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# Wave propagation characteristics in the cavity with hyperbolic medium

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### Wave Propagation Characteristics in the Cavity with Hyperbolic Medium

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#### ABSTRACT

Electromagnetic waves propagation in the complex cavity with anisotropic hyperbolic metamaterial are investigated using direct calculation of modal field and dispersion equation. The transfer matrix method was adopted for arbitrary orientation of optical axis according to slab boundary. Increasing of the density of states in the cavity have show.

Keywords: hyperbolic metamaterials, thermal emission, eigenstates, density of states

#### **1. INTRODUCTION**

Metamaterials keep the interest to nest investigations and creation new types of them due to unusual properties of them [1]. One of the promising variant of the metamaterials is hyperbolic metamaterials (HMM) [2]. Hyperbolic medium exhibits hyperbolic-type dispersion in space of wave-vectors and described by the diagonal extremely anisotropic permittivity tensor [3]. As have been shown, exploitation of HMM seems to be promising for enhancement of the near-field thermal radiative heat transfer [4]. There are few realizations of hyperbolic media; most popular are metallic nanowire arrays embedded in the dielectric host matrix [5] and sub-wavelength metal-dielectric alternating multilayer films [6]. Recently we proposed asymmetric hyperbolic metamaterial (AHMM) consisting of periodically arranged layers (or wires) in host media, titled relatively to outer boundary [7]. The most important feature of AHMM is the possibility to excite a very slow wave in AHMM by a plane wave, incoming from free space, while a minimal reflection may be achieved. Exploitation of HMM seems to be promising for enhancement of the near-field thermal radiative heat transfer [8]. Strong directive thermal emissions (exceeding Planck's limit) was predicted in the far-field zone of AHMM consisting of periodically arranged layers in active host media by fluctuation-dissipation theorem [9].

Here we investigated propagation characteristics electromagnetic waves inside complex cavity contains asymmetrical hyperbolic medium (AHM), namely spectral characteristic of electromagnetic waves propagation and variation of eigenstates number. We prove the effect of enhancement of the near-field thermal radiative heat transfer by direct calculation of the number of the field eigenstates in complex cavity with AHMM. If in each field eigenstate oscillator has Planck thermal energy, the radiation density distribution in the cavity can be calculated. The eigenstates determine the spatial and polarization properties, are parameterized by wavevector  $\mathbf{k}$ , having the eigenfrequency  $\omega = \omega(\mathbf{k})$ . We investigated eigenvalues of the system together with Pointing vector in every region of the cavity. Increasing of the numbers of modes in the vacuum regions of the cavity is show. From the other hand the variation of the axis orientation or AHMM slab in the cavity affected strongly on the number of electromagnetic field quantum states. This effect can be used for the control of states number.

#### 2. METHOD AND RESULTS

Geometry of the system considered is shown in Fig.1. We investigated cavity partially filled with AHMM.

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Saratov Fall Meeting 2017: Laser Physics and Photonics XVIII; and Computational Biophysics and Analysis of Biomedical Data IV, edited by Vladimir L. Derbov, Dmitry E. Postnov, Proc. of SPIE Vol. 10717, 1071718 · © 2018 SPIE · CCC code: 1605-7422/18/\$18 · doi: 10.1117/12.2314628 We investigated electromagnetic waves propagation in the complex cavity contains asymmetrical hyperbolic medium, see Fig.1a. The box with dimensions  $l_x \times l_y \times l_z$  has ideally reflecting walls. AHMM has thickness h, vacuum parts have thicknesses  $l_1 \quad l_2$ . The optical axis (denoted O) orientation is given by angle  $\theta$ , see Fig.1b,  $\varphi$  is the angle between x-axis and line of nodes,  $\alpha$  is incidence angle. Incidence wave vector is  $k=(k_x,k_y,k_z)$ , x-component of the incidence wave vector  $k_x=K \sin \alpha$ . AHM consists of periodically arranged layers (or wires) in a host media, titled relatively to outer boundary. Note that AHMM is periodic in the x-direction, infinite in the y-direction and it has a finite-thickness h in the z direction.



