



This is an electronic reprint of the original article. This reprint may differ from the original in pagination and typographic detail.

Saarakkala, Seppo E.; Mukherjee, Victor; Sokolov, Maksim; Hinkkanen, Marko; Belahcen, Anouar

Analytical model including rotor eccentricity for bearingless synchronous reluctance motors

Published in: Proceedings of the 23rd International Conference on Electrical Machines, ICEM 2018

DOI: 10.1109/ICELMACH.2018.8506892

Published: 03/09/2018

Document Version Peer-reviewed accepted author manuscript, also known as Final accepted manuscript or Post-print

Please cite the original version:

Saarakkala, S. E., Mukherjee, V., Sokolov, M., Hinkkanen, M., & Belahcen, A. (2018). Analytical model including rotor eccentricity for bearingless synchronous reluctance motors. In *Proceedings of the 23rd International Conference on Electrical Machines, ICEM 2018* (pp. 1388-1394). IEEE. https://doi.org/10.1109/ICELMACH.2018.8506892

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

# Analytical Model Including Rotor Eccentricity for Bearingless Synchronous Reluctance Motors

Seppo E. Saarakkala, Victor Mukherjee, Maksim Sokolov, Marko Hinkkanen, and Anouar Belahcen

Abstract—This paper deals with modelling of rotor eccentricity in a dual three-phase winding bearingless synchronous reluctance motors (BSyRMs). The motor includes two separate sets of three-phase windings: one for torque production and the other one for radial force production. For this motor, an improved analytical model with linear magnetic material is presented. The accuracy of the model depends on the accuracy of the inverse-airgap function. Typically, a series expansion is used for approximating the inverse-airgap function. In order to make the main-winding inductances depend on the radial position, at least the first two terms have to be included in the expansion, enabling calculation of the radial forces caused by unbalanced magnetic pull. The improved model is applicable, e.g., for stability analysis, time-domain simulations, or developing real-time control methods.

*Index Terms*—Dual-winding motor, eccentricity, inductance, inverse-airgap function, open-loop stability, radial force, unbalanced magnetic pull

#### I. INTRODUCTION

Bearingless motors offer an attractive alternative to conventional electrical machines equipped with mechanical bearings or active-magnetic-bearings (AMBs), especially in highspeed applications [1]. Bearingless drives integrate the functionalities of active magnetic bearings and electrical machine inside a single unit, which reduces the size, complexity and price of the system [2]. Several motor topologies have been proposed in the literature to be used as bearingless motors, e.g., [2]–[5]. In this paper, a dual three-phase winding BSyRM is considered. The first winding set is applied for production of the shaft torque and is referred to as a main winding. The second winding set is for production of the radial force for stable levitation of the rotor and is referred to as a suspension winding. The advantages of BSyRM are, e.g., that it neither needs the permanent magnets (PMs) placed in the rotor, like the PM machines do, nor it produces additional losses because of the rotor currents, like the induction machines do.

In a conventional SyRM, the existence of magnetically non-conducting flux barriers makes the reluctance vary inside the rotor depending upon the total number and the shape of the flux barriers [6]. The reluctance difference creates the operational torque of the motor. The electromagnetic torque may be calculated, if the terminal inductances and the

currents of the motor are available. Similarly, in the case of a BSyRM, the torque and the radial forces can be computed analytically, when the terminal inductances and the currents of both main and suspension windings are known [7]. When a horizontally mounted BSyRM drive is started, the rotor is initially resting on the safety bearing, meaning that the rotor is radially significantly displaced from the center of the stator (magnetic center position). The radial displacement creates an unbalanced magnetic pull, when one of the two windings is energized with current excitation. It is beneficial to have an understanding of the radial forces caused by the unbalanced magnetic pull, and possibly compensate them in the levitation-control system [7], for a smoother operation during the initial lift up of the rotor. Furthermore, the rotor may be subjected to radial displacement variations even during the normal operation, e.g., because of bending modes of the shaft [8]. In addition to the effect of unbalanced magnetic pull, the eccentric rotor position causes inductance variation, which may deteriorate performance of model-based control systems [9], [10] and even lead to instability in the worst case [11].

Typically, the inductances of the bearingless motors are derived by first approximating the inverse-airgap length of an eccentric rotor with a series expansion of cosine function and using it further to define the permeance function of the airgap. The permeance function is finally used to obtain the inductances of the two windings sets [7]. An assumption of cylindrical rotor surface is used in [7], when deriving the inductances of the model. Furthermore, the obtained analytical small-signal model is valid only in the vicinity of centric operating point. On the other hand, the saliency of the rotor is taken into account in [12] by using piecewise defined inverseairgap length, making it more elaborate way to derive the inductances for motors with salient rotor structure (such as BSyRMs). In both of these studies, only the first term is included in the series expansion, making them inaccurate approximations. The inaccuracy further results in an inability to predict the radial force caused by the unbalanced magnetic pull. To cope with the eccentric radial positions, additional second order equations are introduced in [7] to make the mainwinding inductances dependent on the radial displacement of the rotor. However, these additional equations complicates the model and require extra parameters to be tuned.

The existing models for BSyRMs are improved in this paper to inherently deal with the eccentric operation. In accordance with [12], the rotor saliency is taken into account by using piecewise defined inverse-airgap length. On the other hand, the accuracy of the inverse-airgap length approximation is increased by including more terms in the series expansion.

This work was supported by the Academy of Finland and by the Estonian Research Council under grants PUT1260.

