momenta, we clearly see a sharp Anderson-Bogoliubov mode where Re$\chi(q,\omega)$ changes sign. The dispersion of this mode is close to the weak-coupling result $c_s = v_F/\sqrt{3}$. For higher momenta, the dispersion curves downwards when it approaches the quasiparticle continuum starting at energies above 2Δ, with $\Delta \simeq 0.21\epsilon_F$.

Once $\chi(q,\omega)$ is calculated, we use Beliaev theory [36] to describe the effects of the resulting induced Bose-Bose interaction on the excitation spectrum of the atomic BEC. The single-particle propagator $\bar{G}(\mathbf{k},\omega)$ for the BEC is a 2×2 matrix, and the Dyson equation reads

$$\bar{G}(\mathbf{k},\omega) = \bar{G}_0(\mathbf{k},\omega) + \bar{G}_0(\mathbf{k},\omega)\bar{\Sigma}(\mathbf{k},\omega)\bar{G}(\mathbf{k},\omega). \quad (11)$$

The bare propagator is

$$\bar{G}_0(\mathbf{k},\omega) = \begin{bmatrix} G_0(\mathbf{k},\omega) & 0 \\ 0 & G_0(\mathbf{k},-\omega)\end{bmatrix}, \quad (12)$$

and the self-energy is

$$\bar{\Sigma}(\mathbf{k},\omega) = \begin{bmatrix} \Sigma_{11}(\mathbf{k},\omega) & \Sigma_{12}(\mathbf{k},\omega) \\ \Sigma_{21}(\mathbf{k},\omega) & \Sigma_{11}(\mathbf{k},-\omega)\end{bmatrix}, \quad (13)$$

where we have used the inversion symmetry $\mathbf{k} \leftrightarrow -\mathbf{k}$. The effects of interactions are included via the "Hartree-Fock" self-energies illustrated in Fig. 1(b), given by $\Sigma_{11}(\mathbf{k},\omega) = \Sigma_{11}(\mathbf{k},-\omega)^* = n_0V(0,0) + n_0V(\mathbf{k},\omega)$ and $\Sigma_{12}(\mathbf{k},\omega) = \Sigma_{21}(\mathbf{k},\omega) = n_0V(\mathbf{k},\omega)$. Solving these equations for $\bar{G}(\mathbf{k},\omega)$ yields the Green's functions for the diagonal elements,

$$G(\mathbf{k},\omega) = \frac{\omega + \epsilon_k + n_0V(\mathbf{k},\omega)}{\omega^2 - E(k,\omega)^2}, \quad (14)$$

where $E(k,\omega) = \epsilon_k^2 + 2\epsilon_k n_B V(k,\omega)$. The off-diagonal elements are $G_{12}(\mathbf{k},\omega) = G_{21}(\mathbf{k},\omega) = -n_B V(\mathbf{k},\omega)/[\omega^2 - E(k,\omega)^2]$, where n_B is the density of the BEC. The theory satisfies the Hugenholtz-Pines relation for the chemical potential $\mu = \Sigma_{11}(0) - \Sigma_{12}(0) = n_B V(0,0)$. These interacting Green's functions describe excitations with energy dispersion E_k given by solving

$$E_k = \mathrm{Re}\,E(k,\omega = E_k). \quad (15)$$

In the absence of the induced interaction, this results in the usual Bogoliubov dispersion $\varepsilon_k = \sqrt{\epsilon_k^2 + 2n_B\mathcal{T}_B\epsilon_k}$. However, due to the momentum and frequency dependence of $V(\mathbf{k},\omega)$, (15) is implicit and needs to be solved numerically. The equation also yields damping of the excitations given by $\delta_k = \mathrm{Im}\,E(k,E_k)$.

Figures 3–5 show the dispersion E_k obtained from (15), in the BCS ($k_Fa_F = -1$), unitarity $1/k_Fa_F = 0$, and BEC ($k_Fa_F = 1$) regimes of the Fermi gas, respectively. The calculations are performed using parameters corresponding to densities $n_F = n_B = 10^{13}\,\mathrm{cm}^{-3}$ and scattering lengths $a_B = a_{BF} = 400a_0$, and inspired by the superfluid Bose-Fermi mixture experiment [24], we use the masses of ^{6}Li and ^{7}Li atoms. From (7), this yields $\kappa = 4\pi a_{\mathrm{ind}} m_B{}^{-1} v_F^2/c_s^2$ for the strength of the induced interaction with the effective scattering length $a_{\mathrm{ind}} \simeq 70a_0$.

