Sun, B.; le Roux, J. D.; Jämsä-Jounela, S. L.; Craig, I. K.

**Optimal control of a rougher flotation cell using adaptive dynamic programming**

*Published in:* IFAC-PapersOnLine

*DOI:* 10.1016/j.ifacol.2018.09.387

Published: 23/08/2018

*Document Version*
Publisher’s PDF, also known as Version of record

*Please cite the original version:*
Optimal control of a rougher flotation cell using adaptive dynamic programming

B. Sun *•••, J. D. le Roux **••, S.L. Jämsä-Jounela *
I. K. Craig **

* School of Chemical Engineering, Aalto University, Espoo, Finland.
** Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria, South Africa.

Abstract: The operation of a rougher flotation cell affects the performance of mineral processing plant. The dynamics of a rougher flotation cell varies due to the diversity of the feeding conditions. In order to eliminate the dependence of optimal control on a precise model, this paper studies the optimal control of a flotation cell using adaptive dynamic programming which can learn from the operation data and iteratively improve the controller. A classic flotation model is adopted to simulate a rougher flotation cell. The simulation results indicate the effectiveness of the controller.

© 2018, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: froth flotation, process modelling, optimal control, adaptive dynamic programming

1. INTRODUCTION

The operational objective of a mineral processing plant is to maximize the economic value of the flotation concentrate sold to a smelter. Although a variety of factors influence the value of a concentrate (Wills, 2006; Cramer, 2008), the net smelter return (the price the smelter pays for the concentrate) is predominantly determined by the concentrate grade and recovery (Sosa-Blanco et al., 2000). Locating the optimal operating point on the grade-recovery curve and determining the optimal manipulated variables are challenging for the maximization of net smelter return (Hodouin et al., 2001). Studies concerning this challenge have been mainly focusing on:

- design of supervisory controller which can optimize the economic return of a flotation plant, and
- development of a realistic simulation environment (based on a reliable model) for control testing.

The grade and recovery of the concentrate is a complex function of an extraordinary number of interacting variables in the froth flotation process (Jovanovic and Miljanovic, 2015). Even if only the key set of variables necessary to control a flotation circuit is considered, it remains an appreciable challenge to model the interaction between these variables. A degree of modeling accuracy can be sacrificed for control purposes to reduce the size of the parameter set, but the model still has to provide at least qualitatively accurate dynamic responses (Oosthuizen et al., 2017). Sufficient instrumentation and reasonable on-line measurements should be available to support the parameter set to ensure parameters are not based on estimates which cannot be calculated with appropriate ease and updated within a reasonable time-frame. Bascur (1982) derives a simplified flotation model with a reduced parameter set from a validated detailed phenomenological population balance model. Jämsä-Jounela (1992) used the simplified model to develop and test a controller for an industrial flotation rougher circuit. The simplified model’s parameters are obtained through an industrial experimental campaign.

For the operation of a flotation plant, Proportional-Integral-Derivative (PID) control remains sufficient to maintain the stability, while more advanced control methods are required to achieve the economic objectives (Jovanovic and Miljanovic, 2015). Model Predictive Control (MPC) is widely used in the process industry as a supervisory controller. It generally make use of a precise system model and state feedback to ensure integral action. However, a realistic dynamic model that cover the whole operation range of a flotation plant (Bergh and Yianatos, 2011) and appropriate instrumentation (Bouchard et al., 2009) are not always available. Thus, in a flotation circuit, system states and model parameters are difficult to observe from measurements (Le Roux et al., 2016), which limits the implementation of predictive control in some cases.

In order to provide an alternative control when the model parameters are unknown, this paper studies the optimal control of flotation process based on Adaptive Dynamic Programming (ADP) (Wang et al., 2009). ADP is an integration of adaptive control, dynamic programming and reinforcement learning. It is a ‘model-free’ optimal control approach which can, starting from an initial admissible controller, iteratively approach the optimal control by learning from the interactions with the controlled object in an ‘actor-critic’ manner. To start with, the flotation process and its optimal operation problem are briefly introduced and analyzed in Section 2. The ADP-based
data-driven optimal control is presented in Section 3. To test the proposed controller in simulation rigorously, a testing environment which represent the dynamics of the plant reasonably and accurately is developed in Section 4 based on a simplified flotation model proposed and verified by Bascur (1982) and Jämsä-Jounela (1992). Section 5 concludes the study.

