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Use of a Fuzzy Decision-making Approach in an Analysis of the Vulnerability of Street Networks for Disaster Management

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Abstract. Disaster management with respect to urban structures has received more attention in recent years. In disaster management, the most vulnerable structures in a modern society are the critical networks, such as transportation networks. The vulnerability analysis of spatial networks should not depend only on the topological structure; some non-topological attributes, such as population information, should also be considered. In a rescue operation, decision-making problems are very often uncertain or vague because of the lack of information. Therefore, the classification of a high or low-risk area on the basis of spatial information should not have crisp boundaries and it would be more reasonable to use a fuzzy approach. In this paper, population information and a betweenness centrality measure of the road network were used as the evaluation criteria, and a fuzzy multiple-attribute decision-making (MADM) approach was used to support a vulnerability analysis of the road network of Finland for disaster management. In order to validate the model, results were compared with original population information and a betweenness attribute map. The validation results showed the hotspots in a fuzzy MADM vulnerability map have a similar pattern to an original input attributes map and the number of hotspots were reduced to a reasonable scale in order to improve rescue efficiency.

Keywords: Fuzzy multiple-attribute decision-making, spatial network vulnerability analysis, GIS, disaster management.

1 Introduction

Natural man-made disasters cause insecurity for people and society. Therefore, disaster management with respect to urban structures has become one of the core tasks of governments and various literary sources deal with emergency and disaster management (Alçada-Almeida 2009, Golant and Burton 1970). A review of the literature indicates that the most vulnerable and also vital structures in a modern society are the various networks such as the electricity, water, and transportation networks. What happens to a city if the electricity

network is down or if no water is available or if the transportation network is not functioning? When disaster strikes, how many people are in danger and what is the most efficient way to save people's lives and property? These are vitally important questions which need to be answered in advance and not when the situation is actually occurring.

There are two types of decision-making processes in rescue operations: rapid decision making and long-term decision making (Castrén 2007). Rapid decision making happens in the preliminary phase. A decision will be made on the basis of how well the persons in charge of the situation know the area(s) at risk and it forces the decision to be made by combining the rescue personnel's knowledge and intuition. Long-term decision making can be involved if the rescue mission is not extremely urgent and there is time to analyse the situation and areas. In the long-term decision-making process, the rescue personnel will consider many issues and it can be very complex and time-consuming because of the difficulties involved in combining all the different kinds of information. Therefore, multi-criterial decision making analysis for disaster management is needed.

A critical location in the infrastructure can be defined as an object whose removal or destruction changes the structure of the infrastructural network in terms of flow and connectedness. Much research has been done to analyse the vulnerability of road networks on the basis of an analysis of the topological structure of the network using only topological attributes. For instance, line graph modelling with connectivity analysis and topological measures were combined to identify critical locations on a road network (Demšar 2008). The centrality measures were the core method used in that research study. The vertices that correspond to critical locations have one or more of the following three properties: they are cut vertices and they have a high betweenness value or a low clustering coefficient. It was not always sufficient to base network vulnerability analysis only on the topological structure for disaster management; some non-topological attributes, such as population information, should also be considered (Zhang, Z. and Virrantaus 2010). For instance, saving people's lives is always the most important task in a disaster; therefore, people make the location critical. Crisp multiple-attribute decision making (MADM) was used to combine not only the topological attributes of a road network but also non-topological attributes to create a vulnerability map of a transportation network for disaster management. For instance, the road segments that correspond to critical locations should have one or more of the following properties: they have cut edges (topological attribute), they have a high betweenness value (topological attribute), and they have the most people living nearby (non-topological attribute) (Zhang and Virrantaus 2010).

Crisp MADM decision analysis cannot solve the uncertainty or vagueness of geographical information in vulnerability analysis. For instance, it is difficult to say exactly how many people living near a road segment constitute a high-risk area in disaster management. Therefore, the classification of a high or low-risk area on the basis of population information should not have crisp boundaries and

it would be more reasonable to use a fuzzy approach. Zhang et al. (2014) used fuzzy MADM to analyse the vulnerability of buildings on the basis of population information for disaster management. In their work, road networks and their corresponding topological attributes were not considered. In this study, we propose a fuzzy MADM approach to identify vulnerable locations on a road network for the purposes of disaster management. Population information and topological attributes of the road network were used as evaluation criteria. The results were visualised with the help of GIS tools. The article is organised as follows: Section 2 gives an overview of the background and methodology, such as mathematical terms and definitions. The methods used in this research are presented in Section 3. The testing of the methods and results are presented in Section 4. Section 5 presents some conclusions and discussion.