S. E. Saarakkala, V. Mukherjee, M. Sokolov, M. Hinkkanen and A. Belahcen are with the Department of Electrical Engineering and Automation, Aalto University, Finland, P.O.Box 15500, FI-00076 Aalto, Finland (e-mail: firstname.lastname@aalto.fi).

A. Belahcen is also with Department of Electrical Power Engineering and Mechatronics, Tallinn University of Technology, 19086 Tallinn, Estonia.



Fig. 1. BSyRM studied in this paper. (a) Salient 4-pole rotor structure and the coordinate systems of the main and suspension windings, marked with (d, q) and (x, y), respectively. (b) Geometry and the mesh used in FEA. The suspension winding can be seen at the bottom of the slots. (c) Magnetic flux density at the rated torque and levitation forces.

To concentrate solely on the effects of rotor eccentricity, a simple textbook-type salient-pole reluctance rotor is considered and the magnetic saturation is ignored. The main contributions of this paper are:

- 1) It is demonstrated that the approximation of the inverseairgap length significantly influences the capability of the resulting model to approximate the radial forces of the motor. This is especially evident for larger eccentricity values.
- 2) It is shown that at least the first two terms have to be included in the series expansion for the inverse-airgap function in order to make the main-winding inductances dependent on the radial position and, further, to include the effect of the unbalanced magnetic pull in the calculated radial forces.
- It is demonstrated that the open-loop stability of a dualwinding motor model depends not only on the model structure, but also on its parameter values.
- 4) Unlike in the earlier works, a straightforward method to find the parameter values of the models is presented in this paper.

The improved model is applicable, e.g., for a stability analysis, time-domain simulations, or developing real-time control methods for BSyRMs. Unlike the earlier small-signal models [7], [12], the improved model can be used with higher rotor eccentricities (e.g., during the start up of a BSyRM drive).

#### II. ANALYTICAL FLUX-LINKAGE MODELS INCLUDING ROTOR ECCENTRICITY

As depicted in Fig. 1, the studied BSyRM has a salient 4-pole rotor. A 4-pole main winding for the torque production and a 2-pole suspension winding for the radial-force production are sinusoidally distributed in the stator. In the following, the system model is analyzed in stationary coordinates, having set the rotor angle  $\vartheta_m = 0$ . Nevertheless, because sinusoidal flux-linkage distributions are assumed, the presented models

are directly applicable in rotating dq-coordinates, after completing the necessary coordinate transformations. The flux linkages of the BSyRM are analysed taking into account variations in the radial position of the rotor. The analysis is carried out by assuming linear magnetics both in the stator and rotor iron. The nominal airgap length of the motor is  $g_0 = 1$  mm.

When a cylindrical rotor surface is assumed and the rotor is displaced to the positive x and y directions from its centric point [cf. Fig. 1(a)], the airgap length can be given as a cosine function within a complete revolution ( $\theta = 0...2\pi$ )

$$g(\theta, x, y) = g_0 - x\cos(\theta) - y\sin(\theta) \tag{1}$$

Then, the permeance function is given as

$$P(\theta, x, y) = \mu_0 R l g^{-1}(\theta, x, y)$$
<sup>(2)</sup>

where R is the rotor radius and l is the axial length. Permeability of the air is denoted by  $\mu_0$ . By using the permeance function, the airgap flux distributions generated by the main and suspension windings can be given in  $(\alpha,\beta)$  and (x,y)directions

$$\phi_{\mathbf{x}} = P\left(\frac{A_{\mathbf{x}}}{2} + V_{\mathbf{x}}\right) \quad \phi_{\mathbf{y}} = P\left(\frac{A_{\mathbf{y}}}{2} + V_{\mathbf{y}}\right) \tag{3}$$

$$\phi_{\rm d} = P\left(\frac{A_{\rm d}}{2}\right) \quad \phi_{\rm q} = P\left(\frac{A_{\rm q}}{2}\right) \tag{4}$$

where  $A_d$ ,  $A_q$  and  $A_x$ ,  $A_y$  are the magneto motive force (MMF) space distributions of the main and suspension windings, respectively. The magnetic potentials of the rotor in the x and y directions are denoted by  $V_x$  and  $V_y$ , respectively [7].

Under these assumptions, the flux linkages of the main winding  $\psi_{\rm m}$  and the suspension winding  $\psi_{\rm s}$  can be presented in matrix format [2]:

$$\begin{bmatrix} \boldsymbol{\psi}_{\mathrm{m}} \\ \boldsymbol{\psi}_{\mathrm{s}} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{L}_{\mathrm{m}}(x,y) & \boldsymbol{M}(x,y) \\ \boldsymbol{M}^{\mathrm{T}}(x,y) & \boldsymbol{L}_{\mathrm{s}}(x,y) \end{bmatrix}}_{\boldsymbol{L}_{\Sigma}(x,y)} \begin{bmatrix} \boldsymbol{i}_{\mathrm{m}} \\ \boldsymbol{i}_{\mathrm{s}} \end{bmatrix}$$
(5)

$$g^{-1}(\theta, x, y) = \frac{K(\theta)}{g(\theta, x, y)} \approx \frac{K(\theta)}{g_0} \left( 1 + \underbrace{\left[\frac{x}{g_0}\cos(\theta) + \frac{y}{g_0}\sin(\theta)\right]}_{\text{first term}} + \underbrace{\left[\frac{x}{g_0}\cos(\theta) + \frac{y}{g_0}\sin(\theta)\right]^2}_{\text{second term}} + \cdots \right)$$
(7)