2. PROCESS ANALYSIS

Flotation is a complex physico-chemical separation process involving three phases: solids, liquid and gas. The aim of flotation is to separate valuable minerals from gangue minerals. The separation is based on differences in the surface properties of the minerals. In general, the surface of minerals should be hydrophobic, and the surface of gangue should be hydrophilic. When air is pumped through the pulp, the hydrophobic particles attach to the bubbles and float to the top, and the hydrophilic particles attach to the liquid and flow to the bottom. A more complete review of froth flotation can be found in Bascur (1982), Jämsä-Jounela (1992), Wills (2006) and Oosthuizen et al. (2017).

The flotation mechanism can be illustrated by Fig. 1. The two distinguishable sections in the flotation cell is the pulp and froth volume. Each section is subdivided into a liquid and an air (bubble) phase such that pulp and froth volume. Each section is subdivided into two distinguishable sections in the flotation cell is the pulp, the hydrophobic particles attach to the bubbles and leave the cell in the concentrate stream ($Q_C$) remain attached to bubbles and leave the cell in the froth stream ($Q_T$). In the froth phase, particles either attach to the bubbles and flow from the pulp volume to the froth volume, or attach to the liquid and flow to the bottom. A more complete review of froth flotation can be found in Bascur (1982), Jämsä-Jounela (1992), Wills (2006) and Oosthuizen et al. (2017).

Slurry containing valuable minerals and gangue ($Q_{Feed}$) is pumped into the pulp volume. Air ($Q_A$) is also pumped into the pulp volume from the bottom of the cell. Particles can either attach to the bubbles and flow from the pulp volume to the froth volume, or attach to the liquid and leave the cell through the tailings stream ($Q_T$). Particles can also flow from the pulp to the froth through entrainment ($Q_E$). In the froth phase, particles either remain attached to bubbles and leave the cell in the concentrate stream ($Q_C$), or detach from the bubbles and drain back to the pulp volume ($Q_R$).

According to the type of mineral, following three types of chemicals are added to facilitate flotation:
- Collectors: chemicals which cause a mineral’s surface to be hydrophobic so that the mineral can be ‘collected’.
- Frothers: chemicals which reduce the surface tension of water to stabilize the bubbles into a froth layer at the top of the flotation cell to ease concentrate removal.
- Modifiers: chemicals which modify pulp conditions to enhance the surface chemistry differences between valuable and gangue minerals.

In practice, the chemical dosages as well as the flow rates of air input and tailing are adjusted to achieve desired grade ($j_γ$) and recovery ($j_υ$) of the target species $j$. Since grade and recovery are inversely proportional (Bauer and Craig, 2008), a trade-off exists between these variables. The control challenge is to achieve the optimal relation between $γ$ and $υ$.

3. DATA-DRIVEN OPTIMAL CONTROL USING ADAPTIVE DYNAMIC PROGRAMMING

Assume the dynamics of a flotation cell can be expressed in state-space format:

$$\dot{x} = f(x) + g(x)u$$  \hspace{1cm} (1)

in which, $x$ is a vector of system states indicating the evolution of the concentration of mineral species and different volumes against time, $u$ is the vector of manipulated variables, $f(x)$ and $g(x)$ are locally Lipschitz functions of states.

The optimal control objective is to find an optimal control law which can force the practical recovery and/or grade to the desired set-points. Therefore, the objective function $J$ is:

$$J(x(0), u) = \int_0^T [H(x) + u^T Ru]dt$$  \hspace{1cm} (2)

where $H(x)$ is a function of the states measuring the difference between the actual and the setting value of the recovery and/or grade of target mineral, $R$ is weighting matrix for the controlled variables.