2 Background and Theory

The idea for this research work comes from a project called “Critical Infrastructure Protection”, funded by the Scientific Committee of National Defence of Finland. The goal of the project is to analyse the vulnerability of the critical networks (transportation networks, water pipe networks, and natural gas networks) for disaster management purposes. A vulnerability map of the critical networks can be used for preparedness planning, for instance, where to set up a rescue facility so that rescue personnel can save more lives and resources. It can also be used for supporting city planning, for instance, where to set up a critical new network so it will not increase the number of vulnerable locations relative to the current network in terms of disaster management.

The concept of vulnerability has many definitions and the meaning of the term may depend on the context. Vulnerability studies of spatial networks have attracted a lot of attention recently. Berdica (2002) defines the vulnerability of a transportation system as susceptibility to incidents that can result in considerable reductions in road network serviceability. A similar definition was introduced by Nicholson and Du (1994). Taylor and D’Este (2007) state that “a network link is critical if loss (or substantial degradation) of the link significantly diminishes the accessibility of the network or of particular nodes, as measured by a standard index of accessibility”. Wakabayashi and Iida (1992) and Bell and Iida (1997) think the vulnerability analysis of complex spatial networks is very closely related to minimal paths and cuts. The minimal routes refer to the minimum number of successive links needed to connect a pair of nodes. For instance, the network will function well if any one of the routes functions. The minimum cut sets are the minimum number of links needed to disconnect a pair of nodes. If any one of the cut sets fails, the networks fail. Lei (2003) thought hubs play a key role in the proper functioning of transportation and communication systems. Therefore, two hub protection problems were defined that aim to minimise the system costs associated with the worst-case facility loss. In this article, the term vulnerability was applied to spatial networks such as transportation networks to evaluate the importance of a road segment in connection with a disaster on the basis of its topological location and population information.

2.1 Basic definitions of graph theory

Jungnickel (2005) has introduced a basic definition of graph theory. A graph $G = G(V, E)$ is a pair of two disjointed finite sets V and E , where E is a subset of $V \times V$. The elements of V are *vertices*, the elements of E are *edges*. An edge $e = uv$ from the set E connects vertices u and v . When representing this structure graphically, the vertices are usually drawn as dots and edges as lines connecting each two respective vertices. Two vertices u, v of graph G are *adjacent* or *neighbours* if uv is an edge of G . Two edges of G are *adjacent* if they share a common end vertex. If no direction is specified on the edges, such as the edge uv is considered the same as the edge vu , then the graph is *undirected otherwise is directed*. The vertices and the edges are considered as a weighted graph if numerical or other values assigned to them and these values are called the *weights*. In this research work, a road network can be modelled as graphs, where road segments can be modelled as edges of the graph, starting or ending points of the road segments or intersection points of the road segments can be modelled as vertices. At this point in the development of our method we model the road network as undirected and unweighted graphs even though the method could eventually be extended to graphs that are either weighted or directed or both.

The properties of edges can be translated into properties of vertices, a so called line graph or an edge dual of its original graph. Given a graph G , the line graph $L(G)$ takes the edges of G as its vertices, such as $V(L(G)) = E(G)$. Two vertices u and v in the line graph are connected if and only if the respective edges u and v are adjacent in G .

2.2 Centrality Measures

The idea of network centrality was first introduced by Bavelas for social networks (Bavelas 1948). It describes the structural importance of each vertex in the graph so vertices with higher centrality have a larger impact on other vertices. Centrality measures have often been used in graph analysis and modelling (Freeman 1977, Freeman et al. 1991, Klein 2010). Degree centrality is based on the idea that the most important nodes are those with the largest number of ties to other nodes in the graph.

Betweenness centrality can be defined on a vertex or an edge, and it measures how many times a vertex or an edge falls on the shortest path between other vertices in comparison with the total number of shortest paths. For the graph $G = (V, E)$ with n vertices, the betweenness $C_B(v)$ for the vertex v is:

$$C_B(v) = \sum_{\substack{s,t \in V \\ s,t \neq v}} \frac{\mathfrak{g}_{st}(v)}{\mathfrak{g}_{st}} \quad (1)$$

where $\mathfrak{g}_{st}(v)$ is the number of shortest paths passing across v and is the total number of shortest paths (Barthélemy 2004).