Consider first the BCS regime with $k_Fa_F = -1$ shown in Fig. 3. Comparing the Bogoliubov spectrum ϵ_k for the atomic BEC decoupled from the Fermi gas with the Beliaev spectrum E_k for the coupled Bose-Fermi mixture obtained from (15), we see that coupling to the Anderson-Bogoliubov mode results in an avoided crossing. Since we are neglecting backaction effects on the Fermi gas, this avoided crossing becomes a

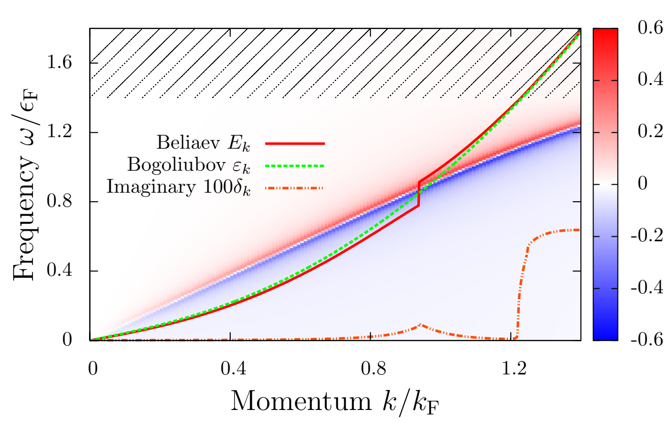

FIG. 4. (Color online) Same as Fig. 3, but for a unitary Fermi gas with $k_Fa_F = \infty$. Here $\Delta \approx 0.69\,E_F$.

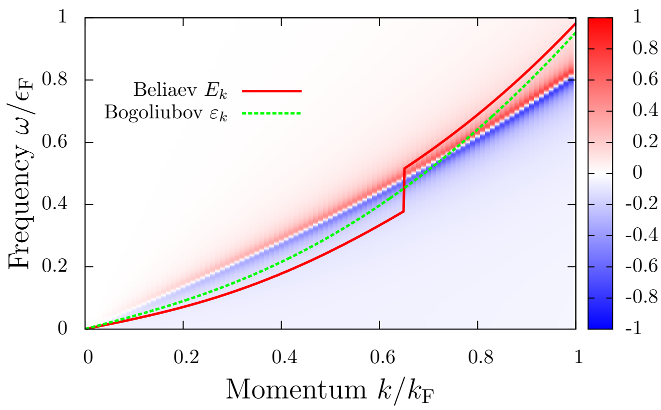

FIG. 5. (Color online) Same as Figs. 3 and 4, but for the Fermi superfluid in the BEC regime with $k_F a_F = 1$. Here $\Delta \approx 1.35\,\epsilon_F$.

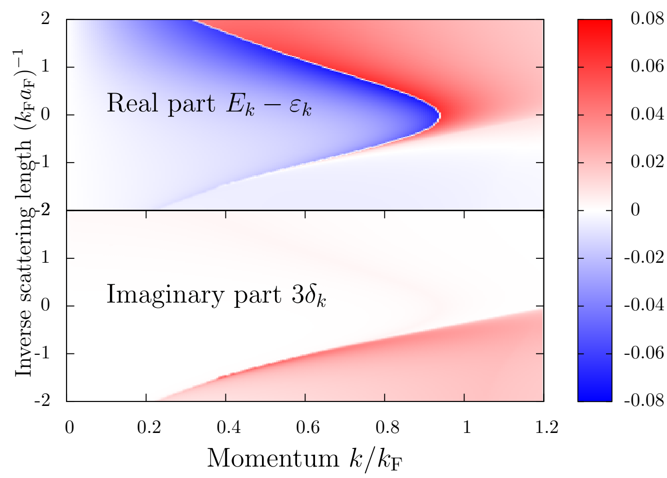

FIG. 6. (Color online) (top) Correction $(E_k - \varepsilon_k)/\epsilon_F$ to the BEC dispersion due to the induced interaction as a function of Fermi-Fermi scattering length $k_F a_F$ and momentum k. (bottom) Decay δ_k/ϵ_F of the excitations in the BEC.

discontinuous jump in the bosonic excitation frequency. We expect this prediction to be qualitatively correct, except very close to the avoided crossing, since the induced interaction diverges when the two excitation frequencies are equal, making the corresponding avoided crossing sharp. Figure 3 also shows that the excitations of the BEC become damped when their energy is inside the quasiparticle continuum of the Fermi gas. This reflects that the excitation dissipates energy by exciting quasiparticles in the superfluid Fermi gas.