The optimal control policy can be obtained by solving following Hamilton-Jacobi-Bellman (HJB) equation

$$\nabla V(x)^T [f(x) + g(x)u] + H(x) + u^T Ru = 0$$  \hspace{1cm} (3)

with $V(0) = 0$. If the solution $V^*(x)$ of (3) exists, the optimal control law is given by

$$u^*(x) = -\frac{1}{2}R^{-1}g^T(x)\nabla V(x)$$  \hspace{1cm} (4)

Policy Iteration (PI) approach has been proposed to approximate the solution of HJB which is generally difficult to solve:

1. Choose an initial stabilizing admissible controller $u_0(x)$
2. For $i \geq 0$, solve following Lyapunov equation for $V(i)$

$$\nabla V_i^T(x)[f(x) + g(x)u_i] + Q(x) + u_i^T Ru_i = 0$$  \hspace{1cm} (5)

3. Update the control law

$$u_{i+1}(x) = -\frac{1}{2}R^{-1}g^T(x)\nabla V_i(x)$$  \hspace{1cm} (6)
In the PI, it is assumed that $f(x)$ and $g(x)$ are known. To eliminate the requirement on the knowledge of $f(x)$ and $g(x)$, a model-free PI technique is adopted in this study. The basic idea of the model-free PI technique is to learn from the input-state information to update the controller $u_i(x)$.

Suppose at the $i$th iteration step, $u(x)$ is decomposed as
$$u(x) = u_i(x) + v_i$$
where $u(x)$ is the practical control law, $u_i(x)$ is the iterative control policy to be updated. Thus for each $u_i$, the time derivative of cost function is
$$V_i = -H(x) - u_i^T R_u - 2 u_i^T R_t v_i$$
Integrate (8) on a time interval $[t, t + \Delta t]$, 
$$V_i(x(t + \Delta t)) - V_i(x(t)) = \int_t^{t + \Delta t} [-H(x) - u_i^T R_u - 2 u_i^T R_t v_i] d\tau$$
(9) indicates when an admissible control policy $u_i(x)$ is given, it can be used as the equation to approximate $V_i(x)$ and $u_{i+1}(x)$. Consider the compact set $\Omega$, assume $\phi_j(x)_{j=1}^{\infty}$ and $\psi_j(x)_{j=1}^{\infty}$ are two infinite sequences of linearly independent smooth basis functions on $\Omega$, where $\phi_j(0) = 0$ and $\psi_j(0) = 0$ for all $j = 1, 2, \ldots$. Then according to approximation theory, for each $i = 0, 1, \ldots$, the cost function $V_i(x)$ and input $u_{i+1}(x)$ can be approximated by
$$\hat{V}_i(x) = \sum_{j=1}^{N_1} \hat{c}_{i,j} \phi_j(x) = \hat{c}_i^T \Phi(x)$$
where
$$\hat{c}_i = [\hat{c}_{i,1} \hat{c}_{i,2} \cdots \hat{c}_{i,N_1}]^T$$
$$\Phi(x) = [\phi_1(x) \phi_2(x) \cdots \phi_{N_1}(x)]^T$$
$$\hat{k}_i = \begin{bmatrix} \hat{k}_{i,1} & \hat{k}_{i,2} & \cdots & \hat{k}_{i,N_2} \\ \hat{k}_{i,1} & \hat{k}_{i,2} & \cdots & \hat{k}_{i,N_2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{k}_{i,m_1} & \hat{k}_{i,m_2} & \cdots & \hat{k}_{i,m_2} \end{bmatrix}$$
$$\Psi(x) = [\psi_1(x) \psi_2(x) \cdots \psi_{N_2}(x)]^T$$
with $N_1 > 0$ and $N_2 > 0$ two sufficiently large integers.