2.2 Fuzzy MADM

Fuzzy set theory was first introduced by Zadeh as a mathematical theory of vagueness (Zadeh 1965). A fuzzy set A in X is characterized by a membership (characteristic) function. If X is the universe of discourse, and its elements are denoted by x , then the fuzzy set A in X is defined as a set of ordered pairs which is called the membership function (MF) of x in A . It maps each element of X to a membership value between 0 and 1. The MF itself can be an arbitrary curve whose shape fits the best of the dataset from the point of view of simplicity, convenience and efficiency in computing. There are many types of MFs, and the simplest MFs are formed using straight lines such as a triangular and trapezoidal MF. A Gaussian curve is another type of MF and a mathematical form of a Gaussian fuzzy MF is shown by (2). The Gaussian membership function depends on two parameters c and σ as given by

$$\text{Gaussian}(x; c, \sigma) = e^{-\frac{1}{2} \left(\frac{x-c}{\sigma} \right)^2} \quad (2)$$

Where c represents the centre of the MF and σ determines its width (Math works 2015). A Gaussian MF achieves smoothness of the curve compared to triangular and trapezoidal MFs. A better way of defining MF curves is by using the adaptive-network-based fuzzy inference system (ANFIS) if the input and output data of the fuzzy rules are available. ANFIS can serve as a basis for constructing a set of fuzzy if-then rules with appropriate membership functions to generate the stipulated input-output pairs (Jang 1993). However, it is not a suitable method for this research work because we don't have output data available. In this case, output data is the vulnerability of the road network and this is created by using input attribute data and self-designed fuzzy rules. Therefore, we used a Gaussian MF so the degree of membership function can change gradually without corner points.

The fuzzy set theory in the field of MADM is justified when the intended goals or their attainment cannot be defined crisply but only as a fuzzy set. Bellman and Zadeh (1970) introduced decision making in a fuzzy environment. It can be summarised as follows:

$$D = G \cap C \quad (3)$$

where G is the fuzzy goal, C is the fuzzy constraint, and D is the fuzzy decision that is characterised by a membership function as follows:

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)) \quad (4)$$

The maximising decision is then defined as follows:

$$\max_{x \in X} \mu_D(x) = \max_{x \in X} \min(\mu_G(x), \mu_C(x)) \quad (5)$$

for k fuzzy goals and m fuzzy constraints, the fuzzy decision is defined as follows:

$$D = G_1 \cap G_2 \cap \dots \cap G_k \cap C_1 \cap C_2 \cap \dots \cap C_m \quad (6)$$

and the corresponding maximising decision is:

$$\max_{x \in X} \mu_D(x) = \max \min (\mu_{G_1}(x), \dots, \mu_{G_k}(x), \mu_{C_1}(x), \dots, \mu_{C_m}(x)) \quad (7)$$

3 Methodology

In this article, fuzzy MADM was used to compute the vulnerability of the road network and the results were compared with an original population size map and a road network betweenness value map. Two attributes; the total number of people living nearby a given road segment and the betweenness value of the road, were used as evaluation criteria. A road which is located in a densely populated area is considered vulnerable because more people will use this road to escape from a disaster. The betweenness value represents the topological importance of the road. A road which has high betweenness connects many of the shortest paths to the destination of a journey. A road which has a high betweenness value is important during the evacuation process because rescue personnel may often use this road to evacuate people.

The national road and street database (Digiroad) was used for the vulnerability analysis of the road network (Finnish Road Administration 2016). Digiroad is a nationwide database which contains accurate data on the location of all roads and streets in Finland, as well as their most important physical features. This research work is the continuation work from Demšar et al. (2008) 's work. Digiroad data was first converted into an ASCII file where the topology information was retained in the form of the numbers of from- and to-nodes for each road segment. Geometrical or attribute information was removed from the data during the transformation. The graph representing the network was then transformed into a line graph using a specially written programme. Betweenness centrality was computed to the line graph by using the Pajek large network analysis software (Nooy et al. 2011).

