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# A New Technique of Error Rate Evaluation in a Wireless Environment with Arbitrary Interference and AWGN

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**Abstract**—In this work, we present a new technique for the M-QAM symbol error rate evaluation in communication systems equipped with conventional detectors designed for additive white Gaussian noise (AWGN) model of interference but operating over a fading environment with arbitrary interference and AWGN. In contrast to many reported methods, the given technique is applicable to arbitrary interference and fading models, and it can efficiently be implemented numerically. These facts make the derived result especially convenient for practical purposes.

**Index Terms**—Error rate, fading channels, interference, M-QAM, non-Gaussian noise.

## I. INTRODUCTION

Interference modeling plays a key role in many tasks related to design and analysis of communication systems. The additive white Gaussian interference model is very convenient for practical purposes, but in many realistic scenarios, statistical distributions of interference amplitude are essentially non-Gaussian [1], [2]. Such are scenarios with natural or man-made impulsive interference, and interference caused by a few spatially distributed nodes [3]- [7].

For known interference statistics, optimal receiver design in the sense of a given criterion may be possible although this is often a very challenging task. Design and analysis of optimal receivers for different interference models were presented, e.g., in [3]- [7], but the considerations were restricted by binary transmission schemes.

However, in practice, interference statistics are often not known, and the receivers are equipped with conventional detectors designed for the additive white Gaussian noise (AWGN) model of interference. Most papers analyzing error rates under different interference types assumed conventional AWGN detectors, e.g., [8]- [16]. Generally, these works considered particular interference types and specific fading conditions. At the same time, generic techniques applicable to miscellaneous operating conditions are required for practical purposes.

In this paper, we present a new technique for symbol error rate (SER) assessment, which in contrast to previously reported results, can be applied to arbitrary fading models and interference statistics. The obtained SER formula involves the products of expectations over the interference distribution and fading statistics of the transmission link. These expectations can therefore be evaluated independently. For the expectations over the fading statistics of the transmission link, we obtain

formulas for some generalized fading models such as mixtures of (generalized) gamma distributions or  $\eta$ - $\mu$  distributions [17]- [19]. In view of approximating abilities of gamma distributions and mixtures of gamma distributions [17], [20], the derived formulas ensure applicability of the SER formula to a large variety of fading and shadowing scenarios. We show that the expectations over the interference statistics can be evaluated by using derivatives of interference moment generating function (MGF), which can efficiently be implemented numerically with the help of modern software (such as Maple or Mathematica) via one-line operator or analytically via Faa di Bruno's formula [21].

## II. SYSTEM MODEL

We consider a single-antenna point-to-point transmission where the transmitter (Tx) communicates with the receiver (Rx) over a fading environment characterized by a channel gain  $h$ . For the sake of simplicity, we assume that an arbitrary square  $M$ -quadrature amplitude modulation (QAM) is used. But the analysis below can directly be extended to an arbitrary rectangular QAM.

Let a transmitted symbol  $s$  be drawn from an M-QAM constellation  $\mathcal{C}$  and be corrupted by the AWGN  $n$  and interference  $\mathcal{I}$ . Then the received symbol  $r$  can be represented as

$$r = \sqrt{\mathcal{E}_s}sh + \mathcal{I} + n \quad (1)$$

where  $\mathcal{E}_s$  is the transmit power,  $E\{|s|^2\} = 1$ , and  $E$  denotes the expectation. The variance of  $n$  is  $2\sigma^2$ . Generally,  $\mathcal{I}$  is a random variable (RV). We assume that the in-phase (I),  $\mathcal{I}_I$ , and quadrature (Q),  $\mathcal{I}_Q$ , components of  $\mathcal{I}$  are independent and identically distributed (i.i.d.). Fading statistics characterizing propagation from interfering nodes to the Rx are taken into account via the statistics of  $\mathcal{I}$  (see section IV below), and they may differ from those of  $h$ .