Figure 4 depicts the spectrum E_k when the Fermi gas is in the unitarity regime with $1/k_F a_F = 0$. We again see that there is an avoided crossing, evidenced by a jump in the Beliaev dispersion E_k, when the Bogoliubov mode approaches the collective mode of the Fermi gas. In fact, the resulting energy shift is larger than in the BCS case since the spectral weight of the collective mode is larger in the unitarity regime. The bosonic excitations are again damped for energies $\omega > 2\Delta$. The small residual damping near the avoided crossing reflects, however, the small imaginary part $i\eta$ that we have built into the Fermi theory to obtain convergence. In the limit $\eta \to 0$, the bosonic excitations are undamped outside the quasiparticle continuum, even at the avoided crossing since it corresponds to the coupling of two undamped excitations.

Finally, Fig. 5 shows the dispersion E_k in the BEC regime of the Fermi gas with $k_F a_F = 1$. The avoided crossing feature and the energy shift in E_k are now even more pronounced due to a smaller sound velocity of the Fermi gas, which approaches the Bogoliubov sound speed of a dimer BEC, thereby making κ larger, as can be seen from (7). The quasiparticle continuum of the Fermi gas is outside the range of the plot due to the large pairing energy in the BEC regime. There is, therefore, no damping of the bosonic modes shown.

The above results show how the coupling between the superfluid bosons and fermions leads to significant effects on the spectrum of the atomic BEC, which depend on the properties of the Fermi gas. In the recent experiment on the superfluid ^{6}Li and ^{7}Li mixture, the Fermi-Fermi scattering length a_F could be tuned using a Feshbach resonance. We therefore plot in Fig. 6 the difference $E_k - \varepsilon_k$ between the Beliaev and Bogoliubov excitation spectra as a function of a_F, keeping all other parameters as in Figs. 3–5. We also plot the damping of the mode. Two effects are apparent. First, since the sound velocity in the Fermi gas depends on a_F, the momentum where the bosonic mode exhibits the avoided crossing depends on a_F. Also, the induced interaction in general decreases (increases) E_k for energies below (above) the avoided crossing, as expected. The magnitude of the energy shift increases towards the BEC regime since spectral weight of the collective mode in the Fermi superfluid increases. Second, Fig. 6 clearly shows the damping caused by the coupling to the quasiparticle excitations of the superfluid Fermi gas. This quasiparticle continuum moves to higher momenta as the system approaches the BEC limit and the pairing gap increases. The residual damping below the quasiparticle continuum shown in Fig. 6 is, as explained above, a result of using a nonzero η in the numerics, and it vanished for $\eta \to 0$. This illustrates how the collective and single-particle spectra of the strongly correlated Fermi gas can be mapped out by measuring its effects on the excitations in the BEC. We note that the effects can be increased significantly by increasing a_{BF} since $\kappa \propto a_{BF}^2$. In addition to varying a_F and a_{BF}, one can also vary a_B, which will increase even further the ways one can probe the excitations in this Bose-Fermi mixture. The excitations of a BEC have already been measured using Bragg spectroscopy [37–44].

In conclusion, we examined a mixture of a BEC and a superfluid Fermi gas using Beliaev theory for the bosons combined with quasiparticle random-phase approximation for the fermions. The fermions were shown to mediate a frequency- or momentum-dependent interaction between the bosons, which leads to two qualitatively new effects. First, the induced interaction diverges at the sound mode of the Fermi gas, which results in a sharp avoided-crossing feature in the excitation spectrum of the BEC. Second, the excitation of quasiparticles in the Fermi gas leads to a damping of the excitations of the BEC. By varying the densities and scattering lengths of the system, these effects can be used to systematically probe the properties of the Fermi gas in the strongly correlated BCS-BEC crossover. Our work may be extended in a number of directions: It would be interesting to include the backaction of the bosons on the superfluid Fermi gas to obtain a detailed description of the avoided crossing of the sound modes. Trapping effects can be included using a local density approximation, which has proven to work well when considering short-wavelength Bragg scattering [39]. Finally, the theory can be extended to finite temperatures, which would result in a damping of BEC excitations for all momenta due to the presence of thermally excited quasiparticles in the Fermi gas.