Denote $t$ and $t + \Delta t$ as $t_k$ and $t_{k+1}$, and consider a time sequence $\{t_k\}_{k=0}^{l}$ with $l > 1$ a sufficient large integer, then according to (9), $\hat{c}_i$ and $\hat{k}_{i+1}$ can be obtained in a least square sense by minimizing the approximation error
$$\begin{bmatrix} \hat{c}_i \\ \vec{\psi}(\hat{k}_{i+1}) \end{bmatrix} = (\Theta^T \Theta)^{-1} \Theta^T \Pi$$
(16) in which, $\Theta = [\Xi \ 2I_{\Psi_0} - 2I_{\Psi_i}]$, $\Pi = [-M_H - M_W]$, and
$$\Xi = \begin{bmatrix} \Phi(x(1))^T - \Phi(x(0))^T \\ \Phi(x(2))^T - \Phi(x(1))^T \\ \vdots \\ \Phi(x(l))^T - \Phi(x(l-1))^T \end{bmatrix} \in \mathbb{R}^{l \times N_1}$$
and
$$\hat{c}_i = \begin{bmatrix} \hat{c}_{0,1} \\ \hat{c}_{0,2} \\ \vdots \\ \hat{c}_{0,N_1} \end{bmatrix}$$
$$\hat{k}_{i+1} = \begin{bmatrix} \hat{k}_{i+1,1} \\ \hat{k}_{i+1,2} \\ \vdots \\ \hat{k}_{i+1,m_2} \end{bmatrix}$$
Then the practical control law $u_{i+1}(x)$ can be written as
$$u_{i+1}(x) = \hat{k}_{i+1}^T \Psi(x) = \hat{c}_i^T \Phi(x)$$

$$I_{\Psi_0} = \begin{bmatrix} \int_{t_0}^{t_1} u^T [R \otimes \Psi(x)^T] dt \\ \int_{t_1}^{t_2} u^T [R \otimes \Psi(x)^T] dt \\ \vdots \\ \int_{t_{i-1}}^{t_i} u^T [R \otimes \Psi(x)^T] dt \end{bmatrix} \in \mathbb{R}^{l \times m \times N_2}$$
(18)

$$I_{\Psi_i} = \begin{bmatrix} \int_{t_0}^{t_1} \psi_1(x)^T \psi_1(x)^T dt \\ \int_{t_1}^{t_2} \psi_1(x)^T \psi_1(x)^T dt \\ \vdots \\ \int_{t_{i-1}}^{t_i} \psi_1(x)^T \psi_1(x)^T dt \end{bmatrix} \in \mathbb{R}^{l \times m \times N_2}$$
(19)

4. EXPERIMENTAL STUDY

4.1 Simulation environment

A simplified phenomenological model developed by Bascur (1982) is selected to simulate the real flotation process. The model nomenclature is shown in Table 1. In this...
study, the manipulated variables available to the controller are the aeration rate \( Q_A \) and the tailings rate \( Q_T \). A constant collector and frother addition rate is assumed. There is generally a linear relationship between the adhesion/detachment rates and chemical additions, and can be modeled if sufficient sampling campaign data is available.

Process Dynamics: The mass balance for mineralogical species \( j \) can be written as:

\[
\frac{dC_j^P}{dt} = \frac{Q_{Feed}C_{j,feed}^P}{V_{LP}} + \frac{Q_Rk_j^R C_j^P}{V_{LP} (1 + \alpha_j^P)} - \frac{Q_E + Q_A \alpha_j^P V_{LP} C_j^P}{V_{LP} (1 + \alpha_j^P)} - \frac{Q_TC_j^P}{V_{LP}} \tag{22a}
\]

\[
\frac{dC_j^F}{dt} = \frac{Q_E + Q_A \alpha_j^F V_{LP} C_j^F}{V_{LP} (1 + \alpha_j^F)} - \frac{Q_Rk_j^R C_j^F}{V_{LP} (1 + \alpha_j^F)} + \frac{Q_CC_j^F}{V_{LP}} \tag{22b}
\]

where \( C_{j,feed}^P, C_j^P, \) and \( C_j^F \) (t/m³) represent the concentration of species \( j \) in the feed, froth, and respectively; \( \alpha_j^P \) and \( \alpha_j^F \) are the attachment/detachment rate constants of the species in the pulp and froth respectively; \( k_j^R \) is the drainage rate constant for species \( j \). The model above assumes ideal conditions such that the bubbles exiting through the tailing stream is zero and the bubbles in the concentrate is equal to \( Q_A \).