Following [8]- [16], we consider the conventional maximum likelihood detector that decides in favor of the symbol  $\hat{s} \in \mathcal{C}$  based on a rule

$$\hat{s} = \arg \min_{s \in \mathcal{C}} |r - \sqrt{\mathcal{E}_s}sh|^2. \quad (2)$$

## III. ERROR RATE ANALYSIS

An  $M$ -QAM symbol can be viewed as a combination of two independent pulse amplitude modulated (PAM) symbols,  $\sqrt{M}$ -PAM representing the respective I and Q symbol components.

Due to the symmetry, the SERs of I and Q components are equal, and the M-QAM SER  $P_s$  can be assessed as

$$P_s = 1 - (1 - P_1)^2 \quad (3)$$

where  $P_1$  is the SER of  $\sqrt{M}$ -PAM.

We assume that the transmitted symbols are equally probable and present below a technique for  $P_1$  evaluation.

*Proposition 1:* The  $\sqrt{M}$ -PAM SER  $P_1$  can approximately be expressed in the form of a finite sum as

$$\begin{aligned} P_1 &\approx \alpha_M \sum_{l=0}^{N_{\max}} \frac{1}{l!} E_{\mathcal{I}_1^2} \left\{ \exp\left(-\frac{\mathcal{I}_1^2}{2\sigma^2}\right) \left(\frac{\mathcal{I}_1^2}{2\sigma^2}\right)^l \right\} \\ &\quad \underbrace{E_{|h|^2} \left\{ \Gamma\left(l + 1/2, \gamma_T |h|^2/2\right) \right\}}_{E_{|h|^2}(l)} \\ &\times \frac{E_{|h|^2}(l)}{\Gamma(l + 1/2)} = \alpha_M \sum_{l=0}^{N_{\max}} \frac{(-1)^l}{l!} \\ &\quad \times \frac{d^l \mathcal{M}_{\frac{\mathcal{I}_1^2}{2\sigma^2}}(t)}{dt^l} \Big|_{t=1} \cdot \frac{E_{|h|^2}(l)}{\Gamma\left(l + \frac{1}{2}\right)} \end{aligned} \quad (4)$$

where  $\alpha_M = \frac{\sqrt{M}-1}{\sqrt{M}}$ ,  $E_x$  means the expectation with respect to (w.r.t.)  $x$ ,  $\Gamma(c)$  is the gamma function,  $\Gamma(c, x)$  is the upper incomplete gamma function, and  $\gamma_T = \mathcal{E}_s/(2\sigma^2)$  is the transmit signal-to-noise ratio (SNR).  $\mathcal{M}_x(t) \triangleq E_x \{ \exp(-tx) \}$  is the moment generating function (MGF) of  $x$ , and  $\mathcal{M}_{\frac{\mathcal{I}_1^2}{2\sigma^2}}(t) = \mathcal{M}_{\mathcal{I}_1^2}\left(\frac{t}{2\sigma^2}\right)$  [22].

*Proof:* See Appendix A.

In (4),  $N_{\max}$  controls the approximation accuracy. Generally, its value is specified by the estimated SER order:  $N_{\max}$  increases as the SER becomes smaller, which is the case of high SNR and low interference levels, as well as small constellation sizes  $M$ . The value of  $N_{\max}$  can be assigned based on the following proposition.

*Proposition 2:* For arbitrary SNR, interference levels, and  $\forall \epsilon > 0$ ,  $N_{\max}$  can be found such that the difference  $r$  between the real value of  $P_1$  and the expression on the right hand side (RHS) of (4) satisfies the following inequality

$$r \leq \epsilon = \alpha_M \left( 1 - \sum_{l=0}^{N_{\max}} \frac{(-1)^l}{l!} \frac{d^l \mathcal{M}_{\frac{\mathcal{I}_1^2}{2\sigma^2}}(t)}{dt^l} \Big|_{t=1} \right). \quad (5)$$

*Proof:* See Appendix A.

