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Influence of Magnetic Saturation on Modeling of an Induction Motor

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Abstract—This paper deals with modeling of a saturated induction machine. In control algorithms, the inverse-$\Gamma$ model is widely used. However, the saturation models are typically based on the $\Gamma$ or $\Gamma'$ model. Transformation between these models is easy if the machine is magnetically linear, but since the motor saturates, the modeling becomes more complex. In this paper, we study the properties of the $\Gamma$ and inverse-$\Gamma$ models and the transformation between these two models when the saturation is taken into account. The phenomena related to model transformation are studied by means of analytical equations. Also, the functionality of the models is studied by means of laboratory experiments.

Index Terms—Induction motor, magnetic saturation, $\Gamma$ model, inverse-$\Gamma$ model.

I. INTRODUCTION

Speed-sensorless, model-based control of an induction motor is widely used in industrial applications. Modeling of an induction motor is discussed, e.g., in [1]–[7], but there are still open issues related to the effect of the saturation characteristics of the motor.

The $\Gamma$ model is often considered as a reference model in literature, where the $\Gamma$ and inverse-$\Gamma$ models are derived from. Typically, the inverse-$\Gamma'$ model is used in motor control due to the implementation aspects, e.g., an easier implementation of the torque equation. However, the saturation phenomena are easier to include in the $\Gamma'$ model. For example, in [8], the inverse-$\Gamma$ model, including the saturation, is used in a controller and observer structure. In [9], the saturating $\Gamma$ model is used. The magnetic saturation phenomenon is present in all electrical machines. In many commercial motors, the main flux (magnetizing branch) is significantly saturated already at the rated operating point. An often used way to model the saturation of the induction motor is to model the stator inductance as a function of the stator flux, e.g. [10]. Furthermore, the low-power induction motors are often equipped with closed rotor slots, making the rotor leakage flux saturate highly as a function of rotor current (i.e., torque) [11]. Thus, both inductances of the $\Gamma$ and inverse-$\Gamma'$ models are nonlinear. These phenomena will affect also the model transformation making the coupling factor depend nonlinearly on the fluxes and currents. Hence, the selection of the motor model for different purposes is not a self-evident issue. In [12], the influence of the motor model selection is studied by means of the finite element method (FEM). However, this paper does not apply any saturation model and the analysis is based only on the FEM-data.

The main contributions of this paper are:

- It is demonstrated that the selection of the motor model structure ($\Gamma$ model or inverse-$\Gamma$ model) is not a trivial issue. Even though it is possible to transform information between the models, the magnetic saturation effect will make the models depend nonlinearly on each other.
- The transformation between the $\Gamma$ and inverse-$\Gamma$ model structures is analyzed. The starting point for the transformation is a saturated $\Gamma$ model.
- It is shown that the saturation effect is easier to take into account in the $\Gamma$ model, making it a preferable choice for the identification and control algorithms.

The properties of the models are compared by means of steady-state experiments using a 2.2-kW induction machine.

II. MOTOR MODEL

The motor is modeled by using complex-valued space vectors, e.g., $\mathbf{u}_s = u_{sd} + j u_{sq}$. The parameters of the inverse-$\Gamma$ model are marked with primes, e.g. $L'_M$. 

Fig. 1. Induction motor models: (a) $\Gamma$-model with saturating $L_M$ and $L_R$; (b) inverse-$\Gamma$-model with saturating $L'_M$ and $L'_R$. These nonlinear parameters cause nonlinearity also in $R'_R$. 

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Index Terms: Induction motor, magnetic saturation, $\Gamma$ model, inverse-$\Gamma$ model.
TABLE I
MAGNETIC MODEL PARAMETERS FOR 2.2-kW MOTOR.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( R_s )</th>
<th>( R_r )</th>
<th>( L_m )</th>
<th>( L_∞ )</th>
<th>( c )</th>
<th>( d )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (p.u.)</td>
<td>0.065</td>
<td>0.04</td>
<td>2.56</td>
<td>0.14</td>
<td>1.06</td>
<td>0.025</td>
<td>6</td>
</tr>
</tbody>
</table>