The liquid volume balance at the pulp and froth interface is given by:

\[
\frac{dV_{LP}}{dt} = Q_{Feed} - Q_T - Q_E + Q_R \tag{23a}
\]

\[
\frac{dV_{LF}}{dt} = Q_E - Q_R - Q_C. \tag{23b}
\]

### 4.2 Parameter Estimation

The model is fitted to the industrial sampling campaign data in Table 2 from Jämsä-Jouhela (1992). The mineral species considered in the campaign are: apatite \((j = 1)\), calcite \((j = 2)\), dolomite \((j = 3)\), and other \((j = 4)\).

#### Cell flow-rates and densities

Given the density and the mass content per mineral species, as well as the solids percentage per stream, it is possible to calculate the density of the feed \( \rho_{Feed} \), tailings \( \rho_T \), and concentrate \( \rho_C \). Since the total mass flow of each stream is known, the densities can be used to calculate \( Q_{Feed}, Q_T, \) and \( Q_C \). At steady-state, the following relationship should be satisfied \( Q_{Feed} = Q_C + Q_T \). The densities and flow-rates were calculated from the data in Table 2 and are shown in Table 3. The air flow \( Q_A \) is assumed to be a measured quantity. The estimation of \( Q_R \) and \( Q_E \) is discussed below.

#### Concentrations

It is assumed that the concentration of each mineral species in the froth \( C_j^F \) is equal to its concentration in the concentrate, and the concentration of each mineral species in the pulp \( C_j^P \) is equal to its concentration in the tailings. Therefore:

\[
C_j^P = \frac{M_j^P}{Q_F}
\]

where \( M_j^P \) (t/h) is the mass flow of mineral species \( j \). \( C_{j,feed}^P \) and \( C_j^F \) can be calculated in a similar manner. The calculated concentration values are shown in Table 4.

#### Volumes

The pulp level is measured as 3.2 m high. Therefore, the total pulp and froth volumes are:

\[
V_{BP} + V_{LP} = 3.2 \times 3.49 \times 3.64 = 40.6515m^3
\]

\[
V_{BF} + V_{LF} = 0.15 \times 3.49 \times 3.64 = 1.9055m^3 \tag{24}
\]

It is assumed that the ratio of the liquid in the froth to the air in the froth is equal to the ratio of the concentrate flow to the air flow, i.e.:

\[
\frac{V_{BP}}{V_{LP}} = \frac{V_T}{V_{LP}}
\]