where the current vectors are defined as  $i_{\rm m} = [i_{\rm md}, i_{\rm mq}]^{\rm T}$  and  $i_{\rm s} = [i_{\rm sx}, i_{\rm sy}]^{\rm T}$ . The flux-linkage vectors are defined similarly and the inductance matrices in (5) are

$$\boldsymbol{L}_{\mathrm{m}} = \begin{bmatrix} L_{\mathrm{d}} & L_{\mathrm{dq}} \\ L_{\mathrm{dq}} & L_{\mathrm{q}} \end{bmatrix} \quad \boldsymbol{L}_{\mathrm{s}} = \begin{bmatrix} L_{\mathrm{x}} & L_{\mathrm{xy}} \\ L_{\mathrm{xy}} & L_{\mathrm{y}} \end{bmatrix} \quad \boldsymbol{M} = \begin{bmatrix} M_{\mathrm{dx}} & M_{\mathrm{dy}} \\ M_{\mathrm{qx}} & M_{\mathrm{qy}} \end{bmatrix}$$
(6)

where the elements may depend on the radial position of the rotor. As an example, the following elements are defined as

$$L_{\rm d} = \frac{1}{2} \int_0^{2\pi} \phi_{\rm d} A_{\rm d} \mathrm{d}\theta \qquad M_{\rm dx} = \frac{1}{2} \int_0^{2\pi} \phi_{\rm x} A_{\rm d} \mathrm{d}\theta \qquad (7)$$

and the remaining elements in (6) can be defined similarly [7].

It can be seen that the inverse-airgap function is required in order to be able to use the permeance function for calculation of the inductance matrices elements. The exact analytical inverse-airgap function inside the integrals (7) would result in complicated integration. Instead, the inverse-airgap function can be approximated using series expansions [cf. (7) at the top of the page]. The more terms are considered in the expansion, the more accurately it models the actual inverse-airgap length. For BSyRMs, it is unrealistic to assume cylindrical rotor surface. Therefore, the rotor saliency is taken into account by using a piecewise defined coefficient similarly to [12]

$$K(\theta) = \begin{cases} 0, & \gamma < \theta < \frac{\pi}{2} - \gamma \\ 0, & \frac{\pi}{2} + \gamma < \theta < \pi - \gamma \\ 0, & \pi + \gamma < \theta < \frac{3\pi}{2} - \gamma \\ 0, & \frac{3\pi}{2} + \gamma < \theta < 2\pi - \gamma \\ 1, & \text{otherwise} \end{cases}$$
(8)

where the constant  $0 < \gamma < \pi/4$  defines the saliency of the rotor. As an example, Fig. 2 shows the inverse-airgap length within a complete revolution of  $\theta$ , when x = y = 0.4 mm and  $\gamma = 25^{\circ}$ . The inverse-airgap length is numerically calulated as an inverse of (1) and compared with approximations from (7). It can be seen that the accuracy of the series expansion substantially increases when more terms are included.

#### A. Textbook Model [7]

In the textbook [7], the inductance matrices for BSyRMs are derived from (2) by assuming a cylindrical rotor and including the first term in the inverse-airgap function (7):

$$\boldsymbol{L}_{\mathrm{m}} = \begin{bmatrix} L_{\mathrm{d}} & 0\\ 0 & L_{\mathrm{q}} \end{bmatrix} \quad \boldsymbol{L}_{\mathrm{s}} = \begin{bmatrix} L_{\mathrm{s}} & 0\\ 0 & L_{\mathrm{s}} \end{bmatrix}$$
$$\boldsymbol{M}(x, y) = \begin{bmatrix} M_{\mathrm{d}}x & -M_{\mathrm{d}}y\\ M_{\mathrm{q}}y & M_{\mathrm{q}}x \end{bmatrix} \tag{9}$$



Fig. 2. Inverse-airgap length calculated numerically as an inverse of (1) and approximated using (7). The nominal airgap length of the motor is  $g_0 = 1$  mm.

where  $L_d$ ,  $L_q$ ,  $L_s$  are inductances and  $M_d$ ,  $M_q$  are radial-force constants. Moreover, the radial-force vector in xy coordinates  $F^s$  is [13]

$$\boldsymbol{F}^{\mathrm{s}} = \begin{bmatrix} F_{\mathrm{x}} \\ F_{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} i_{\mathrm{md}} i_{\mathrm{sx}} & i_{\mathrm{mq}} i_{\mathrm{sy}} \\ -i_{\mathrm{md}} i_{\mathrm{sy}} & i_{\mathrm{mq}} i_{\mathrm{sx}} \end{bmatrix} \begin{bmatrix} M_{\mathrm{d}} \\ M_{\mathrm{q}} \end{bmatrix}$$
(10)

This textbook model is well known and will be used as a benchmark model for comparisons in the following sections.