The population information data came from the building register data of the city of Helsinki in Finland. It contains information about the locations of buildings and the number of persons who registered a building as their home address. In this case, only the population information of residential buildings was used to model the vulnerability of the street network and other types of buildings such as shops, office buildings etc. were ignored due to the lack of population information. We created a 300 metres buffer from each road segment and selected a number of residential buildings which fell inside the buffer. After that, we calculated the total number of people who live inside the selected buildings for each road segment and attached this population information to the segment.

3.1 *Vulnerability Analysis of the Road Network Using the Fuzzy MADM Model*

In the MADM model, each road segment is considered as one alternative. Each alternative was evaluated according to two attributes: the total number of people living near this road (PA) and the betweenness value of the road (BA). A Gaussian membership function was created for each input attribute and is illustrated in Fig. 1.

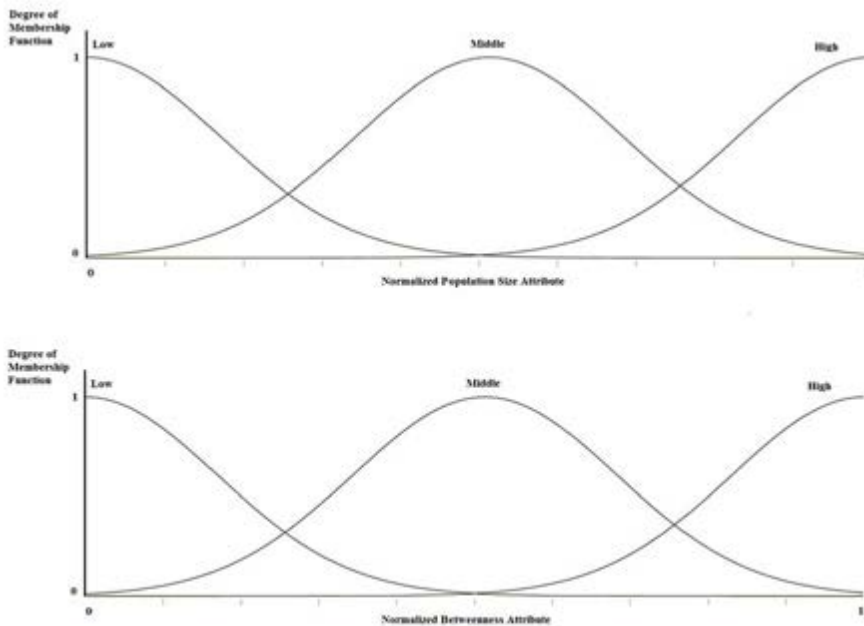


Figure 1. Gaussian membership function curve for population size attribute and betweenness attribute.

The input attributes were first described in terms of linguistic variables. For instance, the population size attribute was described as “a small number of people”, “a medium number of people”, and “a large number of people” living near a road segment. The output vulnerability value can also be described in terms of linguistic variables, such as the “low vulnerability”, “medium vulnerability”, and “high vulnerability” of a road segment. These linguistic variables were represented by a fuzzy MF. The input attribute value is more likely to belong to a certain MF when the degree of the MF μ is closer to 1. Three MFs, low, middle, and high, were defined for each input attribute and output result. Fuzzy rules such as “If X is high and Y is high then Z is high” can be used to obtain the relationship between the input and output. In a fuzzy rule, the if part (antecedent part) partitions the input space into a number of fuzzy regions (MFs), and the then part (consequent part) describes the behaviour of the system in these fuzzy regions (Zimmermann 2001). The word *and* in the fuzzy rule is a fuzzy operator. The operator *and* is used to describe a fuzzy intersection or conjunction of the input MF. It takes the minimum value of the input MFs. In this study, the experts do not make decisions about choosing the most vulnerable road segment in a disaster. Instead, we use a fuzzy logic system to capture the experts’ knowledge, and the experts’ decisions about the vulnerability value of each road segment that are based on input attributes are represented by using fuzzy rules. Nine fuzzy rules were created on the basis of the population size and betweenness attributes and are illustrated in Fig. 2. Fig. 3 illustrates a surface view of nine fuzzy rules.