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[1] J. R. Schrieffer, *Theory of Superconductivity* (Perseus Books, New York, 1999).
[2] H. Heiselberg, C. J. Pethick, H. Smith, and L. Viverit, Influence of induced interactions on the superfluid transition in dilute Fermi gases, Phys. Rev. Lett. **85**, 2418 (2000).
[3] L. P. Gor'kov and T. K. Melik-Barkhudarov, Contribution to the theory of superfluidity in an imperfect Fermi gas, Sov. Phys. JETP **13**, 1018 (1961).
[4] D. J. Scalapino, The case for $d_{x^2-y^2}$ pairing in the cuprate superconductors, Phys. Rep. **250**, 329 (1995).
[5] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory: Concepts and Applications* (Wiley-VCH, Weinheim, 1991).
[6] A. G. Truscott, K. E. Strecker, W. I. McAlexander, G. B. Partridge, and R. G. Hulet, Observation of Fermi pressure in a gas of trapped atoms, Science **291**, 2570 (2001).
[7] F. Schreck, L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, Quasipure Bose-Einstein condensate immersed in a Fermi sea, Phys. Rev. Lett. **87**, 080403 (2001).
[8] G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Fermi-Bose quantum degenerate ^{40}K-^{87}Rb mixture with attractive interaction, Phys. Rev. Lett. **89**, 150403 (2002).
[9] C.-H. Wu, J. W. Park, P. Ahmadi, S. Will, and M. W. Zwierlein, Ultracold fermionic Feshbach molecules of ^{23}Na^{40}K, Phys. Rev. Lett. **109**, 085301 (2012).
[10] J. W. Park *et al.*, Quantum degenerate Bose-Fermi mixture of chemically different atomic species with widely tunable interactions, Phys. Rev. A **85**, 051602 (2012).
[11] M.-S. Heo *et al.*, Formation of ultracold fermionic NaLi Feshbach molecules, Phys. Rev. A **86**, 021602 (2012).
[12] T. D. Cumby, R. A. Shewmon, M. G. Hu, J. D. Perreault, and D. S. Jin, Feshbach-molecule formation in a Bose-Fermi mixture, Phys. Rev. A **87**, 012703 (2013).
[13] S.-K. Tung, K. Jimenez-Garcia, J. Johansen, C. V. Parker, and C. Chin, Geometric scaling of Efimov states in a ^{6}Li-^{133}Cs mixture, Phys. Rev. Lett. **113**, 240402 (2014).
[14] L. Viverit, C. J. Pethick, and H. Smith, Zero-temperature phase diagram of binary boson-fermion mixtures, Phys. Rev. A **61**, 053605 (2000).
[15] M. J. Bijlsma, B. A. Heringa, and H. T. C. Stoof, Phonon exchange in dilute Fermi-Bose mixtures: Tailoring the Fermi-Fermi interaction, Phys. Rev. A **61**, 053601 (2000).
[16] Z.-Q. Yu, S. Zhang, and H. Zhai, Stability condition of a strongly interacting boson-fermion mixture across an interspecies Feshbach resonance, Phys. Rev. A **83**, 041603(R) (2011).
[17] D. Ludwig, S. Floerchinger, S. Moroz, and C. Wetterich, Quantum phase transition in Bose-Fermi mixtures, Phys. Rev. A **84**, 033629 (2011).
[18] E. Fratini and P. Pieri, Mass imbalance effect in resonant Bose-Fermi mixtures, Phys. Rev. A **85**, 063618 (2012).
[19] G. Bertaina, E. Fratini, S. Giorgini, and P. Pieri, Quantum Monte Carlo study of a resonant Bose-Fermi mixture, Phys. Rev. Lett. **110**, 115303 (2013).
[20] T. Sogo, P. Schuck, and M. Urban, Bose-Fermi pairs in a mixture and the Luttinger theorem within a Nozières-Schmitt-Rink-like approach, Phys. Rev. A **88**, 023613 (2013).
[21] J. H. Pixley, X. Li, and S. Das Sarma, Damping of long wavelength collective modes in spinor Bose-Fermi mixtures, arXiv:1501.05015.
[22] D. H. Santamore and E. Timmermans, Fermion-mediated interactions in a dilute Bose-Einstein condensate, Phys. Rev. A **78**, 013619 (2008).
[23] A. P. Albus, S. A. Gardiner, F. Illuminati, and M. Wilkens, Quantum field theory of dilute homogeneous Bose-Fermi mixtures at zero temperature: General formalism and beyond mean-field corrections, Phys. Rev. A **65**, 053607 (2002).
[24] I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, and C. Salomon, A mixture of Bose and Fermi superfluids, Science **345**, 1035 (2014).
[25] W. Zheng and H. Zhai, Quasiparticle lifetime in a mixture of Bose and Fermi superfluids, Phys. Rev. Lett. **113**, 265304 (2014).
[26] P. W. Anderson, Random-phase approximation in the theory of superconductivity, Phys. Rev. **112**, 1900 (1958).
[27] N. N. Bogoliubov, V. V. Tolmachev, and D. V. Shirkov, *A New Method in the Theory of Superconductivity* (Academy of Science, Moscow, 1958).
[28] A. Minguzzi, G. Ferrari, and Y. Castin, Dynamic structure factor of a superfluid Fermi gas, Eur. Phys. J. D **17**, 49 (2001).
[29] P. Nozières and D. Pines, *The Theory of Quantum Liquids* (Perseus Books, New York, 1999), Vol. 2.
[30] D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Weakly bound dimers of fermionic atoms, Phys. Rev. Lett. **93**, 090404 (2004).
[31] R. Côté and A. Griffin, Cooper-pair-condensate fluctuations and plasmons in layered superconductors, Phys. Rev. B **48**, 10404 (1993).
[32] A. J. Leggett, Theory of a superfluid Fermi liquid. II. Collective oscillations, Phys. Rev. **147**, 119 (1966).
[33] J. R. Engelbrecht, M. Randeria, and C. A. R. Sá de Melo, BCS to Bose crossover: Broken-symmetry state, Phys. Rev. B **55**, 15153 (1997).
[34] G. M. Bruun and B. R. Mottelson, Low energy collective modes of a superfluid trapped atomic Fermi gas, Phys. Rev. Lett. **87**, 270403 (2001).
[35] R. Combescot, M. Yu. Kagan, and S. Stringari, Collective mode of homogeneous superfluid Fermi gases in the BEC-BCS crossover, Phys. Rev. A **74**, 042717 (2006).