#### B. Improved Model

In this paper, the inductance matrices in (5) are presented as matrix products of inductance matrices and unitless displacement matrices, which describe the rotor radialdisplacement dependency. The following format of matrices can be obtained by applying (2), (3), (7), and (7)

$$\boldsymbol{L}_{\mathrm{m}}(x,y) = \boldsymbol{L}_{\mathrm{m}0}\boldsymbol{D}_{\mathrm{m}}(x,y) \quad \boldsymbol{L}_{\mathrm{s}}(x,y) = L_{\mathrm{s}0}\boldsymbol{D}_{\mathrm{s}}(x,y)$$
$$\boldsymbol{M}(x,y) = c_{0}\boldsymbol{L}_{\mathrm{m}0}\boldsymbol{D}_{\mathrm{M}}(x,y) \quad \boldsymbol{L}_{\mathrm{m}0} = \begin{bmatrix} L_{\mathrm{d}0} & 0\\ 0 & L_{\mathrm{q}0} \end{bmatrix}$$
(11)

where  $D_{\rm m}(x, y)$ ,  $D_{\rm s}(x, y)$ , and  $D_{\rm M}(x, y)$  are the displacement dependency matrices. The self inductances of the main winding are  $L_{\rm d0}$  and  $L_{\rm q0}$  and the self inductance of the suspension winding is  $L_{\rm s0}$ . Scalar valued coefficient  $c_0 = \sqrt{2L_{\rm s0}/(L_{\rm d0}+L_{\rm q0})}/2$  links these inductances to the mutual-coupling elements [i.e., force constants in (9)]. Furthermore, the saliency ratio is defined as

$$\frac{L_{\rm d0}}{L_{\rm q0}} = \frac{4\gamma + \sin(4\gamma)}{4\gamma - \sin(4\gamma)} \approx \frac{3}{4\gamma^2} - \frac{2}{5}$$
(12)

It is important to notice that if the saliency is modelled differently (e.g., by using trigonometric functions) or if more than three first terms are considered in (7), the previous format [(11) and (12)] may not be directly applicable.

1) Displacement Matrices: The First Term Considered in the Inverse-Airgap Approximation: The inverse-airgap function (7) is approximated with its first term. This leads to the following displacement matrices

$$D_{\rm m} = I \qquad D_{\rm s} = \begin{bmatrix} \frac{2\gamma(2g_0^2 - x^2)}{\pi g_0^2} & -\frac{2\gamma xy}{\pi g_0^2} \\ -\frac{2\gamma xy}{\pi g_0^2} & \frac{2\gamma(2g_0^2 - y^2)}{\pi g_0^2} \end{bmatrix}$$
$$D_{\rm M} = \frac{1}{g_0} \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$
(13)

where I is a 2×2 identity matrix. An important special case is obtained for a centric rotor, making the cross-coupling terms between the main and suspension windings to disappear. In this case, the displacement matrices reduce to

$$\boldsymbol{D}_{\mathrm{m}} = \boldsymbol{I} \quad \boldsymbol{D}_{\mathrm{s}} = \boldsymbol{I} \quad \boldsymbol{D}_{\mathrm{M}} = \boldsymbol{0} \tag{14}$$

where **0** is a 2 × 2 null matrix. It is worth noticing that the textbook model (9) is a combination of (13) and (14), where  $D_{\rm m}$  and  $D_{\rm s}$  are selected from (14) and  $D_{\rm M}$  from (13), further assuming that  $L_{\rm d}/L_{\rm q} = M_{\rm d}/M_{\rm q}$ .

2) Displacement Matrices: The First Two Terms Considered in the Inverse-Airgap Approximation: According to (11) and (13), the radial displacement affects only the inductance matrix  $L_s$  and the mutual-coupling matrix M. On the other hand, according to findings in [7] for BSyRMs, the eccentricity affects also the self inductances of the main winding. Thus, the inverse-airgap function accuracy is increased by including the second term in the series expansion (7). This modification results in the following displacement matrices

$$\boldsymbol{D}_{\rm m} = d_{\rm m} = \left(1 + \frac{x^2 + y^2}{2g_0^2}\right) \quad \boldsymbol{D}_{\rm s} = \begin{bmatrix} d_{\rm x} & d_{\rm xy} \\ d_{\rm xy} & d_{\rm y} \end{bmatrix} \quad (15)$$
$$\boldsymbol{D}_{\rm M} = \frac{1}{g_0} \begin{bmatrix} \frac{2x(g_0^2 + y^2)}{2g_0^2 + x^2 + y^2} & \frac{-2y(g_0^2 + x^2)}{2g_0^2 + x^2 + y^2} \\ \frac{y(2g_0^2 - x^2 + y^2)}{2g_0^2 + x^2 + y^2} & \frac{x(2g_0^2 + x^2 - y^2)}{2g_0^2 + x^2 + y^2} \end{bmatrix}$$

where the elements of  $D_s$  are

$$d_{\rm x} = \frac{\sin(4\gamma)(x^4 - y^4 + 2g_0^2x^2 - 2g_0^2y^2)}{4\pi g_0^2(2g_0^2 + x^2 + y^2)} \\ + \frac{4\gamma(8g_0^4 + 3x^4 + y^4 + 6g_0^2x^2 + 6g_0^2y^2 + 4x^2y^2)}{4\pi g_0^2(2g_0^2 + x^2 + y^2)} \\ d_{\rm y} = \frac{\sin(4\gamma)(y^4 - x^4 - 2g_0^2x^2 + 2g_0^2y^2)}{4\pi g_0^2(2g_0^2 + x^2 + y^2)} \\ + \frac{4\gamma(8g_0^4 + x^4 + 3y^4 + 6g_0^2x^2 + 6g_0^2y^2 + 4x^2y^2)}{4\pi g_0^2(2g_0^2 + x^2 + y^2)} \\ d_{\rm xy} = -\frac{xy[\sin(4\gamma)(2g_0^2 + x^2 + y^2) - 4\gamma(x^2 + y^2)]}{2\pi g_0^2(2g_0^2 + x^2 + y^2)}$$
(16)

From (11) and (15), it is evident that also the elements of inductance matrix  $L_{\rm m}$  depend on x and y.