Eq. (4) involves expectations of channel gains  $E_{|h|^2}(l)$ . They can be evaluated for many generalized fading models with the help of the following proposition.

*Proposition 3:* The expectation  $E_{|h|^2}(l)$  in (4) can be evaluated in closed forms for some generalized fading distributions presented in Table I, where the abbreviation PDF means the probability density function,  $B(\cdot, \cdot)$  is the beta function,  ${}_2F_1(\cdot)$  is the Gauss hypergeometric function,  $H[\cdot]$  is Fox's  $H$  function [23, vol. 3],  $G[\cdot]$  is the Meijer  $G$  function [23, vol. 3],  $\Delta(n, a) = \frac{a}{n}, \frac{a+1}{n}, \dots, \frac{a+n-1}{n}$ ,  $I_\alpha(\cdot)$  is the modified Bessel function of the first kind of the order  $\alpha$ , and  $F_1(\cdot)$  is an Appell hypergeometric function [23, vol. 3, (7.2.4.1)].

*Proof:* See Appendix B.

Routines for implementation of Fox's  $H$  function are well known [25], [26]. The method used in sub-section C

of Appendix B can efficiently be applied to other fading distributions not considered in this work. Fubini's theorem along with results reported in Table I provide techniques for  $E_{|h|^2}(l)/\Gamma(l + \frac{1}{2})$  evaluation for fading models represented via mixtures of (generalized) gamma or  $\eta - \mu$  distributions. All these facts make the presented method applicable to a large variety of real fading distributions.

Eq. (4) represents a general method of SER evaluation. Under specific operational conditions, simplified approximate approaches can be obtained. In particular, the following proposition can be formulated for the low interference regime.

*Proposition 4:* Under the low interference regime, i.e. if  $E\{\mathcal{I}^2\} \ll \mathcal{E}_s$ , the  $\sqrt{M}$ -PAM SER  $P_1$  can approximately be assessed as

$$\begin{aligned} P_1 &\approx \alpha_M \left[ 2E_{|h|^2} \left\{ Q\left(\sqrt{\gamma_T |h|^2}\right) \right\} + \right. \\ &\quad \left. \sqrt{\frac{2\gamma_T}{\pi}} E_{\mathcal{I}_1^2} \left\{ \frac{\mathcal{I}_1^2}{2\sigma^2} \right\} E_{|h|^2} \left\{ \exp\left(-\frac{\gamma_T |h|^2}{2}\right) |h| \right\} \right]. \end{aligned} \quad (6)$$

*Proof:* Eq. (6) immediately follows from (7) (see Appendix A) with  $a \ll b$  that makes possible application of [27, 06.27.06.0011.01] by taking into account that  $Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$ , where  $\operatorname{erfc}(\cdot)$  is the complementary error function.

It can be seen that the first term in (6) represents the SER under noise-limited scenarios while the second term represents the effect of low interference power. It can also be seen that application of (6) is convenient if the interference power can be assessed. An application example is given in Section IV.

The expectation  $E = E_{|h|^2} \left\{ \exp\left(-\frac{\gamma_T |h|^2}{2}\right) |h| \right\}$  has the form of Laplace transform of product of power function and fading PDF. For many practical fading distributions, tabulated formulas can be used. Because of space restrictions, we give here only references to the formulas for fading distributions listed in Table I. For Nakagami- $m$  distribution,  $E$  can be assessed with the help of [23, vol. 4, eq. 2.1.1.1]. Eqs. [23, vol. 4, eq. 3.15.1.2] and [23, vol. 4, eq. 2.2.1.22] can be used for the  $\eta - \mu$  and GG fading distributions, respectively.

#### IV. APPLICATION EXAMPLES

In all considered scenarios we assumed the ordinary distance-dependent path-loss model [28] with the path-loss exponent  $\zeta = 3.8$ .