### Table 1. Nomenclature

<table>
<thead>
<tr>
<th>Parm</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_j )</td>
<td>-</td>
<td>Froth attachment and detachment rate constant for species ( j )</td>
</tr>
<tr>
<td>( \alpha_j^P )</td>
<td>-</td>
<td>Pulp attachment and detachment rate constant for species ( j )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-</td>
<td>Drainage vs entrainment relationship</td>
</tr>
<tr>
<td>( \gamma_j )</td>
<td>-</td>
<td>Grade of species ( j )</td>
</tr>
<tr>
<td>( \upsilon_j )</td>
<td>-</td>
<td>Recovery of species ( j )</td>
</tr>
<tr>
<td>( \rho_j )</td>
<td>t/m³</td>
<td>Density of species ( j )</td>
</tr>
<tr>
<td>( \rho_{Feed} )</td>
<td>t/m³</td>
<td>Density of feed</td>
</tr>
<tr>
<td>( \rho_C )</td>
<td>t/m³</td>
<td>Density of concentrate species</td>
</tr>
<tr>
<td>( \rho_T )</td>
<td>t/m³</td>
<td>Density of tailings</td>
</tr>
<tr>
<td>( C_{Feed}^P )</td>
<td>t/m³</td>
<td>Concentration of species ( j ) in feed</td>
</tr>
<tr>
<td>( C_j^P )</td>
<td>t/m³</td>
<td>Concentration of species ( j ) in pulp</td>
</tr>
<tr>
<td>( k_j^R )</td>
<td>-</td>
<td>Drainage rate constant</td>
</tr>
<tr>
<td>( M_j )</td>
<td>t/h</td>
<td>Mineral flow of species ( j )</td>
</tr>
<tr>
<td>( Q_{coll} )</td>
<td>m³/h</td>
<td>Collector addition rate</td>
</tr>
<tr>
<td>( Q_{froth} )</td>
<td>m³/h</td>
<td>Frother addition rate</td>
</tr>
<tr>
<td>( Q_A )</td>
<td>m³/h</td>
<td>Aeration-rate</td>
</tr>
<tr>
<td>( Q_C )</td>
<td>m³/h</td>
<td>Concentrate flow-rate</td>
</tr>
<tr>
<td>( Q_E )</td>
<td>m³/h</td>
<td>Entrainment flow-rate</td>
</tr>
<tr>
<td>( Q_{Feed} )</td>
<td>m³/h</td>
<td>Pulp feed flow-rate</td>
</tr>
<tr>
<td>( Q_R )</td>
<td>m³/h</td>
<td>Drainage flow-rate</td>
</tr>
<tr>
<td>( Q_T )</td>
<td>m³/h</td>
<td>Tailings flow-rate</td>
</tr>
<tr>
<td>( V_{BP} )</td>
<td>m³</td>
<td>Volume of air (bubbles) in froth</td>
</tr>
<tr>
<td>( V_{BF} )</td>
<td>m³</td>
<td>Volume of air (bubbles) in pulp</td>
</tr>
<tr>
<td>( V_{LF} )</td>
<td>m³</td>
<td>Volume of liquid in froth</td>
</tr>
<tr>
<td>( V_{LP} )</td>
<td>m³</td>
<td>Volume of liquid in pulp</td>
</tr>
<tr>
<td>( V_T )</td>
<td>m³</td>
<td>Total cell volume</td>
</tr>
</tbody>
</table>

### Table 2. Sampling campaign data.

<table>
<thead>
<tr>
<th>Total flow (t/h)</th>
<th>Dry flow (t/h)</th>
<th>Solids (%)</th>
<th>Assay (P₂O₅)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Feed</td>
<td>1885.6</td>
<td>584.0</td>
<td>31.0</td>
</tr>
<tr>
<td>Cell Tail</td>
<td>1811.8</td>
<td>560.0</td>
<td>30.9</td>
</tr>
<tr>
<td>Cell Conc.</td>
<td>73.8</td>
<td>24.0</td>
<td>32.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mineral flows (t/h)</th>
<th>Apatite</th>
<th>Calcite</th>
<th>Dolomite</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed (( M_j^F ))</td>
<td>37.99</td>
<td>133.79</td>
<td>89.26</td>
<td>322.99</td>
</tr>
<tr>
<td>Tail (( M_j^F ))</td>
<td>26.15</td>
<td>127.06</td>
<td>85.91</td>
<td>320.86</td>
</tr>
<tr>
<td>Conc. (( M_j^F ))</td>
<td>11.84</td>
<td>6.73</td>
<td>3.35</td>
<td>2.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mineral distribution or Mass content (%)</th>
<th>Apatite</th>
<th>Calcite</th>
<th>Dolomite</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Feed</td>
<td>6.30</td>
<td>22.91</td>
<td>15.28</td>
<td>55.31</td>
</tr>
<tr>
<td>Cell Tail</td>
<td>4.67</td>
<td>22.69</td>
<td>15.34</td>
<td>57.30</td>
</tr>
<tr>
<td>Cell Conc.</td>
<td>49.23</td>
<td>27.98</td>
<td>13.98</td>
<td>8.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density ( \rho_j ) (t/m³)</th>
<th>3.19</th>
<th>2.71</th>
<th>2.85</th>
<th>2.78</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.49</td>
<td>3.64</td>
<td>3.35</td>
</tr>
</tbody>
</table>
Table 3. Densities, flow-rates, and volumes.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{\text{Feed}} )</td>
<td>1.558</td>
<td>( \rho_T )</td>
<td>1.554</td>
</tr>
<tr>
<td>( \rho_C )</td>
<td>1.643</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_{\text{Feed}} )</td>
<td>12100</td>
<td>( Q_A )</td>
<td>360</td>
</tr>
<tr>
<td>( Q_C )</td>
<td>44.92</td>
<td>( Q_E )</td>
<td>50</td>
</tr>
<tr>
<td>( Q_T )</td>
<td>1166</td>
<td>( Q_R )</td>
<td>5</td>
</tr>
<tr>
<td>( \rho_{\text{Feed}} )</td>
<td>(m^3/h)</td>
<td>( \rho_T )</td>
<td>(m^3/h)</td>
</tr>
<tr>
<td>( Q_{\text{Feed}} )</td>
<td>12100</td>
<td>( Q_A )</td>
<td>360</td>
</tr>
<tr>
<td>( Q_C )</td>
<td>44.92</td>
<td>( Q_E )</td>
<td>50</td>
</tr>
<tr>
<td>( Q_T )</td>
<td>1166</td>
<td>( Q_R )</td>
<td>5</td>
</tr>
<tr>
<td>( \text{Volume} ) (m^3)</td>
<td>1.694</td>
<td>( V_{BP} ), ( V_{LF} )</td>
<td>1.694</td>
</tr>
<tr>
<td>( V_{LF} )</td>
<td>0.2114</td>
<td>( V_{LF} )</td>
<td>38.96</td>
</tr>
</tbody>
</table>