 TABLE I

 PARAMETER ESTIMATES OF THE TEXTBOOK MODEL (9)

	Parameter set 1	Parameter set 2
	(x = 0, y = -0.6  mm)	(x = y = 0)
$L_{\rm d}$ (mH)	17	14.5
$L_q$ (mH)	10.4	8.9
$L_{\rm s}$ (mH)	239	220
$M_{\rm d}~({\rm H/m})$	57	40
$M_{\rm q}~({\rm H/m})$	34	22

 TABLE II

 Parameter Estimates of the Improved Model (11)

	Parameter set 3	Parameter set 4
	(x = 0, y = -0.6  mm)	(x = y = 0)
$L_{d0}$ (mH)	14.4	14.5
$L_{q0}$ (mH)	8.8	8.9
$L_{s0}$ (mH)	281	284
$c_0$	2.46	2.47
$\gamma$ (deg)	34.7	34.9

### C. Radial Forces

The stored magnetic co-energy of the system is

$$W'_{\rm m} = \frac{1}{2} \begin{bmatrix} \boldsymbol{i}_{\rm m}^{\rm T} & \boldsymbol{i}_{\rm s}^{\rm T} \end{bmatrix} \begin{bmatrix} \boldsymbol{L}_{\rm m0} \boldsymbol{D}_{\rm m} & c_0 \boldsymbol{L}_{\rm m0} \boldsymbol{D}_{\rm M} \\ c_0 \boldsymbol{L}_{\rm m0} \boldsymbol{D}_{\rm M}^{\rm T} & \boldsymbol{L}_{\rm s0} \boldsymbol{D}_{\rm s} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_{\rm m} \\ \boldsymbol{i}_{\rm s} \end{bmatrix}$$
(17)

and the radial forces can be calculated as

$$F_{x} = \frac{\partial W'_{m}}{\partial x} = \frac{1}{2} \left( \boldsymbol{i}_{m}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{m}}{\partial x} \boldsymbol{i}_{m} + \boldsymbol{i}_{s}^{T} \boldsymbol{L}_{s0} \frac{\partial \boldsymbol{D}_{s}}{\partial x} \boldsymbol{i}_{s} \right) \\ + \frac{c_{0}}{2} \left( \boldsymbol{i}_{m}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{M}}{\partial x} \boldsymbol{i}_{s} + \boldsymbol{i}_{s}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{M}}{\partial x} \boldsymbol{i}_{m} \right) \\ F_{y} = \frac{\partial W'_{m}}{\partial y} = \frac{1}{2} \left( \boldsymbol{i}_{m}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{m}}{\partial y} \boldsymbol{i}_{m} + \boldsymbol{i}_{s}^{T} \boldsymbol{L}_{s0} \frac{\partial \boldsymbol{D}_{s}}{\partial y} \boldsymbol{i}_{s} \right) \\ + \frac{c_{0}}{2} \left( \boldsymbol{i}_{m}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{M}}{\partial y} \boldsymbol{i}_{s} + \boldsymbol{i}_{s}^{T} \boldsymbol{L}_{m0} \frac{\partial \boldsymbol{D}_{M}}{\partial y} \boldsymbol{i}_{m} \right)$$
(18)

It is worth noticing that the partial derivatives of the displacement matrices in (18) define the x and y dependencies of the radial forces.

#### III. COMPARISON BETWEEN STATIC FEA RESULTS AND ANALYTICAL MODELS

Flux linkages of the studied BSyRM are solved as functions of currents and radial positions, similarly as in [14]. The calculation is based on static finite-element analysis (FEA) in pre-selected operating points. The main-winding current components ( $i_{md}$ ,  $i_{mq}$ ) are varied between 0 and 20 A, the suspension winding current components ( $i_{sx}$ ,  $i_{sy}$ ) are varied between -2 and 2 A, and the radial displacements are varied between -0.6 and 0.6 mm both in the x and y directions. Together with the flux linkages, the FEA solver calculates the radial forces and the motor torque at given operating points. Fig. 1(b) shows the rotor geometry used in FEA. As an example, Fig. 1(c) shows the magnetic-field solution at the rated torque and rated radial force.

#### A. Parameters of the Textbook Model (9)

The textbook flux-linkage model (9) contains five parameters to be decided. Furthermore, it would be easy to interpret these parameters being fully independent from each other. Thus, in order to define the values of the parameters, it is not enough to solve them from (5) and (9) at a given operating point. Instead, also the force equation (10) is needed when solving all the parameters. When the flux linkages, currents, and radial forces are obtained from FEA in different operating points, the parameter values can be easily obtained by two consecutive linear least squares (LLS) fits. The LLS method provides a unique solution and neither initial values nor cost functions are needed.

First, the force coefficients are solved by using (10):

$$\begin{bmatrix}
F_{x}(1) \\
F_{y}(1) \\
\vdots \\
F_{x}(N) \\
F_{y}(N)
\end{bmatrix} = \underbrace{\begin{bmatrix}
i_{md}(1)i_{sx}(1) & i_{mq}(1)i_{sy}(1) \\
-i_{md}(1)i_{sy}(1) & i_{mq}(1)i_{sx}(1) \\
\vdots & \vdots \\
i_{md}(N)i_{sx}(N) & i_{mq}(N)i_{sy}(N) \\
-i_{md}(N)i_{sy}(N) & i_{mq}(N)i_{sx}(N)
\end{bmatrix}}_{X_{F}} \begin{bmatrix}
M_{d} \\
M_{q}
\end{bmatrix}$$
(19)

where N is the total number of operating points used for the parameter fitting. The parameter vector  $[M_d, M_q]^T$  can be then solved from

$$\begin{bmatrix} M_{d} \\ M_{q} \end{bmatrix} = \left( \boldsymbol{X}_{F}^{T} \boldsymbol{X}_{F} \right)^{-1} \boldsymbol{X}_{F}^{T} \boldsymbol{y}_{F}$$
(20)