First, based on (4), we evaluated the SER performance under interference coming from Poisson fields of interferers characterized by the density  $\lambda$  and transmit power  $P_{\text{int}}$  equal to the probe Rx power at SNR=0 dB, i.e. the signal-to-interference power ratio (SIR) was equal to the SNR. We applied an expression for the interference MGF derived in [29, eqs. (5), (6)].

We considered Nakagami- $m$  fading models of interfering links, and Nakagami- $m$  and  $\eta - \mu$  fading scenarios [19] for the useful links. Under the former scenarios, we assumed that the useful and interfering links were modeled by identical Nakagami- $m$  fading distributions, and under the latter scenarios, we considered format 1 of  $\eta - \mu$  fading [19] with  $\eta = 0.3$  and  $\mu = 0.855$ . The SER estimates are shown in Fig.

TABLE I  
FORMULAS FOR  $E_{|h|^2} \{ \Gamma(l + 1/2, \gamma_T |h|^2/2) \}$  FOR DIFFERENT FADING STATISTICS.

Fading distribution	PDF of channel power gains	$E_{ h ^2}(l)/\Gamma(l + \frac{1}{2})$
Nakagami- $m$	$f_{ h ^2_G}(x) = \frac{x^{m-1}}{\Gamma(m)\theta^m} \exp(-\frac{x}{\theta})$ , $m$ - shape param., $\theta$ - scale param.	$1 - \frac{(\gamma_T \cdot \theta/2)^{l+\frac{1}{2}}}{(l+\frac{1}{2})B(m, l+\frac{1}{2})}$ $\times {}_2F_1\left(l + \frac{1}{2}, m + l + \frac{1}{2}; l + \frac{3}{2}; -\gamma_T \frac{\theta}{2}\right)$
Generalized gamma (GG) [18], arbitrary $\nu$	$f_{ h ^2_{GG}}(x) = \frac{\nu(\beta/\bar{\gamma}_{GG})^{m\nu}}{\Gamma(m)} x^{\nu m-1} \exp\left[-\left(\frac{\beta x}{\bar{\gamma}_{GG}}\right)^\nu\right]$ , $m, \nu$ - shape param., $\bar{\gamma}_{GG} = E\{ h ^2_{GG}\}$ $\beta = \Gamma(m + 1/\nu)/\Gamma(m)$	$\frac{1}{\Gamma(m)\Gamma(l+\frac{1}{2})}$ $\times H_{2,2}^{1,2}\left[\frac{2\beta}{\gamma_T \bar{\gamma}_{GG}} \middle  \begin{matrix} (1/2 - l, 1), (1, 1) \\ (m, 1/\nu), (0, 1) \end{matrix} \right]$
Generalized gamma, rational $\nu = \frac{j}{k}$		$\frac{j^l k^{m-1/2} (2\pi)^{1-(j+k)/2}}{\Gamma(m)\Gamma(l+\frac{1}{2})}$ $\times G_{2j, k+j}^{k, 2j} \left[ \left(\frac{2j\beta}{k^k j \gamma_T \bar{\gamma}_{GG}}\right)^j \middle  \begin{matrix} \Delta(j, 1/2 - l), \Delta(j, 1) \\ \Delta(k, m), \Delta(j, 0) \end{matrix} \right]$
$\eta$ - $\mu$ model was introduced in [19] in two formats with different physical interpretation of the parameter $\eta$	$f_Z(x) = \frac{2\sqrt{\pi}\mu^{\mu+\frac{1}{2}} H_1^\mu x^{\mu-\frac{1}{2}} \exp(-2\mu x H_1)}{\Gamma(\mu)H^{\mu-\frac{1}{2}}} I_{\mu-\frac{1}{2}}(2\mu Hx)$ $Z =  h ^2_{\eta-\mu}/\bar{\gamma}_{\eta-\mu}$ ; $\bar{\gamma}_{\eta-\mu} = E\{ h ^2_{\eta-\mu}\}$ ; $H_1, H$ , and $\mu$ are parameters; $H_1$ and $H$ are functions of $\eta$ [19] $\alpha_1 = \frac{\gamma_T \bar{\gamma}_{\eta-\mu}}{4\mu(H_1+H)}$ ; $\alpha_2 = \frac{\gamma_T \bar{\gamma}_{\eta-\mu}}{4\mu(H_1-H)}$ .	$\frac{\Gamma(2\mu+\frac{1}{2})(1+\alpha_1)^{-\mu}}{\sqrt{\pi}\Gamma(\frac{1}{2})(1+\alpha_2)^\mu} \times$ $F_1\left(\frac{1}{2}, \mu, \mu, 2\mu+1; \frac{1}{1+\alpha_1}, \frac{1}{1+\alpha_2}\right) + \frac{(16\mu^2 H_1)^\mu}{\Gamma(2\mu)} \times$ $\sum_{k=1}^l \frac{\Gamma(2\mu+k-\frac{1}{2})(\gamma_T \bar{\gamma}_{\eta-\mu})^{k-\frac{1}{2}}}{\Gamma(k+\frac{1}{2})(\gamma_T \bar{\gamma}_{\eta-\mu} + 4\mu H_1)^{2\mu+k-\frac{1}{2}}} \times$ ${}_2F_1\left[\mu + \frac{k-0.5}{2}, \mu + \frac{k+0.5}{2}; \mu + 0.5; \left(\frac{4\mu H}{\gamma_T \bar{\gamma}_{\eta-\mu} + 4\mu H_1}\right)^2\right]$