Figure 1. Structure under investigation. (a) - cavity partially filled with AHMM (blue area). (b) – AHMM with details of the characteristics,  $\theta$ ,  $\varphi$ ,  $\psi$  –Euler angles.

As known, hyperbolic medium described by the effective diagonal extremely anisotropic permittivity tensor.

$$\overline{\varepsilon} = \begin{bmatrix} \varepsilon_{\perp} & 0 & 0\\ 0 & \varepsilon_{\perp} & 0\\ 0 & 0 & \varepsilon_{\parallel} \end{bmatrix}$$
(1)

where  $\varepsilon_{\parallel} = \varepsilon_h$  is the permittivity of the host medium. The principal components of the permittivity tensor have opposite signs which results in a hyperbolic shape of the isofrequency contours. For the transverse tensor component  $\varepsilon_{\epsilon}$  we use the homogenization model [8].

Quantization of the field in transverse plane x,y is not difficult:  $k_x = (\pi/l_x) n_x$ ,  $k_y = (\pi/l_y) n_y$ ,  $n_{x,y} = 0,\pm 1,\pm 2,...$  For z-dependence we used the solution of Maxwell equation based on the Berreman 4x4 matrix  $\Delta$  which is convenient for the investigation of the propagation of polarized light in anisotropic media. We have adopted this method for the calculations of light propagation in AHMM slabs [7]. The components of the electric field *E* and the magnetic field *H* in the plane of the slab can be written as  $\Psi \exp(ikr-i\omega t)$ , where  $\Psi$  is a column vector and where the angular frequency  $\omega = cK = 2\pi c/\lambda$ ,  $K = \omega/c = 2\pi/\lambda$  is the wave vector in vacuum; k = (kx, ky, kz). The column vector  $\Psi$  satisfies the equation:

$$\frac{\partial}{\partial z}\Psi = \frac{i\omega}{c}\Delta\Psi \tag{2}$$

Berreman 4x4 matrix  $\Delta$  includes the matrix elements  $\Delta_{ij}$  which generally determined by main components of the dielectric tensor { $\varepsilon_{a}, \varepsilon_{a}, \varepsilon_{a}, \varepsilon_{ij}$ }, Euler angles  $\theta, \varphi, \psi$ , which describes the orientation of optical axis,  $\Psi = \{E_x, H_y, E_y, -H_x\}$  [10,11]. If the medium has the losses or gain than the components  $\varepsilon_{a}$  and  $\varepsilon_{ij}$  are complex.

The eigenvalues of the matrix  $\Delta$  can be written as follows [12]:

$$\lambda_{1,2} = \pm \sqrt{\varepsilon_{\perp} - \left(\frac{ck_x}{\omega}\right)^2}, \qquad \lambda_{3,4} = -\frac{1}{\varepsilon_{33}} \left[ \varepsilon \frac{ck_x}{\omega} \cos\theta \sin\varphi \pm \sqrt{\varepsilon_{\perp} (\varepsilon_{\parallel} \varepsilon_{33} - (\varepsilon_{\parallel} - \varepsilon \sin\theta \cos^2\varphi)) \left(\frac{ck_x}{\omega}\right)^2} \right]. \tag{3}$$

Here  $\varepsilon_{33} = -\varepsilon_{13} \sin 2\theta + \varepsilon_{11} \cos 2\theta$ ,  $\varepsilon = (\varepsilon_{11} - \varepsilon_{13}) \sin \theta$ .

Electromagnetic fields of the transmitted, incident and refracted waves on the slab with the thickness *h* are related by the equation

$$\Psi_{\rm T} = \mathbf{P}(h)(\Psi_{\rm I} + \Psi_{\rm R}) \tag{4}$$

where P(h) is the propagation matrix for layer with thickness h [12],  $\Psi_T$ ,  $\Psi_I$  and  $\Psi_R$  are vectors of the transmitted, incident and refracted waves.