A. Voltage and Flux Equations

Fig. 1(a) shows the dynamic \( \Gamma \) model of the induction motor in stator coordinates [1]. The voltage equations of an induction motor are

\[
\frac{d\psi_s}{dt} = u_s - R_s i_s \tag{1a}
\]

\[
\frac{d\psi_R}{dt} = -R_R i_R + j\omega_m \psi_R \tag{1b}
\]

where the stator voltage vector is denoted by \( u_s \), the stator current vector by \( i_s \), stator resistance by \( R_s \), and the electrical angular speed of the rotor by \( \omega_m \). The rotor current vector is \( i_R \) and the rotor resistance is \( R_R \). The stator and rotor flux linkages, respectively, are given by

\[
\psi_s = L_M(i_s + i_R) \tag{2a}
\]

\[
\psi_R = \psi_s + L_s i_R \tag{2b}
\]

where the stator inductance is \( L_M \) and the leakage inductance \( L_s \). The electromagnetic torque is

\[
T = \frac{3p}{2} \ln \{i_s \psi_s^*\} \tag{3}
\]

where \( p \) is the number of pole pairs of the motor.

B. Magnetic Saturation

The stator and leakage inductances depend on the flux linkages (or the currents) due to the magnetic saturation. Because the amplitudes of the flux linkages change (e.g. due to the field weakening or loss-optimizing control), the magnetic-saturation effects should be modeled and taken into account in the control algorithms.

Fig. 1(a) shows the \( \Gamma \) model that is a preferred starting point for modelling a saturated induction machine [1]. The saturation characteristics of the \( \Gamma \) model (unlike those of the T model) can be, in theory, uniquely defined based on measurements from the stator terminals. Furthermore, since the effects of the closed rotor slots are to be modelled, the leakage inductance should be located at the rotor side.

The flux densities in the stator core depend mainly on the stator flux linkage [1]. The stator flux saturation characteristic of a 2.2-kW machine, computed with FEM, are shown in Fig. 2(a). The leakage flux saturation characteristics are shown in Fig. 2(b). Due to the closed rotor slots, the leakage flux has an easy route via the slot bridges at low rotor currents. When the rotor current (and the rotor leakage flux) increases, the thin bridges become fully saturated.

The saturation characteristics could be modelled using measured or computed look-up tables. Alternatively, explicit functions can be used, which typically simplifies the implementation of the model and allows extrapolation beyond the data range used in the fitting procedure. We apply a saturation model similar to [13], [14]

\[
L_M(\psi_s) = \frac{L_u - L_∞}{1 + (\psi_s/c)^s} + L_∞ \tag{4a}
\]

\[
L_σ(\psi_σ) = \frac{L_u - L_∞}{1 + (\psi_σ/d)^s} + L_∞ \tag{4b}
\]

where \( \psi_s = |\psi_s| \) and \( \psi_σ = |\psi_σ| \) are the flux magnitudes, \( L_u \) is the unsaturated inductance, \( L_∞ \) is the fully saturated inductance, and \( c, d, r, \) and \( s \) positive constants. Fig. 2(a) also shows the curves corresponding to (4) parametrized by Table I. Induction machines may show some cross-saturation effects, especially if the rotor slots are skewed. If needed, these effects could be included in the saturation model [14], at the expense of more detailed characterization of the machine and increased number of model parameters.

III. ANALYSIS OF THE MODEL TRANSFORMATION

In this section, we analyze, how the coupling factor is influenced when the operating point of the motor changes. The analysis is based on the equations (1) and (2), in which the saturation is implemented using (4). The equations are parametrized according the Table I. Steady state operation is considered.

The parameters and variables of the \( \Gamma \) model can be transformed to those of the inverse-\( \Gamma \) model using the coupling factor [1]

\[
γ = \frac{L_M}{L_M + L_σ} \tag{5}
\]

giving the following parameters for the inverse-\( \Gamma \) model:

\[
L′_σ = γL_σ \quad L′_M = γL_M \quad R′_R = γ^2 R_R \tag{6}
\]

When this transformation is used, the models are mathematically equivalent in steady state (and in transients as well if the magnetics are linear). It can be seen that due to the magnetic saturation, the equivalent inverse-\( \Gamma \) model parameters depend on the stator flux and the rotor current. The rotor flux can be transformed to the inverse-\( \Gamma \) rotor flux as \( \psi'_R = γ \psi_R \).
Fig. 3. Transformation of the $\Gamma$-model parameters to the inverse-$\Gamma$-model when the stator flux is kept constant and torque (and inherently rotor current) changes: (a) fluxes; (b) magnetizing inductance; (c) leakage inductance (d) rotor resistance.