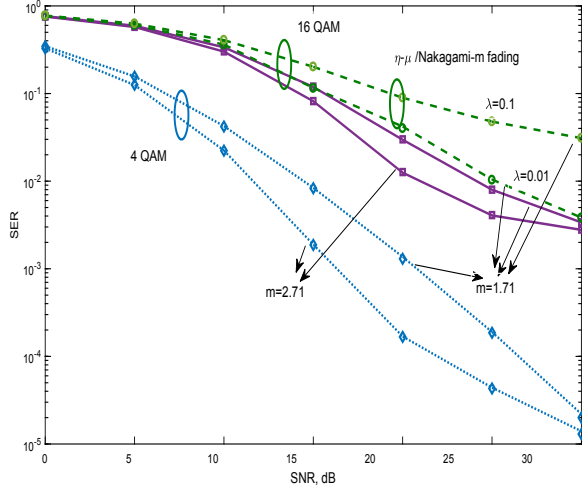


Fig. 1. 4 QAM and 16 QAM SER performance under AWGN and Poisson interferer fields of density  $\lambda$ . Interfering links follow Nakagami- $m$  fading distributions, and useful links follow Nakagami- $m$  (solid and dotted lines) and  $\eta$ - $\mu$  (dashed lines) fading models. Single points report simulation results.

1. Effects of different fading parameters can conveniently be analyzed with the help of the proposed technique. In particular, it can be seen that a power imbalance between the I and Q components of scattered waves in multipath clusters (not observed in Nakagami- $m$  fading, but inherent to  $\eta$ - $\mu$  fading and expressed via the parameter  $\eta = 0.3$ ) has a deteriorating effect on the SER.

The presented SER estimation method is applicable to arbitrary and generally different fading models of useful and interfering links. Effects of fading coefficients of interfering links are inherently included into the MGF expression  $\mathcal{M}_{\mathcal{I}_1^2}(t)$ , and effects of fading coefficients of fading links are evaluated

via  $E_{|h|^2}(l)$ . In Fig. 2, we show 16 QAM SER estimates for scenarios where the useful and interfering links were modeled by different GG fading distributions. Useful link parameters were  $m = \{1.71; 2.71\}$ ,  $\nu = \{1; 4\}$ , and interfering link parameters were  $m = \{1.71; 2.71\}$ ,  $\nu = \{6; 6\}$ . Both shape parameters  $m$  and  $\nu$  are inversely proportional to the amount of fading [18], and thus their increasing boosts both the useful and interfering powers. For the considered parameters, we observe SER decreasing as  $m$  and  $\nu$  increase.