The main and suspension winding inductances can be then solved from (5) and (9) by using the known force coefficients. The parameters are solved again by using LLS method

$$\begin{bmatrix} \boldsymbol{\psi}_{\mathrm{m}}(1) - \boldsymbol{M}\boldsymbol{i}_{\mathrm{s}}(1) \\ \vdots \\ \boldsymbol{\psi}_{\mathrm{m}}(N) - \boldsymbol{M}\boldsymbol{i}_{\mathrm{s}}(N) \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{\mathrm{md}}(1) & 0 \\ 0 & \boldsymbol{i}_{\mathrm{mq}}(1) \\ \vdots \\ \boldsymbol{i}_{\mathrm{md}}(N) & 0 \\ 0 & \boldsymbol{i}_{\mathrm{mq}}(N) \end{bmatrix} \begin{bmatrix} L_{\mathrm{d}} \\ L_{\mathrm{q}} \end{bmatrix}$$
$$\begin{bmatrix} \boldsymbol{\psi}_{\mathrm{s}}(1) - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{m}}(1) \\ \vdots \\ \boldsymbol{\psi}_{\mathrm{s}}(N) - \boldsymbol{M}^{\mathrm{T}}\boldsymbol{i}_{\mathrm{m}}(N) \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{\mathrm{s}}(1) \\ \vdots \\ \boldsymbol{i}_{\mathrm{s}}(N) \end{bmatrix} L_{\mathrm{s}}$$
(21)

As examples, the parameters of the textbook flux-linkage model are solved in two different radial-position operating points. The first operating point is chosen to resemble the start-up of the motor, meaning that the rotor would rest on the safety bearings and the y-direction radial position has a high value: x = 0 and y = -0.6 mm. In the second operating point, a centric rotor is assumed, meaning that x = y = 0. The resulting parameter values are given in Table I and referred to as parameter sets 1 and 2, respectively. Table I demonstrates that when the textbook model is used, the radial-position operating point clearly affects to the results of the LLS parameter fit.

#### B. Parameters of the Improved Model (11)

When using the inductance matrices (11), all the necessary parameters can be solved at any operating point just by using (5), (11) and (15), without having to know the calculated radial forces from FEA. The data received from FEA is now used in two consecutive LLS fits, as explained in the following. First, only the main-winding is supplied with current, having set the suspension-winding current to zero. The elements of the main-winding inductance matrix can be solved from

$$\begin{bmatrix} \boldsymbol{D}_{m}^{-1}\boldsymbol{\psi}_{m}(1) \\ \vdots \\ \boldsymbol{D}_{m}^{-1}\boldsymbol{\psi}_{m}(N) \end{bmatrix} = \begin{bmatrix} i_{md}(1) & 0 \\ 0 & i_{mq}(1) \\ \vdots & \vdots \\ i_{md}(N) & 0 \\ 0 & i_{mq}(N) \end{bmatrix} \begin{bmatrix} L_{d0} \\ L_{q0} \end{bmatrix}$$
(22)

using the LLS method, similarly as in (19) and (20). The main-winding inductances are then used to calculate  $\gamma$  from (12).

In the next stage, the main-winding current is set to zero having non-zero values in the suspension winding current. The elements of the suspension-winding inductance matrix can be solved from

$$\begin{bmatrix} \boldsymbol{D}_{s}^{-1}\boldsymbol{\psi}_{s}(1) \\ \vdots \\ \boldsymbol{D}_{s}^{-1}\boldsymbol{\psi}_{s}(N) \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{s}(1) \\ \vdots \\ \boldsymbol{i}_{s}(N) \end{bmatrix} L_{s0}$$
(23)

by using the LLS method. Finally, the scalar valued coefficient  $c_0$  can be solved as  $c_0 = \sqrt{2L_{s0}/(L_{d0} + L_{q0})}/2$ .

As examples, the parameters of the improved model are solved in the same radial-position operating points as with the textbook model in the previous section. The obtained parameter values are given in Table II and referred to as parameter sets 3 and 4. Table II shows that when the improved model is used, the fitted parameter values are almost independent of the radial-position operating point.

## C. Flux Linkages and Radial Forces as a Function of Radial Position

Once the parameters of the models are defined, the flux linkages and the radial forces are calculated using (9), (10), (11), and (18). The values are defined by using parameter set 2 for the textbook model and parameter set 4 for the improved model. The flux linkages and currents are calculated in the same current and radial position operating points, which were defined when calculating the FEA results.

Fig. 3(a) shows the flux-linkage component  $\psi_{md}$  as a function of radial position x at y = 0,  $i_{md} = 20$  A, and  $i_{mq} = i_{sx} = i_{sy} = 0$ . Fig. 4(a) shows the corresponding radial force  $F_x$ . It is evident that the textbook model fails to take into account the main-winding flux-linkage variation, caused by the radial displacement. This results in an inability to predict the radial force caused by the unbalanced magnetic pull as can be seen in Fig. 4(a). On the other hand, the improved model can inherently take into account the main-winding flux-linkage variation and, thus, improve the accuracy of the radial force calculation without a need to use additional functions to deal with the eccentric rotor positions.