Table 4. Concentrations and rate constants with dimensionless units.

<table>
<thead>
<tr>
<th>Apatite</th>
<th>Calcite</th>
<th>Dolomite</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{jF,\text{Feed}}^{\text{P}} )</td>
<td>0.0314</td>
<td>0.111</td>
<td>0.0738</td>
</tr>
<tr>
<td>( C_{jF}^{\text{P}} )</td>
<td>0.0224</td>
<td>0.109</td>
<td>0.0737</td>
</tr>
<tr>
<td>( C_{jF}^{\text{P}} )</td>
<td>0.263</td>
<td>0.150</td>
<td>0.0744</td>
</tr>
<tr>
<td>( \alpha_{jF}^{\text{P}} )</td>
<td>0.420</td>
<td>0.0271</td>
<td>0.0192</td>
</tr>
<tr>
<td>( \alpha_{jF}^{\text{P}} )</td>
<td>0.0360</td>
<td>0.0709</td>
<td>0.0042</td>
</tr>
<tr>
<td>( k_{Rj}^{F} )</td>
<td>9.92</td>
<td>9.70</td>
<td>9.77</td>
</tr>
<tr>
<td>( \beta )</td>
<td>3.57</td>
<td>3.57</td>
<td>3.57</td>
</tr>
</tbody>
</table>

\[
\frac{V_{LF}}{V_{BF}} = \frac{Q_C}{Q_A} \tag{25}
\]

With these assumptions and the values for \( Q_C \) and \( Q_A \), the volumes can be calculated using (24). These values are shown in Table 3. Given the assumptions and ratios in (25) it is possible to express \( Q_C \) in terms of \( Q_A \) and \( Q_T \):

\[
Q_C = \frac{V_{LF}}{V_{BF}} (Q_A - \frac{V_{BP}}{V_{LF}} Q_T) \tag{26}
\]

where \( V_T = 42.557 \) m^3 is the total cell volume.