The evaluation of (4) requires high-order MGF derivatives. They can efficiently be evaluated numerically since many modern software packages have built-in derivation operators. In this work, we applied the derivation operator implemented in Mathematica as  $D[MGF[t], \{t, l\}]/t \rightarrow 1$ .

Then we analyzed the M-QAM SER in the low interference regime with the help of (6). We considered a Poisson field of interferers of density  $\lambda = 0.01$  and transmit power  $P_{\text{int}} = 0.1 \mathcal{E}_s|_{SNR=0dB}$  operating outside of a guard circle of the radius  $R_g$  around the probe Rx. Under these conditions, taking into account that the interference power is split equally between the I and Q components,  $E_{\mathcal{I}_1^2} \left\{ \frac{\mathcal{I}_1^2}{2\sigma^2} \right\} = \frac{\pi \lambda P_{\text{int}}}{2\sigma^2(\zeta-2)R_g^{\zeta-2}}$  [30, eq. (14)]. We assumed identical Nakagami- $m$  fading conditions for both useful and interfering signals and assessed interference effects in terms of difference  $\Delta$  SER between the SERs with and without interference. The graphs of 16 QAM  $\Delta$  SER versus the radius of guard zone  $R_g$  for a few values of parameter  $m$  are shown in Fig. 3 for  $\gamma_T = 10$  dB.

## V. CONCLUSION

Estimation of error probability under non-Gaussian interference is of interest in different operational scenarios. For example, providing of acceptable error probability levels is an important task in spectrum sharing wireless systems [31].

Practical receivers are very often equipped with conventional detectors designed for AWGN since real interference

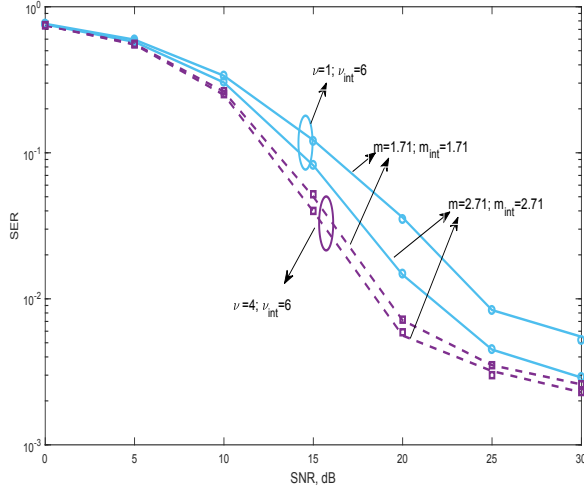


Fig. 2. 16 QAM SER performance under AWGN and Poisson interferer fields of density  $\lambda = 0.01$ . Interfering links are modeled by GG fading distributions with  $\nu_{\text{int}} = 6$  and  $m_{\text{int}} = \{1.71, 2.71\}$ . Useful links are modeled by GG fading distributions with  $m = \{1.71; 2.71\}$  and  $\nu = \{1; 4\}$ . Single points report simulation results.

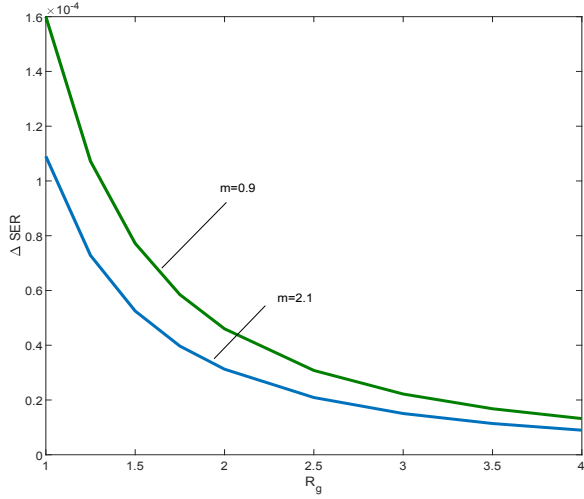


Fig. 3. Effects of low interference power on 16 QAM SER,  $\gamma_T = 10$  dB.

statistics or optimal detectors may be unavailable. Although many works analyzed such detectors under various interference types, the considerations were restricted by concrete interference and fading statistics.