Figs. 3(b), 3(c), and 4(b) show the flux-linkage components  $\psi_{md}$ ,  $\psi_{sx}$ , and the radial force  $F_x$ , respectively, as a



Fig. 3. Flux linkages as a function of radial position x at y = 0: (a)  $\psi_{md}$  at  $i_{md} = 20$  A and  $i_{mq} = i_{sx} = i_{sy} = 0$ ; (b)  $\psi_{md}$  at  $i_{md} = 20$  A,  $i_{mq} = 0$ , and  $i_{sx} = i_{sy} = 2$  A; (c)  $\psi_{sx}$  at  $i_{md} = 20$  A,  $i_{mq} = 0$ , and  $i_{sx} = i_{sy} = 2$  A.



Fig. 4. Radial force  $F_x$  as a function of radial position x at y = 0: (a)  $i_{md} = 20$  A and  $i_{mq} = i_{sx} = i_{sy} = 0$ ; (b)  $i_{md} = 20$  A,  $i_{mq} = 0$ , and  $i_{sx} = i_{sy} = 2$  A.

function of radial position x, at y = 0,  $i_{md} = 20$  A,  $i_{mq} = 0$ , and  $i_{sx} = i_{sy} = 2$  A. Similar behaviour as in the previous example can be observed also in this case.

If more than three terms are included in (7), the radial force calculation using (18) can be improved. This comes at the price of significant increase in complexity of the model. Thus, the selected analytical model is always a compromise between the complexity and accuracy.

#### IV. APPLICATION EXAMPLE: STABILITY OF AN OPEN-LOOP DUAL-WINDING MOTOR MODEL

The voltage equations of the main winding (marked with subscript m) and the suspension winding (marked with subscript s) can be given as a state-space representation [2]

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\psi_{\mathrm{m}}\\\psi_{\mathrm{s}}\end{bmatrix}}_{\psi} = \underbrace{\begin{bmatrix}u_{\mathrm{m}}\\u_{\mathrm{s}}\end{bmatrix}}_{u} - \underbrace{\begin{bmatrix}R_{\mathrm{m}}I & \mathbf{0}\\\mathbf{0} & R_{\mathrm{s}}I\end{bmatrix}}_{R}\underbrace{\begin{bmatrix}i_{\mathrm{m}}\\i_{\mathrm{s}}\end{bmatrix}}_{i} \qquad (24)$$

where the voltage vectors are defined as  $\boldsymbol{u}_{\rm m} = [u_{\rm md}, u_{\rm mq}]^{\rm T}$ and  $\boldsymbol{u}_{\rm s} = [u_{\rm sx}, u_{\rm sy}]^{\rm T}$ . The resistances of the windings are  $R_{\rm m}$ and  $R_{\rm s}$ . The current vector  $\boldsymbol{i}$  can be solved from (5) and substituted to (24), leading to

$$\frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}t} = \boldsymbol{u} - \boldsymbol{R}\boldsymbol{L}_{\Sigma}(x,y)^{-1}\boldsymbol{\psi}$$
(25)

The open-loop stability of a dual-winding motor model can be now analyzed by calculating the eigenvalues of the matrix  $-\mathbf{RL}_{\Sigma}(x, y)^{-1}$ . If all the eigenvalues have negative real parts, the system is stable, otherwise the system is unstable.

The stability of the system is studied by varying the rotor displacements in x and y directions between -1 mm and 1 mm. The elements of  $L_{\Sigma}(x, y)$  are defined by using (9) and the resistance of the windings are  $R_{\rm m} = 0.1 \Omega$  and  $R_{\rm s} = 2.9 \Omega$ . Fig. 5(a) shows the stability regions when parameter set 1 from Table I is applied. Fig. 5(b) shows the stability regions when parameter set 2 from Table I is applied. The available radial direction movement is limited inside the circle marked with black dashed line (because the nominal airgap of the motor is 1 mm). It is evident from Fig. 5(a) that the open-loop motor model does not remain stable within the whole operation region. Thus, the combination of the textbook model with parameters set 1 from Table I does not represent a physically consistent system.

When the improved model is applied, a stable system is obtained for the whole operating region regardless of which parameter set is selected from Table II.

#### V. CONCLUSIONS

An improved analytical model for BSyRMs is proposed. The model is applicable, e.g., for a stability analysis, time-



Fig. 5. Dual-winding motor model (25) open-loop stability, when the textbook model (9) is applied and the rotor displacement is varied: (a) parameter set 1; (b) parameter set 2. The light gray area represents a stable system, whilst the white area represents an unstable system.

domain simulations, or developing real-time control methods. The paper shows that the rotor eccentricity has a significant effect on the inductances and the model accuracy depends on of the inverse-airgap length approximation accuracy. It is shown that at least the first two terms have to be included in the inverse-airgap length series expansion in order to be able to make the main-winding inductances depend on the radial positions, and consequently, see the effect of unbalanced magnetic pull on the calculated radial forces. By examining the open-loop system pole locations, it is shown that the stability of a dual-winding motor model depends not only on the model structure, but also on its parameter values. Finally, a straightforward LLS method, to fit the parameter values of the models, is proposed.