A. Rotor Branch Saturation

As explained, the motor model can be transformed between $\Gamma$ and inverse-$\Gamma$ models using the coupling factor (5). The coupling factor, however, includes nonlinear inductances. As a result, the parameters in (6) will become nonlinear functions of the fluxes or currents.

The effect of model transformation is investigated using the $\Gamma$ model [Fig. 1(a)] as a reference model, in which the magnetic model including the saturation phenomena is modeled using (4).

As an example, the effect of the coupling between the $\Gamma$ and inverse-$\Gamma$ model is shown in Fig. 3. The magnitude of the stator flux is kept constant and the motor is at steady state.

Fig. 3(a) shows that the rotor flux linkage drops as a function of torque due to the rotor current. Since the stator flux stays constant, the $\Gamma$ model main inductance $L_M$ is also constant as can be seen in Fig. 3(b). The leakage inductance in Fig. 3(c) drops rapidly as a function of the torque due to the rotor current. The same behaviour can also be seen in Fig. 2(b). Since the coupling factor (5) depends on the both inductances, the flux and inductance values at the inverse-$\Gamma$ model differ from the ones in the $\Gamma$ model in highly nonlinear way. Without the saturation phenomenon in the rotor branch, the coupling factor, and inherently the transformed parameters, would be linear in the case of constant stator flux.

The rotor resistance $R'_R$ also depends on the torque [cf. Fig. 3(d)]. The rotor resistance increases approximately 30% from its no-load value when the torque is increased to 1.5 p.u. It can be noticed that the model transformation will make the modeling and identification of these nonlinear phenomena more complex.

B. Magnetizing Branch Saturation

The main (magnetizing) inductance depends on the stator flux amplitude according to (4a). Since the coupling factor (5) includes this term, the nonlinearity originates also from there. Fig. 4 presents an example of situation in which the torque is kept constant and the stator flux changes. Fig. 4(a) shows that the magnetizing inductance depends highly on the stator flux linkage. In Fig. 4(b), we can see that the coupling of the leakage inductance behaves nonlinearly as well.

IV. Fitting the Saturating Motor Data in the Models

The dynamic $\Gamma$ model is presented in Fig. 1(a). The stator inductance of this model is directly the magnetizing inductance $L_M$ and its nonlinearity can be explicitly defined as a function of the amplitude of the stator flux as demonstrated in (4a) and the numerical values are easily obtained by means of measurements from the motor terminals, e.g., by applying a no-load test.
the quadrature component of the stator current as follows: proportional to the magnitude of the rotor flux linkage and of the rotor flux oriented control, the torque becomes directly algorithm implementation point of view. For example, in case can be done through the following steps:

\[ \text{Inverse-}\frac{1}{\Gamma} \]

having the magnetizing inductance transformed to inverse-\( \frac{1}{\Gamma} \)

- \( L \) is the model rotor flux linkage \( \psi \)
- \( \text{Inverse-}\frac{1}{\Gamma} \) model 

Thus, the physical nature of the motor makes the \( \frac{1}{\Gamma} \) model a more natural choice for the modeling purposes. However, the \( \frac{1}{\Gamma} \) model [Fig. 1(b)] has advantages from the control algorithm implementation point of view. For example, in case of the rotor flux oriented control, the torque becomes directly proportional to the magnitude of the rotor flux linkage and the quadrature component of the stator current as follows:

\[ T = \frac{3p}{2} \psi_t s \]

In [1], the model transformation using the coupling factors is introduced. However, it is possible to transform the saturating magnetizing inductance using several approaches. Two different approaches are introduced to transform the information of a saturating stator inductance \( L_M \) of the \( \Gamma \) model to the corresponding saturating magnetizing inductance \( L'_M \) of the inverse-\( \Gamma \) model.