In this work, we presented a new method for the SER assessment, which can be applied to arbitrary interference and fading statistics. The method is especially convenient if the MGF of interference is known. The generality of new technique was proven by its efficiency under a few practical scenarios where previously reported results could not be applied, and Monte Carlo simulations were the only alternative to the presented technique. Vice versa, under the considered scenarios, the given method seems to be the only possibility to check results of Monte Carlo simulations. The presented method can easily be implemented numerically because required high-

order MGF derivatives can be evaluated via a one-line operator realized in many modern software packages.

In this letter, we also presented an approximate simplified SER formula for low interference regime.

## APPENDIX A

### PROOFS OF PROPOSITION 1 AND PROPOSITION 2

It can be seen from (1) that the I component of the received symbol conditioned on  $h$  and  $\mathcal{I}_I$  is corrupted by Gaussian noise with a non-zero mean  $\mathcal{I}_I$ . Thus, the PAM SER averaged over the transmitted symbols and conditioned on the channel gain  $h$  and  $\mathcal{I}_I$ ,  $P_I(|h|, \mathcal{I}_I)$ , can be expressed in terms of Gaussian  $Q$  function as [28]

$$\begin{aligned}
 P_I(|h|, \mathcal{I}_I) &= \alpha_M \left[ Q \left( \frac{\sqrt{\mathcal{E}_s/2}|h| + \mathcal{I}_I}{\sigma} \right) + Q \left( \frac{\sqrt{\mathcal{E}_s/2}|h| - \mathcal{I}_I}{\sigma} \right) \right] \\
 &= \alpha_M \left[ Q \left( \underbrace{\frac{\sqrt{\gamma_T}|h|^2}{b}} + \underbrace{\frac{\mathcal{I}_I}{a}} \right) + Q \left( \underbrace{\frac{\sqrt{\gamma_T}|h|^2}{b}} - \underbrace{\frac{\mathcal{I}_I}{a}} \right) \right] \stackrel{(I)}{=} Q_{\frac{1}{2}}(a, b) \\
 &\stackrel{(II)}{=} \underbrace{\sum_{l=0}^{\infty} \exp \left( -\frac{a^2}{2} \right) \frac{a^{2l} \Gamma \left( l + 1/2, \frac{b^2}{2} \right)}{2^l l! \Gamma(l + 1/2)}}_S \quad (7)
 \end{aligned}$$

where  $Q_{\mu}(a, b)$  is the generalized Marcum  $Q$  function [32], (I) is due to [32, eq. (7)], and (II) is due to [33, eq.(16)].

The arguments of Gaussian  $Q$  function in (7) are expressed via the sum of independent RVs  $|h|^2$  and  $\mathcal{I}_I^2$ , which makes averaging over the corresponding statistics practically impossible while the representation in terms of Marcum  $Q$  function overcomes this problem. It can be seen that the expectations w.r.t. the independent random variables  $a$  and  $b$  in (7) can be evaluated separately. Evaluating  $E_{|h|^2, \mathcal{I}_I} \{S\}$  by applying Fubini's theorem [24], we obtain a series  $S_1 = \sum_{l=0}^{\infty} s_{1l}$  similar to (4) with the difference that  $N_{\text{max}}$  is changed to  $\infty$ . Based on a Laplace transform property [23, vol. 4, eq. (1.1.2.9)], we can find that

$$\begin{aligned}
 E_{\mathcal{I}_I^2} \left\{ \exp \left( -\frac{\mathcal{I}_I^2}{2\sigma^2} \right) \left( \frac{\mathcal{I}_I^2}{2\sigma^2} \right)^l \right\} &= \\
 &= \frac{(-1)^l}{l!} \frac{d^l \mathcal{M}_{\frac{\mathcal{I}_I^2}{2\sigma^2}}(t)}{dt^l} \Big|_{t=1}. \quad (8)
 \end{aligned}$$