[36] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[37] M. Kozuma *et al.*, Coherent splitting of Bose-Einstein condensed atoms with optically induced Bragg diffraction, Phys. Rev. Lett. **82**, 871 (1999).

[38] J. Stenger *et al.*, Bragg spectroscopy of a Bose-Einstein condensate, Phys. Rev. Lett. **82**, 4569 (1999).

[39] D. M. Stamper-Kurn *et al.*, Excitation of phonons in a Bose-Einstein condensate by light scattering, Phys. Rev. Lett. **83**, 2876 (1999).

[40] J. Steinhauer, R. Ozeri, N. Katz, and N. Davidson, Excitation spectrum of a Bose-Einstein condensate, Phys. Rev. Lett. **88**, 120407 (2002).

[41] R. Ozeri, N. Katz, J. Steinhauer, and N. Davidson, Colloquium: Bulk Bogoliubov excitations in a Bose-Einstein condensate, Rev. Mod. Phys. **77**, 187 (2005).

[42] S. B. Papp *et al.*, Bragg spectroscopy of a strongly interacting ^{85}Rb Bose-Einstein condensate, Phys. Rev. Lett. **101**, 135301 (2008).

[43] J. J. Kinnunen and M. J. Holland, Bragg spectroscopy of a strongly interacting Bose-Einstein condensate, New J. Phys. **11**, 013030 (2009).

[44] S. Ronen, The dispersion relation of a Bose gas in the intermediate- and high-momentum regimes, J. Phys. B **42**, 055301 (2009).