Transmission and reflection coefficients can be found as:

$$T = \frac{|T_x/\cos\alpha|^2 + |T_y|^2}{|E_x/\cos\alpha|^2 + |E_y|^2}, \qquad R = \frac{|R_x/\cos\alpha|^2 + |R_y|^2}{|E_x/\cos\alpha|^2 + |E_y|^2}$$
(5)

where  $T_x$ ,  $T_y$ ,  $R_x$ ,  $R_y$  can be calculated from (4) at given  $E_x$ ,  $E_y$ .

First of all we calculated a spectral characteristic of electromagnetic waves propagation in the cavity contains a hypothetical hyperbolic structure with parameters:  $l_1 = 100 \ \mu m$ ,  $l_2 = 120 \ \mu m$ ,  $h = 50 \ \mu m$ . As follow from the theory which have been used, number of mode propagated in the cavity are proportional the number of maximums of the reflection coefficient *R* (red curves) and transmission coefficient *T* (green curves) on the Fig.2 and Fig.3. Here *R* and *T* are average coefficients concluded ordinary and extraordinary waves. The arrows on indicate the longitudinal modes locations for the total length 220 and 540  $\mu m$ . Red arrows indicated ordinary waves in the cavity, black arrows indicate waves of free space regions which can't propagate in AHMM. All other fluctuation of the *R* and *T* correspond to extraordinary waves, which exists due to hyperbolic media presence. We investigated the effect of optical axis orientation on the variation of number of the modes.



Figure.2. The transmission (green curves) and reflection (red curves) coefficients vs  $k_z$ .  $l_1 = 100 \ \mu\text{m}$ ,  $l_2 = 120 \ \mu\text{m}$ ,  $h=50 \ \mu\text{m}$ ,  $T=300^{\circ}\text{K}$ ,  $\varphi=\pi/4$ ;. The arrows indicate the longitudinal modes locations for the total length 220 and 540  $\ \mu\text{m}$ . (a)  $\theta=45^{\circ}$ . (b)  $\theta=40^{\circ}$ .

We observed spectral characteristic of the cavity for variation  $0\mathbb{C}\theta\mathbb{C}90^\circ$ . As we have shown early [13], for AHMM there is special value of optical axis orientation angle when radiation comes through boundary AHMM-vacuum without of reflection. This value of angle  $\theta$  depends of parameters of AHMM. For structure with parameters given above the best result was observed for  $\theta=45^\circ$ , Fig.2a. For comparison we give spectral characteristic of the same cavity for  $\theta=40^\circ$ , Fig.2b. Easy to see, that changing optical angle  $\theta$  to affect number of modes inside cavity. So tuning of the parameters of the AHMM can provide increasing density of states inside the cavity outsides AHMM. For simplicity every calculation here have done for =0.

Then we estimated affect of the nodes angle  $\varphi$ , see Fig.3. As easy to see on graphics, the changing of angle  $\varphi$  has a slight effect on the spectral characteristics behaviors, but number of the modes inside the cavity remains preliminary the same.



Figure.3. The transmission (green curves) and reflection (red curves) coefficients vs  $k_z$ .  $l_1 = 100 \ \mu\text{m}$ ,  $l_2 = 120 \ \mu\text{m}$ ,  $h=50 \ \mu\text{m}$ ,  $T=300^{\circ}\text{K}$ ,  $\varphi=\pi/3$ ;. The arrows indicate the longitudinal modes locations for the total length 220 and 540  $\ \mu\text{m}$ . (a)  $\theta=45^{\circ}$ . (b)  $\theta=40^{\circ}$ .