If the series  $S_1$  is truncated by  $N_1 = N_{\text{max}} + 1$  terms, the residual  $r_{S_1}(N_{\text{max}})$  must be assessed. One can note that the expectation of regularized gamma function  $E_{|h|^2}(l)/\Gamma(l + \frac{1}{2}) < 1$ , and thus  $S_1 < S_2 = \sum_{l=0}^{\infty} \frac{1}{l!} E_{\mathcal{I}_I^2} \left\{ \exp \left( -\frac{\mathcal{I}_I^2}{2\sigma^2} \right) \left( \frac{\mathcal{I}_I^2}{2\sigma^2} \right)^l \right\} =$

$\sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \frac{d^l \mathcal{M} \frac{x^2}{2\sigma^2}(t)}{dt^l} \Big|_{t=1} = 1$ . Since  $0 \leq s_{1l} \leq s_{2l}$  for  $\forall l$ , the residuals  $r_{S_1}$  and  $r_{S_2}$  resulting from truncation of  $S_1$  and  $S_2$ , respectively, by  $N_1$  terms, are related as  $r_{S_1} < r_{S_2} = \sum_{l=N_{\max}+1}^{\infty} s_{2l} = 1 - \sum_{l=0}^{N_{\max}} \frac{(-1)^l}{l!} \frac{d^l \mathcal{M} \frac{x^2}{2\sigma^2}(t)}{dt^l} \Big|_{t=1} = \delta$  since  $S_2$  is a convergent series. Due to this fact, for  $\forall \delta > 0$ ,  $N_{\max}$  can be found such that  $r_{S_1} \leq \delta$ , and  $r = \alpha_M r_{S_1} \leq \epsilon = \alpha_M \delta$ . ■

## APPENDIX B

### PROOF OF PROPOSITION 3

#### A. Nakagami- $m$ Fading

$E_{|h|^2}(l)$  can be evaluated via [23, vol. 4, eq. (3.10.1.2)].

#### B. Generalized Gamma Distribution

The result for arbitrary  $\nu$  was derived in [18, eq. (10)].  $E_{|h|^2}(l)$  for rational  $\nu$  results from [23, vol. 3, eq. (8.3.2.22)] providing reduction of Fox's  $H$  function to the Meijer  $G$  function for rational values of  $\nu = \frac{j}{k}$ . An identical result was reported in [18].

#### C. $\eta$ - $\mu$ Fading

We take into account that  $\frac{\Gamma(1/2+l, x)}{\Gamma(l+1/2)} = \frac{\Gamma(1/2, x)}{\sqrt{\pi}} + \exp(-x) \sum_{k=1}^l \frac{x^{k-1/2}}{\Gamma(k+1/2)}$  [27] and that  $\Gamma(1/2, x) = 2\sqrt{\pi}Q(\sqrt{2x})$  [34, eq. (4.71)], [32, eq. (7)]. Then based on Fubini's theorem,  $\frac{E_{|h|^2}(l)}{\Gamma(l+1/2)} = 2E_x\{Q(\sqrt{\gamma_T x})\} + \sum_{k=1}^l \left(\frac{\gamma_T}{2}\right)^{k-\frac{1}{2}} \frac{E_x\{\exp(-\frac{\gamma_T}{2}x)x^{k-1/2}\}}{\Gamma(k+1/2)}$ . We obtain  $E_{|h|^2}(l)$  by evaluating the first term via [35, eq. (10)] and the second term via [23, vol. 4, eq. (3.15.1.2)]. ■

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