### A. Saturation Model Transformation: Case A

The \( \Gamma \) model magnetizing inductance \( L_M \), which depends on the amplitude of the stator flux linkage [cf. (4a)], is transformed to inverse-\( \Gamma \) model. The transformation results in having the magnetizing inductance \( L'_M \) as a function of the inverse-\( \Gamma \) model rotor flux linkage \( \psi'_R \). The transformation can be done through the following steps:

1. Solve the stator flux as a function of the magnetizing current: \( \psi_s = \psi_s(i_s) \), cf. Fig. 5(a).
2. Next, in Fig. 5(b), calculate the rotor flux \( \psi'_R \) as:
3. Finally, calculate the inverse-\( \Gamma \) model magnetizing inductance as

\[ L'_M(\psi'_R) = \frac{\psi'_R}{i_s(\psi_s)} \]

\[ \psi_s = u_s - R_s i_s \]

\[ \omega_s = \frac{u_s - R_s i_s}{\psi_s} \]

\[ \psi_s = \frac{u_s - R_s i_s}{\omega_s} \]

where \( \omega_s \) is the stator angular frequency. Using different stator voltage magnitudes, the stator flux values can be obtained as a function of stator current.

\[ \psi'_R(i_s) = \psi_s(i_s) - L'_a i_s \]

Because the saturation characteristics of the leakage inductance \( L'_a \) are relatively complex to model, select a constant \( L'_a \). Alternatively, selecting \( L'_a \) relatively close to the fully saturated inductance \( L_\infty \) should lead to reasonable results, because according to Fig. 2(b), it represents the behaviour of \( \psi'_R \) relatively well.

\[ L'_M(\psi'_R) = \frac{\psi'_R}{i_s(\psi_s)} \]

\[ \psi_s = \frac{u_s - R_s i_s}{\omega_s} \]

\[ \psi_s = \frac{u_s - R_s i_s}{\omega_s} \]

The value of the constant \( L'_a \) can be experimentally obtained, e.g., by completing a pulse test or a high frequency sinusoidal excitation test with a stand-still rotor. The rotor deep-bar effect and the leakage flux saturation effect (e.g. due to closed rotor slots) is not considered. These assumptions are typical implementing the motor control algorithms [15].
Fig. 6. Transformation of the saturating magnetizing flux and inductance from $\Gamma$ to inverse-$\Gamma$ model. (a) shows the stator flux linkage $\psi_s$ as a function of the $\Gamma$ model magnetizing current $i_M$ and the inverse-$\Gamma$ model leakage flux $\psi'_s$ as a function of the stator current $i_s$. The inverse-$\Gamma$ rotor flux linkage $\psi'_R$ is presented as a function of inverse-$\Gamma$ magnetizing current $i'_M$. (b) shows the $\Gamma$ model magnetizing inductance $L_M$ (stator inductance) as a function of stator flux linkage and the transformed inverse-$\Gamma$ model magnetizing inductance $L'_M$ as a function of rotor flux linkage. The $L'_M$ does not correspond the real motor at higher flux amplitude values (dashed line).

Fig. 6 shows an example of the transformation. The stator flux is calculated as a function of magnetizing current using (2a) and (4a) parametrized by Table I, as shown in Fig. 2. Due to the assumption of the constant leakage inductance ($L_\sigma = L_\infty = 0.14$ p.u.), the inverse-$\Gamma$ model does not correspond the real motor at higher flux linkage values. However, the model is valid at the normal operating range. In the Fig. 6, the scale is extended from the normal operating range to show the behaviour of the model at higher flux amplitudes (dashed line).

B. Saturation Model Transformation: Case B

Another approach to the transformation is to directly substract the leakage inductance from the stator inductance:

$$L'_M(\psi_s) = L_M(\psi_s) - L_\sigma$$  \hspace{1cm} (11)
$L_M(\psi_s)$ is fitted to the FEM data, as shown in Fig. 2. The leakage inductance and the rotor resistance are set to constant values, $L_\sigma = L_\infty = 0.14$ p.u. (highly saturated value) and $R_R = 0.04$ p.u.