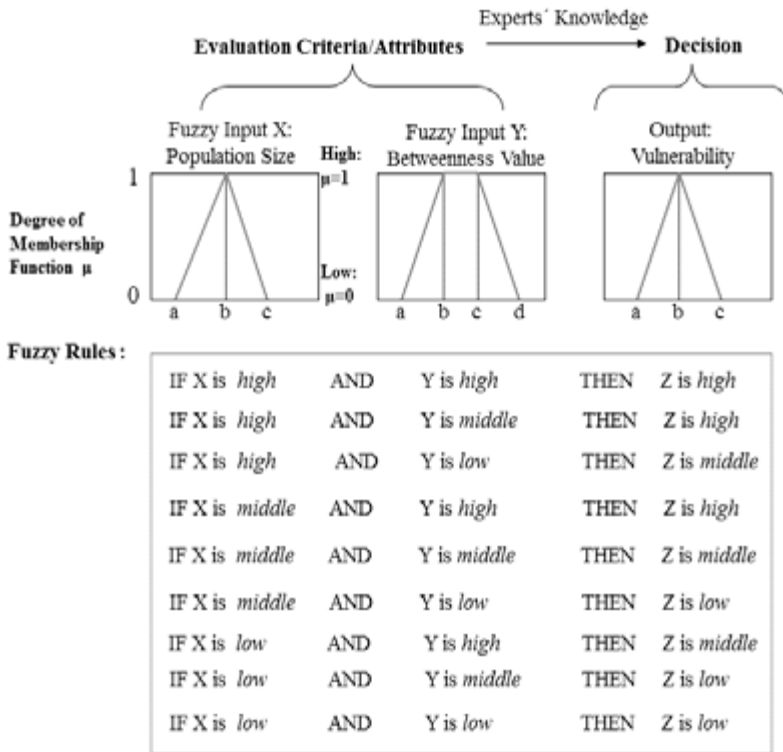


Figure 2: an example of a fuzzy membership function and fuzzy rules.

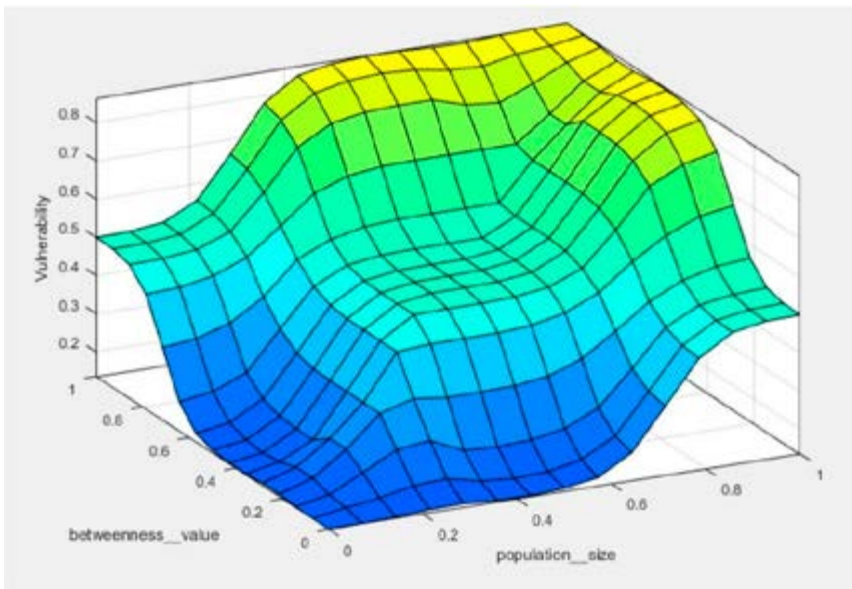


Figure 3. Surface view of the fuzzy rules. The X and Y axis of the fuzzy curve represents the normalized population size and betweenness attribute value and the Z axis represents the fuzzy output as the road vulnerability value.

4 Results

The results of using the fuzzy MADM model to analyse the vulnerability of the road network are shown in Fig. 4. The most vulnerable roads are represented in red on the vulnerability map. The most vulnerable locations are parts of the road segments in Ruoholahti, Herttoniemenranta, Vallila, and Kiviruukki. The most vulnerable roads also include a part of Turunväylä, Kehä ring No. 1, Vihdintie, and the intersection point of roads Nos. E75 and 7.