Rate constants At steady-state the derivatives in (22) and (24) are zero. Therefore:

\[
C_j^{\text{P}} = \frac{Q_{\text{Feed}} C_{jF,\text{Feed}}^{\text{P}} (1 + \alpha_j^{\text{P}})}{\Delta} (Q_R k_{Rj}^{F} + \bar{Q}_C (1 + \alpha_j^{\text{P}})) \tag{27a}
\]

\[
\Delta = \left[ (1 - \beta R) \left( C_j^{\text{P}} \frac{V_{LF}}{V_{BP}} + Q_T \right) \right] + \left( C_i^{\text{P}} \frac{V_{LF}}{V_{BP}} + Q_E + Q_T + Q_R \right) (Q_R k_{Rj}^{F} + \bar{Q}_C + \bar{Q}_C \alpha_j^{\text{P}})
\]

\[
Q_C = Q_E - Q_R = (\beta - 1) Q_R \tag{27b}
\]

where a linear relationship of \( Q_E = \beta Q_R \) is assumed between drainage and entrainment. Given \( \beta \), (27) can be used to calculate parameters \( k_{Rj}^{F} \), \( \alpha_j^{\text{P}} \) and \( \alpha_j^{\text{P}} \). The parameters values are shown in Table 1 for \( \beta = 3.57 \). The acceptable range for \( k_{Rj}^{F} \) is given in Bascur (1982).

4.3 Simulation results

A single flotation cell in the rougher flotation circuit was selected as the testing object. The flotation cell was simulated using the model presented in Section 4.1. The control algorithm was implemented using the following steps:

i. The basis functions \( \hat{k}_i \) and the exploration noise were designed to form an initial admissible controller.

ii. The initial admissible controller was applied to the flotation cell. Matrices (17)-(21) were obtained.

iii. If the rank condition was satisfied, set \( i = 1 \), go to step iv, otherwise go to step ii to obtain more information.

iv. The weighting vector \( k_i \) was updated using Eq.(16).

v. The matrices (19) and (21) were recalculated using the updated weighting vector \( k_i \), let \( i = i + 1 \), go to step iv.

vi. Step iv and v were conducted iteratively until the stopping criterion was met.

The simulation result is shown in Fig. 3. In this simulation study, the stopping criterion is \( i = 11 \). \( C_i^{\text{P}} V_{LP}, C_i^{\text{P}} V_{LF}, V_{LP} \) and \( V_{LF} \) were used as states. The system model is first shifted to the equilibrium point. According to the site experience, the initial controller was designed as

\[
Q_T = 1165 + 0.5x_1 + 0.5x_2 + 0.5x_3 + 0.5x_4
\]

\[
Q_A = 360 + 0.5x_1 + 0.5x_2 + 0.5x_3 + 0.5x_4 \tag{28}
\]

The resulted controller was obtained as

\[
Q_T = 1165 + 18.831 x_1 - 23.316 x_2 - 17.641 x_3 - 32.435 x_4
\]

\[
Q_A = 360 - 5.074 x_1 + 18.087 x_2 - 5.920 x_3 - 1.766 x_4 \tag{29}
\]

The basis functions for the approximation of value function, which are usually polynomials, were selected as \( \{x_1, x_2, x_3, x_4\} \).

The result indicates the effectiveness of the computational algorithm. Compared with the initial admissible controller, the approximated optimal controller can drive the flotation cell to the desired working point within a shorter response time. As the recovery rate is the main control objective, a higher recovery rate was obtained via the approximated optimal control. In this sense, the performance of the approximated optimal control is better. However, an inversely proportional relationship between grade and recovery was observed. Integrate the constraints on the grade into the development of the control algorithm may help keeping the grade in an acceptable range. Although the value of model parameters are not required, the structure of the model, the knowledge of designing an admissible controller as well as the experience in the selection of excitation noise and basis functions are vital in the implementation of this algorithm.

5. CONCLUSION

This paper studies the optimal control of a rougher flotation cell using ADP. Compared with existing control solutions (Miettunen (1983)), ADP eliminates the requirement on the full knowledge of the process model, which is an advantage of data-driven optimal control methods. The effectiveness of the algorithm indicates that it can serve as an alternative when the model parameters are unknown or the model accuracy is not satisfactory. In order to increase the utilization rate of both the process model and the operation data, a future extension could be the integration of ADP and model-based control methods, e.g., MPC. In addition, the system states are assumed online measurable in this study, so observer-based ADP for flotation control is another potential research direction.

REFERENCES

Fig. 3. Performance comparison between initial controller and approximated optimal controller