Secondly, we calculated the eigenwaves of the structure vacuum gaps+AHM assuming that the incident wave is absent. AHMM based on the wire media was observed. We have used slightly another approach. Let  $P_0(l_i)$  is the matrix describing the propagation in air gap with the length  $l_i$ . The total transformation matrix of the structure is  $P_i = P_0(l_1 + l_2)P(h)$ . The eigenvalues  $\lambda_{i,k}$  of this matrix characterizes the phase delay at one pass  $(D=l_1+l_2+h)$  and given by (3). Thus  $\operatorname{Re}(\lambda_{i,k})=2\pi m$ ,  $m=0,\pm 1,\pm 2,\ldots$  determines the eigenwaves. For the simplicity we assume that the lengths of the box  $(l_1 + l_2 + h)$  is sufficiently small:  $l_1 = 100 \mu m$ ,  $l_2 = 120 \mu m$ ,  $h=1 \mu m$  and longitudinal modal index is about 200. Early we presented calculation of the eigenwaves as  $\operatorname{Det}(P)$  [14]. We have shown that each coinciding zeroes of  $\operatorname{Re}(\operatorname{det}(P))$  and  $\operatorname{Im}(\operatorname{det}(P))$  corresponds to one of the set of longitudinal modes. The number of zeroes per unit of  $k_z$  is DOS. Here we calculated eigenvalues of the matrix  $P_t$  and plotted them vs  $k_z$ . The eigenvalues of the ordinary  $\lambda_{1,2}$  (solid curves) and extraordinary  $\lambda_{3,4}$  (dotted curves) waves vs z-component of wavenumber  $k_z$  are present on Fig.4. Blue curves correspond to real parts, while red curves correspond to imaginary parts of the eigenvalues. As it was mention the eigenwaves are determines by the values of  $k_z$ , which give zeroes  $\operatorname{Re}(\lambda_{3,4}) = \operatorname{Im}(\nu_{3,4})$  when  $\operatorname{Im}(\lambda_{1,2})=0$ .

As we have shown before [14] the angle  $\theta = 0$  corresponds to high reflection on the boundary thus the spectrum corresponds to independent resonances in volumes 1 and 3. DOS in this case is the same as in vacuum and thermal emission obeys Planck law. Strong increase of DOS in vacuum parts means that when the orientation of the AHMM axis is close to  $\pi$  /4 which corresponds to the infinite increase of wave-number in AHMM at given frequency of the field  $\omega = cK(k_x, k_y, k_z(k_x, k_y, l_1, l_2, h, \varphi, \theta))$  DOS in vacuum is much larger. This effect depends on the transverse wavenumbers, i.e. from the direction of emission. Assuming the thermal equilibrium of quanta in the field modes, the super-Planck radiation in these directions can expected.



Figure.4. (a) Real (blue) and imaginary (red) parts of eigenstates vs  $k_z$ .  $l_1 = 100 \mu m$ ,  $l_2 = 120 \mu m$ ,  $h=1 \mu m$ .  $\varphi=\pi/2$ ,  $\theta=30^{\circ}$ .

Finely, we present in detail one root of the eigenvalue together with Poynting vector. Here we output eigenstates directly from  $P_t$  (Fig.5a.) and checked Poynting vector value (for extraordinary waves) on the same frequencies (Fig.5b). The value of eigenstates of the ordinary (solid blue lines) and extraordinary (dashed blue lines) have show on the Fig.4a. Easy to see, that Poynting vector has nonzero value at the same frequencies as eigenvalue of extraordinary wave is observed.



Figure 5. (a) Real (blue) and imaginary (red) parts of eigenstates vs  $k_z$ ; ordinary  $\lambda_{1,2}$  (solid curves) and extraordinary  $\lambda_{3,4}$  (dotted curves) waves. (b) Real (blue) part of the Poynting vector of extraordinary wave.  $l_1 = 100 \mu m$ ,  $l_2 = 120 \mu m$ ,  $h=1 \mu m$ .  $\varphi=\pi/2$ ,  $\theta=30^\circ$ .

#### 3. CONCLUSION

Concluding, we have found the modes of the composite cavity with AHMM, calculate longitudinal mode wavenumbers. Spectral characteristics reflection and transmission had presented. We have shown affect optical axis orientation to density of states in the structure. High DOS in AHMM appears as additional longitudinal resonances due to zero of systems determinant. We have shown that the high DOS in AHMM appears as spreading of the longitudinal resonance  $n_z$ 

 $c/2l_z$ , which leads to the increase in DOS in vacuum parts, and, correspondingly, to super-Planck radiation near the angle  $\theta = \pi/4$  in AHMM.

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