The model results were validated against the original population information attribute and betweenness attribute maps (Figs. 5 and 6) in order to see if the roads' vulnerability values match the original input attribute information. For instance, it is not correct if the model shows a road segment which has high vulnerability but has a low population size and betweenness value on the original input attribute maps. The comparison results showed that with the same normalisation and visualisation methods, the vulnerability map has similar patterns to the betweenness value attribute map. For instance, in the middle of the vulnerability map, the red lines which represent part of Turunväylä, Kehä ring No. 1, and their connecting roads also have a high value on the betweenness attribute map. The betweenness value map shows that Haltialantie, Fallintie, and Kuninkaantammentie have the highest betweenness value, but most of these roads received the second lowest vulnerability value because of the low population size attribute of the roads. The same idea also applies to the other major roads such as part of Kehä ring No. 3 and the E75. On the other hand, the vulnerability map also has similar patterns to the population attribute map. For instance, the three star-shaped red spots in the southern part of the vulnerability map refer to the places with a high population density. For instance, the hot spot in Herttoniemenranta refers to road segments which have the highest population size and second highest betweenness value on the original input attribute maps. Herttoniemenranta is a critical location



Figure 4: Vulnerability of road network using the fuzzy MADM model.



Figure 5: Population size attribute map of road networks.



Figure 6: Betweenness value attribute map of road network.

because it connects the eastern part of Helsinki to the city centre. The Finnish government planned to build a new bridge over the sea in Kruunuvuori to more evenly distribute the heavy traffic load of this area. Länsiväylä received the lowest vulnerability value because most of the road segments have the second lowest values on both the population and betweenness attribute maps.

By using fuzzy modelling, the numbers of hot spots from the population size and betweenness maps were reduced to a reasonable scale, and the places which have both a high population size and betweenness values were clearly highlighted.

5 Conclusion and Discussion

In this paper, a fuzzy MADM model was used to compute a vulnerability map for the road networks in the area of the capital of Finland. Comparing the results with the original input attribute maps showed that a fuzzy model is a suitable method to identify the vulnerable locations on a road network for disaster management. Hot spots on the vulnerability map refer to the road segments which have a high population size value and/or a high betweenness value. The biggest benefit of combining several attributes is to save time and rescue resources in preparedness for disaster management. After the attributes have been combined into an overall evaluation score (similarities to the ideal solution), the number of roads that need to be protected is reduced, compared with adding the results together from every single attribute value vulnerability analysis map.

The betweenness computation has low accuracy because it is assumed that the length of each graph segment is the same and the shortest path between each pair of locations (vertex) is the path which contains the fewest locations (vertices). In reality, the shortest path between two locations in a spatial network should have the shortest length; therefore in the future betweenness analysis should be applied to a weighted graph, where the weight refers to the length of each graph segment.

Limitations of this research work come from the population information estimation of road segments. The population dynamics were not considered in this research, because only residential building population information register data were used. This will limit the use of the results because it was assumed that all the people are at home all of the time, which is usually only true at night. This is the reason the Helsinki city centre area doesn't have a high population density hotspot – Helsinki city centre is the business centre of Finland and there are not many residential buildings due to the high real estate price. In the future, we plan to consider the temporal dynamics as well as to include information on other building types such as office buildings, shops etc. in order to make the model more realistic and useful. We created a 300 metres buffer from each road segment and a select number of people who live inside the buffer as population size information data because we assumed that people will use the nearest roads for evacuation in the event of an emergency. However, the buffer analysis of road segments will cause the redundancy of the population information data. For instance, one building might fall inside two road segments' buffers if these roads are located close to each other. It is hard to say exactly which road a person will use if his/her home has several roads nearby with almost equal distances to the home. The person might not choose the road which has the shortest distance to his/her home but the road which has the easiest connection to a major road. On the other hand, it is also difficult to say exactly how many people living nearby constitute a high-risk of a road network. We address this problem by using a fuzzy inference system. The main idea of a fuzzy inference system is that instead of defining crisp boundaries between values of variables, the variable values are defined to be imprecise and with a degree of membership functions. In the future, we can also improve the accuracy of population information calculation by using

more advanced spatial algorithms and take into consideration how people behave in emergency situations.

In this article, our main focus is on proposing a fuzzy MADM approach in combining topological (betweenness attribute) and non-topological (population information) measures of spatial networks for crisis management applications. It was not our intention to produce high quality case study results in this article, which means the fuzzy rules and fuzzy MFs are not produced in the real domain. It was instead, our intention to introduce our methods, which can then be utilised by experts to produce accurate scientific results using their own knowledge and study data. For instance, in the future more evaluation criteria (attributes), such as the number of children and elderly people living near the roads, the cut edge value of the roads, and other centrality measure indices, can also be added to the analysis.

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