The inverse-$\Gamma$ model is parametrized using the $\Gamma$ model as a starting point. The saturating magnetizing inductance $L'_M(\psi'_R)$ is transformed using the guidelines given in Section IV-A. The parameters $L'_R$ and $R'_R$ are transformed from the $\Gamma$ model using the coupling factor (5). The coupling factor is calculated by using the unsaturated magnetizing inductance value and highly saturated leakage inductance value ($L_M = 2.42$ p.u. and $L_\sigma = L_\infty$). The stator resistance value $R_s = 0.065$ p.u. is used in both models.

Fig. 9. The measured and estimated stator current magnitudes. $U = 180$ V, $f = 25$ Hz. (a) shows the measured current magnitude and the estimated magnitudes as in Fig. 8. (b) shows the estimation error $i_s - \hat{i}_s$. The coupling factor for inverse-$\Gamma$ model parameters is set to correspond the unsaturated magnetizing inductance ($L_M = 1.5$ p.u.) and highly saturated leakage inductance ($L_\sigma = L_\infty$).

In Fig. 9, we compare the current magnitudes as a function of the slip (i.e., torque), estimated by the $\Gamma$ and inverse-$\Gamma$ models, to the measured current magnitude. It can be seen that both the $\Gamma$ and inverse-$\Gamma$ models correspond to the measured data relatively fine. According to Fig 9(b), the current estimation error stays reasonably low even with the increasing slip. However, it is important to notice that the selection of the coupling factor influences on the functionality of the inverse-$\Gamma$ model.

To demonstrate the importance of the proper coupling factor selection, the coupling factor (5) is calculated by chancing the magnetizing inductance to correspond the nominal stator flux (i.e., $L_M = 1.5$ p.u.) and having the other parameters unchanged. Fig. 10 shows the resulting current estimation from the $\Gamma$ and inverse-$\Gamma$ models. The current estimation error of the inverse-$\Gamma$ model increases slightly when going to higher currents, being closest to the $\Gamma$ model at relatively small currents. If the magnetizing inductance is selected to correspond a higher saturation state, then the current estimation of the inverse-$\Gamma$ model deviates more from the measured values. It is worth noticing that if the coupling factor separately for corresponding operating point, the $\Gamma$ and inverse-$\Gamma$ models become equal.

VI. CONCLUSIONS

In this paper, the modeling aspects of the induction motor for control and identification purposes are analyzed. Typically, the stator flux linkage is first defined as a function of the magnetizing current by means of no-load test. Then, the resulting data is fitted to either $\Gamma$ or inverse-$\Gamma$ models. The coupling factor for inverse-$\Gamma$ model parameters is set to correspond the nominal magnetizing inductance ($L_M = 1.5$ p.u.) and highly saturated leakage inductance ($L_\sigma = L_\infty$). The inverse-$\Gamma$ current estimation is slightly influenced especially at higher current values.

Fig. 10. The measured and estimated stator current magnitudes. $U = 180$ V, $f = 25$ Hz. (a) shows the measured current magnitude and the estimated magnitudes as in Fig. 8. (b) shows the estimation error $i_s - \hat{i}_s$. The coupling factor for inverse-$\Gamma$ model parameters is set to correspond the nominal magnetizing inductance ($L_M = 1.5$ p.u.) and highly saturated leakage inductance ($L_\sigma = L_\infty$). The inverse-$\Gamma$ current estimation is slightly influenced especially at higher current values.
hence, defined by means of straightforward measurements from the motor terminals. Furthermore, the leakage inductance is at the rotor side, which enables modeling of the rotor leakage inductance saturation as a function of rotor current. These saturation models can be implemented using analytical functions.

The inverse-$\Gamma$ model is, however, easier to implement in the control algorithms, since the torque and slip equations become simpler. Also, the measurement of the leakage inductance using a pulse test is easier. To include the non-linearities modeled with the $\Gamma$ model, the attention has to be paid on the saturation model implementation and the coupling factors have to be calculated at all operating points. Thus, it is demonstrated that the saturation effect is easier to take into account in the $\Gamma$ model, making it a more preferable choice.

REFERENCES


BIographies

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