#### REFERENCES

- H. Mitterhofer, W. Gruber, and W. Amrhein, "On the high speed capacity of bearingless drives," *IEEE Trans. Ind. Electr.*, vol. 61, no. 6, pp. 3119–3126, June 2014.
- [2] A. Chiba, T. Deido, T. Fukao, and M. A. Rahman, "An analysis of bearingless ac motors," *IEEE Trans. Energy Convers.*, vol. 9, no. 1, pp. 61–68, Mar. 1994.
- [3] W. Gruber, W. Amrhein, and M. Haslmayr, "Bearingless segment motor with five stator elements: Design and optimization," *IEEE Trans. Ind. Appl.*, vol. 45, no. 4, pp. 1301–1308, July/Aug. 2009.

- [4] T. Nussbaumer, P. Karutz, F. Zurcher, and J. W. Kolar, "Magnetically levitated slice motors – an overview," *IEEE Trans. Ind. Appl.*, vol. 47, no. 2, pp. 754–766, Mar./Apr. 2011.
- [5] J. Asama, Y. Hamasaki, T. Oiwa, and A. Chiba, "Proposal and analysis of a novel single-drive bearingless motor," *IEEE Trans. Ind. Electr.*, vol. 60, no. 1, pp. 129–138, Jan. 2013.
- [6] N. Bianchi, S. Bolognani, D. Bon, and M. Dai Pre, "Rotor fluxbarrier design for torque ripple reduction in synchronous reluctance and pm-assisted synchronous reluctance motors," *IEEE Trans. Ind. Appl.*, vol. 45, no. 3, pp. 921–928, May/June 2009.
- [7] A. Chiba, T. Fukao, O. Ichikawa, M. Oshima, and M. Takemoto, Magnetic bearings and bearingless drives. Burlington, MA: Newnes, 2005.
- [8] A. Arkkio, B. R. Nepal, and A. Sinervo, "Electromechanical interaction in a synchronous reluctance machine," in *Proc. SPEEDAM*, Pisa, Italy, Aug. 2010, pp. 501–506.
- [9] L. Harnefors and H.-P. Nee, "Model-based current control of ac machines using the internal model control method," *IEEE Trans. Ind. Appl.*, vol. 34, no. 1, pp. 133–141, Jan./Feb. 1998.
- [10] M. Hinkkanen, H. A. A. Awan, Z. Qu, T. Tuovinen, and F. Briz, "Current control for synchronous motor drives: Direct discrete-time pole-placement design," *IEEE Trans. Ind. Appl.*, vol. 52, no. 2, pp. 1530–1541, Mar./Apr. 2016.
- [11] S. E. Saarakkala, M. Sokolov, M. Hinkkanen, J. Kataja, and K. Tammi, "State-space flux-linkage control of bearingless synchronous reluctance motors," in *Proc. IEEE ECCE'16*, Milwaukee, WI, Sept 2016.
- [12] W. Bu, S. Huang, S. Wan, and W. Liu, "General analytical models of inductance matrices of four-pole bearingless motors with two-pole controlling windings," *IEEE Trans. Magn.*, vol. 45, no. 9, pp. 3316– 3321, Sept 2009.
- [13] C. Michioka, T. Sakamoto, O. Ichikawa, A. Chiba, and T. Fukao, "A decoupling control method of reluctance-type bearingless motors considering magnetic saturation," *IEEE Trans. Ind. Appl.*, vol. 32, no. 5, pp. 1204–1210, Sep./Oct. 1996.
- [14] S. E. Saarakkala, M. Sokolov, V. Mukherjee, J. Pippuri, K. Tammi, A. Belahcen, and M. Hinkkanen, "Flux-linkage model including crosssaturation for a bearingless synchronous reluctance motor," in *Proc. ISMB15*, Kitakyushu, Japan, Aug. 2016.

#### VI. BIOGRAPHIES

**Seppo E. Saarakkala** received the M.Sc.(Eng.) degree in electrical engineering from the Lappeenranta University of Technology, Lappeenranta, Finland, in 2008, and the D.Sc (Tech.) degree in electrical engineering from Aalto University, Espoo, Finland, in 2014. He has been with the School of Electrical Engineering, Aalto University, since 2010, where he is currently a Postdoctoral Research Scientist. His main research interests include control systems and electric drives.

Victor Mukherjee received the B.Sc. (Tech.) degree in electrical engineering from National Institute of Technology, Durgapur, India, and the M.Sc. (Tech) degree in industrial electronics from Lappeenranta University of Technology, Lappeenranta, Finland, in 2009 and 2014, respectively. He is currently working toward the D.Sc. degree in Aalto University, Espoo, Finland.

**Maksim Sokolov** received his B.Sc. (Tech.) and M.Sc. (Tech.) degrees from ITMO University, Saint Petersburg, Russia in 2012 and 2014, respectively. Since 2015 he is working toward his doctoral degree in the Electric Drives research group at Aalto University, Finland. His main research interests include control of electric drives, bearingless machines and magnetic levitation.

**Marko Hinkkanen** (M'06-SM'13) received the M.Sc.(Eng.) and D.Sc.(Tech.) degrees in electrical engineering from the Helsinki University of Technology, Espoo, Finland, in 2000 and 2004, respectively. He is an Associate Professor with the School of Electrical Engineering, Aalto University, Espoo. His research interests include control systems, electric drives, and power converters.

Anouar Belahcen (M'13-SM'15) received the M.Sc. (Tech.) and Doctor (Tech.) degrees from Helsinki University of Technology, Finland, in 1998, and 2004, respectively. He is now Professor of electrical machines at Tallinn University of Technology, Estonia and Professor of Energy and Power at Aalto University, Finland. His research interest are numerical modeling of electrical machines, magnetic materials, coupled magneto-mechanical problems, magnetic forces, magnetostriction, and fault diagnostics.